

Article

New Version of Fractional Pachpatte-type Integral

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Article **New Version of Fractional Pachpatte-Type Integral Inequalities** via Coordinated h-Convexity via Left and Right Order Relation **New Version of Fractional Pachpatte-Type Integra Article INEW VERSION OF FRACTIONAL PACNPATTE-Type Integral Inequal Order Relation Order Relation** *Neticle* ,,,,,,,
New Version of Fractional Pachpatte-Type Integral Inequalities *r***ia Coordinated** *h***-Convex**

New Version of Fractional Pachpatte-type Integral

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Imatics and Computer Science 3 Department of Mathematics and Computer Science, Transilvania University of Brasov,
- 1 Department of Mathematics, Faculty of Science, Jazan University, Jazan 45142, Saudi Arabia; and Department of Arabia; Faculty of Science, Jazan 45142, Saudi Arabia; Jazan 45142, Saudi Arabia; Jazan 4514
Arabia; Romania (1992), Romania (1993), Romania (1993), Romania (1993), Romania (1993), Romania (1993), Romani en correspondence: entrepreneur (E.R.N.); muhammad.bilance: environmental parameters (M.B. Correspondence) bakay, komana
khanka Mathamatian Fagu \mathbb{R} eni@isasu.edu (E.R.N.); muhammad.bilan \mathbb{R} ⁴ Department of Mathematics, Faculty of Science, Jazan University, Jazan 45142, Sau khakami@jazanu.edu.sa 2020 Eroil Constanting of Mathematics, Faculty of Science, Jazan University khakami@jazanu.edu.sa $\frac{1}{2}$ eroide
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	- * Correspondence: enwaeze@alasu.edu (E.R.N.); muhammad.bilal@unitbv.ro (M.B.K.) **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly khakami@jazanu.edu.sa khakami@jazanu.edu.sa 2011 Correspondence: enwaeze@alasu.edu (E.R.N.);

Abstract: In particular, the fractional forms of Hermite-Hadamard inequalities for the newly defined class of convex mappings proposed that are known as coordinated left and right \hbar -convexity (LR- \hbar convexity) over interval-valued codomain. We exploit the use of double Riemann–Liouville fractional integral to derive the major results of the research. We also examine the key results' numerical validations that examples are nontrivial. By taking the product of two left and right coordinated \hbar -convexity, some new versions of fractional integral inequalities are also obtained. Moreover, some n conventigy come new versions or memorial analysis mequanties are also columnal increased; come new and classical exceptional cases are also discussed by taking some restrictions on endpoint functions of interval-valued functions that can be seen as applications of these new outcomes. restrictions on endpoint functions of interval-valued functions that can be seen as applications of obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some restrictions on \mathbf{r}_1 intervalse functions that can be seen as applications that can be seen as applications of \mathbf{r}_1 obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some restrictions on endpoint functions of intervalse functions that can be seen as applications of \mathbf{r}_1 obtained functioned that can be seen as annihistions of these news sutcomes. restrictions on endpoint functions of interval-valued functions that can be seen as applications of

Keywords: interval-valued mappings over coordinates; left and right ħ-convexity; double Riemann-Liouville fractional integral operator; Pachpatte-type inequalities **Keywords:** $\mathbf{e}^{\mathbf{e}}$ Riemann-Liouvi **Keywords: intervalued mapping over coordinates; left and right** $\frac{1}{\sqrt{2}}$ **coordinates; left and right** $\frac{1}{\sqrt{2}}$ \mathcal{C} intervalse over convexity; \mathcal{C} over \mathcal{C}

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1. Introduction Khan, M.B.; Hakami, K.H. New The The There
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Prophys.of Integral Inequalities via Coordinated realms C *h*-Convexity via Left and Right Order tight Relation. *Fractal Fract.* 2024, 8, 125. **year** https://doi.org/10.3390/ $frac{1}{2}$ $frac{1}{$ C_H atto. $V6$ $\frac{n-1}{2}$ Academic Editor: Bruce Henry \overline{V} $h_{\rm m}$ Academic Editor: Bruce Henry $Rearriac10030123$ Revised: 5 February 2024

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Integral Inequalities via Coordinated
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^{2e, E.R.;} **1. Introduction**

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There are many uses for the concepts of convex sets and convex functions in the realms of applied and pure sciences. Furthermore, because of its many applications and
realms of applied and pure sciences. Furthermore, because of its many applications and t and Right Order tight relationship to the theory of inequalities, convexity has advanced quickly in recent the Entertain Agency of the search of the search of the Search of the Search Search and Fract. 2024, 8, 125. Years. When determining exact values for a mathematical problem proves to be challenging, requalities can be used to approximate the solution. Since many inequalities can be directly derived from convex functions, there is a close relationship between convexity and the theory of inequalities. challenging the used to approximate the solution. Since the solution of the solution. Since many intervals of the solution. Since the solution of the solution. Since the solution of the solution. Since the solution of the challenging, including to approximate the solution. Since many interesting the solution. Since many interesting the solution. Since many interesting the solution of the solution. Since many interesting the solution of the c equalities. \overline{c} ${\rm des.}$ challenging, including the solution \mathcal{L} approximate the solution. Since \mathcal{L}

The Hermite–Hadamard inequality is one of the most well-known findings in the November 2023 category of classical convex functions, according to Dragomi[r a](#page-21-0)nd Pearce [1]. This inequality has several applications and a straightforward intrinsic geometric explanation. The result February 2024 was mainly credited to Hermite (1822–1901), even though Hadamard (1865–1963) was the
Natural 2021 was not belief that identified it fold The following is been this inconstitution to the d D February 2024 one who first identified it $[2,3]$. The following is how this inequality is [s](#page-21-1)tated: The Hermite-Hadamard inequality is one of the most well-known findings in rovember 2025 category of classical has several applications and a straightforward intrinsic geometric explanation. The result
and the contract of The Hermite–Hadamard inequality is one of the most well-known findings in the

1 Theorem 1. Assume that the convex mapping $J : [\sigma, i] \to \Re$. Then, the following double \blacksquare inequality holds: **Theorem 1.** Assume that the convex mapping $Jj : [\sigma, i] \to \Re$. Then, the following
inequality holds: inequality has several applications and a straightforward intrinsic geometric explanation. **Theorem 1.** Assume that the convex mapping $J: [\sigma, \iota] \to \mathfrak{R}$. Then, the following do 1963 was the one who first identified it following it follows in the following is how this include it in the following is stated: **Theorem 1** Accurate that the convex manning π , [32] θ Then the following double **Theorem 1.** Assume that the convex mapping $\mathbf{J} : [\sigma, \mathfrak{i}] \to \mathfrak{R}$. Then, the following double inequality holds:

Licensee MDPI, Basel, Switzerland. **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality* Attribution (CC BY) license (https://creativecommons.org/license Attribution (CC BY) license **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality* + i 2 ≤ 1 i − **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality* Z ⁱ **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality* **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality* (*x*)*dx* ≤ **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality* (i) + **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality holds:* (**Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality holds:*) 2 , (1)

holds: where R *is a set of real numbers. One can check the concavity of the mappings by replacing the symbol "*≤*" with "*≥*" in double inequality (1).*

The midpoint and trapezoidal-type inequalities, which are the two sides of the The midpoint and trapezoidal-type inequalities, which are the two sides of the Hermite–Hadamard inequality, are used to estimate error boundaries for specific quadra-Hermite–Hadamard inequality, are used to estimate error boundaries for specific ture rules. The origin[al](#page-21-3) derivation of these inequal[iti](#page-21-4)es was in $[4,5]$.

symbol "≤" *with* "≥" *in double inequality (1).*

New extended versions of Simpson-type inequalities were derived by Awan et al. [\[6\]](#page-21-5) New extended versions of Simpson-type inequalities were derived by Awan et al. [6] using differentiable, strongly (s, m) -convex maps. Simpson's integral inequality has been further expanded upon, refined, and generalized in [7–10]. further expanded upon, refined, and generali[ze](#page-21-6)[d in](#page-22-0) [7–10].

In the course of research, various variations of the Hermite–Hadamard inequality In the course of research, various variations of the Hermite–Hadamard inequality have been derived by expanding the definition of convex functions. Conversely, the notion of 5}-convexity [\[11](#page-22-1)[,12](#page-22-2)] is divided into two halves, as follows, with the fundamental requirement that $1 \ge s > 0$. The follo[wing](#page-22-3) [r](#page-22-4)eferences [13–17] contain more generalizations and expansions of classical convex functions.

Fractional calculus is the study of integrals and derivatives of any real order. Fractional calculus is the study of integrals and derivatives of any real order. Fractional integrals are used to solve a wide range of problems involving mathematical science's special functions, as well as their generalizations and extensions to one or more variables. Furthermore, compared to traditional derivatives, fractional-order derivatives describe the memory and hereditary characteristics of distinct processes far better. Actually, current applications in fluid mechanics, mathematical biology, electrochemistry, physics, differential and integral equations, signal processing, and fluid mechanics have been the driving forces behind the recent developments in fractional calculus. Without a doubt, fractional calculus can be used to solve a wide range of diverse problems in science, engineering, and mathematics [\[18–](#page-22-5)[20\]](#page-22-6). The reference [\[21\]](#page-22-7) provides a thorough history of fractional calculus.

Creating different kinds of integral inequalities is a modern issue. Utilizing a range of integrals, including the Sugeno integral [\[22](#page-22-8)[,23\]](#page-22-9), the pseudo integral [\[24\]](#page-22-10), the Choquet integral [\[25\]](#page-22-11), and others, a significant amount of important work has been accomplished in recent years. As a notion of generalization of functions and a significant mathematical subject, interval-valued functions [\[26\]](#page-22-12) have grown in importance as a tool for resolving real-world problems, especially in mathematical economics [\[27\]](#page-22-13). Certain classical integral
. inequalities have been expanded to the domain of interval-valued functions through recent
integral integral integral integral integral integral integral integral integral integral integrals in the lim inequalities have been expanded to the domain of interval-valued functions through studies. New interval variations of Minkowski and Beckenbach's integral inequalities were introduced by Costa et al. [\[28\]](#page-22-14). This generalization established Jensen, Ostrowski,
were introduced by Costa et al. [28]. This generalization established Jensen, Ostrowski, inequalities were introduced by Costa et al. [28]. This generalization established Jensen, and Hermite–Hadamard-type inequalities [\[29\]](#page-22-15). Fractional integrals of Riemann–Liouville with interval values were also used to solve Hermite–Hadamard and Hermite–Hadamard-
tractionalities [29]. The case decilier was [21, 22] and hered the cH, differentially and type inequalities [\[30\]](#page-22-16). Zhao and colleagues [\[31](#page-22-17)[–33\]](#page-22-18) employed the gH -differentiable or
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integral inequalities, and Chebyshev-type inequalities for interval-valued functions. Bu-megral inequalities, and EnerysiteV type inequalities for interval variated raneators. But daka et al. [\[34\]](#page-22-19) used the definitions of gH-derivatives to develop new fractional inequalities and comp_{leta} as the demanders of get derivatives to develop here independent mequanties of the Ostrowski type for interval-valued functions. Log-h-convex fuzzy-interval-valued functions are a new class of convex fuzzy-interval-valued functions that were introduced functions are a new class of convex fuzzy-interval-valued functions that were introduced by Khan et al. [\[35\]](#page-22-20) using a fuzzy order relation. The Jensen and Hermite–Hadamard inequalities were established in this class. To include the Ostrowski-type inequality in the domain of fuzzy-valued functions, the Hukuhara derivative had to be applied, as Anas-tassiou [\[36\]](#page-22-21) showed. Anastassiou's research focused heavily on fuzzy-valued functions, commonly referred to as functions with an interval value. An interesting finding is that Anastassiou's fuzzy Ostrowski-type inequalities could also be used for interval-valued functions. Bede and Gal's [\[37\]](#page-22-22) and Chalco-Cano et al.'s [\[38\]](#page-22-23) publications should be studied in order to gain a thorough grasp of the limitations placed on interval-valued functions by the idea of the H-derivative. Significantly, Chalco-Cano et al.'s recent work [\[39\]](#page-22-24) has produced an Ostrowski-type inequality that is tailored to generalized Hukuhara differen-tiable interval-valued functions. On the other hand, Lupulescu [\[40\]](#page-22-25) introduced the concept h-convex notion to study Jensen and Hermite–Hadamard-type inequalities, Opial-type of left-fractional integral in interval-valued calculus. Then, right fractional integrals are proposed by Budak et al. [\[41\]](#page-22-26), as well as providing the fractional Hermite–Hadamard-type inequalities for interval-valued mappings over coordinates. Zhao et al. [\[42\]](#page-22-27) generalized the Riemann integral inequalities for coordinated convex interval-valued mappings and produced the inequalities for the product of coordinated convexity. After that, Khan et al. [\[43\]](#page-23-0)

provide a new direction in interval-valued calculus by introducing new versions of coordinated integral inequalities via double Riemann integrals and left and right relations. Budak and Sarıkaya [44] and Khan [et](#page-23-2) al. [45] defined Pachpatte's inequalities for the product of coordinated and left and right coordinated convex mappings via fractional integrals, respectively. By using this approach, Khan et [al.](#page-23-3) $[46]$ obtained the left- and right-coordinated interval-valued functions and acquired some integral inequalities in interval fractional calculus for left- and right-coordinated interval-valued functions. Zhang et al. [47] first defined the up and down relations and then discussed some of the properties of these relations. Moreover, he defined the new versions of Jensen-type inequalities using up and down relations. down relations. \mathbf{r} is a new definition of convexity over intervals mapping mapping \mathbf{r} is a new definition of convexity over \mathbf{r} is a new definition of convexity over \mathbf{r} is a new definition of \mathbf{r} coordinated intervalse integral-valued integrals and accuracy integral integral integral integral integral in respectively. By using this approach, Khan et al. [46] obtained the left- and rightrespectively. By using this approach, Khan et al. [46] obtained the left- and rightrespectively. By using this approach, Khan et al. [46] obtained the left- and rightinext of coordinated convexity. After the product of coordinated convexity. After that, Khan et al. \mathcal{A} nctions and acquired some integral inequalities in interval fractional
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functions. On the other hand, Lupulescu [40] introduced the concept of left-fractional

The structure of this research article is as follows. Some classical notions, definitions, and results are recalled and a new definition of convexity over interval-valued mapping is also introduced, which is known as coordinated LR- \hbar -convexity in Section 2. We also report several other findings that follow from this definition of convexity. Using coordinated LR-h-convexity defined in [Se](#page-2-0)ction 2, some new and classical exceptional cases are also obtained in Section 2. In Section 3, involving in[te](#page-2-0)r[va](#page-6-0)l fractional integrals for LR- \hbar -convexity, some well-known inequalities have been generalized, as well as nontrivial examples have also been provided to validate the main outcomes of this paper. Section 4 concludes this study and discusses future work. C^c concludes the study and discusses functions function \mathcal{L} and discusses functions functions \mathcal{L} work. $WOLA$. concludes this study and discusses future work. of this research article is as follows. Some classical notions, definitions,
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substi cusses future work. et al. [41], as well as well as providing the fractional Hermite–Hadamard-type inequalities for μ et al. [41], as well as providing the fractional Hermite–Hadamard-type inequalities for $S₁$ and $S₂$ and $S₃$ has produced an Ostrowski-type and $S₃$ has produced and $S₄$ and $S₅$ and $S₆$ and $S₇$ and $S₈$ and $S₇$ and $S₈$ and $S₉$ a Significantly, Chalco-Cano et al.'s recent work [39] has produced an Ostrowski-type and Ostrowski-type and Ostrowskiintegral in interval-valued calculus. Then, right fractional integrals are proposed by Budak integral in interval-valued calculus. Then, right fractional integrals are proposed by Budak \mathbf{I} functions. On the other hand, Lupulescu \mathcal{A} introduced the concept of left-fractional the co

2. Preliminaries examples have also been provided to validate the main outcomes of this paper. Section 4 of this paper. Section 4 ℏ-convexity, some well-known inequalities been generalized, as well as nontrivial 2. Preliminaries have also been provided to validate the main outcomes of the main outcomes of this paper. Section 4. $\overline{}$ up and down relations. $T_{\rm s}$ 2. Preliminaries up and down relations. coordinated intervals-valued intervalsed integral-valued some integral integral integral integral integral in α -dinated intervalses and accuracy integral integral integral integral integral integral integral integral in $a_{\rm{in}}$ new version in α a new direction in intervalue direction in intervalse calculus by intervalue of coordinated calculus by int ies for coordinated convex intervalses for coordinated mappings and produced mappings and produced the produced the set of α integral integral integrals and left and left and right relations. But \mathcal{L} , refining alles integral integral integrals and left and left and right relations. But \mathcal{L} , including Factor integration \mathcal{L} in the product of the product of \mathcal{L} integral integral intervals in the coordinated convex integral-valued mapping and produced mapping and produced the produced the coordinated map produced the coordinated map produced the coordinated map produced the coordi integral integral includes for coordinated convex integral-valued mapping and produced mappings and produced the produced the second the second mapping and produced the second the second mapping and produced the second the $\frac{1}{\sqrt{2}}$ integral in integral in integral in integral integrals are proposed by Budaking fractional integrals are propos

Let $\mathbb R$ be the set of real numbers and $\mathbb R_I$ containing all bounded and closed intervals within \mathbb{R} . $i \in \mathbb{R}_I$ should be defined as follows: Let $\mathbb R$ be the set of real numbers and $\mathbb R_I$ containing all bounded and closed intervals
within $\mathbb R$. $i \in \mathbb R_I$ should be defined as follows: α should be defined as follows: $\frac{1}{\sqrt{2}}$ **2. Preliminaries** within \mathbb{R} . $i \in \mathbb{R}$ should be defined as follows: $\frac{1}{2}$ α shown. **2. Preliminaries** Let Set of real numbers and closed intervals and set of real numbers and closed intervals α is follows: **2. Preliminaries** $i \in \mathbb{R}_I$ should be defined as follows:
 $i \in \mathbb{R}_I$ should be defined as follows:
 $i = [i_*; i^*] = \{y \in \mathbb{R} | i_* \le y \le i^* \}, (i_*; i^* \in \mathbb{R}).$ (2) ω follows: μ examples have also been provided to validate the main outcomes of this paper. Section 4. examples have also been provided to validate the main outcomes of this paper. Section 4 α The structure of this research article is as follows. Some classical notions, definitions, and α are results and α is a new definition of an interval mapping and convex intervals. The structure of this research article is as follows. Some classical notions, definitions, Let $\mathbb R$ be the set of real numbers and $\mathbb R_I$ containing all bounded and closed intervals The structure of this research article is as follows. Some classical notations, definitions, definitions The structure of this research article is as follows. Some classical notions, definitions, δ follows: α follows: $\sum_{i=1}^{\infty}$ and $\sum_{i=1}^{\infty}$ and $\sum_{i=1}^{\infty}$ defined Pachpathur et al. $\sum_{i=1}^{\infty}$ integral inequalities via double Riemann integrals and left and right relations. Budak and Let $\mathbb R$ be the set of real numbers and $\mathbb R_I$ containing all bounded and closed intervals R be the set of real numbers and \mathbb{R}_I containing all bounded and closed intervals the set of real numbers and \mathbb{R}_I containing all bounded and closed intervals of real numbers and \mathbb{R}_I containing all bounded and closed intervals I numbers and \mathbb{R}_I containing all bounded and closed intervals α and α 17.67 first defined the up and then down relations and then discussed some of the properties d \mathbb{R}_I containing all bounded and closed intervals α and α and α and α fractional integrals, α ontaining all bounded and closed intervals coordinated and left and right coordinated convex mappings via fractional integrals, ing all bounded and closed intervals I bounded and closed intervals inequalities for the product of coordinated convexity. After that, Khan et al. [43] provide the set of real numbers and \mathbb{R}_I containing all bounded and closed intervals within \mathbb{R} . $i \in \mathbb{R}_I$ should be defined as follows: and a new direction intervalued calculus by intervalued calculus by intervalued new versions of containing new versions of containing new versions of containing α $\frac{1}{2}$ integral integrals and left and right relations. But and right relations. inequalities for the product of coordinated convexity. After that, Khan et al. [43] provide Let $\mathbb R$ be the set of real numbers and $\mathbb R_I$ containing all bounded and closed intervals i integrals and r ϵ_t and set al. ϵ as provided the fractional Hermite–Hadamard-type inequalities for ϵ ies
the set of real numbers and \mathbb{R}_I containing all bounded and closed intervals
 \mathbb{R}_I should be defined as follows: ϵ ¹, as well as providing the fractional Hermite–Hadamard-type inequalities for ϵ is and \mathbb{R}_I containing all bounded and closed intervals ϵ as tends the fractional Hermite–Hadamard-type inequalities for ϵ \sqrt{v} Fortunality and contribute are proposed by Fractional integrals are proposed by Budaking and proposed by Budaking are proposed by Budaking and the proposed by Budaking and the proposed by Budaking and the proposed by Budak et al. \mathcal{A} as well providing the fractional Hermite–Hadamard-type inequalities for \mathcal{A} **E** Profitering the set of real numbers and \mathbb{R}_I containing all bounded and closed inter Let $\mathbb R$ be the set of real humbers and $\mathbb R_I$ containing an bounded and closed fine. Significantly, Chalco-Cano et al.'s recent work [39] has produced an Ostrowski-type Let $\mathbb R$ be the set of real numbers and $\mathbb R_I$ containing all bounded and closed intervals be the set of real numbers and \mathbb{R}_I containing an bounded and closed mervals
 $i \in \mathbb{R}_I$ should be defined as follows:
 $i = [i_*, i^*] = \{y \in \mathbb{R} | i_* \le y \le i^* \}, (i_*, i^* \in \mathbb{R}).$ (2)

$$
i = [i_*, i^*] = \{ y \in \mathbb{R} | i_* \le y \le i^* \}, (i_*, i^* \in \mathbb{R}). \tag{2}
$$

It is argued that i is degenerate if $i_* = i^*$. The interval $[i_*, i^*]$ is referred to as positive if $i_* \geq 0$, \mathbb{R}^+_I represents the set of all positive intervals and is defined as $\mathbb{R}_l^+ = \{ [i_*, i^*] : [i_*, i^*] \in \mathbb{R}_l \text{ and } i_* \geq 0 \}.$ as positive if $i_* \ge 0$, \mathbb{R}_I^+ represents the set of all positive intervals and is defined as $\mathbb{R}_I^+ = \{ [i_*, i^*] : [i_*, i^*] \in \mathbb{R}_I \text{ and } i_* \ge 0 \}.$ $\mathbf{z} \cdot [\mathbf{i} \cdot \mathbf{i}^*]$ and $\mathbf{i} = [\mathbf{i} \cdot \mathbf{i}^*]$ and we may $I_{\alpha_1} = \begin{bmatrix} [t_{*}, t_{*}] & [t_{*}, t_{*}] & \in \mathbb{R}_1 \end{bmatrix}$. The isometric interval $t_{*} \geq 0$. $i_* \geq 0$. **2. Preliminaries** $\frac{1}{\pi}$ L be the set of $\lceil \nu_{\ast}, \nu_{\perp} \rceil$ be referred to with produce anter candidate as well need as $\text{Re } \text{Hilb}$ and $\text{Im } \text{Lilb}$ is received to with produce the metal be defined as χ_* is the convexity of χ_* integrating χ_* , χ is defined we $e^{-\theta}$ is $e^{-\theta}$ and $e \ge 0$ \mathbb{R}^+ provided to valid to valid to valid the main outcomes \mathbb{R}^+ , \mathbb{R}^+ is consistent to value to valid to value this paper. concludes this study and discusses future work. and ℓ is degenerate in $\ell_{*} = \ell$. In Section $[\ell_{*}, \ell_{*}]$ is referred to $\ell_{*} = \ell_{*}$. $\begin{bmatrix} -\nu_r & w_l \ w_l & w_l \end{bmatrix}$ represents the set of an positive intervals and is defined as $\begin{bmatrix} w_l & w_l \end{bmatrix}$ $\tau_{\rm k}$ is according in $\tau_{\rm k} = \tau$. In Section $\tau_{\rm k}$, $\tau_{\rm j}$ is return to $\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{D}$ and $\cdot > 0$ itive if $i_{\ast} \geq 0$, \mathbb{R}^+_I represents the set of all positive intervals and is defined as $\{[\iota_*, \iota_+] : [\iota_*, \iota_-] \in \mathbb{R}_l \text{ and } \iota_* \leq 0\}.$
et $o \in \mathbb{R}, i, i \in \mathbb{R}_l$ be defined by with $i = [i, i^*]$ and $i = [i, i^*]$ and we may are recalled and a new definition of $u_* = u$. The interval $[u_*; u_*]$ is referred to The structure of this research article is as follows. Some classical notions, definitions, that i is degenerate if $i_* = i^*$. The interval $[i_*, i^*]$ is referred to $\lbrack t_{*}, t_{*} \rbrack$ $\in \mathbb{N}$ and $t_{*} \leq 0$. expresents the set of all positive intervals and is defined as
 \mathbb{R}_t and $i > 0$ generate if $i_* = i^*$. The interval $[i_*, i^*]$ is referred to as positive if $i_* \ge 0$, \mathbb{R}_I^+ represents the set of all positive intervals and is defined as $\mathbb{R}_I^+ = \{ [i_*, i^*] : [i_*, i^*] \in \mathbb{R}_I \text{ and } i_* \ge 0 \}$.
Let $o \in \mathbb{R}$, $i_* i \in \mathbb{R}_I$ be defined by, with $i = [i, i^*]$ an The structure of this research article is as follows. Some classical notions, definitions, The structure of this research article is as follows. Some classical notions, definitions, It is argued that χ is degenerate if $\chi_* = \chi$. The interval $[\chi_* , \chi]$ The structure of this research article is as follows. Some classical notions, definitions, It is argued that i is degenerate if $i_* = i^*$. The interval $[i_*, i^*]$ is referred to argued that i is degenerate if $i_* = i^*$. The interval $[i_*, i^*]$ is referred to ed that i is degenerate if $i_* = i^*$. The interval $[i_*, i^*]$ is referred to ued that i is degenerate if $i_* = i^*$. The interval $[i_*, i^*]$ is referred to The structure of this research article is as follows. Some classical notions, definitions, The structure of this research article is as follows. Some classical notions, definitions, ≥ 0 . $T_{\rm eff}$ structure of this research article is as follows. Some classical notions, definitions, definiti The structure of this research article is as follows. Some classical notions, definitions, $T_{\rm eff}$ structure of this research article is as follows. Some classical notions, definitions, definiti It is argued that i is degenerate if $i_* = i^*$. The interval $[i_*, i^*]$ is referred to $\mathbf{z}_n \geq 0$. these relations. Moreover, he defined the new versions of Jensen-type inequalities using these relations. Moreover, he defined the new versions of Jensen-type inequalities using and results are recalled and a new of convexity over interval-valued mapping $S_{\rm eff}$ and $K_{\rm eff}$ and $K_{\rm eff}$ defined $P_{\rm eff}$ defined $P_{\rm eff}$ in equalities for the product of It is argued that i is degenerate if $i_* = i$. The interval $[i_* , i]$ is refer respectively. By using the left- and $\ell_* - \ell$. The literval $\lbrack \ell_* , \ell \rbrack$ is referred to $I = \left\{ \begin{bmatrix} 4x & 4 \\ 4x & 1 \end{bmatrix} : \begin{bmatrix} 4x & 4 \\ 4x & 1 \end{bmatrix} \right\}$ interval et al. $\left\{ \begin{bmatrix} 4x & 4 \\ 4x & 1 \end{bmatrix} \right\}$ coordinated and left and right coordinated convex mappings via fractional integrals, It is argued that i is degenerate if $i_* = i^*$. The interval $[i_*, i^*]$ is referred to $\lceil \lfloor t_{*'} t \rfloor \cdot \lfloor t_{*'} t \rfloor \rceil \leq \mathbb{N} \lceil \ln \lfloor t_{*'} t \rfloor \leq 0 \rceil$ rate if $i_* = i^*$. The interval $[i_*, i^*]$ is referred to It is argued that i is degenerate if $i_* = i^*$. The interval $[i_{*}, i^*]$ is referred to respectively. By using this approach, Khan et al. [46] obtained the left- and rightrespectively. By using this approach, \mathcal{A} obtained the left- and right- and right- and right-

Let $\varrho \in \mathbb{R}$, $i, j \in \mathbb{R}$ be defined by, with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may define the interval arithmetic as follows:
 $\sum_{n=1}^{\infty} a_n x^n$ and $\sum_{n=1}^{\infty} a_n x^n$ and $\sum_{n=1}^{\infty} a_n x^n$ and $\sum_{n=1}^{\infty} a_n x^n$ $\ddot{\mathbf{S}}$: $\mathcal{L}_{\mathbf{z}}$ $\mathbf{R} \cdot \mathbf{R} \cdot \mathbf{$ $\mathcal{L} = \mathcal{L} \times \mathcal{L} \times \mathcal{L}$ $\begin{bmatrix} a & b & c \\ c & d & d \end{bmatrix}$ and $\begin{bmatrix} a & b & c \\ c & d & d \end{bmatrix}$ $\begin{bmatrix} a & v & o \end{bmatrix}$ and $\begin{bmatrix} v & v & o \end{bmatrix}$, and $\begin{bmatrix} a & v & o \end{bmatrix}$ Let $\ell \subset \mathbb{R}^n$, $\ell, \ell \in \mathbb{R}^n$ be defined by, which $\ell = [\ell * \ell \ell]$ and $\ell = [\ell * \ell \ell]$, and we may $\frac{1}{2}$ n represents the set of all positive intervals and int r_{in} represents the set of all positive intervals and isomorphism. $r_{\rm reion}$ is the set of all positive intervals and is defined as $r_{\rm so}$ Let $\varrho \in \mathbb{R}$, $i, j \in \mathbb{R}$ be defined by, with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may
ne the interval arithmetic as follows: .
pltinli estien. $r_{\rm F}$ represents the set of all positive intervals is defined as $\frac{1}{2}$ It is a $\frac{1}{2}$ of $\frac{1}{2}$ is degenerated to $\frac{1}{2}$ is referred to $\frac{1}{2}$ is referred to as $\frac{1}{2}$ is referred to $\frac{1}{2}$ is referred to $\frac{1}{2}$ is referred to as $\frac{1}{2}$ is referred to as $\frac{1}{2}$ is Δ define the interval arithmetic as follows: Ω concludes the conclusion $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ α , ι , $\jmath \in \mathbb{R}$ be defined by, with $\jmath = [\jmath_{*}, \jmath_{*}]$ and $\iota = [\iota_{*}, \iota_{*}]$, and we may define the interval arithmetic as follows: $\{u_*, i\} \in \mathbb{R}_I$ and $i_* \geq 0$.
 $\boldsymbol{j} \in \mathbb{R}_I$ be defined by, with $\boldsymbol{j} = [i_*, j^*]$ and $i = [i_*, i^*]$, and we may efined by, with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may α as follows: $\mathbb{R}_I = \{ [a_*, i_-] : [a_*, i_-] \in \mathbb{R}_I \text{ and } a_* \geq 0 \}.$
Let $\varrho \in \mathbb{R}, i, j \in \mathbb{R}_I$ be defined by, with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may \mathbf{s} : $i, j \in \mathbb{R}_I$ and $i \in \{i_*, j^*\}$.
 $i, j \in \mathbb{R}_I$ be defined by, with $j = [i_*, j^*]$ and $i = [i_*, i^*]$, and we may $\begin{bmatrix} \lambda & \lambda' \end{bmatrix} \in \mathbb{R}_I$ and $\lambda_* \geq 0$.
 \vdots \mathbb{R}_I be defined by, with $\boldsymbol{j} = [\boldsymbol{j}_*, \boldsymbol{j}^*]$ and $\lambda = [\lambda_*, \lambda^*]$, and we may α are also obtained in Section 3, in Section 4, i.e., α $\begin{bmatrix} \n\pi_1 \text{ and } \lambda_* \geq 0 \end{bmatrix}$.

reported by, with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may $2aS$ follows: follows: $\frac{1}{\sqrt{2\pi}}$ ² U}.
with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may $\boldsymbol{\psi} = \left[\boldsymbol{j}_*, \ \boldsymbol{j}^* \right]$ and $\boldsymbol{i} = \left[\boldsymbol{i}_*, \ \boldsymbol{i}^* \right]$, and we may $\left[\epsilon_{*}, \, \textit{i}^{*}\right]$ and $\textit{i} = \left[\textit{i}_{*}, \, \textit{i}^{*}\right]$, and we may $\mathbb{R}, i,j \in \mathbb{R}_l$ be defined by, with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may $\mathcal{U}_1 \in \mathbb{R}_I$ and $\mathcal{U}_* \geq 0$.
 \mathbb{R}_I be defined by, with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may represent several other findings that follows.
tion: t relations. Moreover, he defined the new versions of J \mathcal{W} s: $\left[\mathcal{J}_{*}, \mathcal{J}\right]$ and $\mathcal{J}=\left[\mathcal{J}_{*}, \mathcal{J}\right]$, and we may Let $\rho \in \mathbb{R}$, $i, j \in \mathbb{R}_I$ be defined by, with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may define the interval arithmetic as follows: ϵ . Coolor multiplication ϵ $\text{Let } \mathbf{e} \in \mathbb{R}$, $\gamma_i \mathbf{f} \in \mathbb{R}$ for a connect by, which $\mathbf{f} = [\mathbf{f}_*, \mathbf{f}_*]$ and $\mathbf{f} = [\mathbf{f}_*, \mathbf{f}_*]$, and we may the relationship relations. Moreover, ds follows. Let $\varrho \in \mathbb{R}$, $i, j \in \mathbb{R}$ be defined by, with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may the finer variant more as follows. d by, with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may t_{1UV} α convex mapping via fractional integrals, α Let $\varrho \in \mathbb{R}$, $i, j \in \mathbb{R}$ be defined by, with $j = [j_*, j^*]$ and $i = [i_*, i^*]$, and we may

. \mathbf{r} , \mathbf{r} , • Scaler multiplication: • Scaler multiplication: • Scaler multiplication: • Scaler multiplication: ϵ define the interpretation: α interval arithmetic as follows: $\frac{1}{\sqrt{2}}$ \bullet Scaler mumpheation: positive if $\frac{1}{2}$ \bullet > \bullet scaler mumplical. positive if θ and θ θ and θ • α scaler mumplication: positive if ϵ ϵ ϵ ϵ ሼሾ∗, ∗ሿ:ሾ∗, ∗ሿ ∈ ℝூ and ∗ ≥ 0ሽ. • Scaler multiplication: ι represents the set of all positive intervals and is defined as α and is defined as α $\mathbf p$ between $\mathbf p$ \mathbb{R} . ∈ \mathbb{R} \mathbb{R}^n . \mathbb{R}^n **2. Preliminaries 2. Preliminaries** • Scaler multiplication: $\mathbf b$ $\mathbf b$ α -convexition inequalities have been generalized, as well as α well as non- $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$ \mathbf{p} is convexity, some well-known inequalities have been generalized, as well as non-rivial as \mathbf{p} and results are recalled and a new definition of convexity over intervals of conve $\mathbf i$ λ inequalities have been generalized, as well-known in equalities have been generalized, as λ ation recalled and a new definition of convexity over intervals over intervals over intervals over intervals o representative finding that follow from the follow from the follow from the follow from the convexity. reproduce the finding that follow from the follow from the follow from the convexity. $\frac{1}{\sqrt{2}}$ \mathbf{r} \mathbf{A} and \mathbf{A} $\mathbf{1}_{4}$ and $\mathbf{1}_{4}$ defined $\mathbf{1}_{5}$ integral inequalities via double Riemann integrals and left and right relations. Budak and \mathbf{A} and \mathbf{A} and \mathbf{A} \mathbf{a}_{in} \mathbf{A}_1 and \mathbf{A}_2

$$
\varrho \cdot i = \begin{cases}\n\left[\varrho_{i_{*'}} \varrho_{i}^*\right] & \text{if } \varrho > 0, \\
\{0\} & \text{if } \varrho = 0, \\
\left[\varrho_{i^*}, \varrho_{i_*}\right] & \text{if } \varrho < 0.\n\end{cases}
$$
\nAddition:

 \bullet Addition: \bullet Addition: ● Addition: \bullet **have also been provided to validate the main outcomes of the main outcomes of this paper.** \bullet Addition: \mathbb{R}^n is the main outcomes of this paper. Section 4 concludes this study and discusses future work. Section 4 concludes this study and discusses future work. Section 4 concludes this study and discusses future work. are also obtained in Section 2. In Section 2. In Section 3, involving integrals for α ϵ Δ ddition ϵ \ldots convexity defined in Section 2, some new and classical exception 2, some new and cases \ldots \mathbf{A} definition \mathbf{A} \ldots convexity defined in Section 2, some new and classical exception 2, some new and cases \ldots Addition \overline{a} coordinated - α -convexity defined in Section 2, some new and cases α and convexity defined in Section 2, some new and cases α λ definition λ $\frac{1}{2}$ report section that follow from this definition of convexity. λ definition λ representative that follow from this definition of convexity. Using the convexity of convexity. Using the convexity. Using the convexity of convexity. Using the convexity of convexity. Using the convexity of convexity. Usi these relations. Moreover, he defined the new versions of Jensen-type inequalities using these relations. Moreover, he defined the new versions of Jensen-type inequalities using \bullet **definitions** and \bullet **down relations** and the properties of the proper \bullet Addition: Δ definition of convexity over intervalse mapping Δ $i₁$ is also introduced, which is known as coordinated $i₂$. and results are recalled and a new definition of convexity over interval-valued mapping $\sum_{i=1}^{n}$ $\frac{1}{2}$ first defined the up and down relations and then down relations and then discussed some of the properties of the properti

$$
[\dot{\boldsymbol{j}}_*, \ \boldsymbol{j}^*] + [\dot{\boldsymbol{i}}_*, \ \boldsymbol{i}^*] = [\dot{\boldsymbol{j}}_*, \ \boldsymbol{i}_*, \ \boldsymbol{j}^* + \boldsymbol{i}^*]. \tag{4}
$$

ሾ∗, ∗ሿ × ሾ∗, ∗ሿ = ሾሼ∗∗, ∗∗, ∗∗, ∗∗ሽ, ሼ∗∗, ∗∗, ∗∗, ∗∗ሽሿ. (5) • Multiplication: • Multiplication: • Multiplication: • Addition: • Addition: • Addition: • Multiplication: • Scaler multiplication: • Addition: define the interval arithmetic as follows: the interval arithmetic as follows: the interval arithmetic as follows: define the interval arithmetic as follows: · Multiplication: define the interval arithmetic as follows: \mathbf{I} : $\mathbf{I} = \mathbf{I} \cdot \mathbf{I} + \mathbf{I} \cdot$ **2. Preliminaries** $\mathbf b$ be the set of real numbers and closed intervals and closed interval \mathbf{b} $\sum_{i=1}^{n}$ is degenerate if $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ is $\sum_{i=1}^{n}$ is referred to as $\sum_{i=1}^{n}$ \mathbb{R}^n be the set of real numbers and \mathbb{R}^n containing all bounded and closed intervals and closed interval **2. Preliminaries** c stigs c \mathbf{A} be the set of real numbers and containing and containing and containing and containing and containing \mathbf{A} \mathbf{A} Multiplication r_{in} indication for the follow from the follow from the follow from the convexity. Using α ϵ ^{-convexity defined in Section 2, some new and classical exception 2, some new and cases ϵ} reproduce that follow from the follow from the follow from the follow from the convexity. Using U complete - α -convexity defined in Section 2, some new and classical exception 2, some new and cases in Section 2, and ca \bullet introduced, which is known as coordinated \bullet \bullet introduced, which is known as coordinated \bullet \bullet requirements and a new definition of convexity over \bullet \bullet recalled and a new definition of convexity of convexity over \bullet \mathbf{r} \mathcal{L}_{max} α results are recalled and a new definition of convexity over α and results are recalled and a new definition of convexity over \mathcal{L}

$$
\begin{aligned}\n\left[\dot{\mathbf{j}}_{*}, \ \dot{\mathbf{j}}^{*}\right] \times \left[\dot{\mathbf{i}}_{*}, \ \dot{\mathbf{i}}^{*}\right] &= \left[\min\{\dot{\mathbf{j}}_{*}\dot{\mathbf{i}}_{*}, \ \dot{\mathbf{j}}^{*}\dot{\mathbf{i}}_{*}, \ \dot{\mathbf{j}}^{*}\dot{\mathbf{i}}^{*}, \ \dot{\mathbf{j}}^{*}\dot{\mathbf{i}}^{*}\right\}, \ \max\{\dot{\mathbf{j}}_{*}\dot{\mathbf{i}}_{*}, \ \dot{\mathbf{j}}^{*}\dot{\mathbf{i}}_{*}, \ \dot{\mathbf{j}}^{*}\dot{\mathbf{i}}^{*}\}\right].\n\end{aligned}
$$
\nThe inclusion " \sup " means that $\mathbf{i} \supset \mathbf{i}$ if and only if $\left[\mathbf{i}, \ \dot{\mathbf{i}}^{*}\right] \supset \left[\dot{\mathbf{i}} \ \dot{\mathbf{i}}^{*}\right]$ and if and only.

The inclusion " \supseteq " means that $i\supseteq j$ if and only if, $[i_*,i^*]\supseteq[j_*,j^*]$, and if and only if $i < i, i^* < i^*$ that $i \supseteq j$ if and only if, $[i_*, i^*] \supseteq [j_*, j^*]$, and if and only nly if, $[i_*, i^*] \supseteq [j_*, j^*]$, and if and only if ∗ ≤ ∗, [∗] ≤ ∗. if, $[i_* , i^*] \supseteq [j_* , j^*]$, and if and only i ∗ ≤ ∗. ∗ ≤ ∗. ∟ The inclusion " \supset " means that $\mapsto i$ if and only if $\left[\cdot \right]^* \supset \left[\cdot \right]^*$ and if i if $i_* \leq j_*, j^* \leq i^*$. The inclusion \supseteq means that $\psi \supseteq \psi$ if and only ψ , $[\psi, \psi] \supseteq [\psi, \psi]$, and if and only $\text{if } \alpha_* \geq \text{if } \beta_* \neq \text{if } \alpha_* \geq \text{if } \alpha_* \neq \text{if } \alpha_* \geq \text{if } \alpha_* \geq$ $\text{Tr} \alpha * \geq \gamma * \gamma \gamma \geq 1$. $T \mathcal{U}^* \geq \mathcal{J}^* \mathcal{U} \geq \mathcal{U}$ only if and $\mathcal{U}^* \geq \mathcal{U}^* \mathcal{U}$ \geq if *, if \geq if and only if and o The inclusion " \supseteq " means that $i \supseteq j$ if and only if, $[i_*, i^*] \supseteq [j_*]$ \mathbf{v} j = $\begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$ $\lbrack 0 \; \cdot \cdot \cdot \; 0 \; \rbrack$, \ldots only if, $[i_*, i^*] \supseteq [i_*]$ usion " \supseteq " means that $i \supseteq$ n " \supseteq " means that $i\supseteq j$ if and only if, $[i_*,i^*]\supseteq [j_*,j^*]$, and if and on ns that $i\supseteq j$ if and only if, $\bigl[\,i_*,i^*\bigr]\supseteq \bigl[\,j_*,j^*\bigr]$, and if and only The inclusion " \supseteq " means that $i \supseteq j$ if and only if, $[i_*, i^*] \supseteq [j_*, j^*]$, and if positive if $\frac{1}{2}$ $\frac{1}{2}$ 0, $\frac{$ $\liminf_{n \to \infty}$ interns that $\ell \geq \ell$ if and only if, $[\ell^*, \ell] \geq [\ell^*, \ell]$, and is if $i_* \leq j_*, j \leq i$. The inclusion " \supset " means that $i \supseteq i$ if and only if $[i \mid i \mid \supseteq i]$ and if and $\frac{1}{2}$ $\frac{1}{2}$ nclusion " \supseteq " means that $i \supseteq j$ if and only if, $[i_*, i^*] \supseteq [j_*, j^*]$, and if and only if $i_* \leq j_*, j^* \leq i^*$. inclusion " \supseteq " means that $i \supseteq j$ if and only if, $[i_*, i^*] \supseteq [j_*, j^*]$, and if and only ion " \supseteq " means that $i \supseteq j$ if and only if, $[i_*, i^*] \supseteq [j_*, j^*]$, and if and only \hat{i} . \supseteq " means that $i \supseteq j$ if and only if, $[i_*, i^*] \supseteq [j_*, j^*]$, and if and only eans that $i\supseteq j$ if and only if, $[i_*,i^*]\supseteq[i_*,j^*]$, and if and only \boldsymbol{j} if and only if, $[i_{*}, i^{*}] \supseteq [j_{*}, j^{*}]$, and if and only nd only if, $\left[\left. i_*, i^* \right] \supseteq \left[\left. j_*, j^* \right] \right]$, and if and only y if, $[i_*, i^*] \supseteq [j_*, j^*]$, and if and only $i \supseteq j$ if and only if, $[i_*,i^*] \supseteq [j_*,j^*]$, and if and only \ln and only eans that $i\supseteq j$ if and only if, $[i_*,i^*]\supseteq [j_*,j^*]$, and if and only if, $[i_* , i^*] \supseteq |j_*, j^*|$, and if and only define the interval arithmetic as follows: $\left[\left. i_\ast , i^\ast \right] \, \supseteq \big[\left. \left. j_\ast , j^\ast \right] \right\rangle$, and if and only define the interval arithmetic as follows: The inclusion " \supseteq " means that $i \supseteq j$ if and only if, $[i_*, i^*] \supseteq [j_*, j^*]$, and $\frac{1}{1}$ ne inclusion $\frac{1}{1}$. positive intervals the set of all positive intervals $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ The inclusion $\theta^* \geq 0$ The inclusion " \supseteq " means that $i \supseteq j$ $\vec{i} \cdot \vec{r}$ $\text{max max } \gamma \geq \gamma \text{ max only } \gamma, [\gamma, \gamma] \geq [\gamma, \gamma]$, and if and only $\int u^{*}v^{*}$ = $\int v^{*}$. $\lim_{\lambda \to 0} \lim_{\lambda \to 0} \frac{1}{\lambda} \sum_{i=1}^N \lim_{\lambda \to 0} \lim_{$ $\begin{array}{lll} \n\frac{\partial}{\partial t} & \mu \n\end{array}$. $(\mathcal{V}^*, \mathcal{V}) \geq [\mathcal{V}^*, \mathcal{V}]$, and if and only $\lbrack \theta^{*}, \theta \rbrack$, and if and only The inclusion " \supseteq " means that $i \supseteq j$ if and only if, $[i_*,i^*] \supseteq [j_*,j^*]$, and if and only The inclusion \geq means that $\chi \geq \chi$ if and only μ , $[\chi_*, \chi_-] \geq [\chi_*, \chi_-]$, and if and The inclusion " \supseteq " means that $i \supseteq j$ if and only if, $[i_*,i^*] \supseteq [j_*,j^*]$, and if and only \mathbb{R} . ∈ ∈ should be defined as follows: $\mathbf{E} = \mathbf{I}$ The inclusion $\mathbb{T} \supseteq \mathbb{T}$ means that $i \supseteq j$ if and only if, $[i_*,i_-] \supseteq [j_+$ The inclusion " \supseteq " means that $i\supseteq j$ if and only if, $[i_*,i^*]\supseteq [j_*,j^*]$ let ₿ be the set of real numbers and αll bounded and αll bounded and closed intervals and closed intervals and
Intervals and closed intervals and closed intervals and closed intervals and closed intervals and all bounded $\alpha^* \geq \beta^* \cdot \beta^* \geq \alpha$. α inclusion $^{\mu}$ \supset $^{\mu}$ Let ℝ the set of real numbers and ℝூ containing all bounded and closed intervals $\mathcal{U}^* \mathcal{U} = \mathcal{U}$. rclusion " \supseteq " means that $i\supseteq\emptyset$ if and only if, $[i_*,i_']\supseteq [i_*,j_']$, and if and only $\hat{p} \leq \hat{i}$. sion \mathbb{Z}^{\times} means that $i\supseteq\mathbb{Z}$ if and only if, $[i_{*},i_{*}]\supseteq[i_{*},\mathbb{Z}^{\times}]$, and if and only ϵ is the main outcomes of the main outcomes of this paper. Section 4.

Remark 1 ([47]) (i) The relation " \leq ." is defined on \mathbb{R}_{\geq} by $\frac{1}{\sqrt{2}}$ $\mathcal{L}_{\mathcal{A}}$ on $\mathbb{R}_{\mathcal{I}}$ hu \mathbb{R} inclusion \mathbb{R} if and only i T $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ (i) The relation " \leq_p " is defined on \mathbb{R}_I by $\mathcal{L} = \mathcal{L} \times \mathcal{L} \times \mathbb{R}$, with $\mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathbb{R}$ **Remark 1** ($\left[\frac{4}{1}\right]$). (*i*) The retainors Let ∈ℝ , , ∈ ℝூ be defined by, with = ሾ∗, ∗ሿ and = ሾ∗, ∗ሿ , and we may **Remark 1** ([47]). (i) The relation " \leq_p " is defined on \mathbb{R}_I by \mathbb{E} , \mathbb{E} , \mathbb{E} , \mathbb{E} and \mathbb{E} and \mathbb{E} and \mathbb{E} and \mathbb{E} , \mathbb **1** ([47]). (i) The relation " \leq_p " is defined on \mathbb{R}_I by μ , (*i*) The retation μ_{p} is aefined on \mathbb{R} by $\mathcal{L} = \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L}$ $\mathcal{L} = \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L}$, with $\mathcal{L} = \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L}$ $\mathbf{L} = \mathbf{R} \cdot \mathbf{R}$ $\mathbf{F} \bullet \mathbf{I}$ \mathcal{U} by interval arithmetic as follows: $p(\mathbf{x}) = \mathbf{y} \mathbf$ $(Y|)$, $(U|I)$ is a relation $\alpha^* \leq p^*$ is define ℓ μ \cdot μ α μ \mathbb{R} μ ା represents the set of all positive intervals and is defined as \mathbb{R}^n \leq_p " is defined on \mathbb{R}_I by $p \cdot 1 \cdot 1$ $\mathbb{D} \cdot 1$ α is defined on \mathbb{R}_I by **Remark 1** ([47]), (i) The relation " \lt_{n} " is defined on \mathbb{R}_{r} by **Remark 1** ([47]). (i) The relation " \leq_p " is defined on \mathbb{R}_1 by $\frac{1}{\sqrt{2}}$ μ represents the set of all positive intervals and is defined as μ **Remark 1** ([47]). (i) The relation " \leq_p " is defined on \mathbb{R}_I by ሼሾ∗, ∗ሿ:ሾ∗, ∗ሿ ∈ ℝூ and ∗ ≥ 0ሽ. $\frac{1}{\sqrt{1-\frac{1}{n}}}\frac{1}{\sqrt{1-\frac{1}{n}}}}$ $\lim_{t \to \infty} \int f(t) \, dt$ reduction $\sum p$ is defined on \mathbb{R} by \hat{E} The set of \hat{E} and \hat{E} is the set of \hat{E} contained and \hat{E} containing intervals in the set of \hat{E} **Remark 1** ([47]). (i) The relation " \leq_p " is defined on \mathbb{R}_I by ν $-r$, ∗ $\frac{1}{\sqrt{2}}$ L^{final} and R containing and containing and containing and containing and containing intervalse int \ddot{r} , ∗ \ddot{r} CD be the set of real numbers and containing and co

examples have also been provided to validate the main outcomes of this paper. Section 4

examples have also been provided to validate the main outcomes of this paper. Section 4

$$
\left[\dot{\boldsymbol{j}}_{*}, \, \boldsymbol{j}^{*}\right] \leq_{p} \left[\dot{\boldsymbol{i}}_{*}, \, \boldsymbol{i}^{*}\right] \text{ if and only if } \boldsymbol{j}_{*} \leq \dot{\boldsymbol{i}}_{*}, \, \boldsymbol{j}^{*} \leq \boldsymbol{i}^{*}, \tag{6}
$$

 $\{a^*\}\in\mathbb{R}_I$, and it is a pseudo-order relation. The relation $\{a^*, a^*\}\leq n$ \mathbb{R} ent to $[\dot{\pmb{j}}_{*}, \ \dot{\pmb{j}}^{*}] \leq [\dot{\pmb{i}}_{*}, \ \dot{\pmb{i}}^{*}]$ on \mathbb{R}_{I} when it is $'' \leq_{p}''$. $\exists j \in \mathbb{R}_I$, and it is a pseudo-order relation. The relation $\lfloor j_*, j^* \rfloor \leq_p$ to $[i_*, j^*] \leq [i_{*'}]$ \mathbb{R}_I , and it is a pseudo-order relation. The relation $\lfloor j_{*}, j^* \rfloor \leq_p$ $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ is $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$, \begin $\{t_{i}, i^{*}\} \in \mathbb{R}_{I}$, and it is a pseudo-order relation. The relation $[i_{*}, j^{*}] \leq p$ nclusion to $[\mathcal{J}_*, \mathcal{J}^*] \leq [\mathcal{J}_*, \mathcal{J}^*]$ on \mathbb{R}_I when it is \mathbb{R}_P^* . \mathbb{R}^* $\vert \in \mathbb{R}_I$, and it is a pseudo-order relation. The relation $\vert j_*, j^* \vert \leq_p$ T ent to $[\mathcal{J}_*, \mathcal{J}^*] \leq [\mathcal{J}_*, \mathcal{J}^*]$ on \mathbb{R}_I when it is \leq_p . $n\mathbb{R}_I$ when it is " \leq_p ". when it is " $\leq p$ ". a pseudo-order relation. The relation $[i_*, i^*] \leq p$ i_* , i^* on \mathbb{R}_I when it is \leq_p . $\leq p^{\prime\prime}$. α -order relation. The relation $\left[j_*, j^*\right] \leq_p$ \mathfrak{m}_I when it is $\leq_{p} \infty$. \mathbb{R}^* , $[i_*^{\ast}, i^*] \in \mathbb{R}_I$, and it is a pseudo-order relation. The relation $[i_*^{\ast}, i^*] \leq_p$ cident to $[i_*^{\ast}, i^*] \leq [i_*^{\ast}, i^*]$ on \mathbb{R}_I when it is " \leq_p ". is a pseudo-order relation. The relation $[j_*, j^*] \leq_p$
] on \mathbb{R}_I when it is " \leq_p ". eudo-order relation. The relation $[j_*, j^*] \leq_p$
_I when it is " \leq_p ". rder relation. The relation $[j_*, j^*] \leq_p$
i it is " \leq_p ". lation. The relation $[j_*, j^*] \leq_p$
 \leq_p ". elation $[j_*, j^*] \leq_p$ $\left[j_{*}, j^{*}\right] \leq_{p}$ $\{a^*\}\leq_{p}$ $[\phi^*]$, $[\phi^*] \in \mathbb{R}$, and it is a pseudo-order relation. The relation $[\phi]$ $[\boldsymbol{j}_*, \ \boldsymbol{j}^*] \leq [\boldsymbol{i}_*]$ rd it is a pseudo-order relation when it is " \leq_p ". for all $\left[\dot{\mathbf{j}}_{*}, \dot{\mathbf{j}}^{\mathrm{T}}\right]$, $\left[\dot{\mathbf{i}}_{*},\dot{\mathbf{k}}\right]$ $\left\lfloor i_{\ast}\right\rfloor$ \mathbb{R}^* = \mathbb{R}_I , and it is a pseudomental \mathbb{R}^* = \mathbb{R}_I , and it is a pseudomental $\leq \left[\begin{smallmatrix} i & * \end{smallmatrix}\right]$ on \mathbb{R}_I t $\begin{aligned} \mathbb{E} \in \mathbb{R}_I, \text{ and it is a pseudo-order relation. The relation } \mathbb{E} \end{aligned}$ $\left[i_*, i^*\right]$ on \mathbb{R}_I whe \dot{x} for all $[i_*, j^*]$, $[i_*, i^*] \in \mathbb{R}_I$, ar $\lfloor i_{*'}\right. i^{*} \rfloor$ coincident define the interval arithmetic as follows: or all $[\mathcal{J}_*, \mathcal{J}]$, $[\mathcal{I}_*, \mathcal{I}] \in$ define the interval arithmetic as follows: $ll [j_*, j^*], [\iota_*, i^*] \in \mathbb{R}_l$, and it is a pseudo-order relation. The relation $[j_*, j^*] \leq_{p}$ p incident to $[i_*, j^*] \leq [i_*, i^*]$ on \mathbb{R}_I when it is " $\leq n$ ". $\mathbb{R}^* \times \mathbb{R}^n \to \mathbb{R}^n \times \mathbb{R}^n$ $\{x, i^*\} \in \mathbb{R}_I$, and it is a pseudo-order relation. The relation $[i, i^*] \leq n$ $\left[\begin{array}{c} \mathbf{z} \end{array} \right]$ if $\left[\begin{array}{c} \mathbf{z} \end{array} \right]$ if \mathbb{R} if \mathbb{R} as \mathbb{R} \mathbb{R}^* , \mathbb{R}^* , \mathbb{R}^* = \mathbb{R}^* , \mathbb{R}^* define the interval arithmetic as follows: *id it is a pseudo-order relation. The relation* $[j_*, j^*] \leq_p$ \hat{a} if all positive intervals the set of all positive intervals and intervals are \hat{a} in \hat{a} in \hat{a} in \hat{b} is defined as \hat{a} is defined as \hat{a} is defined as \hat{a} is defined as \hat{a} is def $\ddot{\bullet}$, $\ddot{\bullet}$, $\ddot{\bullet}$ define the interval arithmetic as follows: \mathcal{L}_{p} is equaled obtained that $\left[\dot{\boldsymbol{j}}_{*}, \, \dot{\boldsymbol{j}}^{*} \right] \leq p$ $\sum_{i=1}^{\infty}$ when it is " $\leq n$ ". $\ddot{}$, for all $[j_*, j^*]$, $[i_*, i^*] \in \mathbb{R}_I$, and it is a pseudo-order relation. The relation $[i_*]$ $[i_*, i^*]$ coincident to $[i_*, j^*]$ ∗ , for all $[i_*, j^*]$, $[i_*, i^*] \in \mathbb{R}_I$, and it is a pseudo-order relation. The relation $[i_*, i_*]$ $[i_*, i^*]$ coincident to $[i_*, j^*] \leq [n]$ *], $[i_*, i^*] \in \mathbb{R}_I$, and it is a pseudo-order relation. The relation $[i_*, i^*]$ $\lbrack t_{*}, t_{*} \rbrack$ comment to $\lbrack t_{*}, t_{*} \rbrack = \lbrack t_{*}, t_{*} \rbrack$ on as when to as $\lbrack t_{*} \rbrack$. $\{x^*, i^*\}\in \mathbb{R}_I$, and it is a pseudo-order relation. The relation $[i_*, i^*]\leq 1$ $\lceil \sqrt{2} \rceil$ concurred to $\lceil \sqrt{s} \sqrt{d} \rceil = \lceil \sqrt{s} \sqrt{d} \rceil$ on as where the as $\lceil \sqrt{2} \rceil$. $\begin{aligned} \n\mathbb{R}^* \leq \mathbb{R}^n, \quad \text{and it is a pseudo-order relation.} \n\mathbb{R}^* \leq \mathbb{R}^n, \quad \text{and it is a pseudo-order relation.} \n\end{aligned}$ The relation $[j_*, j^*] \leq_p$ $\frac{m}{p}$. ∗ , $\{ x \in \mathcal{B} \mid \mathcal{A}_{*}, x^* \} \leq_{p} \mathcal{B}$ ∗ ≤*^p* for all $[i_*, j^*]$, $[i_*, i^*] \in$
 $[i_*, i^*]$ coincident to $[i_*, j_*.$ i_* , i^*] coincident to $[j_*$, $j^*] \leq [i_*$, i^*] on \mathbb{R}_I when it is " \leq_p ". for all $\begin{bmatrix} i & i^* \end{bmatrix}$ $\begin{bmatrix} i & i^* \end{bmatrix} \in \mathbb{R}$ $\{i^*\}$ coincident to $\left[i_*,\; j^*\right] \leq \left[\overline{i_*},\; i^*\right]$ on \mathbb{R}_I when it is " \leq_p ". all $[i_*, j^*]$, $[i_*, i^*] \in \mathbb{R}_I$, and coincident to $[j_*, j^*] \leq [i_*, i^*]$ on \mathbb{R}_I when it is " \leq_p ". $[i_*, j^*], [i_*, i^*] \in \mathbb{R}_I$, and it is $\begin{aligned} \mathbf{A}^*], \ [\mathbf{i}_*, \ \mathbf{i}^*] \in \mathbb{R}_I, \ \text{and} \ \text{it is a pseudo-order relation}. \ \text{The relation} \ [\mathbf{j}_*, \ \mathbf{j}^*] \leq_p \ \text{incident to} \ [\mathbf{j}_*, \ \mathbf{j}^*] \leq [\mathbf{i}_*, \ \mathbf{i}^*] \ \text{on} \ \mathbb{R}_I \ \text{when} \ \text{it is} \ \text{or} \ \leq_p \text{``}. \end{aligned}$ i_* , i^* on \mathbb{R}_I when it is " \leq_p ". $\lceil i \rceil \leq \mathbb{R}$, and it is a nseudo-order relation \mathcal{A}^* on \mathbb{R}_I *when it is* " \leq_p ".

integral inequalities for coordinated convex interval-valued mappings and produced the

 $x \mapsto \frac{1}{2} \int_{0}^{1} \frac{1}{\sqrt{2}} \, \mathrm{d}x \, \mathrm{d}x$

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(ii) It can be easily seen that " \leq_p " looks like "left and right" on the real line $\mathbb R$, so we call " \leq_p " "left and right" (or "LR" order, in short). (x, α) , ∗ (x, α) ω_j , ω_i , ω_j , ω_i , ω_i , ω_i , ω_i , ω_i , ω_j , ω_i Let b be the set of line $\mathbb R$ so we $\sum_{i=1}^{n}$ n , that $^{\prime\prime}$ $\lt\prime$ $^{\prime\prime}$ looks like "left and right" on the real line \mathbb{P} so we **2. Preliminaries 2. Preliminaries** \ddot{r} and \ddot{r} ight", on the real line \mathbb{R} , eq. \ddot{r} $\sum_{\mu} p$ *looks like* $''$ on the real line \mathbb{P} , eq zue $\mathcal{I}(t)$. **2. Preliminaries** (ii) It can be expite easy that $\mu > \mu$ looks like "left and work" an the real line \mathbb{R} so rue $T_{\rm eff} = \frac{H_{\rm eff}}{2}$ is one of the Hadamard internal integration integrate $\left[1 + \frac{H_{\rm eff}}{2}\right]$. cho and the theory of the teat and us, The Hermite–Hadamard inequality is one of the most well-known findings in the (ii) It can be easily seen that " \leq_p " looks like "left and right" on the real line \R , so we α be directly derived from convex functions, there is a close relationship between $\frac{1}{\alpha}$ close relationship between $\frac{1}{\alpha}$ close relationship between $\frac{1}{\alpha}$ close relationship between $\frac{1}{\alpha}$ close rela convexity and the theory of interests. *coincident to* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ *on* ℝூ *when it is* " ≤ "*. (ii) It can be easily seen that* " ≤ " *looks like "left and right" on the real line* ℝ, *so we call* $\begin{aligned} \n\text{curl} \quad \sum p \quad \text{let } \text{unit } \text{ right}} \quad \text{(or } \text{LR} \text{ outer}, \text{at } \text{s} \text{th} \text{]} \n\end{aligned}$ realms of applied and pure sciences. Furthermore, because of its many applications and (ii) It can be easily seen that " \leq_p " looks like "left and right" on the real line R, so we Order Relation. *Fractal Fract.* **2024**, *8*, $\mathcal{C}a$ μ and the easily seen in a se de la construcción de la cons **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double n be easily seen that \leq_p tooks like roks uke veji unu ri (*a)* It can be easily seen that \leq_p books like tell that fight on the real line \mathbb{R} , so a easily seen that \leq_p looks like left and right on the real line $\mathbb R$, so we \leq_p tooks the teft and right for the real time \mathbb{R} , so we Liouville fractional integral to derive the major results of the research. We also examine the key $r(u)$ it can be easily seen that $\leq p$ tooks the text and tight of (ii) It can be easily seen that $l' < l'$ looks like "left and right" on the real line \mathbb{R} so call $u < u$ "left and right" (or "I R" order in short) $r(t)$ it can be easily seen that $\leq p$ tooks the tep and right on the real the \mathbb{R} , so (ii) It can be easily seen that $\ell' < \ell'$ looks like "left and right" on the real line $\mathbb R$ so zue $\frac{1}{\alpha}$ is the process of the product of the product $\frac{1}{\alpha}$ is the product and product $\frac{1}{\alpha}$ to the product and $\frac{1}{\alpha}$ is the product of two left and the product and $\frac{1}{\alpha}$ is the product and $\frac{1}{\alpha}$ easily seen that " \leq_p " looks like "left and right" on the real line \R , so we \mathcal{L}_t , in short). **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly (*a)* it can be easily seen that \leq_p -tooks like left and right and right and right and right and right \leq_p call \leq_p left and right (or LR order, in short). **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly (a) it can be easily seen that \leq_p -looks like left and right ∞ * Correspondence: enwaeze@alasu.edu (E.R.N.); muhammad.bilal@unitbv.ro (M.B.K.) * Correspondence: enwaeze@alasu.edu (E.R.N.); muhammad.bilal@unitbv.ro (M.B.K.) (*ii*) It can be easily seen that \leq_p tooks like $k = 1$ μ) it can be easily seen that \geq p - tooks like ι efu and right on the real khakami@jazanu.edu.sa Thus \leq_p wors the step and right on the real time \mathbb{R} , so we $\frac{4}{3}$ μ is μ and μ call \leq_p ight and right (or EN order, in short). (ii) It can be easily seen that " \leq_p " lock call " \leq_p " "left and right" (or "LR" order, in short).
The Hausdorff-Pompeju distance between intervals $\begin{bmatrix} i & i^* \end{bmatrix}$ $\begin{bmatrix} i & i^* \end{bmatrix} \in \mathbb{R}$, is 2 Department of Mathematics and Computer Science, Alabama State University, (ii) It can be easily seen that " \leq_p " looks like "left and right" on t 2 Department of Mathematics and Computer Science, Alabama State University, (ii) It can be easily seen that " \leq_p " looks like "left and right" on the real line \R , so we (*ii*) It can be easily seen that $\alpha \leq p$ α looks it
call $\alpha' \leq p$ α' left and right" (or "LR" order, in short, (ii) It can be easily seen that " \leq_p " looks like "left and right" on the real line \mathbb{R} , so we

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for all ሾ∗, ∗ሿ,ሾ∗, ∗ሿ ∈ ℝூ, *and it is a pseudo-order relation. The relation* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ

right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also

3 Department of Mathematics and Computer Science, Transilvania University of Brasov,

The Hausdorff-Pompeiu distance between intervals $[\dot{\boldsymbol{j}}_*, \dot{\boldsymbol{j}}^*], [\dot{\boldsymbol{i}}_*, \dot{\boldsymbol{i}}^*] \in \mathbb{R}_I$ is given by the contract of the c $\mathbf y$ $\frac{1}{2}$ eff-Pom (c) ER enterpresenting. $discrete$ state $iscrete$ showled as $[i_*, j^*], [i_*, i^*] \in \mathbb{R}_l$ is and $discrete$ between intervals $[i_*, j^*], [i_*; i^*] \in \mathbb{R}_l$ ሼሾ∗, ∗ሿ:ሾ∗, ∗ሿ ∈ ℝூ and ∗ ≥ 0ሽ. distance between intervals $[j_*, j^*]$, $[i_*, i^*] \in \mathbb{R}_l$ is ሼሾ∗, ∗ሿ:ሾ∗, ∗ሿ ∈ ℝூ and ∗ ≥ 0ሽ. $\left[\begin{array}{ccc} a & b \\ c & d \end{array}\right] \left[\begin{array}{ccc} a & b \\ c & d \end{array}\right] \left[\begin{array}{ccc} a & b \\ c & d \end{array}\right] \left[\begin{array}{ccc} a & b \\ c & d \end{array}\right]$ category of competitions, and $\left[\frac{d^2y}{dx^2}\right]$. $T_{\rm eff} = \frac{1}{2}h^2$ by $m_{\rm eff}$ is one of the most well-known findings integrals $\begin{bmatrix} i & i^* \end{bmatrix}$. category of classical convex functions, and $\left[\begin{matrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{matrix}\right]$. Then The Frausdom-Fompera distance between microals $[\mathcal{J}_*, \mathcal{J}_* | \cdot [\mathcal{I}_*, \mathcal{I}_*] \in \mathbb{R}$ is
tived by $\frac{1}{2}$ is one of the most well-known finding in the most well-k The Fourier distance between the road $\left[1\ast 1\right]$ of $\left[1\ast 1\right]$ is the most most well-known finding in the most model in the m The Hausdorff–Pompeiu distance between intervals ሾ∗, ∗ሿ, ሾ∗, ∗ሿ ∈ ℝூ is given by The Hausdorff-Pompeiu distance between intervals $[j_*, j^*], [i_*, i^*] \in$ can be directly derived from convex functions, there is a close relationship between $\mathcal{L}(\mathbf{r}, \mathbf{r}, \mathbf{r})$ can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathbf{v}_i changing, including to a solution to approximate the solution. Since $\begin{bmatrix} a & b & c \end{bmatrix}$ is $\begin{bmatrix} c & d & d \end{bmatrix}$ The Hausdorff-Pompeiu distance between intervals $[j_*, j^*]$, $[i_*, i^*] \in \mathbb{R}_l$ is *(ii) It can be easily seen that* " ≤ " *looks like "left and right" on the real line* ℝ, *so we call* t_i relationship to the theory of inequalities, convexity has advanced α in recent α tight relationship to the theory of inequalities, convexity has advanced α The Hausdorff-Pompeiu distance between intervals $[\hat{\jmath}_*, \hat{\jmath}_*]$, $[\hat{\imath}_*, \hat{\imath}^*] \in \mathbb{R}_I$ is \mathcal{Y} \mathcal{E}^{IV} and \mathcal{E}^{IV} given by $R = \frac{1}{2} \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right)$ in the set of the set Riemann–Liouville fractional integral operator; Pachpatte-type inequalities R iemann–Liouville fractional integral operator; Pachpatte-type integral operator; Pachpatte-type inequalities **Keywords: intervalued mappings over convexity; left and right** $\mathcal{L}(\mathbf{r}, \mathbf{r}, \mathbf{r})$ **; double** $\mathcal{L}(\mathbf{r}, \mathbf{r}, \mathbf{r})$ **; double** $\mathcal{L}(\mathbf{r}, \mathbf{r})$ **; double** $\mathcal{L}(\mathbf{r}, \mathbf{r})$ **; double** $\mathcal{L}(\mathbf{r}, \mathbf{r})$ **; double Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double $\alpha u = \alpha_p$ refunding the convertional cases are also discussed by the management of the Hausdorff-Pompeiu distance between intervals $\begin{bmatrix} i & i^* \end{bmatrix}$ $\begin{bmatrix} i & i^* \end{bmatrix} \in \mathbb{R}$ α on endpoint functions of intervalse functions of intervalse functions that can be seen as applications of α $\int_{a}^{b} [f(x)]^2 dx$ and competential exceptional cases are also discussed by $\int_{a}^{b} [f(x)]^2 dx$ or $\int_{a}^{b} [f(x)]^2 dx$ $\begin{array}{lllll} \mu & \leq_p & \text{left unit right (or \ & LN \ & \text{outer, in short).} \end{array}$
The Hausdorff–Pompeiu distance between intervals $\begin{array}{lllll} \lambda & \lambda^* \end{array} \begin{bmatrix} \lambda & \lambda^* \end{bmatrix} \in \mathbb{R}$, is $r_{\rm{even}}$ by seen as applications of intervalse functions that can be seen as applications of $r_{\rm{off}}$ deduced the distance between intervals $\left[\mathbf{y}_{*}, \mathbf{y}_{\perp}\right]$ ($\mathbf{y}_{*}, \mathbf{y}_{\perp}\right]$) $\left[\mathbf{y}_{*}, \mathbf{y}_{\perp}\right]$ The Hausdorff–Pompeiu distance between intervals $[i_*, j^*]$, $[i_*, i^*] \in \mathbb{R}_l$ is restrictions on endpoint functions of interval-valued functions that can be seen as applications of right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also r_{S} is that examples are non-trivial. By taking the product of two left and product of two left and product and product and product of two left and product of two left and product of two left and product and product r_{S} resultsty that examples are non-trivial. By taking the product of the product and product and product and product and product of two left and product of the product and product and product and product and product convexity over intervalse convexity) over intervalse convexity the use of double Riemann– convexity (- $\frac{1}{2}$ -convexity) over intervalse codomain. We exploit the use of double Riemann– defined convex mapping proposed that are known as coordinated that are known as coordinated left and right α -dimensional right α defined convex mapping proposed that are known as coordinated that are known as coordinated left and right α -dimensional right α defined convex mapping proposed that are known as coordinated that are known as coordinated left and right α -dinamiking α -dinamiking α α defined that are known as coordinated that are known as coordinated left and right α -dinamiping α -dinamiping α defined class of convex mappings proposed that are known as coordinated that are known as coordinated left and right α **Absorption In particular, the fractional forms of Hermite–Hadamard inequalities for the newly stated in the new ly-the new ly-the newly stated in the newly stated in the new ly-the newly stated in the newly stated in th Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly 4 Department of Mathematics, Faculty of Science, Jazan University, Jazan 45142, Saudi Arabia; α Department of Mathematics, β khakamid yazar
Sanada yazar y $\frac{1}{2}$ $\frac{1}{2}$ The Hausdorff-Pompeiu distance between interval
given by 4 Department of Mathematics, Faculty of Science, Jazan University, Jazan 45142, Saudi Arabia; 1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, 1 \geq Ine Hausdorff-Pompeiu distance between intervals \mathcal{J}_{*} , a
given by **Incometer 19 Integral Coordinates via Coordinate School intervals** $[i_{*}, j^{*}]$, $[i_{*}, i^{*}] \in \mathbb{R}_{I}$ is **Order Relation New Version of** $\frac{d}{dt}$ **New Version of Fractional Pachpatte-type Integral** given by $\frac{1}{2}$ **New Version of Fraction of Fraction**

given by
\n
$$
d([j_*, j^*], [i_*, i^*]) = max\{|j_* - i_*|, |j^* - i^*|\}.
$$
\n
$$
(7)
$$
\nIt is a familiar fact that (\mathbb{R}_I, d) is a complete metric space.

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There are many uses for the concepts of convex sets and convex functions in the

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right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also

There are many uses for the convex sets and convex sets and convex $\mathcal{L}_{\mathcal{A}}$

It is a familiar fact that (\mathbb{R}_I, d) is a complete metric space. • Scaler multiplication: depending that $(\mathbb{D} \cdot d)$ is a complete is a familiar fact that (\mathbb{R}_I, d) is a complete metric space. I_I , *d*) is a complete metric space. It is a familiar fact that (\mathbb{R}_I, d) is a complete metric space. \mathbf{R} is a failure It is a familiar fact that (\mathbb{R}_I, d) is a complete metric space. It is a familiar fact that (\mathbb{R}_I, d) is a complete metric space. T_{total} is one of the H (\mathbb{F}_{n-1}) is one of the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most well-kn α category of complete them of α complete them by α . $\text{Cov}(x) = \frac{1}{2} \int_{0}^{\infty} \int_{0$ T_{max} distance between intervals T_{max} It is a familiar fact that (\mathbb{R}/μ) is a complete metric space. It is a familiar fact that (\mathbb{R}_I, d) is a complete metric space. \mathbf{r} is a let $\overline{\mathcal{R}}$ to a familiar ray $\frac{1}{2}$ Integral Inte \sim Community vice and \sim $\sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n}$ Integral Inequalities via Coordinated \mathbb{P}^1 and \mathbb{P}^1 \mathbb{P}^1 \mathbb{P}^1 $\binom{1}{1}$ It is a familiar fact that (\mathbb{R}_I, d) is a complete metric space. convexity (-ℏ-convexity) over interval-valued codomain. We exploit the use of double Riemann– It is a familiar fact that (\mathbb{K}_I, a) is a complete metric space. Γ Financial Mathematics and Actual Science (FMAS)-Research Γ It is a familiar fact that (\mathbb{R}_I, d) is a complete metric space.

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1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of Mathematics, Faculty

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

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Remark 1 ([47])**.** *(i) The relation* " ≤ " *is defined on* ℝூ *by*

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Inequalities via Coordinated ℏ-Convexity via Left and Right

3 Department of Mathematics and Computer Science, Transilvania University of Brasov,

There are many uses for the convex sets and convex sets and convex sets and convex \mathcal{A}

(ii) It can be easily seen that " ≤ " *looks like "left and right" on the real line* ℝ, *so we call*

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

tight relationship to the theory of inequalities, convexity has advanced quickly in recent

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Version of Fractional Pachpatte-type

Academic Editor: Bruce Henry

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Accepted: 6 February 2024

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 $interval$ -valued mapping (IVM) and $J \in$ $\mathcal{L} = \mathcal{L} \times \mathcal{L} \times \mathcal{L}$, with $\mathcal{L} = \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L}$ $jR_{[\sigma,i]}$. Then, interval Riemann–Liouville-type integrals of J are defined as $\mathcal{L} = \mathcal{L} \times \mathcal{L} \times \mathcal{L}$, with $\mathcal{L} = \mathcal{L} \times \mathcal{L} \times \mathcal{L} \times \mathcal{L}$ $p \equiv \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ \mathbf{n} 1 ([30,40]). Let $J\mathbf{j}: [\sigma, \mathfrak{i}] \to \mathbb{R}^{\mathfrak{m}}_{I}$ positive if $\mathbf{r} = \mathbf{r} - \mathbf{r} - \mathbf{r}$ represents the set of all positive intervals and is defined as $\mathbf{r} = \mathbf{r} - \mathbf{r}$ $(30,40)$. Let $\mathbf{J}: [\sigma,\mathfrak{i}] \to \mathbb{R}^+_I$ be positive if $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ ା represents the set of all positive intervals and is defined as \mathbb{R}^n α , i $\rightarrow \mathbb{R}^+_I$ be an interval-valued p $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ ା represents the set of all positive intervals and is defined as $\mathcal{L}_{\mathcal{A}}$ **Definition 1** ([30,40]). Let $JJ:[\sigma,i]\to\mathbb{R}_I^+$ be an interval-valued mapping (IVM) and $JJ\in$ $\mathcal{JK}_{[\sigma, i]}$. Then, interval Kiemann–Liouville-type integrals of JJ are aefined as **Dennition 1** ([30,40]). Let j : $[\sigma, 1] \rightarrow \mathbb{R}_i$ be an interval-valued mapping (1VM) are **Definition 1** ([30,40]). Let $J: [\sigma, i] \to \mathbb{R}^+_I$ be an interval-valued mapping (IVM) and $J \in \mathbb{R}^+$
and Then interval Riemann-Liouville-type integrals of J are defined as T_{max} is T_{max} and T_{max} is the result was mainly T_{max} (1822–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1911), 1963 was the one who first identified it 3 $\frac{2}{3}$. The following is stated: \mathcal{R} . Then interval Riemann-Liouville-tupe integrals of \mathbb{R} are defined as $\mathcal{F}^{\text{u}}(\mathcal{O})$ $\frac{1}{2}$ was the following inequality in $\frac{1}{2}$. $\mathcal{D} = \mathcal{D}$ $\mathcal{F}[\mathcal{O},t]$ \mathbf{D}_s Convexity and the theory of integration \mathbf{D}_s (1) \mathbf{D}_s in \mathbf{D}_s $\sum_{i=1}^{n}$ The integral Discussor I is one of the most well-known finding $\frac{1}{n}$ $\mathcal{R}_{[\sigma, i]}$. Then, interval Riemann–Liouville-type integrals of J are defined as **Definition 1** ([30,40]). Let $\mathbf{J} : [\alpha, \mathbf{i}] \to \mathbb{R}_I^+$ be an interval-valued mapping (IVM) and $\mathbf{J} \in \mathbb{R}$. realms of applied and pure sciences. Furthermore, because of its many applications and **Definition 1** ([30,40]). Let $\mathbf{J} : [\mathbf{\sigma}, \mathbf{i}] \to \mathbb{R}_I^+$ be an interval-valued mapping (IVM) and $\mathbf{J} \in$ **Definition 1** ([30,40]). Let $\mathfrak{I}: [\alpha, \mathfrak{i}] \to \mathbb{R}_I^+$ be an interval-valued mapping (IVM) and $\mathfrak{I} \in$ **Definition 1** ([30,40]). Let π : $[\sigma, i] \rightarrow \mathbb{R}^+_I$ be an $j \mathcal{R}_{[\sigma,i]}$. Then, interval Riemann–Liouville-type integrals of J are defined as **efinition 1** ([30,40]). Let $\mathbf{J}: [\alpha, \mathbf{i}] \to \mathbb{R}_I^+$ be an interval-Order Relation. *Fractal Fract.* **2024**, *8*, **Definition 1** ([30,40]). Let \mathbf{J} : $[\mathbf{\sigma}, \mathbf{i}] \rightarrow \mathbb{R}_I^+$ be $\mathcal{R}_{[\sigma,i]}$. Then, interval Riemann–Liouville-type integra Order Relation. *Fractal Fract.* **2024**, *8*, $R = \frac{1}{2}$ **Definition 1** ([30,40]). Let $J: [\sigma, i] \to \mathbb{R}^+_I$ be an interval-valued R is a free set integral operator; Pachpatte-type integral operator; Pachpatte-**Definition 1** ([30,40]). Let $JJ : [\sigma, i] \to \mathbb{R}^+_I$ be an interval-valued mapping (IVM) and J **Definition 1** ([30,40]). Let $\textbf{J} : [\sigma, \textbf{i}] \to \mathbb{R}^+_I$ be an interval-valued mapping (IVM) and $\textbf{J} \in$
 $\mathcal{R}_{[\sigma, \textbf{i}]}$. Then, interval Riemann–Liouville-type integrals of \textbf{J} are defined as **Definition 1** ([30,40]). Let $\mathbf{J}: [\alpha, \mathfrak{i}] \to \mathbb{R}_I^+$ be an interval-valued mapping (IVM) and $\mathbf{J} \in$ ϵ is the use of double ϵ $c_{\rm ref}$ intervalse convexity) over intervalse convexity the use of double Riemann– $J^{\prime\,c}[\sigma,\iota]$ $3\mathcal{R}_{\mathcal{C}}$ and Then, interval Riemann–Liouville-type integrals of \mathcal{R} are defined as $\mathcal{O}[\mathcal{O}/4]$ $[0,1]$ aR_i , and Then interval Riemann–Liouville-type integra \mathcal{S} $\mathcal{J}\mathcal{K}_{\left[{\bm\sigma},\mathfrak{i}\right]}.$ Then, interval Kiemann–Liouville-type integrals of JJ are defined as Γ ([50,70]). Let ι] \cdot [σ , ι] \rightarrow \mathbb{R} _I be an interval-valued mapping (1 ν) **Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4** $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ **Definition 1** ([30,40]). Let $J : [\sigma, i] \to \mathbb{R}_I^+$ be an interval-valued mapping (IVM) and $J \in \mathbb{R}$ $\mathcal{F}[\mathcal{O}_L]$ T_{av} , $\left[\boldsymbol{v}^{\prime}\right]$ $\mathcal{O}(\mathcal{O},1)$

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

$$
g_{\boldsymbol{\sigma}^+}^{\alpha} J(y) = \frac{1}{\Gamma(\alpha)} \int_{\boldsymbol{\sigma}_+}^y (y - \mathbf{t})^{\alpha - 1} J(\mathbf{t}) d\mathbf{t} \ (y > \boldsymbol{\sigma}), \tag{8}
$$

$$
g_{\mathbf{i}-}^{\alpha} \mathbf{J}(y) = \frac{1}{\Gamma(\alpha)} \int_{y}^{\mathbf{i}} (\mathbf{t} - y)^{\alpha - 1} \mathbf{J}(\mathbf{t}) d\mathbf{t} \quad (y < \mathbf{i}),\tag{9}
$$

where $\alpha > 0$ and Γ is the gamma function. ℐ where $\alpha > 0$ and Γ is the gamma function. $\mathcal{L}(\mathcal{W})$ ∂n . $n.$ μ ion. μ where $\alpha > 0$ and Γ is the gamma function. where $\alpha > 0$ and Γ is the gamma function. ϵ inequalities can be used to approximate the solution. Received: 14 November 2023 where $\alpha > 0$ and Γ is the gamma function. \mathcal{C} Version of Fractional Pachpatte-type $\sum_{i=1}^{n}$ μ integral Indian Indian Componented via Componente via Componente via Componente via Componente via Componente v where $\alpha > 0$ and Γ is the gamma function. $\frac{d}{dt}$ cannot mapping proposed that are known as coordinated that are known as $\frac{d}{dt}$ \ddot{c} mapping convex mapping proposed that are known as coordinated left and \ddot{c} -dimensional right \ddot{c} ω response ω . ω cannot be the gamma junction. where $\alpha > 0$ and Γ is the gamma function.

The inclusion "⊇" means that ⊇ if and only if,ሾ∗, ∗ሿ ⊇ ሾ∗, ∗ሿ, and if and only egrals are defined T^* means that T^* if and only if and ls are defined as follows Interval and fuzzy Riemann-type integrals are defined as follows for coordinated y). $VM J(x, y).$ $F/M \pi(x, y)$ \mathbf{Y} *Fract.* **2024**, \mathbf{Y} , $\mathbf{Y$ $F(M, \pi(x, y))$ $F/M \pi(r, u)$ Interval and fuzzy Riemann-type integrals are defined as follows for coordinated $V^M \pi(x, y)$ can be directly derived from convex functions, there is a close relationship between Interval and fuzzy Kiemann-type integrals are defined as follows for coordina
 $U^{\mathcal{M}}$ $\mathcal{I}(\alpha,\alpha)$ $\lim_{x \to \infty} \frac{1}{\sqrt{f(x)}}$ can be directly derived from convex functions, there is a close relationship between Interval and fuzzy Kiemann-type integrals are defined as follows for coordinated $M \pi(\omega, \omega)$ $T(\mathbf{r},\mathbf{y})$. can be directly derived from convex functions, there is a close relationship between Interval and fuzzy Riemann-type integrals are defined as follows for coordinated Accepted: 6 February 2024 V M $J(x, y)$. V M J (x, y) . $VM \, \mathfrak{J}(x,y).$ T Order Relation. *Fractal Fract.* **2024**, *8*, Interval and fuzzy Riemann-type integrals are defined as follows for coordinated $\prod_{i=1}^{n}$ V *IVI* J (λ, y) . $U\tilde{M} \tilde{\mathbf{u}}(\mathbf{x}, u)$ $U\mathcal{M}$ $\mathcal{I}(\gamma, \nu)$ results' numerical validations that examples are nontrivial. By taking the product of two left and V M J $J(x, y)$. abstract the fractional forms of Hermite extension of Hermite–Hadamard inequalities for the new little ∂ and ∂ convex mapping proposed that are known as coordinated left and right ∂ -dimensional Interval and fuzzy Riemann-type integrals are defined as follows for coordinated 4 Department of Mathematics, Faculty of Science, Jazan University, Jazan 4 \mathbf{v} iversity, \mathbf{v} , $\$ $V\Lambda A$ $\frac{1}{4}$ Department of Science, $\frac{1}{2}$ $\mathcal{L}_{\mathcal{P}}$ eroilor Brasov, $\mathcal{L}_{\mathcal{P}}$

The result was mainly credited to \mathcal{H}_c

Theorem 2 ([42]). Let $JJ : [\sigma, i] \subset \mathbb{R} \to \mathbb{R}$ be an IVM, given by $JJ(x) = [J_*(x), J^*(x)]$ for all $x \in [\sigma, i]$. Then, \iint_S is Riemann integrable (IR-integrable) over $[\sigma, i]$ if and only if $\iint_x (x)$ and $\iint_x (x)$ both are Riemann integrable (R-integrable) over $[\sigma, \iota]$ Moreover, if J is IR-integrable over $[\sigma, \iota]$, then
both are Riemann integrable (R-integrable) over $[\sigma, \iota]$. Moreover, if J is IR-integrable over $[\sigma, \iota]$, both are Riemann integrable (R-integrable) over $[\sigma,\mathfrak{i}].$ Moreover, if \mathfrak{I} is IR-integrable over $[\sigma,\mathfrak{i}]$, then 1963 was the one who first identified it first identified it following is how this inequality is stated: The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known **category of classical convex functions**, and $e^{i\theta}$, $e^{i\theta}$ and Pearly M, given by $j(x)$ **Theorem 2** ([42]) Let $\overline{\mathbf{u}} \cdot [\cdot, \cdot] \subset \mathbb{P}$ be an IVM circular $\overline{\mathbf{u}}(x) = [\overline{\mathbf{u}}(x), \overline{\mathbf{u}}^*(x)]$ \overline{d} is \overline{d} in \overline{d} in \overline{d} into \overline{d} is \overline{d} intrinsic geometric explanations and a straightforward intrinsic geometric explanation. $\frac{1}{\sqrt{1-\frac{1$ $\begin{bmatrix} 9 & 1 \\ 2 & 3 \end{bmatrix}$ who first identified it is how the following is stated: **category of classical convex functions**, $[\sigma, 1] \subseteq \mathbb{R} \to \mathbb{R}$ be an IVM, given by $J(x) = [J_*(x), J_1(x)]$ in $x \in [\sigma, 1]$. Then, if is kiemann integrable (TK-integrable) over $[\sigma, 1]$ if and only if $J]_*(x)$ and if both are Klemann integrable (K-integrable) over $[\sigma, 1]$. Nioreover, if j is IK-integrable over $[\sigma, 1]$, **begiven** 2 ([42]) Let $\pi \cdot [\cdot, \cdot] \subset \mathbb{P}$ $\setminus \mathbb{P}$ be an IVM given by $\pi(x) = [\pi(x), \pi^*(x)]$ for $\mu \in$ [s, i] Then Π is Riemann integrable (IR-integrable) over [s, i] if and only if $\P(\mathbf{x})$ and $\P^*(\mathbf{x})$ $T_{\rm eff}$ are \overline{B} iomann integrable (R-integrable) gree $\left[\frac{1}{2} \right]$. Moreover if $\overline{\Pi}$ is IR-integrable gree $\left[\frac{1}{2} \right]$ then 196 wing identified it is how the following it is how the following is stated: $[42]$. Let $J: [\sigma, 1] \subseteq \mathbb{R} \to \mathbb{R}$ be an IVM, given by $J(x) = [J_*(x), J_-(x)]$ for **Theorem 2** ([42]). Let J : $[\sigma, \iota] \subset \mathbb{R} \to \mathbb{R}$ be an IVM, given by J $(x) = [J_*(x), J^*(x)]$ for all $x \in [\sigma, \iota]$. Then, J is Riemann integrable (IR-integrable) over $[\sigma, \iota]$ if and only if $J_*(x)$ and $J^*(x)$ both are Riemann integrable (R-integrable) over $[\sigma, i]$. Moreover, if J is IR-integrable over $[\sigma, i]$, then **Theorem 2** ([42]). Let $JJ : [\sigma, i] \subset \mathbb{R} \to \mathbb{R}$ be an IVM, given by $JJ(x) = [J_*(x), J^*(x)]$ for 196 identified in the following identified in the following is 12 . The following is stated: **can be directly definitions** of $(\lfloor 4\angle \rfloor)$, Let J : $[\sigma,1] \subset \mathbb{R} \to \mathbb{R}$ be an 1V M, given challenging, inequalities can be used to approximate the solution. Since many inequalities **can be directly derived from convex functions**, the directions of α convex functions α close relationship between α or α close relationship between α close relationship between α close relationship betw tight relationship to the theory of inequalities, convexity has advanced α **Theorem 2** ([42]). Let $JJ: [\sigma, i] \subset \mathbb{R} \to \mathbb{R}$ be an IVM, given by $JJ(x) = [J_*(x)]$ realms of applied and pure sciences. Furthermore, because of its many applications and **Theorem 2** ([42]). Let $JJ : [\sigma, i] \subset \mathbb{R} \to \mathbb{R}$ be an IVM, given by $JJ(x) = [J_*(x), J^*(x)]$ for these new outcomes. obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some these new outcomes. obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some **Theorem 2** ([42]). Let $\mathbf{J}: [\sigma, \mathbf{i}] \subset \mathbb{R} \to \mathbb{R}_I$ be an IVM, given by $\mathbf{J}(\mathbf{x}) = [\mathbf{J}_*(\mathbf{x}), \mathbf{J}^*(\mathbf{x})]$ for $\prod_{i=1}^n$, $\prod_{i=1}^n$, $\prod_{i=1}^n$ ($\prod_{i=1}^n$ or an iv M , $\sum_{i=1}^n$ ($\prod_{i=1}^n$ ($\sum_{i=1}^n$ ($\sum_{i=1}^n$) $\prod_{i=1}^n$ ($\sum_{i=1}^n$) the *the gradie* (It in convexity) over intervalse intervalse codomain. We exploit the use of double \mathbb{R} **Theorem 2** ([42]). Let $JJ: [\sigma, i] \subset \mathbb{R} \to \mathbb{R}$, be an IVM, given by $JJ(x) = [J_*(x), J^*(x)]$ for $\sum_{i=1}^{n}$ numerical values are non-trivial. By taking the product of two left and product of two l an $x \in [0, t]$. Then, f is Kiemann integrable (R-integrable) over $[\alpha, t]$ if β and only η η _{*}(x) and η (x) α) both are Riemann integrable (R-integrable) over $[\alpha, t]$. Moreover, if η is IR-integrable over $r_{\rm c}$ -convexity, some new versions of fractional integral integral integral integral integral in **Absorber** \mathcal{L} $[\mathcal{I}_1]$, $\mathcal{L}(\mathcal{I}_2)$, $[\mathcal{O}, \mathcal{V}] \subseteq \mathbb{R}$ is the new lattice for \mathcal{I}_3 , $(\mathcal{S}, \mathcal{V})$ ω and the mannihilar codomain. The use of double ω , ω , ω , ω , ω , ω , ω **Theorem 2** ([42]). Let $JJ : [\sigma, i] \subset \mathbb{R} \to \mathbb{R}$ be an IVM, given by $JJ(x) = [J_*(x), J^*(x)]$ for both are Riemann integrable (R-integrable) over $[\sigma,\mathfrak{i}]$. Moreover, if J is IR-integrable over $[\sigma,\mathfrak{i}]$, then ali $k \cdot \pi$ i.e.m. $\mathbb{E}[\mathcal{F}_2]$. Euroj. $[\mathcal{O},\mathcal{F}] \subseteq \mathbb{R}$ is a substitute of \mathcal{V}_1 , $\mathcal{S}(\mathcal{O})$ if $\mathcal{O}(\mathcal{M}) = [\mathcal{V}_1, \mathcal{N}_2]$.

$$
(IR)\int_{\sigma}^{i} J(x)dx = \left[(R)\int_{\sigma}^{i} J_{*}(x)dx, (R)\int_{\sigma}^{i} J^{*}(x)dx \right].
$$
 (10)

The family of all IR-integrable of IVMs over coordinates and R-integrable functions over $[\sigma,$ $\lim_{(a, i]} \lim_{(a, i]} \lim_{n \to \infty} \lim_{(a, i]} \lim_{n \to \infty} \lim_{n \to$ ℴ = ቈ() න Ԓ∗() r coordinates and R-integra
 $100 \mu m$ may be an $110 \mu m$ of $111 \mu m$ over every mass with $100 \mu m$ over $100 \mu m$ *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract The family of all TR-integrable of TV Ms over coordinates and R-integrable functions over $[\sigma, \iota]$ are
denoted by $IR_{[\sigma, \iota]}$ and $R_{[\sigma, \iota]},$ respectively. *holds:* The family of all IR-integrable of IVMs over coordinates and R-integrable functions over $[\sigma,\mathfrak{i}]$ are The family of all TR-integrable of TV INS over coorainates and R-integrable inequality has several applications and a straightforward intrinsic geometric explanation. The family of all IR-integrable of TV Ms over coordinates and R-integrable functional conditions of the condition of the category of classical convex functions, according to Dragomir and Pearce [1]. This The family of all IR-integrable of IV Ms over coordinates and R-integrable functions over $[\sigma, \mathfrak{i}]$ are $\mathfrak{g}_{\mathfrak{p}}$ and $\mathfrak{g}_{\mathfrak{p}}$ and $\mathfrak{g}_{\mathfrak{p}}$ The respectively. challenging, including integration of \overline{V} and solution \overline{V} integrated to approximate the solution. Since \overline{V} The family of all IR-integrable of IV Ms over coordinates and R-integrable functions over $[\sigma, \mathfrak{i}]$ are σ challenging, including c and c approximate the solution of \mathbf{p} internality functions solution. Since \mathbf{p} The family of all IR-integrable of IVMs over coordinates and R-integrable functions over $[\sigma, i]$ are Revised: 5 February 2024 \mathcal{L} θ denoted by IR_{tot} and R_{tot} respectively $\begin{bmatrix} V^{r+1} \\ V^{r+1} \end{bmatrix}$ is many applications and its many app denoted by $IR_{[\sigma,i]}$ and $R_{[\sigma,i]}$, respectively. Γ rte denoted by $IR_{[\sigma,i]}$ and $R_{[\sigma,i]}$, respectively. R_{rel} obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some right coordinated of the convenience of fractional integration of fractional integral integral integral in a s results' numerical validations that examples are nontrivial. By taking the product of two left and y of an IX integrable of TV I wis over coordinates and R-integrable fanctions over $[\sigma, v]$ are

 $($ $($ $)$ **Theorem 3** ([\[31\]](#page-22-17)). Let $\mathbf{J}: \Omega = [\sigma, \mathbf{i}] \times [\varepsilon, \mathbf{v}] \subset \mathbb{R}^2 \to \mathbb{R}$, be an IVM on coordinates, given by $\mathbf{J}(x, y) = [\mathbf{J}_*(x, y), \mathbf{J}^*(x, y)]$ for all $(x, y) \in \Omega = [\sigma, i] \times [\varepsilon, \mathbf{v}]$. Then, \mathbf{J} is double integrable $\langle \alpha, \beta \rangle = [\beta, \langle \alpha, \beta \rangle]$, $\langle \alpha, \beta \rangle$ for an $\langle \alpha, \beta \rangle \subset \mathbb{Z}^2 = [0, \cdot] \wedge [\infty]$. Then, β is active integrable over Ω . Moreover Ω . Moreover Ω . *The family of a strategieor of the family of a strategieor coordinates is denoted by a strategieor of the s* if \iint is FD-integrable over Ω , then $(1D\text{-}integi\omega t)$ been s 2 y and only y $J_{\frac{1}{2}}(x,y)$ and $J_{\frac{1}{2}}(x,y)$ both are D-integrable over s 2. Normally *y s f D -megrable boer s z*, *men* **EXECT** Fractional Fraction Fraction Fraction Constant π (x x) and $\pi^*(x, y)$ both are D-integrable $\frac{1}{2}$, $\frac{1}{2}$, *(ID-integrable)* over Ω if and only if $\Pi(x, y)$ and $\Pi^*(x, y)$ both are D-integrable over Ω . Moreover *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract (ID-integrable) over Ω if and only if $J_*(x,y)$ and $J^*(x,y)$ both are D-integrable over Ω . Moreover, 1963 was the one who first identified it first identified it following is how this inequality is stated: **Theorem 3** ([31]). Let $\mathbf{J} : \Omega = [\alpha, \mathfrak{i}] \times [\varepsilon, \mathfrak{v}] \subset \mathbb{R}^2 \to \mathbb{R}$ be an IVM on coordinates, given \overline{U} T Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known f **Theorem 3** ([31]). Let $\mathbf{J} : \Omega = [\boldsymbol{\sigma}, \mathbf{i}] \times [\boldsymbol{\varepsilon}, \mathbf{v}] \subset \mathbb{R}^2 \to \mathbb{R}$, be an IVM on coordinates, given **Copyright:** © 2024 by the authors. Submitted for possible open access $T = \alpha + 1 + 1 + m^2$ is one of the most well-known findings in the most well-known finding in the mos category of classical convex functions, according to Dragomir and Pearce [1]. This $\frac{1}{2}$ was the following inequality in $\frac{1}{2}$. **Theorem 3** ([31]). Let $J: \Omega = [\sigma, i] \times [\epsilon, \nu] \subset \mathbb{R}^2 \to \mathbb{R}$ be an IVM on coordinates, given by $J(x,y) = [J_*(x,y), J^*(x,y)]$ for all $(x,y) \in \Omega = [\sigma, i] \times [\epsilon, \nu]$. Then, J is double integrable α is $c_1(\alpha, y) = [J_*(\alpha, y), J_*(\alpha, y)]$ for an $(\alpha, y) \in \mathbb{R}^2 = [0, 1] \wedge [0, 0]$. Then, if $J_*(\alpha, y)$ σ and the theory of inequalities. \mathcal{L}_{max} and \mathcal{L}_{max} and \mathcal{L}_{max} and \mathcal{L}_{max} problem proves to be a mathematical problem p c_1 is the used to approximate the solution. $t_{\text{U}}(x, y) = [y]_{\text{S}}(x, y)$ for y in the theory of \mathbb{R}^n in recent \mathbb{R}^n in recent \mathbb{R}^n is advanced \mathbb{R}^n in recent \mathbb{R}^n in recent \mathbb{R}^n is a convexity in recent \mathbb{R}^n is a convexi $\left(\text{ID-integrable}\right)$ over Ω if and only if $J_{*}(x,y)$ and $J^{*}(x,y)$ both are D-integrable over Ω . Moreover, c_i is the used to construct the solution. **1. Introduction** *over* Ω , then *f (iii) It can be easily seen that it call and right and right and right seen that* α , *so we call an* $\mathcal{L}_{\mathcal{A}}$ *for all* ሾ∗, ∗ሿ,ሾ∗, ∗ሿ ∈ ℝூ, *and it is a pseudo-order relation. The relation* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ $T_{\rm c}$ are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex sets and convex sets and convex functions in the convex sets and convex sets and convex sets an σ are α the convex sets and convex sets and convex sets and convex α functions in the convex \overline{a} \sim \sim *remarks 1 D* the relation over *s s*, then **KEYWORDS:** $\begin{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} x \\ z \end{bmatrix}, \begin{bmatrix} y \\ z \end{bmatrix}, \begin{bmatrix} y$ **Theorem 3** ([31]), Let $\Pi: \Omega = [\alpha, i] \times [\epsilon, \mathfrak{v}] \subset \mathbb{R}^2 \to \mathbb{R}$, be an IVM on coordinates, given if \iint is FD-integrable over Ω , then $I₁$ Integral Int $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Let $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Let $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ χ ^l and χ integral operator; Pach package integral operator; Pachpathe-type integral operator; Pachpathe-

$$
(ID)\int_{\sigma}^{i} \int_{\varepsilon}^{v} J(x, y) dy dx = (IR)\int_{\sigma}^{i} (IR)\int_{\varepsilon}^{v} J(x, y) dy dx.
$$
 (11)

The family of all ID-integrable of IVMs over coordinates over coordinates is denoted by $\mathfrak{TD}_\Omega.$ The family of all ID-integrable of IVMs over coordinates over coordinates is denoted by $\mathfrak{D}_{\Omega}.$ The family of all ID-integrable of IVMs over coordinates over coordinates is denoted by $\mathfrak{TD}_{\Omega}.$ " ≤ " *"left and right" (or "LR" order, in short).* can be directly derived from convex functions, there is a close relationship between \mathcal{L} challenging, incorporation in the solution of the solution of the solution of the solution. Since many includes The family of all ID-integrable of IVMs over coordinates over coordinates is denoted by $\mathfrak{TD}_{\Omega}.$ can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L} I he family of all ID-integrable of IV Ms over coordinates over coordinates i. realment of a property of a pure sciences. Furthermore, and pure sciences is depended by TO to the theory of τ relationship to the theory of intervals indicated τ in recent τ \mathbf{F}_{eff} able of IVMs oner soordinates oner soordinates is denoted by $\mathcal{F}\Omega$ The family of all ID-integrable of IV Ms over coordinates over coordinates is denoted by $\mathfrak{TD}_\Omega.$ \sim $\sum_{i=1}^{n}$

Definition Here is the main definition of fuzzy Riemann–Liouville fractional integral on the main definition of fuzzy Riemann–Liouville fractional integral on the main definition of fuzzy Riemann–Liouville fractional int Here is the main definition of fuzzy Kiemann–Liouville fractional integral on the coordinates of the function $J(x, y)$ by: Here is the main definition of f Here is the main definition of fuzzy Riemann-Liouville fractional integral on the $\mathbf{p} \cdot \mathbf{p}$ coordinates of the function $I(x, y)$ by: $T_{\rm eff}$ and $T_{\rm eff}$ **DEFINITION** 11 (*Definition 1* $\mathbf{y}_j(.,y_j)$, \mathbf{y}_j). coordinates of the function $I(x, y)$ by $\frac{1}{2}$ result was mainly contained to Hermite (1822–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1 coordinates of the function $J(x, y)$ by: **Definition** $\mathbf{y}_1(x, y, y) = \mathbf{y}$. **Copyright:** © 2024 by the authors. Here is the main definition of fuzzy Riemann-Liouville fractional convexity and the theory of inequalities. The theory of inequalities of inequalities. The theory of inequalities of inequalities of \mathbb{H} $\frac{1}{\sqrt{N}}$ The function $\frac{1}{N}(x, y)$ by: coordinates of the function $J(x, y)$ by: $\frac{1}{2}$ by: can be directly derived from convex functions, there is a close relationship between $\mathcal{L}(\mathcal{X})$ Here is the main definition of fuzzy Kiemann–Liouville fractional coordinates of the function $J(x, y)$ by: Here is the main definition of fuzzy Riemann-Liouville fractional integral on the coordinates of the function $J(x, y)$ by: can be directly derived from convex functions, there is a close relationship between $\mathcal{L}(\mathcal{A})$ (x, u) by used to approximate the solution. Since (x, u) is (u, u) can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L} \int *i* iii) is easily seen that **"** \int and *right"* on the real line \int $\mathcal{M}^{\text{max}}_{\text{max}}$

Definition 2 ([41]). Let $\mathcal{J} : \Omega \to \mathbb{R}$ and $\mathcal{J} \in \mathfrak{SD}_\Omega$. The double fuzzy interval Rieman– $J_{\sigma^+,\varepsilon^+}$, $J_{\sigma^+,\varepsilon^-}$, $J_{\varepsilon^-,\varepsilon^+}$, $J_{\varepsilon^-,\varepsilon^-}$, $J_{\varepsilon^-,\varepsilon^-}$, $J_{\varepsilon^-,\varepsilon^-}$, $J_{\varepsilon^-,\varepsilon^-}$, $J_{\varepsilon^-,\varepsilon^-}$ **Dennition** 2 ([41]). Let $J = \alpha \beta$ **2** ([41]). Let $J: \Omega \to \mathbb{R}_I$ and $J \in \mathfrak{TD}_{\Omega}$. The double fuzz $\frac{1}{2}$ and the ground $\frac{1}{2}$ $\frac{1}{$ **Definition 2** ([41]). Let $J : \Omega \to \mathbb{R}$ and $J \in \mathfrak{TD}_{\Omega}$. The double fuzzy interval Rieman— Liouville-type integrals $g^{\alpha, \beta}_{\alpha^+, \varepsilon^+}$, $g^{\alpha, \beta}_{\alpha^+, \nu^-}$, $g^{\alpha, \beta}_{i^-, \varepsilon^+}$, $g^{\alpha, \beta}_{i^-, \nu^-}$ of J order $\alpha, \beta > 0$ are defined by: Liouville-type integrals $g^{\alpha, \beta}_{\alpha^+, \varepsilon^+}$, $g^{\alpha, \beta}_{\alpha^+, \nu^-}$, $g^{\alpha, \beta}_{i^-, \varepsilon^+}$, $g^{\alpha, \beta}_{i^-, \nu^-}$ of JJ order $\alpha, \beta > 0$ are defined by: Liouville-type integrals $\int_{0^+ \kappa^{+}}^{\alpha, \beta} \int_{0^+ \kappa^{+}}^{\alpha, \beta} \int_{0^+ \kappa^{+}}^{\alpha, \beta} \int_{i^+ \kappa^{+}}^{\alpha, \beta} \int_{i^-, \mathfrak{v}^{-}}^{\alpha, \beta}$ of J order $\alpha, \beta > 0$ are defined by: **Definition 2** ([41]). Let $\mathbf{J} : \Omega \to \mathbb{R}$ _I and \mathbf{J} \in **Definition 2** ([41]). Let $J : \Omega \to \mathbb{R}$ and $J \in \mathfrak{TD}_{\Omega}$. The double fuzzy interval Rieman— $\sigma_{\alpha',\beta}$. The following is $\sigma_{\alpha',\beta}$. The following is stated: $\begin{bmatrix}\n\cdots \\
\cdots\n\end{bmatrix}$ and a straightforward intrinsic geometric explanations and a straightforward intervalse α , β Elouvalle-type integrals $J_{\sigma^+, \varepsilon^+}$, $J_{\sigma^+, \varepsilon^-}$, J_{i^-, ε^+} , J_{i^-, ε^-} by J order $\alpha, \beta > 0$ and integrals $a^{\alpha, \beta}$ $a^{\alpha, \beta}$ $a^{\alpha, \beta}$ $a^{\alpha, \beta}$ of Horder $\alpha, \beta > 0$ are defined by σ_{α}^{3} was the one who first identified it follows in σ_{α}^{3} . The following is stated: $\sum_{i=1}^{\infty}$ p ublication under the terms and the terms and ter $T^{\alpha, \beta}$ and $T^{\alpha, \beta}$ of Horder $\alpha, \beta > 0$ are defined by: α_{1}^{3} , who first identified it following it following it following it follows: I journille-type integrals $a^{\alpha, \beta}$ σ Liouville-type integrals $\int_{0}^{\alpha,\beta}$ + $\int_{0}^{\alpha,\beta}$ + $\int_{0}^{\alpha,\beta}$ + $\int_{0}^{\alpha,\beta}$ + $\int_{0}^{\alpha,\beta}$ of J order α,β \int is a familiar fact that \int ϵ functions, according to Dragomir and ϵ Liouville-type integrals $\jmath_{\sigma^+,\varepsilon^+}$, $\jmath_{\sigma^+,\mathfrak{v}^-}$, $\jmath_{\mathfrak{i}^-,\varepsilon^+}$, $\jmath_{\mathfrak{i}^-,\mathfrak{v}^-}$ of JJ order α , $\beta > 0$ and **Demitton 2** ($\left[\frac{41}{1}\right]$). Le **Definition 2** ([41]). Let $\Pi: \Omega \to \mathbb{R}$, and $\Pi \in \mathfrak{TD}_{\Omega}$. The double fuzz μ_{α} α, β α, β α, β α, β α β α α β α α β α β categories $J_{\sigma^+, \varepsilon^{++}} J_{\sigma^+, \varepsilon^{-}} J_{\varepsilon, \varepsilon^{++}} J_{\varepsilon^-, \varepsilon^{+-}} J_{\varepsilon^-, \varepsilon^{--}}$ by solutions, p $>$ convenience by: $\alpha, \beta, \alpha, \beta$ of π order α, β, γ are defined by: Liouville-type integrals $g^{\prime\prime}_{\sigma^+,\varepsilon^+}$, $g^{\prime\prime}_{\sigma^+,\mathfrak{v}^-}$, $g^{\prime\prime}_{\iota^-, \varepsilon^+}$, $g^{\prime\prime}_{\iota^-,\mathfrak{v}^-}$ of J order α , $\beta > 0$ are defined by: **Definition** 2 ([41]). Let $\iint_R f(x) dx = \iint_R f(x) dx$ in $\iint_R f(x) dx = \iint_R f(x) dx$. years. When determining exact values for a mathematical problem proves to be ([41]). Let $j: \Omega \to \mathbb{R}_I$ and $j \in \Omega$ Ω . The double fuzzy interval Kieman— Revised: 5 February 2024

convexity and the theory of inequalities.

$$
g_{\boldsymbol{\sigma}^{+},\varepsilon^{+}}^{\alpha,\beta}J(x,y)=\frac{1}{\Gamma(\alpha)\Gamma(\beta)}\int_{\boldsymbol{\sigma}}^{x}\int_{\varepsilon}^{y}(x-t)^{\alpha-1}(y-s)^{\beta-1}J(t,s)dsdt,\ \ (x>\boldsymbol{\sigma},y>\varepsilon),\ \ (12)
$$

$$
g_{\boldsymbol{\sigma}^{+},\mathbf{b}}^{\alpha,\beta} - \mathcal{J}(x,y) = \frac{1}{\Gamma(\alpha)\Gamma(\beta)} \int_{\boldsymbol{\sigma}_{\alpha}}^{x} \int_{y}^{\mathbf{b}} (x-\mathbf{t})^{\alpha-1} (\mathbf{s}-y)^{\beta-1} \mathcal{J}(\mathbf{t},\mathbf{s}) d\mathbf{s} d\mathbf{t}, \quad (x > \boldsymbol{\sigma}, y < \mathbf{b}), \tag{13}
$$

$$
g_{\mathfrak{i}^{-},\varepsilon^{+}}^{\alpha,\,\beta}J(x,y)=\frac{1}{\Gamma(\alpha)\Gamma(\beta)}\int_{x}^{\mathfrak{i}}\int_{\varepsilon}^{y}\left(\mathbf{t}-x\right)^{\alpha-1}\left(y-\mathbf{s}\right)^{\beta-1}J(\mathbf{t},\mathbf{s})d\mathbf{s}d\mathbf{t},\ \, (x<\mathfrak{i},y>\varepsilon),\ \ \, (14)
$$

Abstract: In particular, the fractional forms of Hermite–Hadamard inequalities for the newly

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also

The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–

can be directly derived from convex functions, there is a close relationship between

Khan, M.B.; Hakami, K.H. New York (1988)

ℏ-Convexity via Left and Right

Order Relation

Khan, M.B.; Hakami, K.H. New

New Version of Fractional Pachpatte-type Integral

New Version of Fractional Pachpatte-type Integral

Received: 14 November 2023

Order Relation

<u>, convexity via Left and Right</u>

New Version of Fractional Pachpatte-type Integral

 $\mathcal{L}_{\mathcal{A}}$ are determining exact values for a mathematical problem problem problem problem problem problem problem proves to be

category of classical convex functions, according to Dragomir and Pearce [1]. This

Order Relation. *Fractal Fract.* **2024**, *8*,

Khan, M.B.; Hakami, K.H. New York (1985)

Definition 1 ([30,40])**.** *Let* Ԓ:ሾℴ, ሿ → ℝூ

Received: 14 November 2023

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

can be directly derived from convex functions, there is a close relationship between

1. Introduction

Liouville fractional integral to derive the major results of the research. We also examine the key

results' numerical validations that examples are nontrivial. By taking the product of two left and

Order Relation

 \mathcal{A}_max Department of Mathematics, \mathcal{A}_max are substituting to \mathcal{A}_max

2 Department of Mathematics and Computer Science, Alabama State University,

 \sim Correspondence: enwaneze. Enwaneze: enwaneze. enwaneze. En water of \sim

New Version of Fractional Pachpatte-type Integral

Tareq Saeed 1*,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4**

inequality has several applications and a straightforward intrinsic geometric explanation.

$$
\mathcal{J}_{\mathfrak{i}^{-},\mathfrak{v}}^{\alpha,\beta} - J(\alpha,y) = \frac{1}{\Gamma(\alpha)\Gamma(\beta)} \int_{x}^{\mathfrak{i}} \int_{y}^{\mathfrak{v}} (\mathsf{t} - x)^{\alpha-1} (\mathsf{s} - y)^{\beta-1} J(\mathsf{t}, \mathsf{s}) d\mathsf{s} d\mathsf{t}, \quad (x < \mathfrak{i}, y < \mathfrak{v}). \tag{15}
$$

 $\overline{}$ years. When determining exact values for a mathematical problem p

realms of applied and pure sciences. Furthermore, because of its many applications and

ା *be an interval-valued mapping () and* Ԓ ∈ ℐℛሾℴ,ሿ*.*

New Version of Fractional Pachpatte-type Integral

3 Department of Mathematics and Computer Science, Transilvania University of Brasov,

 \sim Correspondence: enwaneze. enwaneze \sim

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

can be directly derived from convex functions, there is a close relationship between

3 Department of Mathematics and Computer Science, Transilvania University of Brasov,

 \sim Correspondence: enwaneze. enwaneze \sim

Here is the classical and newly defined concept of coordinated LR- \hbar -convexity over fuzzy number space in the codomain via fuzzy relation given by: **Copyright:** © 2024 by the authors. $\frac{1}{\sqrt{1}}$ convexity and the theory of interest in the theory of interest in the contenum of the ϵ challenging in the used to approximate the solution. Since ϵ is approximate the solution. can be directed from convex functions, the interest relationship between challenging in the challenging into a contract the solution of \mathbf{r} and $\$ $\frac{1}{2}$ can be directed from convex functions, there is a close $\frac{1}{2}$ functions, $\frac{1}{2}$ ϵ challenging in the canonical the solution of the solution. Since ϵ the solution of the solution. Since ϵ many $\frac{1}{2}$ be directed from convex functions, the convex functions $\frac{1}{2}$ challenging in the canonical the solution of the use of α is α in α in α in α is α can be directed from convex functions, the convex functions \mathbf{g} and \mathbf{g} fuzzy number space in the codomain via fuzzy relation given by: Here is the classical and newly defined concept of coordinated LR - \hbar -convexity over Here is the classical and newly defined concept of coordinated LR- \hbar -convexity over Free is the chassical and newly defined concept of coordinated ER n convexity over
fuzzy number space in the codomain via fuzzy relation given by: $\overline{\text{SUS}}$ right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also There is the classical and newly defined concept of coordinate right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also Here is the classical and newly defined concept of coordinated *LK-n-*convexity over results' numerical validations that examples are nontrivial. By taking the product of two left and Liouville fractional integral to derive the major results of the research. We also examine the key There is the classical and newly defined concept of coordinated *EK-n*-convexity over defined class of convex mappings proposed that are known as coordinated left and right ℏ - Here is the classical and newly defined concept of coordinated LR-h-convexity over 1 ere is the C fuzzy number space in the codomain via fuzzy relation given by: 3 Department of Mathematics and Computer Science, Transilvania University of Brasov, 1 Ere is the classical and for space in the codomain via fuzzy relation given by: Here is the classical and newly defined concept of coordinated LR- \hbar -convexity over
fuzzy number space in the codomain via fuzzy relation given by: 3.3 Department of Mathematics and Computer Science, Transilvania University of Bras α z y number space in the codomani via $\mathcal{L}_{\text{M223}}$ fuzzy number space in the codomain via fuzzy relation given by: 2 Department of Mathematics and Computer State University, Alabama State University, Alab 2 Department of Mathematics and Computer Science, Alabama State University, $\frac{1}{2}$ Department of Mathematics and Computer State University, Alabama State University 2 Department of Mathematics and Computer Science, Alabama State University, \mathcal{L} department of Mathematics and Computer State University, Alabama State University, fuzzy number space in the codomain via fuzzy relation given by: 2 Department of Mathematics and Computer State University, Alabama State University, Alab **Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4 Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4 Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4** Here is the classical and newly defined concept of coordinated $LR-\hbar$ Here is the classical and newly defined concept of coordinated LR- \hbar -convexity over
fuzzy number space in the codomain via fuzzy relation given by: Here is the classical and newly defined concept of coornel and **Pack**
 New Your State Integration *Stype In the codomain via fuzzy relation give* fuzzy number space in the codomain via fuzzy relation given by:

Definition 3 ([45]). The IVM $JJ : [\varepsilon, \mathfrak{v}] \to \mathbb{R}^+_I$ is referred to be LR- \hbar -convex IVM on $[\varepsilon, \mathfrak{v}]$ if publication under the terms and The result was mainly contributed to \mathcal{L} and \mathcal{L} and \mathcal{L} are even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even thou category of convex functions, according to $\frac{1}{2}$. This is the Pearce $\frac{1}{2}$ convexity and the theory of inequalities. $\frac{1}{\sqrt{1-\frac{1$ $\sum_{i=1}^{\infty}$ convexity and the theory of inequalities. $\frac{1}{\sqrt{k}}$ is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most w Γ exity and the theory of (15) . The theory of Γ $\frac{1}{\sqrt{1+\frac{1$ ion 3 ([45]) The IVM $\bar{\mathbf{u}}$ is $\mathbf{v} = \mathbf{v} + i\mathbf{v}$. T Hermite–Hadamard inequality is one of the most well-known findings in the most well-known finding in the most well-known **Definition 3** ([45]). The IVM \mathbf{J} : $[\varepsilon, \mathfrak{v}] \to \mathbb{R}_1^+$ is referred to be LR- \hbar -convex IVM on $[\varepsilon, \mathfrak{v}]$ if **Definition 3** ([45]). The IVM $\boldsymbol{\rm J}$: [ε , $\boldsymbol{\rm v}$] $\rightarrow \mathbb{R}_I^+$ is referred to be LR- \hbar -conve R_{c} restrictions on $I(A|E)$. The IVM π , $\left[\alpha \right]$ π is negleculated to be IR is courrent IVM on $\left[\alpha \right]$ if $\mathbf{y} \cdot [\mathbf{y}, \mathbf{y}]$ respectively. \overline{D} obtained and classical exceptional cases are also discussed by taking some \overline{C} **Deminism** on $(T\omega)$, the tv m y_j of ω_j as ω_j \overline{D} and classical exceptional cases are also discussed by taking some \overline{D} **PERIMENTAL-VALUE FUNCTIONS ON ENDPOINT FUNCTIONS OF INTERVALLATIONS OF INTERVALLATIONS THAT CAN BE SEEN ASSESS** obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some **Definition** σ ([10]). The TV be \mathbf{y} , [c, c] \rightarrow \mathbb{R} to rejerve to be ER *n* con $\sum_{i=1}^{n}$ $\binom{n}{i}$ Moreover, some new and cases are also discussed by taking some $\binom{n}{i}$ and $\binom{n}{i}$ and $\binom{n}{i}$ $r_{\text{transverse}}$ (respectively) that is the set q_{ref} interval q_{ref} is regarded to max in bench obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some **PERIMPHONE OF EXTRA LATE FUNCTIONS ON EXTRA CONCERT FUNCTION** $\left[\mathbf{c}, \mathbf{d}\right]$ obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some restrictions $(\lceil x \rceil)$. The total j . $[\lceil x \rceil]$ is negative to be seen as applications to the set $[\lceil x \rceil]$ of $\lceil y \rceil$ $\mathbf{P}_{\mathbf{r}}(C_{\mathbf{r}}(t), \mathbf{r}, \mathbf{q})$ ([AFI) \mathbf{F}^{1} , $\mathbf{I}^{1}(\mathbf{M}, \mathbf{r}, \mathbf{q})$, \mathbf{r}^{1} , \mathbf{r}^{2} , \mathbf{r}^{2} , \mathbf{r}^{3} , \mathbf{r}^{4} , \mathbf{r}^{5} , \mathbf{r}^{6} , \mathbf{r}^{7} , \mathbf{r}^{8} , \mathbf{r}^{7} $\sum_{i=1}^n$ coordinated $\sum_{i=1}^n$, some new versions of $\sum_{i=1}^n$ integrals are also convexity (-ℏ-convexity) over interval-valued codomain. We exploit the use of double Riemann– **Demittion 3** ([45]). The TV M J]: [ε , v] $\rightarrow \mathbb{R}$ is referred to be LK-n-convex TV I convexity (-ℏ-convexity) over interval-valued codomain. We exploit the use of double Riemann– **Demittion** 3 ([45]). The TV M J]: [ε , v] $\rightarrow \mathbb{R}$ is referred to be LK-n-convex TV M on [ε , v] convexity (-ℏ-convexity) over interval-valued codomain. We exploit the use of double Riemann– **Definition 3** ([45]). The IVM $\boldsymbol{\rm J}$: [ε , $\boldsymbol{\rm v}$] $\rightarrow \mathbb{R}^+_I$ is referred to be LR- \hbar -convex IVM on [ε , $\boldsymbol{\rm v}$] if 4 Department of \mathcal{B} Department of Science, Jazan University, Ja \mathcal{B} efinition 3 $([45])$. The \overline{z} 3 Department of Mathematics and Computer Science, Transilvania University of Brasov, \overline{D} department of \overline{D} \sum $\sum_{i=1}^{n}$ Montgomery, AL 36101, USA \mathbf{D} \mathcal{C} and \mathcal{C} and \mathcal{C} 36101, \mathcal{C} 36101, USA \sum dependent of $\left(\lfloor x \rfloor\right)$, The TV M $\left(y \right)$ is $\lfloor y \rfloor$ of $\lfloor y \rfloor$ to referred to be ER to control. $M_{\rm H}$, $L_{\rm 20}$ \sim 1×10^{14} \rm{m} \pm \sim \sim \sim $\frac{3}{3}$ Department of Mathematics and Computer Science, Transilvania University of Brasilvania University of Brasil **Definition 3** ([45]). The IVM J χ Science, χ $\frac{1}{2}$ of Science, $\frac{1}{2}$ 589, $\frac{1}{2}$ 589, Saudi Arabia; tsalmalki $\frac{1}{2}$ 1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of Mathematics, Faculty $1 - 1$ Financial Mathematics and Actuarial Science (FMAS)-Research Group, Γ 1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of Mathematics, Faculty 1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of Mathematics, Faculty **Definition 3** ([45]). The IVM π is n $\rightarrow \mathbb{R}^+$ is referred to be LR-h-convex IVM on [s, n] if **Definition 3** ([45]). The IVM $\boldsymbol{\Pi}$: [ε , $\boldsymbol{\nu}$] $\rightarrow \mathbb{R}^+_I$ is referred to be LR- \hbar -convex IVM on [ε , $\boldsymbol{\nu}$] if

$$
JJ(\kappa \varepsilon + (1-\kappa)\mathfrak{v}) \leq_p \hbar(\kappa)J(\varepsilon) + \hbar(1-\kappa)J(\mathfrak{v}), \qquad (16)
$$

inequality has several applications and a straightforward intrinsic geometric explanation.

years. When determining exact values for a mathematical problem proves to be

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

2 Department of Mathematics and Computer Science, Alabama State University, Alabama State University, Alabama S
2 Department of Mathematics and Computer Science, Alabama State University, Alabama State University, Alabama

can be directly derived from convex functions, there is a close relationship between

where $\hbar:[0,\,1]\to\mathbb{R}^+$. If inequality (16) is reversed, then J is referred to be \hbar -concave IVM on $[\varepsilon,\, \mathfrak{v}].$ ω ncre $n : [0, 1] \rightarrow \mathbb{R}$. The Quantity (1) \mathbb{R}^n , \mathbb{R}^n , $[0, 1]$, \mathbb{D}^+ , \mathbb{R}^n is usuality (16) is usuave of the most well-known finding in the concerner \mathbb{R}^n ω iere $u: [0, 1] \rightarrow \mathbb{R}$. If inequality (10) is reversed, then If is referred \mathbb{R}^{n} \mathbb{R}^{n} , \mathbb{R}^{n} , \mathbb{R}^{n} + \mathbb{R}^{n} \mathbb{R}^{n} \mathbb{R}^{n} \mathbb{R}^{n} \mathbb{R}^{n} and \mathbb{R}^{n} \mathbb{R}^{n} \mathbb{R}^{n} \mathbb{R}^{n} \mathbb{R}^{n} \mathbb{R}^{n} \mathbb{R}^{n} \mathbb{R}^{n} \mathbb{R}^{n where u , $[v, 1] \rightarrow \infty$ and v inequality (10) is reversed, then if is referred to be n-conc $T \times$ For $\frac{1}{2}$ in the most well-known findings in the mo ω and α , β , γ \rightarrow ∞ . The quality (10) is reversed, then η is referred to be n-concave TV IVI of T_{ref} is the Hermite–Hadamard inequality include integration in the most well-known findings in the most well-known findi where $\hbar:[0,\,1]\to\mathbb{R}^+$. If inequality (16) is reversed, then J is referred to be \hbar -concave IVM on $[\varepsilon,\, \mathfrak{v}].$ tight relationship to the theory of inequalities, convexity has advanced α tight relationship to the theory of inequalities, convexity has advanced α in recent α tight relationship to the theory of inequalities, convexity has advanced quickly in recent tight relationship to the theory of inequalities, convexity has advanced α Version of Fractional Pachpatte-type Khan, M.B.; Hakami, K.H. New restrictions on endpoint functions of interval-valued functions that can be seen as applications of restrictions on endpoint functions of interval-valued functions that can be seen as applications of restrictions on endpoint functions of interval-valued functions that can be seen as applications of r_0 right coordinated r_1 -convexiting r_2 integrating r_1 is fractional integration in the also see the also results that examples are non-trivial. By taking the product of the product of the product of two left and the product of the product of two left and the product of the product of the product of two left and the product a rof where α , and η when integrated integration integrals are also integrated in α , α $\mathbf{L} \in \left[0, 1\right]$ integral to derive the major results of the results of the key also examine the key also examin r_{rel} is the product of two left and the product of two left and product of two left and r_{rel} and r defined class of convex mappings proposed that are known as coordinated that are known as coordinated left and right α where $\hslash:[0,\,1]\to\mathbb{R}^+$. If inequality (16) is reversed, then J is referred to be \hbar -concave IVM on [ɛ, ʊ where $\hslash:[0,\,1]\to\mathbb{R}^+$. If inequality (16) is reversed, then J is referred to be \hbar -concave IV M on $[\varepsilon,\,\mathfrak{v}].$ where $n : [0, 1] \rightarrow \mathbb{R}^n$. If they daily (10) is reversed, then If is referred to be n-concube TV ivi on [ε, b \sim Correspondence: enwanetedu (E.R.N.); muhammad.bilal. \sim \sim Correspondence: enwaneze.enwater.enwaki.edu (E.R.N.); muhammad.bilal. \sim khakami@jazanu.edu.sa where $n : [0, 1] \rightarrow \mathbb{R}^n$. If inequality (10) is reversed, then if is referred to be no $k = k \cdot \frac{1}{2}$ α corresponding (E.R.); including the entiremporal of the model of α is the entries of α . $\frac{1}{1}$ Department of Mathematics and Computer Science, Transilvania University of Bras $\frac{1}{2}$ $3 \text{div} \cdot \text{K}$ Separation of K ω i.e. ω is ω is ω is ω is ω is ω is the sequence of ω 3 Department of Mathematics and Computer Science, Transilvania University of Brasov, \overline{a} ω M org k , $[0, 1]$, \mathbb{D} $+$ \mathbb{H} \mathbb{H} $\sum_{i=1}^{\infty}$ $\sum_{i=1}^{\in$ molity (16) is remembed than $JJ(\kappa \varepsilon + (1 - \kappa)\mathfrak{v}) \leq_p \hbar(\kappa)JJ(\varepsilon) + \hbar(1 - \kappa)JJ(\mathfrak{v})$, (16)
where $\hbar : [0, 1] \to \mathbb{R}^+$. If inequality (16) is reversed, then JJ is referred to be \hbar -concave IV M on [ε , \mathfrak{v}].

Theorem 4 ([45]). Let $\hbar : [0, 1] \to \mathbb{R}^+$ and $JJ : [\varepsilon, \mathfrak{v}] \to \mathbb{R}^+$ be a LR- \hbar -**Theorem 4** ([45]). Let $\hbar : [0, 1] \to \mathbb{R}^+$ and $JJ : [\varepsilon, \mathfrak{v}] \to \mathbb{R}^+_I$ be a LR- \hbar -convex IVM on $[\varepsilon, \mathfrak{v}]$, given by $J(y) = [J_*(y), J^*(y)]$ for all $y \in [\varepsilon, \mathfrak{v}]$. If $J \in L([\varepsilon, \mathfrak{v}], \mathbb{R}_I^+)$, then **Theorem 4** ([45]). Let $\hbar : [0, 1] \to \mathbb{R}^+$ and $JJ : [\varepsilon, \mathfrak{v}] \to \mathbb{R}^+_I$ be a LR- \hbar -convex IVM on $[\varepsilon, \mathfrak{v}]$, given by $J(y) = [J_*(y), J^*(y)]$ for all $y \in [\varepsilon, \mathfrak{v}]$. If $J \in L([\varepsilon, \mathfrak{v}], \mathbb{R}_I^+)$, then given by $J(y) = [J_*(y), J^*(y)]$ for all $y \in [\varepsilon, \mathfrak{v}]$. If $J \in L([\varepsilon, \mathfrak{v}], \mathbb{R}_I^+)$, then $\mathcal{S}_{\text{total}}$ one one one who first identified it is the following internal inequality is not the following inequality is stated. $T(t) = \frac{\pi(t)}{2\pi} \int_{0}^{t} \frac{\mu(t)}{2\pi} \int_{0}^{t} \frac{\mu(t)}{2\pi} \frac$ Species of \mathcal{G}_1 , \mathcal{G}_2 , \mathcal{G}_3 . The following is the following inequality is stated. The result of $\begin{bmatrix} \Gamma(\omega) & \Gamma$ given by $\mathfrak{J}(y) = [\mathfrak{J}_*(y), \mathfrak{J}^*(y)]$ for all $y \in [\varepsilon, \mathfrak{v}]$. If $\mathfrak{J} \in L([\varepsilon, \mathfrak{v}], \mathbb{R}_I^+)$, then can be directly derived from convex functions, there is a close relationship between titudent $\left[\begin{matrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{matrix}\right], \text{ Let } n : [n] \rightarrow \infty$ and $j : [c, 0] \rightarrow \infty$ realms of applied and pure sciences. Furthermore, because of its many applications and \mathbf{t} (\mathbf{t} + \mathbf{t}), Let \mathbf{u} , \mathbf{v} , \mathbf{v} in \mathbf{u} in \mathbf{u}), \mathbf{t} , \mathbf{v} is \mathbf{v} in recent \mathbf{v} in \mathbf{v} in \mathbf{v} , \mathbf{v} , \mathbf{v} , \mathbf{v} , \mathbf{v} , \mathbf{v} , \mathbf{v} Academic Editor: Bruce Henry T there are many uses for the convex sets and convex sets and convex α functions in the convex f **Theorem 4** ([45]). Let $\hbar : [0, 1] \to \mathbb{R}^+$ and $\mathbf{J} : [\varepsilon, \mathfrak{v}] \to \mathbb{R}^+$ be a LR- \hbar -convex IVM on $[\varepsilon, \mathfrak{v}]$, α ², β , β $\liminf_{\epsilon \to 0} \frac{1}{\pi}$ In $\left(\epsilon \right)$ Integration given by $\mathfrak{J}(y) = [\mathfrak{J}_*(y), \mathfrak{J}^*(y)]$ for all $y \in [\varepsilon, \mathfrak{v}]$. If $\mathfrak{J} \in L([\varepsilon, \mathfrak{v}], \mathbb{R}_I^+)$, then Version of Fractional Pachpatte-type $\overline{0}$ **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **Keywords:** $\frac{1}{2}$ intervalse over coordinates; left and right $\frac{1}{2}$ **Keywords: intervalse over coordinates; left and right** $\mathcal{L}(\mathcal{A})$ these new outcomes. obtained $M(u) = \begin{bmatrix} \pi(u) & \pi^*(u) \end{bmatrix}$ for all $u \in [a, b]$ r_{c} restrictions on endpoint functions that can be seen as applications that can be seen as applications of the see results that $\left[\frac{1}{2}x\right]$ is taking that examples are non-trivial. By taking the product of two left and $\left[\frac{1}{2}x\right]$ given by $J(y) = [J_*(y), J^*(y)]$ for all $y \in [\varepsilon, \mathfrak{v}]$. If $J_1 \in L([\varepsilon, \mathfrak{v}], \mathbb{R}_I^+)$, then Theorem \mathbf{F} ([+0]), Let $n : [0, 1] \to \mathbb{R}$ and $j : [\varepsilon, 0] \to \mathbb{R}$ be a LK-n-concex 1 V N1 on $[\varepsilon, 0]$,
given by $\mathbf{J}(y) = [\mathbf{J}_*(y), \mathbf{J}^*(y)]$ for all $y \in [\varepsilon, \mathfrak{v}]$. If $\mathbf{J} \in L([\varepsilon, \mathfrak{v}], \mathbb{R}^+_I)$, then **defined convert mapping proposed that are known as coordinated that are known as coordinates proposed that Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly **Theorem 4** ([45]), Let $h : [0, 1] \to \mathbb{R}$ and $j : [\varepsilon, 0] \to \mathbb{R}$ be a LK-n-convex 1 V N1 on $[\varepsilon, 0]$, Hieorem 4 (1451), Let u_{1} , $[v_{2}]$ 29 Eroilor Boulevard, 500036 Brasov, Romania **HEOLENT 4** ($[45]$). Let $[n]$, $[0, 1] \rightarrow \mathbb{R}$ and J . $[2, 0]$ Eroilor Boulevard, 500036 Brasov, Romania **HEORETH** $\left(\frac{1}{2}\right)$ **, EEL N**, $\left[\frac{0}{4}\right] \rightarrow \infty$ university, $\left[\frac{1}{2}\right]$ 29 Eroilor Boulevard, 500036 Brasov, Romania **THEOTEM 4** ([45]). Let $n : [0, 1] \rightarrow \mathbb{R}$ and $j : [e, 0] \rightarrow \mathbb{R}$ _I be a LK-n-convex. $29 \text{ F} \cdot 125$ $[0, 1] \rightarrow \mathbb{R}$ and J . $[\varepsilon, 0] \rightarrow \mathbb{R}$ be a LN-n-convex TV in on $[\varepsilon, 0]$, **2 Theorem 4** ([45]). Let $h : [0, \cdot]$ of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa **2 Lineorem** 4 ($[45]$). Let $n:$ $[0, 0]$ 1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of Mathematics, Faculty **THEOTEIN 4** ([45]). Let $h : [0, 1] \rightarrow \mathbb{R}$ and $J : [\epsilon, 0] \rightarrow \mathbb{R}$ be a \mathbf{r} financial Mathematics and Actual Science (FMAS)-Reserves (Group, \mathbf{r} and \mathbf{r} \mathbf{r} **THEOLETT 4** $([T_1]_1,$ Let $[n_1]_1 \rightarrow \infty$ $[nn_1]_1$, $[c, v] \rightarrow \infty$ _I be a 1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of Mathematics, Faculty **of Science, City, Abdulation** $J \colon [e, v] \to \mathbb{R}$, Box 802020203, IV IVI ON $[e, v]$, 1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, \mathbb{R}^n $\lim_{t \to \infty} \left[\frac{1}{t} \right]$, Let $\left[\frac{1}{t} \right]$, $\left[\frac{1}{t} \right]$ and $\left[\frac{1}{t} \right]$, $\left[\frac{1}{t} \right]$, 1 Financial Mathematics and Actuarial Science ($\frac{1}{2}$ $\frac{$ given by $\mathbf{J}(\mathbf{y}) = [J_*(\mathbf{y}), J_*(\mathbf{y})]$ for an $\mathbf{y} \in [\varepsilon, \mathbf{v}]$. If $J_1 \in L([\varepsilon, \mathbf{v}], \mathbb{R}_1)$, then 1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of $\frac{1}{2}$ $\frac{1}{2}$ given by $j_j(y) = [j_{*}(y), j_{*}(y)]$ for an $y \in [\varepsilon, \mathfrak{v}]$. If $j_j \in L([\varepsilon, \mathfrak{v}], \mathbb{R}_j)$, then

$$
\frac{1}{\alpha\hbar\left(\frac{1}{2}\right)}J\left(\frac{\varepsilon+\mathfrak{v}}{2}\right)\leq_{p}\frac{\Gamma(\alpha)}{(\mathfrak{v}-\varepsilon)^{\alpha}}\left[\mathfrak{I}_{\varepsilon^{+}}^{{\alpha}}J(\mathfrak{v})+\mathfrak{I}_{\mathfrak{v}}^{{\alpha}}-J(\varepsilon)\right]\leq_{p}\left[J(\varepsilon)+J(\mathfrak{v})\right]\int_{0}^{1}v^{\alpha-1}[\hbar(v)+\hbar(1-v)]dv.
$$
\n(17)

Fractal Fract. **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract \mathbb{F}^1 , \mathbb{F}^1 , \mathbb{F}^2 , $\$ **Definition 4.** The IVM $J: \Omega \to \mathbb{R}^+_I$ is referred to be coordinated LR- \hbar -convex IVM on Ω if \sim semesters at sine at six sjons and a straightforward is even category of classical convex functions, and \mathcal{L}_{max} functions, and \mathcal{L}_{max} and \mathcal If \mathbf{r} in the restriction \mathbf{r} as several applications and a straightforward interior \mathbf{r} **Copyright:** © 2024 by the authors. $\mathbf{D}_{\mathbf{u}}(\mathbf{t},\mathbf{t})$ is a $\mathbf{H}=\mathbf{H}(\mathbf{t},\mathbf{t})$ is one of the most \mathbf{t}_1 in the most \mathbf{t}_2 $\sum_{i=1}^{n}$ functions, and Pearly $\sum_{i=1}^{n}$ functions, and $\sum_{i=1}^{n}$ challenging, including the solution of the solution. Since α **Cannon 4.** The tv tvt $f(x) \propto z \rightarrow \infty$ _I is referred to be coordinated E. challenging, including the solution. Since α $c_1 \ldots$ c_{max} is referred to be coordinated c_{max} functions for the on-sizing between c_{max} $DeIIIIIOII$ 4. $\frac{1}{2}$ years. When determining exact values for a mathematical problem prob \rightarrow \mathbb{R}_I is referred to be coordinated L N-n-convex IV in on Δ 1 y \mathcal{L}_{L} $t_{\rm c}$ relationship to the theory of inequalities, convexity has advanced σ **Definition 4.** The LV M $j: \Omega \to \mathbb{R}_j$ is referred tight relationship to the theory of inequalities, convexity has advanced α **Definition 4.** The LV M J : $\Omega \to \mathbb{R}$ is referred to be coord tight relationship to the theory of inequalities, convexity has advanced α **Demition 4.** The LV M J : $\Omega \rightarrow \mathbb{R}$ is referred to be coordinated EK-h- $\sum_{i=1}^{n} a_i t_i$ to the theory of $\sum_{i=1}^{n} a_i t_i$ in recent $\sum_{i=1}^{n} a_i t_i$ **Demitton 4.** The TV bit J_1 , $\lambda_2 \rightarrow \mathbb{R}$ is referred to be coordinated EK-n-co tight relationship to the theory of inequalities, convexity has advanced quickly independent q **Demition 4.** The LV M j : $\Omega \rightarrow \mathbb{R}_j$ is referred to be coordinated LK-h-convex LV M on Ω tight relationship to the theory of inequalities, convexity has advanced quickly independent α **http://www.fractionalisty.** When \mathbb{R}^n is referred to be coordinated LK-h-convex TV M on \mathbb{R}^n if $\sum_{i=1}^{n} C_i$ is a T_i with $\sum_{i=1}^{n} C_i$ of $\sum_{i=1}^{n} C_i$ functions in the convex sets and convex functions in the conve **Definition 4.** The IVM $J \textrm{j}: \Omega \to \mathbb{R}^+_I$ is referred to be coordinated I Order Relation. *Fractal Fract.* **2024**, *8*, $\sum_{i=1}^{n}$ **Negotianal** The IVM $\mathbf{u} \cdot \mathbf{O} \rightarrow \mathbb{R}^+$ is referred to be conveniented $\mathbf{I} \cdot \mathbf{R}$ -K \mathcal{L} integral operator; Pachpathe-type integral operator; Pachpathe-type inequalities in *D* **EXERCISE 1.** *The IVIII*</sup> J₁. a *a* α is performance *b* decommendation of convex IVIII on an **Negation 4.** The IVM $\pi \cdot \Omega \rightarrow \mathbb{P}^+$ is referred to be coordinated LR-5-convex IVi **Definition 1.** The LV III j_1 , \mathbf{r}^2 is \mathbf{r}^2 to referred to be contained LK to concentration \mathbf{r}^2 **Definition** Λ The U/M $\pi \cdot \Omega$ \mathbb{P}^+ is ref $r =$ can be seen as an endpoint $\sum_{i=1}^n r_i$ intervalse that can be seen as applications of $\sum_{i=1}^n r_i$ intervalse of $\sum_{i=1}^n r_i$ intervalse of $\sum_{i=1}^n r_i$ in $\sum_{i=1}^n r_i$ in $\sum_{i=1}^n r_i$ **Dofinition** d. The IVM $\pi \cdot Q \rightarrow \mathbb{R}^+$ is reference restrictions on endpoint functions of interval-valued functions that can be seen as applications of $\mathbf{D}_{\mathbf{c}}\mathbf{C}_{\mathbf{c}}$ (i.e. \mathbf{A} and cases are also discussed by taking some are also discussed by taking some \mathbf{A} in \mathbf{D} $r =$ can be seen as applied functions of $r = r - \frac{1}{2}r$ and $\frac{1}{2}r = r - \frac{1}{2}r$ **Definition** d. The U/M $\pi \cdot \Omega \rightarrow \mathbb{D}^+$ is referred to be coordinated by \mathbf{r} on endpoint functions that can be seen as applications that can be seen as applications of intervals of \mathbf{r} $\mathbf{D}_k\mathbf{C}_k(t)$ or \mathbf{A} , \mathbf{F}^{\dagger}_{k} , $\mathbf{U}(M,\pi,\mathbf{C})$ as \mathbb{R}^+ is actual to be conditively $I(D,k)$ cases $\mathbf{U}(M,\omega,\mathbf{C})$ if $r = \frac{1}{2}$ restrictions on endpoints $\frac{1}{2}$ intervalse functions that can be seen as applications of $\frac{1}{2}$ \overline{O} of \mathbb{R}^+ is solvential to be considered to by taking some HIM so, \overline{O} if **Definition 4.** The IVM $JJ: \Omega \to \mathbb{R}^+_I$ is referred to be coordinated LR-h-convex IVM on Ω if convexity (-ℏ-convexity) over interval-valued codomain. We exploit the use of double Riemann– convexity) over intervalse intervalse codomain. We exploit the use of double R intervalse R $\mathbf{D} \cdot \mathbf{C} \cdot \mathbf{B}$ is the UAL $\mathbf{E} \cdot \mathbf{D}$ in $\mathbf{D} \cdot \mathbf{E}$ is a set of the second to the LHD \mathbf{E} entropy UA \sum Correspondence: $\sum_{i=1}^{n}$ is $\sum_{i=1}^{n}$ if $\sum_{i=1}^{n}$ is regular to be con- R_{α} * Correspondence: enwaeze@alasu.edu (E.R.N.); muhammad.bilal@unitbv.ro (M.B.K.) $R_1, G_2, H_1, \ldots, A_n$ \sum Correspondence: The TV by $\sum_{i=1}^{n}$ is $\sum_{i=1}^{n}$ to referred to be coordinated. $Ref}_1$ ition 4. The * Correspondence: enwaeze@alasu.edu (E.R.N.); muhammad.bilal@unitbv.ro (M.B.K.) $k \pi \Omega$, \mathbb{R}^+ is set of \mathcal{L} $\mathcal{$ $M \pi \cdot Q \longrightarrow \mathbb{D}^+$ is noted \ldots . The correspondence of the correspondence \ldots is constant \ldots $k \in \mathbb{R}^+$ is a constant of $k \in \mathbb{R}$ \sim Correspondence: entrancement is concerned by $\frac{1}{N}$ \mathbb{R}^+ is ushaved to be a \mathbb{Z} correspondence: engagement en \mathbb{Z} is denoted that \mathbb{Z} . $\mathbf{D}_{\mathbf{S}}\mathbf{G}_{\mathbf{S}}$ it is an \mathbf{A} The UVM π , \mathbf{O} \rightarrow \mathbb{R}^+ is urfamed to be coordinated I E chhakon $\mathbf{D}_{\mathbf{z}}\mathbf{G}_{\mathbf{z}}$ is the Hallith π , Ω , \mathbb{D}^+ is ushound to be soculinated I.D. $\mathbf{\hat{z}}$ saudi k $\pi: \Omega \to \mathbb{R}^+$ is ushaved to be seen installed I.D. is severe, J.W.M. see Ω if \cdots \mathbb{R}^+ is ushowed to be seentimated I.D. E severe, JVM on O. if k_1 to referred to be econd

$$
J[(v_{\mathcal{O}} + (1 - v)i, \kappa \varepsilon + (1 - \kappa)v)]
$$

\n
$$
\leq_{p} \hbar(v)\hbar(\kappa)J[(\sigma,\varepsilon) + \hbar(v)\hbar(1 - \kappa)J[(\sigma,\mathfrak{v}) + \hbar(1 - v)\hbar(\kappa)J[(i,\varepsilon) + \hbar(1 - v)\hbar(1 - \kappa)J[(i,\mathfrak{v})]
$$
\n(18)

for all (σ, i) , $(\varepsilon, \mathfrak{v}) \in \Omega$, and $v, \kappa \in [0, 1]$, where $J(x) \geq_p 0$. If inequality (18) is reversed, for all (σ, i) , $(\varepsilon, \mathfrak{v}) \in \Omega$, and $\upsilon, \kappa \in [0, 1]$, where $\mathfrak{J}(x) \geq_{p} 0$. If inequality (18) is reversed,
then \mathfrak{J} is referred to be coordinate h-concave IVM on Ω . If h is the identity function, we r then f is referred to be coordinate n-concave 1 V ivi on s.2. f_j it is the taentity function, we recover $\mathbf f$ τ in [46], even though Hadamard (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1875–1901), even though Hadamard (1875–1901), even though Hadamard (1875–1901), even though Hadamard (1875–1901), eve **Copyright:** © 2024 by the authors. \mathcal{H} \mathcal{L} \overline{a} and pure sciences. Furthermore, because of its many applications and pure sciences. Furthermore, and pure sciences. Furthermore, \overline{a} realms of applied and pure sciences. Furthermore, because of its many applications and $\frac{1}{2}$ there are many uses $\frac{1}{2}$ the convex sets and convex sets and convex sets and convex $\frac{1}{2}$ en in $[46]$. \mathfrak{m} in [46]. By taking that examples are non-trivial. By taking the product of two left and two le α [46].

Lemma 1. Let $J \rvert : \Omega \to \mathbb{R}^+_I$ be a coordinated IVM on Ω . Then, $J \rvert$ is coordinated LR- \hbar - $J_{\mathcal{J}_x}(w) = J(x, w)$ and $J_{\mathcal{J}} : [\sigma, i] \to \mathbb{R}_I^+$, $J_{\mathcal{J}}(z) = J(z, y)$. convex IVM on Ω if and only if there exist two coordinated LR- \hbar -convex IVMs $J_x : [\varepsilon, \mathfrak{v}] \to \mathbb{R}_+^+$, , $[v \rightarrow e]$ \rightarrow $[v \rightarrow e]$ \mathbb{R}_{I}^{+} , $\mathcal{J}_{y}(z) = \mathcal{J}(z,y)$. $\int y \cdot [0,1]$ $\int x \cdot y$ $\mathcal{L}(\mathbf{z}, \mathbf{y}) = \mathcal{L}(\mathbf{z}, \mathbf{y})$ $J_{x}(w) = J(x, w)$ and $J_{y}: [\sigma, i] \to \mathbb{R}_{I}^{+}$, $J_{y}(z) = J(z, y)$. **Lemma 1.** Let $J\!J:\Omega\to\mathbb{R}_I^+$ be a coordinated IVM on Ω . Then, $J\!J$ is coordinated LR-hpublication under the terms and $\bf L$ conditions of the Creative Commons of the Creative Com A ttribution (CC By) license \overline{A} category of classical convex functions, according to Dragomir and Pearce [1]. This **Lemma 1.** Let $J \rvert : \Omega \to \mathbb{R}^+_I$ be a coordinated IVM on Ω . Then, $J \rvert$ is coordinated LR-h- \ldots of classical convex functions, according to Dragomir and Pearce \ldots ϵ functions, according to Dragomir and ϵ T is one of the most well-known finding inequality is one of the most well-known findings in the most well-known finding in th can be directly derived from convex functions, there is a close relationship between $\frac{1}{2}$ $J_{\mathcal{I}_\alpha}(w) = J_{\mathcal{I}}(x, w)$ and $J_{\mathcal{I}_\alpha} : [\alpha, \mathbf{i}] \to \mathbb{R}_I^+$, $J_{\mathcal{I}_\alpha}(z) = J_{\mathcal{I}}(z, y)$. Ԓ௫() = Ԓ(,) *and* Ԓ௬:ሾℴ, ሿ → ℝூ ା*,* Ԓ௬() = Ԓ(,)*.* Π (*u*) = $\Pi(x, y)$ and Π \cdot [\star i] $\to \mathbb{R}^+$ \cdot Π (*z*) = $\Pi(z, y)$ ϵ_A be directly derived from convex functions, there is a close relationship between ϵ (x, w) and \mathcal{J}_{1} , \vdots $[\alpha, i] \rightarrow \mathbb{R}^+$, \mathcal{J}_{1} , $(z) = \mathcal{J}_{1}(z, y)$. $\frac{1}{\sqrt{2}}$ ା*,* Ԓ௬() = Ԓ(,)*.* $\P(\mathbf{w}) = \Pi(\mathbf{x}, \mathbf{w})$ and $\Pi : [\alpha, \mathbf{i}] \to \mathbb{R}^+$, $\Pi(\mathbf{z}) = \Pi(\mathbf{z}, \mathbf{u})$. c_n inequalities can be used to approximate the solution. Since $\frac{1}{2}$ in $\frac{1}{2}$ $\pi(x_0) = \pi(x_0)$ and $\pi(\cdot|_{\infty}$ its many applications and $\pi(\cdot|_{\infty}$ $\mathbf{f}_{\mathbf{x}}(t)$ is the theory of inequalities, convexity has advanced quickly in recent $\mathbf{f}_{\mathbf{x}}(t)$ π (*v*) = $\pi(x, y)$ and π · [\approx i] = $\frac{d}{dx}$ relationship to the theory of inequalities, convexity has advanced $\frac{d}{dx}$ in recent $\frac{d}{dx}$ in recent $\frac{d}{dx}$ π (*z*₁) = π (*x*₂) and π · [, i] π \mathbb{P}^+ π ($\mathcal{F}_{\mathcal{X}}(x)$ because of its manufacture, because of its many applied its many applications and $\mathcal{F}_{\mathcal{X}}(x)$ π (*zy*) = $\pi(x, y)$ and π i. [α , i] $\rightarrow \mathbb{R}^+$ π ($\frac{d}{dx}$ applied and pure sciences. Furthermore, because of its many applications and its many applications and $\frac{d}{dx}$ π (cn) = $\pi(x, y)$ and π is in the convex sets and convex π (cn) = $\pi(x, y)$ $r_{\mathbf{y}}(x)$ because of and pure sciences. Furthermore, because of its many applications and \mathbf{y} $\pi(\tau v) = \pi(x, w)$ and $\pi(\tau v) = \mathbb{R}^+$ $\pi(\tau) = \pi(z, u)$ $\frac{d}{dx}$ applied and pure sciences. Furthermore, because of its many applications and its many applications and $\frac{d}{dx}$ $T_n = \frac{\pi}{2} \left[\frac{1}{2} \arctan \frac{1}{2} \ar$ $\mathcal{L}_{\mathcal{J}_X}$. Furthermore, because of applications and pure sciences. Furthermore, because of its many applications and pure sciences. convex IVM on Ω if and only if there exist two coordinated LR- \hbar -convex IVMs $J_X : [\varepsilon, \mathfrak{v}] \to \mathbb{R}_I^+$,
 $J_X : [\varepsilon, \mathfrak{v}] \to J(X, \mathbb{Z})$ and $J_X : [\varepsilon, \mathfrak{v}] \to \mathbb{R}^+$, $J_X : [\varepsilon, \mathfrak{v}] \to J(X, \mathbb{Z})$ Π (*u*) = $\Pi(x, y)$ and $J_{J_x}(w) = J(x, w)$ and $J_{J_y}: [\sigma, 1] \to \mathbb{R}^+_I$, $J_{J_y}(z) = J(z, y)$. **Lemma 1.** Let $J: \Omega \to \mathbb{R}^N_I$ be a coordinated TVM on *S1*. The obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some **Lemma 1.** Let $f: \mathbb{R}^n \to \mathbb{R}^n$ be a coordinated TV M on Ω . Then **1.** Let $J: \Omega \to \mathbb{R}^N_I$ be a coordinated TVM on Ω . Then, J_J is coordinated LK-hobtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some **Lemma 1.** Let $J_j: \Omega \to \mathbb{R}_j^*$ be a coordinated IVM on Ω . Then, J_j is coordinated LR-hright coordinated ℏ -convexity, some new versions of fractional integral inequalities are also **Lemma 1.** Let J : $\Omega \to \mathbb{R}^+_I$ be a coordinated IVM on Ω . Then, J is coordinated right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also **Lemma 1.** Let J : $\Omega \to \mathbb{R}^+_I$ be a coordinated IVM on Ω . Then, J is coordinated LR-hright coordinated ℏ -convexity, some new versions of fractional integral inequalities are also : $\Omega \to \mathbb{R}^+_I$ be a coordinated IVM on Ω . Then, IJ is coordinated LR-hright coordinated ℏ -convexity, some new versions of fractional integral inequalities are also **Lemma 1.** Let $J\!J: \Omega \to \mathbb{R}^+_I$ be a coordinated IVM on Ω . Then, $J\!J$ is coordinated LR-h-

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

 $\mathcal{L} = \sum_{i=1}^{n} \mathcal{L}^{(i)}$ and $\mathcal{L}^{(i)}$ and $\mathcal{L}^{(i)}$ interval $\mathcal{L}^{(i)}$ interval $\mathcal{L}^{(i)}$ interval $\mathcal{L}^{(i)}$ *Theorem of all* \int *, since* \int *integrable integrable same considerable functions of* \int **The family of all** *r*_i α ^{*i*} α ^{*i*} α *i over two constants functions of* α \mathbf{F} **Fractal Fractal Fra Theorem 5.** Let $J \in \Omega \to \mathbb{R}^+_I$ be a IVM on Ω , given by $\frac{1}{\sqrt{2}}$ \mathcal{C} result was mainly credit to \mathcal{C} inequality has seen applications and a straightforward intrinsic geometric explanation. T_{in} is one of the most well-known finding in the most given by $\frac{1}{2}$ **Theorem 5** Let $\pi \cdot O \rightarrow \mathbb{R}^+$ l $T_{\text{S}} = \frac{1}{2}$ and $T_{\text{S}} = \frac{1}{2}$ $T_{\text{S}} = \frac{1}{2}$ $T_{\text{S}} = \frac{1}{2}$ $T_{\text{S}} = \frac{1}{2}$ $T_{\text{S}} = \frac{1}{2}$ **Cheorem 5** Let $\overline{\mathbf{u}} \cdot \mathbf{O} \rightarrow \mathbb{R}^+$ be a IVM on \mathbf{O} cirren by \mathcal{L} in the theory of inequalities. The exact Γ and Γ and Γ and Γ $\frac{1}{2}$ channels can be used to approximate the solution. Since $\frac{1}{2}$ in $\frac{1}{2}$ **Theorem 5** Let $\Pi: \mathbb{R}^+$ be challenging can be used to approximate the solution. Since \mathcal{L}_1 \mathbf{L}_{t} relationship to the theory of \mathbf{L}_{t} in recent \mathbf{L}_{\text $\frac{1}{2}$ **Theorem** \mathbf{F} Let $\mathbf{\pi} \cdot \mathbf{O}$ \mathbf{F} be a IVM on $\frac{1}{2}$ is a mathematical problem p \mathbf{T}_{top} relationship to the theory of \mathbf{L}_{top} of \mathbf{R}_{top} in recent \mathbf{L}_{top} in $\frac{1}{2}$ **Theorem E** Let $\pi: \Omega \to \mathbb{R}^+$ has IVM on Ω given by $\frac{1}{2}$ are determinent for a mathematical problem problem problem problem problem problem problem proves to be been proved by $\frac{1}{2}$ The example $I_{\nu}(r, Q)$ of \mathbb{R}^+ has $\mathcal{U}(M, \omega, Q)$ simulars $\frac{1}{2}$ is the determining exact values for a mathematical problem proves to be been problem proves to be been proved to be a mathematical problem proves to be a mathematical problem proves to be a mathematical problem \mathbb{R}^+ theory \mathbf{U}^* to the theory of inequalities, convexity in recent \mathbf{V} in recent \mathbf{V} **Theorem 5.** Let $J \in \Omega \to \mathbb{R}^+_I$ be a IVM on Ω , given by **Theorem 5.** Let $J \textrm{J}: \Omega \to \mathbb{R}^+_I$ be a IVM on Ω , given by

inequality has several applications and a straightforward intrinsic geometric explanation.

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-convex on Ω *given in [46].*

The result was mainly credited to \mathcal{H}_c and \mathcal{H}_c and \mathcal{H}_c

challenging, in the solution can be used to approximate the solution. Since α

 $-$ convex *on* $-$ *given* in *[46]*. *<i> only* $-$ *given* in [46]. *<i> only* $-$ given in [46].

∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and*

can be directly derived from convex functions, there is a close relations, there is a close relationship between α

where >0 *and is the gamma function.*

inequality has several applications and a straightforward intrinsic geometric explanation.

inequality has several applications and a straightforward intrinsic geometric explanation.

 $\overline{}$ years. When determining exact values for a mathematical problem p

3 Department of Mathematics and Computer Science, Transilvania University of Brasov,

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

$$
J(x,y) = [J_*(x,y), J^*(x,y)], \qquad (19)
$$

for all $(x, y) \in \Omega$. Then, J is coordinated LR- \hbar -convex IVM on Ω , if and only if both $J_*(x, y)$ and $J^*(x, y)$ are coordinated LR-h-convex. $($ **i** $($ $($ *and* $($ $*i*)$ *and* $($ $*i*)$ *and* $($ $*i*)$ *and* $*i*)$ *an* $($ **i** $($ $($ *i* $)$ $($ $)$ *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract for all $(x,y)\in \Omega$. Then, J is coordinated LR- \hbar -convex IVM on Ω , if and only if both $J_*(x,y)$ and *holds:* ηv ex. can be directly derived from convex functions, there is a close relationship between \mathbf{r} α directly derived from convex functions, there is a close relationship between relationship between α \mathbf{c}_j interpretation. Since \mathbf{c}_j \mathbf{c} in the used to approximate the solution. Since \mathbf{c} $y^*(x, y)$ are coordinated I B b convex ϵ , in generalities can be used to approximate the solution. Since ϵ for all $(x, u) \in \Omega$ Then Π is coordinated LR-6-convex IVM on Ω if and only if hoth Π (x, u) and $\frac{1}{2}$ (i.e., in the solution of solution. Since $\frac{1}{2}$ $\mathbb{R}^*(x, y)$ are coordinated I B is convergent with the state of a mathematical problem problem proves to be $\mathbb{R}^*(x, y)$ and $\mathbb{R}^*(x, y)$ and $\mathbb{R}^*(x, y)$ are coordinated I B is convergent. challenging, including the solution of the solution. Since \mathcal{L} τ coordinated LD to convex U/M on O if and only if hath Π (x w) and for an $(x, y) \in \Omega$. Then, if is coordinated LK-n-convex TV is on Ω , if and only if both $J_*(x, y)$ and $\mathbb{R}^*(x, y)$ \mathcal{L} February 2024 There are many uses for the convex sets and convex sets and convex sets and convex α ℏ-Convexity via Left and Right f o **1. Introduction** Joi un $(x, y) \in \Omega$ for all $(x, y) \in \Omega$. Inen, if Ω Jor an $(x, y) \in \Omega$. Then, \int for an $(x, y) \in \Omega$. The all $(x, y) \in \Omega$. Then, if is co

 \overline{a} **Proof.** Assume that $J_{*}(x)$ and $J^{*}(x)$ are coordinated LR- \hbar -convex and \hbar -concave on Ω , $(y, \text{ then, from Equation (19)}, \text{ for an } (\vartheta, t), (\epsilon, \theta) \in \Omega, \theta \text{ and } \kappa \in [0, 1], \text{ we have}$ respectively. Then, from Equation (19), for all (γ, i) , $(\epsilon, \mathfrak{v}) \in \Omega$, υ and $\kappa \in [0, 1]$, we have category of classical convex functions, according to Dragomir and Pearce [1]. This $\frac{1}{\sqrt{1-\frac{1$ The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known f T_{eff} is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most wel The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known finding μ $\mathbf{H} = \mathbf{H} \times \mathbf{W}$ is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding i \mathcal{F} Hermite–Hadamard inequality is one of the most well-known findings in the most well-known finding in the most well-k Academic Editor: Bruce Henry \mathbf{E} \mathbf{B}

$$
J_{*}(v_{\mathcal{O}_{+}}+(1-v)i,\kappa\epsilon+(1-\kappa)v)
$$

\n
$$
\leq \hbar(v)\hbar(\kappa)J_{*}(\mathbf{\sigma},\epsilon)+\hbar(v)\hbar(1-\kappa)J_{*}(\mathbf{\sigma},v)+\hbar(\kappa)\hbar(1-v)J_{*}(i,\epsilon)+\hbar(1-v)\hbar(1-\kappa)J_{*}(i,v)
$$
\n(20)\nand

inequality has several applications and a straightforward intrinsic geometric explanation.

Fractal Fract. **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract $\frac{1}{2}$ and $\frac{2}{3}$. The following is how this inequality is stated: $\frac{1}{2}$ and $\frac{1}{2}$. The following is how this inequality is stated. $\frac{1}{2}$ and $\frac{1}{2}$. The following is how this inequality is stated: α iu Published: date

realms of applied and pure sciences. Furthermore, because of its many applications and

$$
J^*(v_{\mathcal{O}} + (1 - v)i, \kappa \varepsilon + (1 - \kappa)v)
$$

\n
$$
\leq \hbar(v)\hbar(\kappa)J^*(\sigma, \varepsilon) + \hbar(v)\hbar(1 - \kappa)J^*(\sigma, v) + \hbar(\kappa)\hbar(1 - v)J^*(i, \varepsilon) + \hbar(1 - v)\hbar(1 - \kappa)J^*(i, v).
$$
\n(21)
\nThen by Equations (19), (3) and (4), we obtain

Then, by Equations (19), (3) and (4), we obtain Then, by Equations (19) , (3) and (4) , we obtain Submitted for possible open access

Then, by Equations (19), (3) and (4), we obtain
\n
$$
\begin{aligned}\nJ(v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-\kappa) v) \\
&= [J_{\ast}(v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-\kappa)v), J^*(v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-\kappa)v)] \\
&\leq_{p} \hbar(v) \hbar(\kappa) [J_{\ast}(\sigma, \varepsilon), J^*(\sigma, \varepsilon)] + \hbar(v) \hbar(1-\kappa) [J_{\ast}(\sigma, v), J^*(\sigma, v)] \\
&+ \hbar(\kappa) \hbar(1-v) [J_{\ast}(i, \varepsilon), J^*(i, \varepsilon)] + \hbar(1-v) \hbar(1-\kappa) [J_{\ast}(i, v), J^*(i, v)]\n\end{aligned}
$$
\nFrom (20) and (21), we have

Fractal Fract. **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract **1. Introduction**

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The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–

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challenging, including, including, inequalities can be used to approximate the solution. Since many inequalities

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Riemann–Liouville fractional integral operator; Pachpatte-type inequalities

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challenging, inequalities can be used to approximate the solution. Since many inequalities

Riemann–Liouville fractional integral operator; Pachpatte-type inequalities

 $J\!J(v_{\mathcal{O}}+(1-v)i,\kappa \varepsilon+(1-\kappa)\,\mathfrak{v})$ $\leq_{p} \hbar(v)\hbar(\kappa)\mathbf{J}(\sigma,\varepsilon)+\hbar(v)\hbar(1-\kappa)\mathbf{J}(\sigma,\mathfrak{v})+\hbar(1-v)\hbar(1-\kappa)\mathbf{J}(\mathfrak{i},\varepsilon)+\hbar(1-v)\hbar(1-\kappa)\mathbf{J}(\mathfrak{i},\mathfrak{v}),$ (https://creativecommons.org/license $\psi_{\mu}(x) = \int_{\mu} \psi(x) \psi(x) \psi(x) dx + \int_{\mu} \psi(x) \psi(x) dx + \int_{\mu} \psi(x) \psi(x) dx + \int_{\mu} \psi(x) \psi(x) dx$ $T(r)E(r)E(1-r)$ $\pi(r, n) + E(1-r)E(1-r)$, $\pi(r, n) + E(1-r)E(1-r)$, $\pi(r, n)$ 19.93 was the one who first identified it follows in $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{3}$ **Copyright:** © 2024 by the authors. \mathbf{p} \overline{a} $J\int (UU - \int (1 - U)t) \kappa(t + (1 - K)U)$ $\sup_{\nu} n(\nu) n(\nu) J(\nu, \nu) + n(\nu) n(1-\nu) J(\nu, \nu) +$ \mathbf{r} and the theory of \mathbf{r} $T_1(UU - \Gamma(T - U)E(\mu) \pi(-\alpha) + E(\mu)E(1 - \mu) \pi(-\mu) + E(\mu)E(\mu)$ $\sum_{i=1}^n n(v)n(x)J(\mathcal{Y},\epsilon) + n(v)n(1-\kappa)J(\mathcal{Y},\mathcal{Y}) + n(1-\epsilon)n(1-\kappa)J(\mathcal{Y},\epsilon) + n(1-\epsilon)n(1-\epsilon)$ $\mathcal{G}[\mathcal{C} \mathcal{C}] = \begin{cases} (-1 - \mathcal{C})^{\dagger} & \text{if } \lambda & \text{if } \lambda \end{cases}$ $\overline{\mathbf{r}}$ derived from convex functions, there is a close relationship between $(1-\alpha)$ convex relationship between $\overline{\mathbf{r}}$ between α $\mathcal{L}(\mathcal{C}\mathcal{C}) = \begin{pmatrix} 1 & \mathcal{C} \\ \mathcal{C} & \mathcal{C} \end{pmatrix}$ \mathbf{r} can be directly derived from convex functions, there is a close relationship between relationship between \mathbf{r} $\mathcal{L}(\mathcal{C}\mathcal{C}) = \mathcal{L}(\mathcal{C})\mathcal{L}(\mathcal{C}) = \mathcal{L}(\mathcal{C})\mathcal{L}(\mathcal{C})$ $\pi(\omega + (4-\omega)\cos(\omega))$ $c_1(\sigma v + (1 - \sigma)t, \kappa) + (1 - \kappa)v$ $\sum_{i=1}^n \mu_i(\sigma_i \mu_i) \mathfrak{z}_j(\sigma_i \sigma_j + \mu_i(\sigma_i \mu_i) \mathfrak{z}_j(\sigma_i \sigma_j + \mu_i(\tau - \sigma_i \mu_i) \mathfrak{z}_j(\sigma_i \sigma_j + \mu_i \sigma_i \sigma_i))$ can be directed from convex functions, there is a close relations, there is a close relationship between $\mathcal{L}(\mathbf{A})$ $(\Gamma(\Gamma - \kappa) \mathbf{v})$ $T_t = \frac{H(t)}{n(1 - \kappa)} \int_{-\infty}^{\infty} (\sigma, \sigma) + \frac{H(t - \sigma)}{n(1 - \kappa)} \int_{-\infty}^{\infty} (\sigma, \sigma) + \frac{H(t - \kappa)}{n(1 - \sigma)} \int_{-\infty}^{\infty} (\sigma, \sigma)$ \mathcal{L} convexity and the theory of integration of $\mathbf{r}(t)$ $T_n(1-v)n(1-\kappa)J(n,\epsilon) + n(1-v)n(1-\kappa)J(n,\nu)$ $JJ(v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-\kappa)v)$ $\langle \xi(t) \rangle_{\mathcal{H}}^{\mathcal{H}}(\mathbf{z}, \mathbf{z}) + \hat{b}(\mathbf{z}) \hat{b}(\mathbf{z} - \mathbf{z}) \mathbf{z}(\mathbf{z}, \mathbf{z}) + \hat{b}(\mathbf{z} - \mathbf{z}) \mathbf{z}(\mathbf{z}, \mathbf{z}) + \hat{b}(\mathbf{z} - \mathbf{z}) \mathbf{z}(\mathbf{z}, \mathbf{z}) + \hat{b}(\mathbf{z} - \mathbf{z}) \mathbf{z}(\mathbf{z}, \mathbf{z})$ $-r$ challenging, inequalities can be used to approximate the solution. Since $\frac{1}{2}$ $\langle \cdot, h(\eta)h(\kappa) \Pi(\kappa \kappa) + h(\eta)h(1-\kappa) \Pi(\kappa \kappa) + h(1-\eta)h(1-\kappa) \Pi(\kappa \kappa) + h(1-\eta)h(1-\kappa)$ $-r$ challenging, inequalities can be used to approximate the solution. Since $\frac{1}{2}$ years. When determining exact values for a mathematical problem proves to be τ_{tot} τ_{tot} years. When determining exact values for a mathematical problem proves to be $r(n)$ $\mathcal{L}_{\mathcal{A}}$ are determining exact values for a mathematical problem problem problem problem problem proves to be been proved to be a mathematical problem proves to be a mathematical problem problem problem proved to be Received: 14 November 2023 $r = \kappa$ in $t_i + \tilde{b}(n)\tilde{b}(1-\kappa)\pi(\alpha, n) + \tilde{b}(1-n)\tilde{b}(1-\kappa)\pi(i,\kappa) + \tilde{b}(1-n)\tilde{b}(1-\kappa)\pi(i,n)$ $y = \sqrt{y}$ and $y = \sqrt{y}$ and $y = \sqrt{y}$ and $y = \sqrt{y}$ and $y = \sqrt{y}$ Received: 14 November 2023 $\leq_p \hbar(v)\hbar(\kappa)\mathbf{J}(\boldsymbol{\sigma},\varepsilon)+\hbar(v)\hbar(1-\kappa)\mathbf{J}(\boldsymbol{\sigma},\mathfrak{v})+\hbar(1-v)$ $\frac{\pi}{4}$. I(1 ii) $\frac{1}{2}$ Order Relation. *Fractal Fract.* **2024**, *8*, $J(x) = (1 - v)i, \kappa \varepsilon + (1 - \kappa)v$ Order Relation. *Fractal Fract.* **2024**, *8*, Integral Inequalities via Coordinated Integral Inequalities via Coordinated $J\left(\frac{v}{v} + (1-v)\right), \kappa \varepsilon + (1-\kappa)v$
 $\langle h(v)h(v)h(v)h(z)h(z) + h(v)h(1-\kappa)h(z)h(z) + h(1-v)h(1-\kappa)h(z) + h(1-v)h(1-\kappa)h(z)h(z) \rangle \rangle$ $-r$ via \mathbf{r} and Right an Integral Inequalities via Coordinated Integral Inequalities via Coordinated Integral Inequalities via Coordinated Integral Inequalities via Coordinated $\mathbf{1}$ these new outcomes. \mathcal{L} $\leq_p h(v)n(k)$ J (σ, ε) + $n(v)n$ $\mathfrak{r}_k \mathfrak{r}_k = \left(1-\kappa\right) \mathfrak{v}_k$ $u(\kappa)$ J (σ, ε) + $n(\nu)n$ $\leq_p \hbar(v)\hbar(\kappa)J(\sigma,\varepsilon) + \hbar(v)\hbar(1-\kappa)J(\sigma,\mathfrak{v}) + \hbar(1-v)\hbar(1-\kappa)J(\mathfrak{i},\varepsilon) + \hbar(v)J(\sigma,\varepsilon)$ $-r$ -coordinated integral integr $\epsilon_{\alpha}(\epsilon) = \frac{\hbar(\mu)\hbar(\kappa)\pi(\alpha,\kappa)}{\hbar(\kappa)\pi(\alpha,\kappa)} + \frac{\hbar(\mu)\hbar(\kappa)\pi(\alpha,\kappa)}{\hbar(\kappa)\pi(\alpha,\kappa)} + \frac{\hbar(\mu)\pi(\kappa)\pi(\kappa)}{\hbar(\kappa)\pi(\kappa)}$ $\frac{1}{\sqrt{2}}$ right coordinated $\frac{1}{\sqrt{2}}$ integral integral integral integral integral integral integral in $\frac{1}{\sqrt{2}}$ results that examples are nontrivial. By taking the product of two left and product and two left and two $\begin{array}{ll}\n\int (v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-\kappa)v) < \frac{\hbar}{2} \int (v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-\kappa)v) \frac{\hbar}{2} \int (v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-v)i, \kappa \varepsilon + (1-v)i, \kappa \varepsilon + (1-v)i, \kappa \varepsilon + (1-v) \frac{\hbar}{2} \int (v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-v)i, \kappa \varepsilon + (1-v) \frac{\hbar}{2} \int ($ results' numerical validations that examples are nontrivial. By taking the product of two left and $\mathcal{L}(\mathbf{1}-\mathbf{r})$ over intervalse the use of double Riemann– $r \rightarrow \infty$ resultsty numerical validations that examples are non-trivial. By taking the product of two left and r $+(1-v)i$, $\kappa \varepsilon + (1-\kappa)v$
 $\langle \kappa \cdot h(n)h(\kappa) \pi(\kappa \cdot \kappa) + h(n)h(1-\kappa) \pi(\kappa \cdot \kappa) + h(1-\kappa)h(1-\kappa) \pi(i\kappa) + h(1-\kappa)h(1-\kappa)h(i\kappa)$ results' numerical validations that examples are nontrivial. By taking the product of two left and $r \leftarrow \frac{1}{2}$ numerical values are non-trivial. By taking the product of two left and product of two left and $\frac{1}{2}$ Γ () () is derived to derive the results of the results of the key also examine the key al \mathbf{r} , $\leq_p\hbar(v)\hbar(\kappa)\jmath(\sigma,\varepsilon)+\hbar(v)\hbar(1-\kappa)\jmath(\sigma,\mathfrak v)+\hbar(1-\upsilon)\hbar(1-\kappa)\jmath(\mathfrak i,\varepsilon)+\hbar(1-\upsilon)\hbar(1-\kappa)\jmath(\mathfrak i,\varepsilon)$ $\left[\begin{array}{cc} v_{\mathcal{O}} & -(1-v)\mathfrak{v}, \kappa \varepsilon + (1-\kappa) \mathfrak{v} \end{array} \right]$ $JJ(v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-\kappa)v)$ 4 Department of Mathematics, Faculty of Science, Jazan University, Jazan 45142, Saudi Arabia; $\sum_{p} n(v)n(h(v))j(v,e) + n(v)n(1 - k)j(v,e) + n(1 - v)n(1 - k)j(v,e) + n(1 - v)n(1 - k)j(v,e)$,
hence, J is coordinated LR- \hbar -convex IVM on Ω . 4 Department of Mathematics, Faculty of Science, Jazan University, Jazan 4 **Include Fract. 2024,** 0, 123 $J[(v_{\alpha} + (1-v)i, \kappa \varepsilon + (1-\kappa)v)]$
 $\leq_p \hbar(v)\hbar(\kappa)J[(\sigma,\varepsilon) + \hbar(v)\hbar(1-\kappa)J[(\sigma,\nu) + \hbar(1-v)\hbar(1-\kappa)J[(i,\varepsilon) + \hbar(1-v)\hbar(1-\kappa)J[(i,\nu) + \hbar(1-\nu)\hbar(1-\kappa)J[(i,\nu) + \hbar(1-\nu)\hbar(1-\kappa)J[(i,\nu) + \hbar(1-\nu)\hbar(1-\kappa)J[(i,\nu) + \hbar(1-\nu)\hbar(1-\kappa)J[(i,\nu) + \hbar($ \sup $\mu(\nu)\mu(\nu)$ \Rightarrow $p \rightarrow e$ \rightarrow $p \rightarrow$ \mathbf{D} **i**, $\kappa \varepsilon + (1 - \kappa) \mathbf{v}$ $\mathbf{E} + (1 - \mathbf{k})\mathbf{n}$ $J[(v_{\alpha} + (1-v)i, \kappa \varepsilon + (1-\kappa)v)]$
 $\langle f, g \rangle = \kappa \int f(x) \, \delta(x) \, \mathbb{I}(\alpha, \varepsilon) + \delta(x) \, \delta(x, \alpha, \varepsilon) + \delta(x) \, \mathbb{I}(\alpha, \varepsilon) + \delta(x, \alpha, \varepsilon) + \delta(x, \$ *Article*

 R iemann–Liouville fractional integral operator; P actional operator; P actional operator; P

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Keywords: interval-valued mappings over coordinates; left and right ℏ -Convexity; double

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results' numerical validations that examples are nontrivial. By taking the product of two left and

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1. Introduction

29 Eroilor Boulevard, 500036 Brasov, Romania

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 R iemann–Liouville fractional integral operator; P actional operator; P actional operator; P

Riemann–Liouville fractional integral operator; Pachpatte-type inequalities

Liouville fractional to derive the major results of the major results of the results of the key also examine the key

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right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also

right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also

realms of applied and pure sciences. Furthermore, because of its many applications and

1. Introduction

hence, J is coordinated LR-h-convex IVM on Ω .
Conversely, let J be coordinated LR-h-convex IVM on Ω . Then, for all (α, i) , Conversely, let J be coordinated LK-n-convex IVM on 11. Then, for all $(\gamma, 1)$,
 $(\varepsilon, \mathfrak{v}) \in \Omega$, v and $\kappa \in [0, 1]$, we have convexity and the theory of inequalities. convexity and the theory of inequalities. Order Relation. *Fractal Fract.* **2024**, *8*, ϵ Order Relation. *Fractal Fract.* **2024**, *8*, ϵ Khan, M.B.; Hakami, K.H. New $r_{\rm r}$ we nave defined class of constant α are known as convex mapping proposed that are coordinated that α -dinamigated α -dinamigated left and right α -dinamigated left and right α -dinamigated left and right α -dina Conversely, let \iint_R be coordinated LR - \hbar -convex IVM on Ω . Then, for all (σ, i) , Montgomery, AL 36101, USA Montgomery, AL 36101, USA Montgomery, AL 36101, USA **Order Relation** $(\varepsilon, \mathfrak{v}) \in \Omega$, v and $\kappa \in [0, 1]$, we have $(E, \mathbf{U}) \in \Omega$, *U* and $K \in [0, 1]$, we have $(E, 0) \in \Omega$, *U* and $K \in [0, 1]$, we have

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 $\leq_{p} \tilde{h}(v)\tilde{h}(\kappa)\tilde{J}(\sigma,\epsilon) + \tilde{h}(v)\tilde{h}(1-\kappa)\tilde{J}(\sigma,\mathfrak{v}) + \tilde{h}(1-v)\tilde{h}(\kappa)\tilde{J}(\mathfrak{i},\epsilon) + \tilde{h}(1-v)\tilde{h}(1-\kappa)\tilde{J}(\mathfrak{i},\mathfrak{v}).$ $J(\nu_{\sigma} + (1-\nu)i, \kappa \varepsilon + (1-\kappa)\nu)$
 $\leq \frac{k(\nu)k(\nu)\pi(-\nu) + k(\nu)k(1-\nu)\pi(-\nu) + k(1-\nu)k(\nu)\pi(-\nu) + k(1-\nu)k(1-\nu)\pi(-\nu)}{2\pi}$ $\sum_{i=1}^n n(v) n(x) J(\sigma, \epsilon) + n(v) n(1-\kappa) J(\sigma, \sigma) + n(1-\nu) n(\kappa) J(\sigma, \epsilon) + n(1-\sigma) J(\sigma, \sigma)$ $\mathcal{L}_1[\mathcal{U}] \propto \mathcal{L}[\mathcal{U}]$ has set $\mathcal{U} = \mathcal{U}[\mathcal{U}]$ in the straightforward integration. $\sum_{i} \frac{\mu_i(\nu_j \mu_i(\kappa) J_i(\nu, \epsilon) + \mu_i(\nu_j \mu_i(\epsilon - \kappa) J_i(\nu, \epsilon)) + \mu_i(\epsilon - \nu_j \mu_i(\kappa) J_i(\nu, \epsilon)) + \mu_i(\epsilon - \nu_j \mu_i(\epsilon - \epsilon))}{\sigma_i(\epsilon - \epsilon)}$ $c_0 = c_1 + (1 - v)$, $\kappa \varepsilon + (1 - \kappa)$ to Dragomir and Pearce κ (1). This property and Pearce κ $\sum_{i=1}^n h(i/n) \mathcal{I}(\mathcal{O}_r \varepsilon) + h(i/n) \mathcal{I}(\mathcal{I} - \kappa) \mathcal{I}(\mathcal{O}_r \mathcal{O}) + h(1-\nu) \mathcal{I}(\kappa) \mathcal{I}(\mathcal{I} \varepsilon) + h(1-\nu)$ $\mathcal{L}_j(\mathcal{U}\mathcal{U} + (1-\nu)\mathbf{I}, \kappa\epsilon + (1-\kappa)\mathbf{V})$ $\sum_{i=1}^n n(v)n(\kappa)$ j(σ , ϵ) + $n(v)n(1-\kappa)$ j(σ , σ) + $n(1-v)n(\kappa)$ j(κ c) + $n(1-v)n$ $T_{\rm eff}$ Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most well-kno $c_1 + (1 - k)0$ $\sum_{i=1}^{\infty} \frac{n(\nu)\mu(\kappa)}{j} \int_{\mathcal{O}} \langle \nu, \varepsilon \rangle + \frac{n(\nu)\mu(1-\kappa)}{j} \int_{\mathcal{O}} \langle \nu, \nu \rangle + \frac{n(1-\nu)\mu(\kappa)}{j} \int_{\mathcal{O}} \langle \nu, \varepsilon \rangle + \frac{n(1-\nu)\mu(1-\kappa)}{j} \int_{\mathcal{O}} \langle \nu, \nu \rangle$ \mathbf{H} Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most well-kno category of convex functions, according to Dragomir and Pearce $\sum_{i=1}^{n}$. This Dragomir and Pearce $\sum_{i=1}^{n}$. This Dragomir and Pearce $\sum_{i=1}^{n}$ $\leq_{p} \hbar(v)\hbar(\kappa)J(\sigma,\varepsilon)+\hbar(v)\hbar(1-\kappa)J(\sigma,v)+\hbar(1-v)\hbar(\kappa)J(\mathfrak{i},\varepsilon)+\hbar(1-v)\hbar(1-\kappa)J(\mathfrak{i},\mathfrak{v}).$
Therefore, again from Equation (20), we have Published: date $-\kappa(\mathfrak{v})$ $-\kappa(\mathfrak{v})$ $-\epsilon$ $-r$ \cdot Convexity via Left and Right and Rig $\mathcal{Y}[(\mathcal{U}\mathcal{U} + (1-\mathcal{U})\mathcal{U}, \kappa \varepsilon + (1-\kappa)\mathcal{U}]$ $\sup_{\nu\in\mathcal{P}}\sum_{i=1}^n\sum_{i=1}^n\sum_{j=1}^n$ $K(x, k) = \frac{1}{\sqrt{K}} \int_{K}^{K} \frac{1}{\sqrt{K}} \left(\frac{1}{\sqrt{K}} \right)^{N/2} \left(\frac{1}{\sqrt{K}} \right)^{N/2}$ $\sum_{i} n(v)n(k)J(\mathcal{O},\varepsilon) + n(v)n(1-\kappa)J(\mathcal{O},\mathfrak{v}) + n(1-v)n(\kappa)J(\mathfrak{v},\varepsilon) + n(1-v)n(1-\kappa)J(\mathfrak{v},\mathfrak{v}).$ $t_0(t_0 + (1 - v)t_0)$ $c_1(x + (1 - v)i \kappa + (1 - \kappa)n)$ $\frac{\partial u}{\partial t} + (1 - v)\psi$, $\kappa c + (1 - \kappa)v$
 $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial t} + \frac{\partial v}{\partial t}$
 $\frac{\partial u}{\partial t} = \kappa \frac{\partial u}{\partial t} + \frac{\partial v}{\partial t}$ $\leq_p h(v)h(\kappa)J(\gamma,\varepsilon)+h(v)h(1-\kappa)J(\gamma,\mathfrak{v})+h(1-v)h(\kappa)J(1,\varepsilon)+h(1-v)h(1-\kappa)J(1,\mathfrak{v}).$ $J\!J(v_{\mathcal{O}}+(1-v)\mathfrak{i},\kappa\boldsymbol{\epsilon}+(1-\kappa)\mathfrak{v})$ $29900 + (1 - \frac{1}{2})$ $h(v)n(N)J(\sigma,\varepsilon) + h(v)n(1 - \kappa)JJ(\sigma,\nu) + h(1 - v)n(\kappa)JJ(\nu,\varepsilon) + h(1 - v)n(1 - \varepsilon)JJ(\nu,\varepsilon)$ $2\mathcal{L} + (1 - \mathcal{R})\mathcal{L}$ (α, ε) + $n(\nu)n(1 - \kappa)$ $\iint(\alpha, \nu)$ + $n(1 - \nu)n(\kappa)$ $\iint(\kappa, \varepsilon)$ + $n(1 - \nu)n(1 - \kappa)$ $\iint(\nu, \nu)$. \mathbf{S} Department of Mathematics and Computer Science, Transilvania University of Bras \mathbf{S} $29 - 29 = 60$ $(1 - \kappa)$ $\iint_V \varphi, \nu) + h(1 - \nu)h(\kappa)J(\nu, \varepsilon) + h(1 - \nu)h(1 - \kappa)J(\nu, \nu).$ $\frac{1}{\epsilon}$ $E(\mu)E(\mu)$ π (a) $E(\mu)E(1 - \mu)$ π (b) $E(1 - \mu)E(\mu)$ π (c) $E(1 - \mu)E(1 - \mu)$ $2\pi\left(\frac{2}{\pi}\right)\log\left(\frac{2}{\pi}\right)$ by $2\pi\left(\frac{2}{\pi}\right)\log\left(\frac{2}{\pi}\right)$ $J(v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-\kappa)v)$
 $\leq_p \hbar(v)\hbar(\kappa)J(\sigma,\varepsilon) + \hbar(v)\hbar(1-\kappa)J(\sigma,v) + \hbar(1-v)\hbar(\kappa)J(i,\varepsilon) + \hbar(1-v)\hbar(1-\kappa)J(i,v).$ 2 Department of Mathematics and Computer Science, Alabama State University, \mathbb{Z} and \mathbb{Z} and 2 Department of Mathematics and Computer Science, Alabama State University, $\mathbf{1}_{\mathbf{1}_{\mathbf{1}}}$ and $\mathbf{1}_{\mathbf{1}_{\mathbf{1}}}$ $\int_{\mathcal{L}_p}$ \mathcal{L} **Order** \leq_{p}

Therefore, again from Equation (20), we have The Hermite–Hadamard inequality is one of the most well-known findings in the $\overline{}$ is one of the most well-known findings in the most well-known findings in the most well-known finding in the most we T_{max} is one of the most well-known findings in the most well-known finding α $\frac{1}{\sqrt{2}}$ is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most T Hermite–Hadamard inequality is one of the most well-known findings in the most well-known finding in the most well-known fin $H_{\text{inter}}(\omega)$ is one of the most well-known findings in the most well-known finding in the most we convexity again them equal Γ berefore again from Equation (20) we have convexity and the theory of \mathbf{r} r_{r} convexity and the theory of interesting $\sum_{i=1}^{n} r_i$ Γ berofore zozin from Equation (20) we between Γ σ in the theory of inequalities. can be directly derived from convex functions, the rate Γ berefore again from Equation. (20) we have $\frac{1}{2}$ can be directly derived from convex functions, there is a close relationship between $\frac{1}{2}$ and $\frac{1}{2}$ can be directly derived from convex functions, the rate α be directly derived from convex functions, there is a close relationship between α Γ bengfore, again from Γ quation. (20) μ approximate the solution. can be directly derived from convex functions, there is a close relationship between ϵ Therefore, again from Equation (20), we have $\frac{1}{2}$ can be directly derived from convex functions, there is a close relationship between $\frac{1}{2}$ and $\frac{1}{2}$ che directly derived from convex functions, there is a close relationship between ϵ α be directly derived functions, there is a close relationship between α close α **Let 2.** The απαγωγή της Στ $\frac{1}{\sqrt{2}}$ there are many uses $\frac{1}{\sqrt{2}}$ convex $\frac{1}{\sqrt{2}}$ convex $\frac{1}{\sqrt{2}}$ convex functions in the convex function \mathcal{L} there are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex sets and convex \mathcal{L} T are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex \mathbb{R} Therefore, again from Equation (20), we have \sim enware \sim \sim enware \sim enware \sim enware \sim \sim enware $\frac{1}{\sqrt{2}}$. 2 Department of Mathematics and Computer Science, Alabama State University, merefore, again fro $\overline{2}$ department of Mathematics and Computer State University, $\overline{2}$ mererore, agam no $Equation (20)$, we have \overline{D} department of Mathematics and Computer State University, Alabama State University, Therefore, again from Equation (20), we have $T_{\text{S}} = \frac{1}{2}$ **Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4 Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4** $\frac{1}{2}$, $\frac{1}{2}$, **Tareq Saeed 1Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4 Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4**

$$
J((v_{\mathcal{O}} + (1 - v)i, \kappa \varepsilon + (1 - \kappa)v))
$$

= $[J_*(v_{\mathcal{O}} + (1 - v)i, \kappa \varepsilon + (1 - \kappa)v), J^*(v_{\mathcal{O}} + (1 - v)i, \kappa \varepsilon + (1 - \kappa)v)].$ (22)

inequality has several applications and a straightforward intrinsic geometric explanation.

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Fractal Fract. **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *holds: holds:* Again, Equations (3) and (4), we obtain **Copyright:** © 2024 by the authors. **Copyright:** © 2024 by the authors. $\Lambda \alpha$ Again, Equations (3) and (4), we obtain challenging, in the used to approximate the solution $\mathcal{L}_{\mathcal{A}}$ approximate the solution. Since $\mathcal{L}_{\mathcal{A}}$ $\mathcal{Y}=\mathcal{Y}$ and a mathematical problem pro $\mathcal{Y}=\mathcal{Y}$ are determining exact values for a mathematical problem proble σ and pure sciences. Furthermore, because of its many applications and its many applications and σ realms of applied and pure sciences. Furthermore, because of its many applications and \mathcal{L} there are many uses \mathcal{L} and convex sets and convex \mathcal{L} and convex functions in the $T_{\rm eff}$ there are many uses for the convex sets and convex sets and convex sets and convex \sim $T_{\rm eff}$ are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex sets and convex $T_{\rm eff}$ There are many uses for the convex sets and convex sets and convex sets and convex sets and convex \mathcal{L} T Again, Equations (3) and (4), we obtain restrictions on endpoint functions of interval-valued functions that can be seen as applications of restrictions on endpoint functions of interval-valued functions that can be seen as applications of these new outcomes. restrictions on endpoint functions of interval-valued functions that can be seen as applications of β ^azanu. edu. sama tamin'ny samana ny taona 2008. Ny haavon'ny tanàna mandritry ny taona 2008–2014. Ny faritr'i Nord-Amerika ny taona 2014. Ny faritr'i Nord-Amerika ny taona 2014. Ny faritr'i Nord-Amerika ny taona $\frac{4}{100}$ and $\frac{4}{10}$ we obtain $\left(\frac{1}{2} \right)$ Again, Equations (3) and (4), we obtain μ $\frac{1}{2}$ Department of Mathematics, $\frac{1}{2}$

 $\hbar(v)\hbar(\kappa)J(\sigma,\epsilon)+\hbar(v)\hbar(1-\kappa)J(\sigma,\mathfrak{v})+\hbar(1-v)\hbar(\kappa)J(\mathfrak{i},\epsilon)+\hbar(1-v)\hbar(1-\kappa)J(\mathfrak{i},\mathfrak{v})$ *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract $h(v)h(\kappa)J(\sigma,\varepsilon)+h(v)h(1-\kappa)J(\sigma,\mathfrak{v})+h(1-v)h(\kappa)J(1,\varepsilon)+h(1-v)h(1-\kappa)J(1,\mathfrak{v})$
= $\hbar(v)\hbar(\kappa)[J_*(\sigma,\varepsilon), J^*(\sigma,\varepsilon)]+\hbar(v)\hbar(1-\kappa)[J_*(\sigma,\mathfrak{v}), J^*(\sigma,\mathfrak{v})]$ (23) $+\hbar(\kappa)\hbar(1-v)[J_*(\sigma,\varepsilon), J^*(\sigma,\varepsilon)] + \hbar(1-v)\hbar(1-\kappa)[J_*(\sigma,\mathfrak{v}), J^*(\sigma,\mathfrak{v})],$ conditions of the \hbar $A\subset \mathbb{R}^n$ is a set of \mathbb{R}^n in the set of \mathbb{R}^n (https://creativecommons.org/license conditions of the \hbar Attribution (CC BY) license (https://creativecommons.org/license *Fractal Fract.* **2024**, *8*, 125
 $\int f(x)dx = f(x)dx$, $\int f(x)dx + f(x)dx + f(x)dx$, $\int f(x)dx + f(x)dx$, $\int f(x)dx + f(x)dx$, **Fractal Fraction Fraction** $\frac{1}{2}$, $\frac{1}{$ (23) $\frac{1}{\sqrt{2}}$ *f*($\left(\begin{matrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \right)$ *(23)* $\mathcal{L}(\cdot)$ in the theory of inequalities. $n(v) n(\kappa) j(\sigma, \varepsilon) + n(v) n(1-\kappa) j(\sigma, v) + n(1-v) n(\kappa) j(\sigma, \varepsilon) + n$ $h(\mathcal{C})[J_{*}(\mathcal{O},\epsilon), J_{*}(\mathcal{O},\epsilon)] + h(\mathcal{O})h(1-\kappa)[J_{*}(\mathcal{O},\epsilon), J_{*}(\mathcal{O},\epsilon)]$
+ $\hbar(\kappa)\hbar(1-\nu)[J_{*}(\mathcal{O},\epsilon), J_{*}(\mathcal{O},\epsilon)] + \hbar(1-\nu)\hbar(1-\kappa)[J_{*}(\mathcal{O},\mathfrak{v}), J_{*}(\mathcal{O},\mathfrak{v})],$ $\mathcal{F}(\lambda \mathcal{F}(\lambda \pi))$ $T_n(v)T_n(x)T_n(y) = T_n(v)T_n(y) = \sum_{k=0}^{n} \frac{(-1)^k}{k!}$ can be directly derived from convex functions, there is a close relationship between $n(v)u(x)$ if $(v, \varepsilon) + n(v)u(x)$ $c \left(\frac{1}{2} \right)$ be directly derived from convex functions, there is a close relationship between $c \left(\frac{1}{2} \right)$ betwee $\mu(\nu) \mu(\kappa)$ j(ν , c) $\pm \mu(\nu) \mu(\mathbf{1})$ can be directly derived from convex functions, there is a close relationship between $J(\mathbf{c}, \varepsilon) + h(\mathbf{c})h(1-\kappa)J(\mathbf{c}, \mathbf{c}) + h(1-\mathbf{c})$ c_1 be directly derived from convex functions, there is a close relationship between c_1 relationship between c_1 $c_1(\mathbf{c}, \mathbf{c}) + n(\mathbf{c})n(\mathbf{1} - \mathbf{c})$ can be directly derived from convex functions, there is a close relationship between $\mathcal{C}(n(\kappa))\mathbf{J}(\boldsymbol{\sigma}, \boldsymbol{\epsilon}) + \mathbf{n}(\boldsymbol{\nu})\mathbf{n}(\mathbf{1} - \kappa)\mathbf{J}(\boldsymbol{\sigma}, \mathbf{v}) + \mathbf{n}(\mathbf{1} - \boldsymbol{\nu})\mathbf{n}(\mathbf{k})\mathbf{J}(\mathbf{0}, \boldsymbol{\epsilon}) + \mathbf{n}(\mathbf{1} - \boldsymbol{\nu})\mathbf{n}(\mathbf{0})$ challenging, inequalities can be used to approximate the solution. Since many inequalities many inequalities m $\hbar(v)\hbar(\kappa)\mathbf{J}(\boldsymbol{\sigma},\boldsymbol{\epsilon})+\hbar(v)\hbar(1-\kappa)\mathbf{J}(\boldsymbol{\sigma},\mathfrak{v})+\hbar(1-v)\hbar(\kappa)\mathbf{J}(\mathfrak{i},\boldsymbol{\epsilon})+\hbar(1-v)\hbar(1-\kappa)\mathbf{J}(\mathfrak{i},\mathfrak{v})$ challenging, including the solution. Since α approximate the solution. Since α challenging, including can be used to approximate the solution. Since α many inequalities α many inequalities α challenging, including to approximate the solution. Since $\frac{1}{2}$ \mathcal{L}_{max} years. When determining exact values for a mathematical problem problem problem problem problem proves to be a mathematical problem problem proves to be a mathematical problem problem problem proves to be a $\hbar(v)\hbar(\kappa)J(\sigma,\varepsilon) + \hbar(v)\hbar(1-\kappa)J(\sigma,\mathfrak{v}) + \hbar(1-v)\hbar(\kappa)J(\mathfrak{i},\varepsilon) + \hbar(1-v)\hbar(1-\kappa)J(\mathfrak{i},\mathfrak{v})$ $+\hbar(\kappa)\hbar(1-v)[J_*(\sigma,\varepsilon), J^*(\sigma,\varepsilon)] + \hbar(1-v)\hbar(1-\kappa)[J_*(\sigma,\mathfrak{v}), J^*(\sigma,\mathfrak{v})],$ $\mathcal{G}_{\ast}(v,v)$ defined that are known as convex mapping (v,v) , (v,v) , (v,v) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **Fractional forms** of H_0 (i.e., $\mathbf{H}^*(t)$ and $\mathbf{H}^*(t)$ and defined convex mapping proposed that are known as coordinated that $\left[\frac{1}{2}\right]$ + $\left[\frac{1}{2}\$ $\begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{B} \\ \mathbf{B} & \mathbf{B} & \mathbf{B} & \mathbf{B} \\ \mathbf{B} & \mathbf{B} & \mathbf{B} & \mathbf{B} \end{bmatrix}$ (Absolutional forms of the newly the newly the newly the newly the newly the newly the new leads of the new leads of the new leads $\left[\frac{n(n+1)(n+1)}{2}\right]$ $\left[\frac{y_1x_1(y_1(y_1), y_1(y_1(y_1)) - n(1+y_1)(y_1(y_1(y_1), y_1(y_1(y_1)))) - n(1+y_1)(y_1(y_1(y_1), y_1(y_1(y_1))))\right]$ khakami@jazanu.edu.sa khakami@jazanu.edu.sa \int_{0}^{1} Correspondence: engage \int_{0}^{1} $\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) - \frac{1$ $k₁$ $\frac{1}{2} \frac{f(x) f(1-x)}{f(1-x)} \frac{\pi}{\pi} \frac{f(x)}{f(1-x)} \frac{\pi^*(x)}{\pi^*(x-x)} \frac{\pi^*(x)}{\pi^*(x-x)} \frac{\pi^*(x)}{\pi^*(x-x)}$ khakami@jazanu.edu.sa $\{k \mid \lambda \in \{1, 3\}, \forall i \in \{1, 3\}$ $\left(\begin{matrix} 2 \\ 1 \end{matrix} \right)$ $\left(\begin{matrix} 2 \\ 3 \end{matrix} \right)$ $\left(\begin{matrix} 0 \\ 2 \end{matrix} \right)$ $\frac{1}{2} \frac{E(x)E(1-x)}{E(x)} = \frac{\pi^*(1-x)}{\pi^*(1-x)} = \frac{1}{2} \frac{E(1-x)}{E(1-x)} = \frac{1}{2} \frac{E(1-x)}{E(1-x)} = \frac{1}{2} \frac{1}{2} \frac{E(1-x)}{E(1-x)} = \frac{1}{2} \frac$ $k \rightarrow k$ idanu. $k \rightarrow k$ $\frac{1}{2}$ $\left[\left(\frac{\alpha}{2}\right)\right]$ in $\left(\frac{\alpha}{2}\right)$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $k(\tau, \alpha)$ $\mathsf{y}_{\mathsf{J}\ast}(\mathsf{v}, \mathsf{v})$

for an $\chi, \omega \in \Omega$ and $\upsilon \in [0, 1]$. Then, from (22) and (25), we have for an $\chi, \omega \in \Omega$ and for all $x, \omega \in \Omega$ and $v \in [0, 1]$. Then, from (22) and (23), we have for all $x, \omega \in \Omega$ and (https://creativecommons.org/license $v \in [0, 1]$, such that t For an χ_t c_{total} derived from convex functions, there is a close relationship between (22) and (22) . For an χ , α $\mathcal{U} \subset [0, 1]$, such that For an $x, w \in \Omega$ and $v \in [0, 1]$. Then, from (22) and (23), we have re challenging, including to approximate the solution. Since (22) in (22) in (22) for an $\alpha \in [0, 1]$. Then, then (22) and (29) , we have n an $\chi_r w \in \Omega$ and $v \in [0, 1]$. Then, from (22) and (25), we have for an $\chi_r w \in \Omega$ and c_1 in the contract can be used to approximate the solution. Since c_1 in the solution. Since c_1 an $\chi_i w \in \Omega$ and $v \in [0, 1]$. Then, from (22) and (23), we have for an $\chi_i w \in \Omega$ and α , α . Then, from (22) and (25), we have for an $\chi, \omega \in \Omega$ and Totall $x, \omega \in \Omega$ for an $\chi, w \in \Omega$ and $v \in [0, 1]$. Then, from (22) and (25), we have for an for all $x, \omega \in \Omega$ and $v \in [0, 1]$. Then, from (22) and (23), we have for all $x, \omega \in \Omega$ and $v \in [0, 1]$, such that $t \cdot \tau$ to the theory of inequalities, convexity has advanced quickly indicated τ $\sigma \subset [0, 1]$, such that \mathcal{A} is a Left and Right and Right and Right and Right and Right and Right and obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some \mathbf{r} -convexity, some new versions of \mathbf{r} right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also $r = r \cdot \frac{1}{2}$ right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also $r = r \cdot \Delta$ right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also results in the sults of two left are non-trivial. By taking the product of two left and product of two left and the product of two left and two for all $x, \omega \in \Omega$ and $v \in [0, 1]$. Then, from (22) and (23), we have for all $x, \omega \in \Omega$ and results' numerical validations that examples are nontrivial. By taking the product of two left and \mathbf{L} that integral to derive the major results of the major results of the key also examine the key also examine

$$
J_{\ast}(v_{\mathcal{O}} + (1-v)i\kappa \varepsilon + (1-\kappa)v)
$$

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$$
\leq \hbar(v)\hbar(\kappa)J_{\ast}(\sigma,\varepsilon) + \hbar(v)\hbar(1-\kappa)J_{\ast}(\sigma,\nu) + \hbar(1-v)\hbar(\kappa)J_{\ast}(i,\varepsilon) + \hbar(1-v)\hbar(1-\kappa)J_{\ast}(i,\nu),
$$

Fractal Fract. **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract and under the terms and th \mathbf{r} and \mathbf{r} is the one who first identified it \mathbf{r} . The following is stated: \mathbf{r} and \mathbf{r} 1 1 \mathbf{C} result was mainly credited to \mathbf{C} and \mathbf{C} \mathbf{u} and \mathbf{v} and \mathcal{L} and \mathcal{L} are mathematical problem pr Riemann–Liouville fractional integral operator; Pachpatte-type inequalities Riemann–Liouville fractional integral operator; Pachpatte-type inequalities $r_{\rm eff}$ end intervalsed functions that can be seen as applications that can be seen as applications of α right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also results' numerical validations that examples are nontrivial. By taking the product of two left and results' numerical validations that examples are nontrivial. By taking the product of two left and results' numerical validations that examples are nontrivial. By taking the product of two left and results' numerical validations that examples are nontrivial. By taking the product of two left and

x. https://doi.org/10.3390/xxxxx

inequality has several applications and a straightforward intrinsic geometric explanation. inequality has several applications and a straightforward intrinsic geometric explanation.

Keywords: interval-valued mappings over coordinates; left and right ℏ -Convexity; double

Keywords: interval-valued mappings over coordinates; left and right ℏ -Convexity; double

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years. When determining exact values for a mathematical problem proves to be

years. When determining exact values for a mathematical problem proves to be

and
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$$
J^*(v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-\kappa)v)
$$
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\leq \hbar(v)\hbar(\kappa)J^*(\sigma,\varepsilon) + \hbar(v)\hbar(1-\kappa)J^*(\sigma,\mathfrak{v}) + \hbar(1-v)\hbar(\kappa)J^*(i,\varepsilon) + \hbar(1-v)\hbar(1-\kappa)J^*(i,\mathfrak{v}),
$$

hence, the result follows. \square $F_{\text{r}}(x) = \frac{1}{2}$ *Fractal* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract $F_{\text{r}}(x,y)$ are found for the motion of \Box *Fractal* x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract F_{r} , $\frac{1}{2}$ $\frac{1}{2}$ category of classical convex functions, according to Dragomir and Pearce [1]. This category of classical convex functions, according to Dragomir and Pearce [1]. This t_{t} relative to the theory of inequalities, convexity has advanced quickly in recent α t_{t} and the theory of inequalities, convexity has advanced quickly in recent α realms of applied and pure sciences. Furthermore, because of its many applications and $t_{\rm H}$ relationship to the theory of inequalities, convexity has advanced quickly in recent α reflict, the result follows. \Box Version of Fractional Pachpatte-type n nence, the result follows. \Box these new outcomes. these new outcomes. these new outcomes.

Example 1. We consider the IVM JJ : [0, 1] \times [0, 1] $\rightarrow \mathbb{R}^+_I$ defined by, The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– The result was mainly contributed to \mathcal{L} $\sum_{i=1}^n$ is the conduct the solution of $\sum_{i=1}^n$ in $\sum_{i=1}^n$ in $\sum_{i=1}^n$ inequalities of $\sum_{i=1}^n$ Example 1 We exact value $\frac{W}{A}$ if $\frac{W}{A}$ is $\frac{W}{A}$ in $\frac{W}{A}$ in $\frac{W}{A}$ and $\frac{W}{A}$ $\sum_{i=1}^n$ integration be used to approximate the solution. Since $\sum_{i=1}^n$ is $\sum_{i=1}^n$ many inequalities of $\sum_{i=1}^n$ $\mathcal{W}_{\mathcal{S}}$ consider the UVM π , $[0, 1] \times [0, 1]$ with defined by conduct the $\{v_1, v_2\}$ is possible, we use the solution of $\{v_1, v_2\}$ **Example 1** Movember the U/M π , $[0, 1] \times [0, 1]$ in \mathbb{R}^+ defined by Example 1. We constant the 1941 y_j (b) y_j (b) x_j mathematics) $T_{\text{max}} = 1.4 \, \text{m}$ uses for the concepts of convex functions in the convex Example 1. We consuler the TV it Γ_{reco} and Γ_{c} and Γ_{c} and Γ_{c} functions in the convex functions in the con **Example 1.** We consuled the TV IVI J]. [0, 1] \wedge [0, 1] \rightarrow \mathbb{R} ^T defined **There is the consuler the convex sets of convex i** $[0, 1] \wedge [0, 1] \rightarrow \mathbb{R}$ is defined by, **All there are many uses of convex sets for the convex functions in the conve Example 1.** We consider the IVM JJ : $[0, 1] \times [0, 1] \rightarrow \mathbb{R}^+_I$ defined by, x. https://doi.org/10.3390/xxxxx **Example 1.** We consider the IVM JJ : [0, 1] \times [0, 1] $\rightarrow \mathbb{R}^+_I$ defined by, **1. 1.** *We constaer the 1V M J* J : $[0, 1] \times$ **Example 1.** We consul Γ 14 W 1 d W 6 al $\begin{bmatrix} 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \end{bmatrix}$ **EXAMPLE 1.** WE CONSULT THE TV IVI J \cdot $\lfloor 0, \rfloor$ **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **Example 1.** We constant the TV IVI J . σ **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **Example 1.** We consult the TV IVI J \cdot [0, 1] \wedge [0, 1] \rightarrow \mathbb{R} ^T **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **Example 1.** We constant the TV IVI $J_1 \cdot [0, 1] \wedge [0, 1]$ and I_2 **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **C 1.** We consuler the TV IVI J $[0, 1] \wedge [0, 1] \rightarrow \mathbb{R}$ ucfined by, **Citation:** Saeed, T.; Nwaeze, E.R.; **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **Example 1.** We consuler the TV IVI JJ , $[0, 1] \times [0, 1] \rightarrow \mathbb{R}_I$ defined by,

$$
J(x) = [xy, (6 + e^x)(6 + e^y)].
$$

 Γ , Γ , Endpoint functions $J_{\mu}(x,y)$, $J^*(x,y)$ are coordinate h-concave functions. Hence, $J(x,y)$ is coordinated LR-h-convex IVM. $VM.\,$ IVM . Enapoint functions $J_{\mu}(x, y)$, $J_{\mu}(x, y)$ are coordinate n-concave functions. Hence, $J_{\mu}(x, y)$ is realms of applied and pure sciences. Furthermore, because of its many applications and Endpoint functions $J_{*}(x,y)$, $J^{*}(x,y)$ are coordinate h-concave functions. Hence, $J(x,y)$ is η -convex TV \dot{M} . realms of applied and pure sciences. Furthermore, because of its many applications and \mathcal{P}_max are determining exact values for a mathematical problem problem problem problem problem problem problem proves to be a mathematical problem problem problem problem problem problem problem problem problem pro Enapoint functions $J_*(x, y)$, $J_j^*(x, y)$ are coordinate h-concave functions. Hence, $J_j(x, y)$ Order Relation. *Fractal Fract.* **2024**, *8*, Integral Inequalities via Coordinated L *napomi* junctions

coordinated LR - \hbar -convex IVM. But the inverse is not true. From Lemma 1 and Example 1, we can easily note that each LR- \hbar -convex IVM is Published: date From Lemma 1 and Example 1, we can easily note that each L_N -*t*-convex 1*V M* is challenging, inequalities can be used to approximate the solution. Since many inequalities can be directly derived from convex from convex from is a close relationship between μ convex from μ challenging, inequalities can be used to approximate the solution. Since many inequalities ample Γ , we can easily note that each LN - η -convex Γ ℓ η is Γ **From Lemma** years. When determining exact values for a mathematical problem proves to be From Lemma 1 and Example 1, we can easily note that each LR - \hbar -convex IVM is

Remark 2. If one assumes that $J_{*}(x,y) = J^{*}(x,y)$, then J is referred to as a classical coordi-*Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract **Remark 2.** If one assumes that $J_{\ast}(x,y) = J^*(x,y)$, then JJ is referred to as a classical coordi-
nated LR-ħ-convex function if JJ meets the stated inequality here: $\frac{1}{2}$ for convex functions, and $\frac{1}{2}$, $\frac{1}{2}$ $\mathbf{H} \mathbf{H} \left(\begin{array}{c} \mathbf{H}^* \\ \mathbf{H}^* \end{array} \right)$ in the most well-known finding in the most well α categories functions, $\int_{\mathbb{R}} \mathbf{w} \cdot \mathbf{y} \cdot \mathbf{r} \cdot d\mathbf{x}$ functions, and $\mathbf{r} \cdot \mathbf{r} \cdot \mathbf{r} \cdot d\mathbf{x}$ $\begin{pmatrix} 1 & \mathbf{m} & \$ $c_{\mathbf{y}_*}(\mathbf{x}, \mathbf{y}) = \mathbf{y}(\mathbf{x}, \mathbf{y})$, and \mathbf{y} is referred to as a classical coordination of \mathbf{y}_* $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ The Hadamard integration into $J_x(x, y) = y(x, y)$ well-known finding into the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding into $J_x(x, y)$ and $J_y(x, y)$ and challenging, inequalities can be used to approximate the solution. Since many inequalities **Remark 2.** If one assumes that $J_{\mu}(x,y) = J'(x,y)$, then challenging, including, inequalities can be used to approximate the solution. Since \mathcal{L} **can be directly definition** density density $\mathbf{u}_1(x, y) = \mathbf{u}_2(x, y)$, then is convexed the theory of the the theory of **challenging channon c** used the used to assume state $J_{\mu}(x, y) = J_{\mu}(x, y)$, then $J_{\mu}(x, y)$ years. When determining exact values for a mathematical problem proves to be **challenging** challenging the used to approximate the solution of $\int_{\mathbb{R}} (x, y) dx = \int_{\mathbb{R}} (x, y) dx$, then if its **Kemark** 2. If one assumes that $J_{\ast}(x, y) = J_{\cdot}(x, y)$, then *J* is referred to as a class years. When determining exact values for a mathematical problem proves to be **Kemark 2.** If one assumes that $J_{\mu}(x, y) = J_{\mu}(x, y)$, then $J_{\mu}(x, y)$ is referred to as a cluster c 2. If one assumes that $J_{\ast}(x, y) = J^{\ast}(x, y)$, then JJ is referred to as a classical coordi- $\mathcal{L}(\mathcal{X}, \mathcal{Y}) = \mathcal{L}(\mathcal{X}, \mathcal{Y})$, then $\mathcal{L}(\mathcal{Y})$ is referred to as a classical coordi-**Remark 2.** *If one a*

$$
J(\nu_{\mathcal{O}} + (1 - \nu)i, \kappa \varepsilon + (1 - \kappa)\mathfrak{v})
$$

\$\leq \hbar(\nu)\hbar(\kappa)J(\sigma,\varepsilon) + \hbar(\nu)\hbar(1 - \kappa)J(\sigma,\mathfrak{v}) + \hbar(\kappa)\hbar(1 - \nu)J(\sigma,\varepsilon) + \hbar(1 - \nu)\hbar(1 - \kappa)J(\sigma,\mathfrak{v})\$. (24)

If one assumes that $\hslash(v)=v$, $\hslash(\kappa)=\kappa$ and $\text{J}_{*}(x,y)=\text{J}^{*}(x,y)$, then J is referred to as a classical coordinated convex function if J meets the stated inequality here: (https://creativecommons.org/license

$$
J(v_{\mathcal{O}} + (1 - v)i, \kappa \varepsilon + (1 - \kappa)v) \n\leq v \kappa J(\mathcal{O}, \varepsilon) + v(1 - \kappa)J(\mathcal{O}, v) + (1 - v)\kappa J(i, \varepsilon) + (1 - v)(1 - \kappa)J(i, v).
$$
\n(25)

inequality has several applications and a straightforward intrinsic geometric explanation.

years. When determining exact values for a mathematical problem problem problem problem problem problem problem

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results' numerical validations that examples are nontrivial. By taking the product of two left and

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Academic Editor: Bruce Henry

Version of Fractional Pachpatte-type

1. Introduction

publication under the terms and

Academic Editor: Bruce Henry

Let one assume that $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $J_*(x, y) \neq J^*(x, y)$, and $J_*(x, y)$ is an affine function and $J^*(x, y)$ is a concave function for the stated inequality here (see [42]) \mathcal{H} **Copyright:** © 2024 by the authors. function and $J^*(x, y)$ is a concave function for the stated inequality here (see [42]) Let one used the solution $f(v) = v$, $h(x) = \kappa$ and $f(x, y) \neq f(x, y)$, and $f(x, y)$, $h(x, y) = h(x, y)$ $\mathbf{y}_i = \mathbf{y}_i + \mathbf{y}_i$ and $\mathbf{y}_i = \mathbf{y}_i$ and $\mathbf{y}_i = \mathbf{y}_i$ and $\mathbf{y}_i = \mathbf{y}_i$ becomes to be becomes to be becomes to be a mathematical problem proves to be a mathematical problem problem problem $\mathbf{y}_i = \mathbf$ Let one assume that $h(v) = v$, $h(x) = \kappa$ and $J_{\frac{1}{2}}(x, y) \neq J_{\frac{1}{2}}(x, y)$, and $J_{\frac{1}{2}}(x, y)$ is an approximate that $y(x, y) = y(x)$ determining exact values for a mathematical problem problem problem problem problem problem proves to be been problem pro Let one assume that $h(v) = v$, $h(\kappa) = \kappa$ and $J_{\kappa}(x, y) \neq J^*(x, y)$, and $J_{\kappa}(x, y)$ is an affine **1. Introduction 1. Introduction 1. Introduction** Riemann–Liouville fractional integral operator; Pachpatte-type inequalities **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double \mathcal{L} iou ussume intergral operator; Pachpatterius integral operator; Pachpathe-type inequalities in $\mathcal{L} = \mathcal{L}$ **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double Eer and Eer integrational integral operator; $P = \text{Eer}(\text{E})$ these new outcomes. Let one assume that $\hslash(v)=v$, $\hslash(\kappa)=\kappa$ and $\text{J}_{*}(x,y)\neq \text{J}^{*}(x,y)$, and $\text{J}_{*}(x,y)$ is an affine defined class of convex mappings proposed that are known as coordinated left and right α -dimensional right α $\mathcal{L} = \frac{1}{\mathbf{N}} \mathbf{x}^{\mu}$ department of \mathbf{r} department of \mathbf{r} department of \mathbf{r} \liminf is generated in \int $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$, $\frac{1}{2}$ $\frac{1}{2}$ on ana s $j(x,y)$ is a con P Europian and $\overline{\mathbf{B}}$ Let one assume that $h(v) = v$, $h(\kappa) = \kappa$ and $J_{\ast}(x, y) \neq J_{\ast}^*(x, y)$, and $J_{\ast}^*(x, y)$ is an affine
function and $J_{\ast}^*(x, y)$ is a concave function for the stated inequality here (see [42])

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Riemann–Liouville fractional integral operator; Pachpatte-type inequalities

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$$
J[(v_{\mathcal{O}} + (1-v)i, \kappa \varepsilon + (1-\kappa)v)]
$$

\n
$$
\supseteq v\kappa J[(\sigma,\varepsilon) + v(1-\kappa)J[(\sigma,\nu) + (1-v)\kappa J](i,\varepsilon) + (1-v)(1-\kappa)J[(i,\nu)].
$$
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The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– category of classical convex functions, according to Dragomir and Pearce [1]. This

The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known finding in the most well-known fin

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Definition 5. Let $J \in \Omega \to \mathbb{R}^+_I$ be a IVM on Ω . Then, we have T is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding T \mathcal{L} challenging, independent to approximate the solution. Since \mathcal{L} **Demittion** d. Let J_j , $\Delta z \rightarrow \mathbb{R}_j$ be a ty *ty* for Δz , then, we have R_{c} character C_{c} in the solution. The solution of many inequalities many independent of many independent of R_{c} **Demittion** derived from α α β or α for a close relationship between relationships between α challenging, inequalities can be used to approximate the solution. Since α **can** be directly dependent from α for a convex functions α and α and α tight relationship to the theory of inequalities, convexity has advanced α **Definition 5.** Let $J \in \Omega \to \mathbb{R}^+$ be a IVM on Ω . Then, we have realms of applied and pure sciences. Furthermore, because of its many applications and realms of applied and pure sciences. Furthermore, because of its many applications and Order Relation. *Fractal Fract.* **2024**, *8*, \mathbf{r} obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some **Definition 5.** Let $J \in \Omega \to \mathbb{R}^+_I$ be a IVM on Ω . Then, we have \sum emitted of \sum $\sum_{j=1}^{n}$ $\sum_{j=1}^{n}$ and $\sum_{j=1}^{n}$ $\sum_{j=1}^{n}$ $\sum_{j=1}^{n}$ **Definition 5** Let $\pi : \Omega \to \mathbb{P}^+$ be a UVM on Ω of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa **Definition 5** Let $\pi \cdot \Omega \rightarrow \mathbb{P}^+$ be a HVM on Ω . Then we have of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa **Definition 5** Let $\pi \cdot \Omega$ $\rightarrow \mathbb{P}^+$ be a HVM an Ω . Then we have of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa 16.6 W Financial Science (FMS)-Research Ω -Research Ω of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa

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restrictions on endpoint functions of interval-valued functions that can be seen as applications of

inequality has several applications and a straightforward intrinsic geometric explanation.

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$$
\mathrm{J}\!\mathfrak{f}(x,y)=\big[\mathrm{J}\!\mathfrak{f}_*(x,y),\ \mathrm{J}\!\mathfrak{f}^*(x,y)\big],
$$

for all $(x,y)\in \Omega$. Then, IJ is coordinated left-LR-h-convex (concave) TV M on Ω , if and only if, $J\!\!J_*(x,y)$ *holds: holds:* for all $(x,y)\in \Omega$. Then, \textsf{J} is coordinated left-LR- \hbar -convex (concave) IVM on Ω , if and only if, $\textsf{J}_*(x,y)$ $\int_{\mathcal{S}} P(x, y) \leq 2\pi$. Then, $\int_{\mathcal{S}} P(x, y) \leq 2\pi$ is extramely to $\int_{\mathcal{S}} P(x, y) \, dx$ (conclude $\int_{\mathcal{S}} P(x, y) \, dx$) and $\int_{\mathcal{S}} P(x, y) \, dx$ and $\int_{\mathcal{S}} P(x, y) \, dx$ ζ category of convex functions, according to Dragomir and Pearce ζ . convexity and the theory of inequalities. $\frac{1}{\sqrt{2}}$ T_{eff} is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most wel and $J^*(x, y)$ are coordinated LR-h-convex (concave) and affine functions on Ω , respectively. for all $(x, y) \in \Omega$. Then, for coordinated ten-th-convex (concuve) TV is on Ω , if and o realms of applied and pure sciences. Furthermore, because of its many applications and for all $(x, y) \in \Omega$. Then, if is coordinated ten-LK-h-convex (concave) TV is on Ω , if and only if, $\mathfrak{I}_{*}(x)$ realms of applied and pure sciences. Furthermore, because of its many applications and for all $(x,y)\in \Omega$. Then, J is coordinated left-LR-h-convex (concave) IVM on Ω , if and only if, $J_*(x,y)$ for all $(x,y)\in \Omega$. Then, \bar{J} is coordinated left-LR- \hbar -convex (concave) IVM on Ω , if and only if, $J_*(x,y)$ \mathcal{L} \sim 32. Then, \mathcal{L} to coordin for all $(x, y) \in \Omega$. Then Π is coordinated left $\overline{I}R$ is convex (concerne) IVM on Ω if and only if Π and $J^*(x,y)$ are coordinated LR-h-convex (concave) and affine functions on Ω , respectively. for an $(x, y) \in x$. Then, sf is coordinated teft-EK-h-convex (concuve) TV INT on x , if and only y , sf for all $(x, y) \in \Omega$. Then, if is coordinated left-LK-h-convex (concave) TV M on Ω , if and only if, $J\!\!\downarrow_\ast$ (x, y) \mathcal{L} University of \mathbb{R}^n department of Mathematics and Computer Science, Transilvania University of Bras \mathcal{L} \int or and $\left(x, \right)$ \mathcal{L} Department of Mathematics and Computer Science, Transilvania University of Bras \mathcal{L} \int or and $(x, y) \subset \Omega$. $\overline{\text{SUSP}}$ μ iefi-Er-h-convex (concuve) TV ivi on Ω , if

Definition 6. Let $J \in \Omega \to \mathbb{R}^+_I$ be a IVM on Ω . Then, we have $T_{\rm eff}$ mainly credited to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), ev $\sum_{i=1}^{n}$ inequality has several applications and a straightforward intrinsic geometric explanation. $\frac{1}{\sqrt{1-\frac{1$ **Demittion 6.** Let f_1 , $\sum_{i=1}^{\infty}$ $\sum_{i=1}^{\infty}$ for every functions $\sum_{i=1}^{\infty}$ close relationship between $\sum_{i=1}^{\infty}$ R_{α} challenging, inequalities can be used to approximate the solution. Since α **Demittion 6.** Eq. $f(x)$ and $f(x)$ is a convex functions of relationship between $f(x)$ challenging, including the solution α approximate the solution. Since α **can** be directly derived from a convex functions of α and α close relationship between α tight relationship to the theory of inequalities, convexity has advanced quickly in recent α **Definition 6.** Let J : $\Omega \to \mathbb{R}^+_I$ be a TVM on Ω . Then, we have tight relationship to the theory of inequalities, convexity has advanced quickly in recent \mathbf{D}_2 Guillemann–Later Ω , \mathbb{D}^+ level Ω in Ω in Ω **Definition 6.** Let $JJ : \Omega \to \mathbb{R}^+_I$ be a IVM on Ω . Then, we have

$$
\mathrm{J}\! \mathrm{J}(x,y) = \big[\mathrm{J}\! \mathrm{J}_*(x,y), \ \mathrm{J}\! \mathrm{J}^*(x,y) \big],
$$

inequality has several applications and a straightforward intrinsic geometric explanation.

if $J_*(x,y)$ and $J^*(x,y)$ are coordinated h-affine and h-(concave) functions on Ω , respectively. *holds: holds:* for all $(x, y) \in \Omega$. Then, J is coordinated right-LR- \hbar -convex (concave) IVM on Ω , if and only $c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_1,$ T_{t} indeed the Hadamard most well-known finding in the most category of classical convex functions, according to Dragomir and Pearce [1]. This category of classical convex functions, according to Dragomir and Pearce [1]. This for all $(x, y) \in \Omega$. Then, η is coordinated right-LR- \hbar -convex (concave) IVM on Ω , if y JJ $_{*}(x,y)$ and JJ (x,y) are coordinated h-affine and h-(concave) functions on S L, respectively. **1. Introduction 1. Introduction** Integral Inequalities via Coordinated $\lim_{n \to \infty} \frac{\ln n}{\ln n}$ is determined to derive the major results of the results of the key also examine the key also defined convex mappings proposed that are known as coordinated that are known as coordinated left and right α -dinated left and right α for all $(x, y) \in \Omega$. Then, if is coordinated right-L defined convex mappings proposed that are known as coordinated that are known as coordinated left and right α -dinated left and right α for all $(x, y) \in \Omega$. Then, if is coordinated right-LN-v-convex defined convex mappings proposed that are known as coordinated that are known as coordinated left and right α for all $(x, y) \in \Omega$. Then, if is coordinated right-LN-te-convex (concave) α convex mappings proposed that are known as coordinated that are known as coordinated left and right α a -convex (concurs) is the use a double and use of double Riemann a

Theorem 6. Let Ω be a coordinated convex set, and let $J: \Omega \to \mathbb{R}_I^+$ be a IVM, defined by $T_{\rm c}$ $\frac{1}{1}$ $\frac{1}{1}$ credit was mainly credit to Hermite (1822–1901), even though Hadamard (1875–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865 $\frac{1}{2}$ challenging, inequalities can be used to approximate the solution. Since $\frac{1}{2}$ is $\frac{1}{2}$ is a $\frac{1}{2}$ in $\frac{1}{2}$ is a $\frac{1}{2}$ interpreted to a proximate the solution concerned to be solution. realms of applied and pure sciences. Furthermore, because of its many applications and **Theorem 6.** Let Ω be a coordinated convex set, and let $J \colon \Omega \to \mathbb{R}^+_I$ be a IVM, defined by obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some The sum ϵ is taking that examples are non-trivial. By taking the product of two left and ρ right convenience integral in The sum ϵ is taken vanished components of the product ϵ righted convexity convergences of fractional integral integral integral integral integral integral integral in The sum ϵ is to λ is a semiinated component of the α sufficient α right convenience integral integral integral integral integral integral integral integral integral in $\frac{1}{2}$ results in the example of the product of the product of two left and product of two left and product of two lef right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also

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\mathrm{J}\! \mathrm{J}(x,y) = \big[\mathrm{J}\! \mathrm{J}_*(x,y), \ \mathrm{J}\! \mathrm{J}^*(x,y) \big],
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 $\lim_{x \to a} \left(x, y, y \right) \in \mathbb{R}^n$. Then, so becommined to conclude 1 T Then \mathbb{R}^n , $\lim_{y \to a} \lim_{y \to$ *holds: holds:* for all $(x,y)\in \Omega$. Then, $\rm J$ is coordinated \hbar -concave IVM on Ω , if and only if $\rm J_{*}(x,y)$ and $\rm J^*(x,y)$ can be directly derived from convex functions, there is a convex functions, the convex functions, α α challenging, in $\mathbb{E}_{\mathbf{z}}$ is callenging, α be used the solution. Since α is α in α is α in α is α in α is α if α is communities to derive the convex functions, there is a convergence relationship between \mathcal{L} y are α mathematical problem and \overline{I} \overline{B} is constant to be a mathematical problem proves to be a m are coordinated ħ-concave and LR-ħ-convex functions, respectively.

Theorem 5. \Box Proof. The demonstration of proof of Theorem 6 is similar to the demonstration proof of Theorem 5 \Box $c_{\rm eff}$ functions, according to Dragomir and Pearce σ of The demonstration of proof of Theory \mathbf{H} $c_{\rm eff}$ functions, according to Dragomir and Pearce σ **Proof.** The demonstration of proof of Theorem 6 is similar to the demonstration proof of $\mathbf{H}_{\mathbf{p}}$

Example 2. We consider the IVMs \mathbf{J} : [0, 1] \times [0, 1] $\rightarrow \mathbb{R}^+_I$ defined by, The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– $T_{\rm eff}$ result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (186 inequality has seen applications and a straightforward intrinsic geometric explanation. The straightforward intrinsic geometric explanation. The straightforward intrinsic geometric explanation. The straightforward intrins

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$$
J(x,y) = [(6-e^x)(6-e^y), 40xy].
$$

Then, we have endpoint functions $J_*(x,y)$, $J^*(x,y)$, which are both coordinated \hbar -concave *holds:* functions. Hence, $J(x, y)$ is coordinated LR-h-concave IVM.

 (ID) before the integral sign. In the next results, to avoid confusion, we will not include the symbols (R) , (IR) , and

3. Main Results

functions are given. We first present an inequality of Hermite–Hadamard via coordinated $F_{\rm R}$, $F_{\rm R}$, $B_{\rm R}$, $B_{\rm R}$, $B_{\rm R}$, $B_{\rm R}$, $C_{\rm R}$ with $T_{\rm R}$ In this section, Hermite–Hadamard and Pachpatte-type inequalities for interval-value LR - \hbar -concave IVMs.

Theorem 7. *Let* $J\mathbf{j}: \Omega$ **Theorem 7.** Let $J: \Omega \to \mathbb{R}_I^+$ be a coordinate LR- \hbar -convex *IVM* on Ω *, where* $J(x,y) = [J_*(x,y), J^*(x,y)]$ for all $(x,y) \in \Omega$ and let $\hbar : [0, 1] \to \mathbb{R}^+$. If $J \in \mathfrak{TD}_{\Omega'}$, then *the following inequalities hold:*

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realms of applied and pure sciences. Furthermore, because of its many applications and

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Keywords: interval-valued mappings over coordinates; left and right ℏ -Convexity; double

realms of applied and pure sciences. Furthermore, because of its many applications and

Abstract: In particular, the fractional forms of Hermite–Hadamard inequalities for the newly

Abstract: In particular, the fractional forms of Hermite–Hadamard inequalities for the newly

right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also

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(ii) It can be easily seen that " ≤ " *looks like "left and right" on the real line* ℝ, *so we call*

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3 Department of Mathematics and Computer Science, Transilvania University of Brasov,

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2 Department of Mathematics and Computer Science, Alabama State University,

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The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–

these new outcomes. The second control is a second control of the second control in the second control in the s

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\frac{\text{Fractal Fract. 2024, 8, 125}}{h^2(\frac{1}{2})} J\left(\frac{\sigma+ i}{2}, \frac{\varepsilon+ v}{2}\right)
$$
\n
$$
\leq p \frac{\Gamma(\alpha+1)}{2h(\frac{1}{2})(i-\sigma)^{\alpha}} \left[\int_{0}^{\alpha} f^{\alpha} \cdot J\left(i, \frac{\varepsilon+ v}{2}\right) + \int_{i-}^{\alpha} J\left(\sigma, \frac{\varepsilon+ v}{2}\right) \right] + \frac{\Gamma(\beta+1)}{2h(\frac{1}{2})(v-\varepsilon)^{\beta}} \left[\int_{\varepsilon}^{\beta} J\left(\frac{\sigma+ i}{2}, v\right) + \int_{v-}^{\beta} J\left(\frac{\sigma+ i}{2}, \varepsilon\right) \right]
$$
\n
$$
\leq p \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha-0)^{\beta}} \left[\int_{0}^{\alpha} f^{\beta} \cdot f^{\alpha} \cdot f^{\beta} \cdot J\left(i, v\right) + \int_{\sigma+1}^{\alpha} f^{\beta} \cdot J\left(i, v\right) + \int_{i-\rho+1}^{\alpha} f^{\beta} \cdot J\left(\sigma, v\right) + \int_{i-\rho+1}^{\alpha} J\left(\sigma, \varepsilon\right) \right]
$$
\n
$$
\leq p \frac{\beta\Gamma(\alpha+1)}{\Gamma(\alpha-0)^{\beta}} \left[\int_{0}^{\alpha} f^{\beta} \cdot J\left(i, \varepsilon\right) + \int_{\sigma+1}^{\alpha} J\left(i, v\right) + \int_{i-\rho+1}^{\alpha} J\left(\sigma, \varepsilon\right) + \int_{i-\rho+1}^{\alpha} J\left(\sigma, v\right) \right] \times \int_{0}^{1} k^{\beta-1} \left[h(\kappa) + h(1-\kappa) \right] d\kappa
$$
\n
$$
+ \frac{\alpha\Gamma(\beta+1)}{\Gamma(\alpha-\varepsilon)^{\beta}} \left[\int_{\varepsilon}^{\beta} J\left(\sigma, v\right) + \int_{\varepsilon}^{\beta} J\left(i, \varepsilon\right) + \int_{\varepsilon+1}^{\beta} J\left(i, v\right) + \int_{\varepsilon}^{\beta} J\left(i, v\right) + \int_{\varepsilon-1}^{\beta} J\left
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(ii) It can be easily seen that " ≤ " *looks like "left and right" on the real line* ℝ, *so we call*

challenging, inequalities can be used to approximate the solution. Since many inequalities

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1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of Mathematics, Faculty

(ii) It can be easily seen that " ≤ " *looks like "left and right" on the real line* ℝ, *so we call*

 $K_{\rm eff}$ intervalse over coordinates; left and right \sim

Keywords: interval-valued mappings over coordinates; left and right ℏ -Convexity; double

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Abstract: In particular, the fractional forms of Hermite–Hadamard inequalities for the newly

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$$
\frac{1}{h^{2}(\frac{1}{2})}J_{j}\left(\frac{\rho+i}{2},\frac{\epsilon+1}{2}\right)
$$
\n
$$
\geq p \frac{\Gamma(\alpha+1)}{2h(\frac{1}{2})(i-\rho)^{8}}\left[\sigma_{\varphi}^{\alpha}+J_{j}(i,\frac{\epsilon+1}{2})+J_{k}^{\alpha}-J_{j}(\sigma,\frac{\epsilon+1}{2})\right]+\frac{\Gamma(\beta+1)}{2h(\frac{1}{2})(i-\epsilon)^{8}}\left[\sigma_{\varphi}^{\beta}+J_{j}(\sigma,\frac{\rho+i}{2},\epsilon)\right]
$$
\n
$$
\geq p \frac{\Gamma(\alpha+1)}{(\Gamma(\sigma)-\Gamma)(\Gamma(\sigma-\epsilon)^{8})}\left[\sigma_{\varphi}^{\alpha}+\beta\left(\Gamma(\nu)+\sigma_{\varphi}^{\alpha}+\beta\right)-J_{k}^{\alpha}(\sigma,\epsilon)+J_{k}^{\alpha}(\sigma,\epsilon)+J_{k}^{\alpha}(\sigma,\epsilon)\right]
$$
\n
$$
\geq p \frac{\beta\Gamma(\alpha+1)}{(\Gamma(\sigma)-\Gamma)^{8}}\left[\sigma_{\varphi}^{\alpha}+J_{j}(\Gamma(\nu)+\sigma_{\varphi}^{\alpha}+J_{j}(\Gamma(\nu)+\sigma_{\varphi}^{\alpha}-J_{j}(\Gamma(\sigma,\epsilon))+J_{k}^{\alpha}-J_{j}(\Gamma(\sigma,\epsilon))\right]\times\int_{0}^{1}N^{\beta-1}[\hbar(\kappa)+\hbar(1-\kappa)]d\kappa
$$
\n
$$
+\frac{\alpha\Gamma(\beta+1)}{(\Gamma(\sigma-\beta)^{8}}\left[\sigma_{\varphi}^{\beta}+J_{j}(\sigma,\nu)+J_{k}^{\beta}-J_{j}(\Gamma(\nu)+J_{k}^{\beta}-J_{j}(\Gamma(\nu)+\beta)-J_{k}(\Gamma(\nu)+\hbar(1-\kappa))\right]d\kappa
$$
\n
$$
\geq p \frac{\alpha\beta[J_{j}(\sigma,\epsilon)+J_{j}(\sigma,\mu)+J_{j}(\sigma,\mu)+J_{j}(\Gamma(\nu)+J_{k}(\Gamma(\nu)+\hbar(1-\kappa))]d\kappa\int_{0}^{1}N^{\alpha-1}[\hbar(\nu)+\hbar(1-\nu)]d\upsilon}{\kappa\alpha}
$$
\nProof. Let $J_{j} : [\sigma, i] \to \mathbb{R}_{I}^{+}$ be a coordinated LR-h=convex IVM. Then, by hypothesis, we have
\n

Proof Let \overline{I} if \overline{A} if $\rightarrow \mathbb{R}^+$ be a coordinated \overline{I} \overline{R} - \overline{L} -convex \overline{I} $\frac{1}{2}$ is $\frac{1}{2}$ if $\frac{1}{2}$ if *Proof* Let $\overline{\Pi} : [\alpha, i] \rightarrow \mathbb{R}^+$ be a coordinated LR-*f*-convex IVM. Then by b $\frac{1}{2}$ is $\frac{1}{2}$ is $\frac{1}{2}$ if $\frac{1}{2}$ is $\frac{1}{2}$ if $(i) \rightarrow \mathbb{R}^+$ be a coordinated *IR-f*₂-convex *IVM* Then by bypothesis *M*^{*+*} $\frac{1}{2}$ is $\frac{1}{2}$ is $\frac{1}{2}$ is $\frac{1}{2}$ is $\frac{1}{2}$ is $\frac{1}{2}$ is $\frac{1}{2}$ if $\frac{1}{2}$ is $\frac{1}{2}$ if $\frac{1}{2}$ if $\frac{1}{2}$ is $\frac{1}{2}$ if $\frac{1}{2}$ if $\frac{1}{2}$ if $\frac{1}{2}$ if $\frac{1}{2}$ if $\frac{1$ *(-integrable) over* Ω *if and only if* Ԓ∗(,) *and* Ԓ∗(,) *both are -integrable over* Ω. *More*over, *if* $\frac{1}{2}$ *M* is $\frac{1}{2}$ is $\frac{1}{2}$ is $\frac{1}{2}$ if $\frac{1}{2}$ **Proof** Let $\overline{\mathbf{I}} \cdot [a, i] \rightarrow \mathbb{R}^+$ be a coordinated $\overline{I} \cdot \overline{R}$ -convex $\overline{I} \vee \overline{I}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot [0, 1] \rightarrow \mathbb{R}$ *be a coordinated EN-R*-CONVEX *IV N* θ if and only if θ if and θ if and θ and θ and θ are θ and θ are θ are θ and θ a **Proof** Let $\mathbb{I} \cdot [a, i] \rightarrow \mathbb{R}^+$ be a coordinated $\mathbb{I} \cdot \mathbb{R}$ -convex $\mathbb{I} \times \mathbb{I}$ Then by by Γ (Γ , Let J *;* $[\theta', \iota]$ \rightarrow \mathbb{R} *_I* be a coordinated EN -*R*-convex *IV NI*. Then, by Π _j α if and only if α if \mathbb{R}^{T} Let \mathbb{R}^{T} i. \downarrow \mathbb{R}^{T} be a coordinated LR-*b*-convex IVM. Then by bypothesis απαισία, *β*^{*γ*}, *γ*^{*γ*}, *γ*^{*γ*}, *γ*^{*γ*}, *γ*^{*γ*}, *γ^{<i>γ*}, *γ*^{*γ*}, $($ **i** $($ $\rightarrow \mathbb{R}^+$ be a coordinated *IR-f*-convey *IVM* Then by bypothesis $\rightarrow \mathbb{R}^n_I$ be a coordinated *ER-R*-convex *IV MI*. Then, by hypothesis, **Proof.** Let $JJ : [\sigma, i] \to \mathbb{R}^+_I$ be a coordinated LR- \hbar -convex IVM. Then, by hypothesis, $\mathsf{W}\mathsf{e}$ nave \mathbf{w} and \mathbf{w} are an \mathbf{w} be an *a* \mathbf{w} *c* and \mathbf{w} *b b* \mathbf{w} *c* and \mathbf **Theorem 3** ([31])**.** *Let* Ԓ: Ω = ሾℴ, ሿ × ሾ, ሿ ⊂ ℝଶ → ℝூ *be an on coordinates, given by* $\frac{1}{2}$ we have γ \blacksquare *is Riemann integrable (right)* over *n* if and \blacksquare **Proof.** Let π : $[\alpha, i] \to \mathbb{R}^+$ be a coordinated LR- \hbar -convex IVM. Then, by h ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* $\mathbf{L}[\mathbf{R}] \to \mathbb{R}^+_I$ be a coordinated LR- \hbar -convex *IVM*. Then, by hypothesis, ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* **Theorem 22** (*LR-h*-convex *IVM*. Then, by hypothesis, **Proof.** Let $JJ : [\sigma, i] \to \mathbb{R}^+_I$ be a coordinated *LR-h*-convex *IVM*. Then, by hypothesis, we have $\mathcal{L}(\mathcal{L})$ **Proof.** Let $JJ : [\sigma, i] \rightarrow \mathbb{R}^+_I$ be a coordinated LR - \hbar -c 1963 we have it is not who first inequality is the following inequality is stated. $\frac{1}{2}$ we have **Proot.** Let $J: [\sigma,1] \to \mathbb{R}_I^+$ be a coordinated LR-h-convex IVM. Then, 1963 we have $123.$ The following it $123.$ The following is stated: inequality has several applications and a straightforward intrinsic geometric explanation. **Proof.** Let $Jj : [\sigma,1] \to \mathbb{R}^+_I$ be a coordinated LK-h-convex IVM. Then, 1963 we have $123.$ The following it $123.$ The following is stated. , i] $\rightarrow \mathbb{R}^+_I$ be a coordinated LR-h-convex IVM. Then, by hypothesis, erroof. Let *i*j: inequality has several applications and a straightforward intrinsic geometric explanation. \mathbb{K}_{I}^{+} be a coordinated LK-h-convex TVM. Then, by hypothesis, **Proof.** Let $\mathbf{J} : [\sigma, \mathbf{i}] \to \mathbb{R}_I^+$ be a coordinated LR- \hbar -convex IVM. Then $\frac{1}{2}$ convex $\frac{1}{2}$ of $\frac{1}{2}$ in $\frac{1}{2}$ H_{max} years. When determining exact values for a mathematical problem proves to be **challenging** in even be used to a coordinated the solution. Let $J: [\sigma, \nu] \to \mathbb{R}_l^+$ be a coordinated the solution α be directed from convex functions, there is a close relationship between α close relationship between α $\mathbf{y} = \mathbf{z} \cdot \mathbf{y}$ is a mathematical problem pro **challenging** in equalities $[\mathcal{C}, \mathcal{C}] \rightarrow \mathbb{R}$ be a coordinated EX-*h*-convex α be directed from convex functions, there is a close relationship between relationship between α years. When determining exact values for a mathematical problem proves to be **or.** Let $\mathbf{J} : [\sigma, \mathbf{u}] \to \mathbb{R}_l$ be a coordinated LK-*h*-convex *IV M*. Then, by hypothesis, α be directly derived from convex functions, there is a close relationship between α close relationship between α $y = f(x)$ m determining exact values for a mathematical problem c j_1 (σ , $i_j \rightarrow \infty$ _I be a coordinated EN -*n*-convex *iv Ni*. Then, by hypothesis, $\mathbf{p}_{\text{max}}(t, \mathbf{I}_{\text{max}} | t, t)$ with the theory of indicated $\mathbf{I}(\mathbf{D}, k)$ convexity $\mathbf{I}(\mathbf{M}, \mathbf{T})$ $\sum_{i=1}^{\infty}$ because for a mathematical problem problem problem problem problem proves to be proves to be proved problem proves to be proved proved problem proves to be proved proved proved proved proved proved proved p α characters can be used to approximate the solution. Since α **Proof.** Let $JJ : [\sigma, i] \to \mathbb{R}^+_I$ be a coordinated LR- \hbar -convex IVM. Then challenging, including to approximate the solution. Since \mathbf{w} is solved to approximate the solution. Since \mathbf{w} t^2 relationship to the theory of inequalities, convexity has advanced quickly in recent α **Proof.** Let π : $[\alpha, i] \rightarrow \mathbb{R}^+_i$ be a coordinated LR- \hbar -convex IVM. Then, by hypothesis, t relationship to the theory of inequalities, convexity has advanced quickly in recent α : π : $[\sigma, i] \rightarrow \mathbb{R}^+_i$ be a coordinated LR- \hbar -convex IVM. Then, by hypothesis, **Integral Integral In Proof.** Let $J: [\sigma, 1] \to \mathbb{R}^+_I$ be a coordinated LK-h-convex IVM. Then, by hypothesis,
we have $\mathbf{W}\mathbf{e}$ have restrictions on endpoint functions of interval-valued functions that can be seen as applications of restrictions on endpoint functions of interval-valued functions that can be seen as applications of these new order the new order restrictions on endpoint functions of interval-valued functions that can be seen as applications of $1001.$ restrictions on endpoint functions of interval-valued functions that can be seen as applications of $J: [\sigma, \iota] \to \iota \kappa_I$ be restrictions on endpoint functions of interval-valued functions that can be seen as applications of **Proof.** Let $JJ : [\sigma, i] \to \mathbb{R}^+_I$ be a coordinated LR- \hbar -convex IVM. Then, by hypc we have $\frac{1}{\sqrt{2\pi}}$ -convexity, some new versions of $\frac{1}{\sqrt{2\pi}}$ if Let π $\left[\alpha \right] \rightarrow \mathbb{R}^+$ be a coordinated LR-*b*-convex *IVM*. Then, by hypothesis $r_{\rm r}$ -convexity, some new versions of fractions of $r_{\rm r}$ **Proof.** Let $JJ: [\sigma, i] \to \mathbb{R}^+_I$ be a coordinated LR-h-convex IVM. Then we have \overline{a} **Proof.** Let $Jj: [\sigma, i] \to \mathbb{R}_I^+$ be a coordinated LR- \hbar -convex IVM. Then, by hypothesis, $r_{\rm max}$ $\mathbf w$ is derive that we also examine the major results of the results of the key also examine **oof.** Let $JJ: [\sigma, i] \to \mathbb{R}_I^+$ be a coordinated LR- \hbar -convex IVM. Then, by hypothesis, Liouville fractional integral to derive the major results of the research. We also examine the key

$$
\frac{1}{\hbar^2\left(\frac{1}{2}\right)} J \left(\frac{\sigma + i}{2}, \frac{\varepsilon + \mathfrak{v}}{2}\right) \leq_p J \left(v_{\mathcal{O}} + (1 - v)i, v\varepsilon + (1 - v)\mathfrak{v}\right) + J \left((1 - v)\sigma + v i, (1 - v)\varepsilon + v \mathfrak{v}\right)
$$

By using Theorem 5, we have

By using Theorem 5, we have By using Theorem 5, we have ising Theorem 5, we have neorem 5, we have \mathbf{e} , By using Theorem 5, we have *over* ሾℴ, ሿ, *then over* ሾℴ, ሿ, *then* Dy using Theorem *J*, we have R_{Y} , P_Y (P_Y) P_Y P_Y for a for P_Y Dy using Theorem *J*, we have R_{V} (using Theorem 5, *v*/ο bevo, Dy using Theorem *J*, we have By using Theorem 5, we have T_f as T_f interests to H and T_f $\mathbf{p}_{\text{truning}}$ Theorem $\mathbf{F}_{\text{trivial}}$ has seen $T_{\rm J}$ case $T_{\rm g}$ credited to μ is though Hadamard (1822–1901), even though Hadamard (1875–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1875–1901), even though H $\sum_{n=1}^{\infty}$ in equality $\sum_{n=1}^{\infty}$ such a straightforward intrinsic geometric explanation. \mathcal{L}_f as \mathcal{L}_f and \mathcal{L}_f are though Hadamard (1822–1901), even though Hadamard (1875–1901), even though Hadamard (1865–1901), \mathcal{L}_f $T_{\rm c}$ and $T_{\rm c}$ and $T_{\rm c}$ and $T_{\rm c}$ **Copyright:** © 2024 by the authors. \mathcal{S}_{S} inequality has seen applications and a straightforward intrinsic geometric explanation. By using Theorem 5, we have \mathcal{L} in \mathcal{L} is approximate the solution. Since \mathcal{L} $\sum_{i=1}^n a_i$ be used to approximate the solution. Since $\sum_{i=1}^n a_i$ \overline{c} in the used to approximate the solution. Since \overline{c} $T_{\rm c}$ are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex sets and convex sets and convex functions in the convex sets and convex sets and convex sets an **1. Introduction** Order Relation. *Fractal Fract.* **2024**, *8*, By using Theorem 5, we have by using inevien

by using Theorem 5, we have
\n
$$
\frac{1}{\hbar^2(\frac{1}{2})} J_* \left(\frac{\sigma + i}{2}, \frac{\varepsilon + \mathfrak{v}}{2} \right)
$$
\n
$$
\leq J_* \left(v_{\mathcal{O}} + (1 - v)i, v\varepsilon + (1 - v)\mathfrak{v} \right) + J_* \left((1 - v)_{\mathcal{O}} + vi, (1 - v)\varepsilon + v\mathfrak{v} \right),
$$
\n
$$
\frac{1}{\hbar^2(\frac{1}{2})} J^* \left(\frac{\sigma + i}{2}, \frac{\varepsilon + \mathfrak{v}}{2} \right)
$$
\n
$$
\leq J^* \left(v_{\mathcal{O}} + (1 - v)i, v\varepsilon + (1 - v)\mathfrak{v} \right) + J^* \left((1 - v)_{\mathcal{O}} + vi, (1 - v)\varepsilon + v\mathfrak{v} \right).
$$

type integrals ℐℴశ,ఌశ $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$ By using Lemma 1, we have R_{V} *Moreover, if* Ԓ *is -integrable over* Ω, *then Moreover, if* Ԓ *is -integrable over* Ω, *then* Ԓ(,) = ሾԒ∗(,), Ԓ∗(,)ሿ *for all* (,) ∈Ω= ሾℴ, ሿ × ሾ, ሿ *. Then,* Ԓ *is double integrable (by using Lemma 1, we have* α Ԓ(,) = ሾԒ∗(,), Ԓ∗(,)ሿ *for all* (,) ∈Ω= ሾℴ, ሿ × ሾ, ሿ *. Then,* Ԓ *is double integrable by using Lemma 1, we have* $\frac{1}{2}$ Ԓ(,) = ሾԒ∗(,), Ԓ∗(,)ሿ *for all* (,) ∈Ω= ሾℴ, ሿ × ሾ, ሿ *. Then,* Ԓ *is double integrable* By using Lemma 1, we have *(-integrable) over* Ω *if and only if* Ԓ∗(,) *and* Ԓ∗(,) *both are -integrable over* Ω. By using Lemma 1, we have T_{S} or a result was matriced to T_{S} we have inequality has several applications and a straightforward intrinsic geometric explanation. D_Y using Lemma 1, we nave inequality has several applications and a straightforward intrinsic geometric explanation. convexity and the theory of inequalities. The theory of inequalities of inequalities of inequalities of inequalities of inequalities of inequalities of inequalities. The inequalities of inequalities of inequalities of ineq By using Lemma 1, we have L_f as L_f in the most well-known finding in the most well-known finding in the most well-known finding in the most well-known L_f Γ convexity and Γ intervention Γ in the theory of individuality of Γ T_f doing behind T_f we have By using Lemma 1, we have T_{Hill} is T_{Hill} in the most well-known finding in the mos T_{max} \sim \sim t_{ij} relationship to the theory of inequalities, convexity α $t_{\rm f}$ relationship to the theory of inequalities, convexity in recent ϵ

$$
\frac{1}{\hbar(\frac{1}{2})} J_{*}(x, \frac{\varepsilon + \mathfrak{v}}{2}) \leq J_{*}(x, v\varepsilon + (1 - v)\mathfrak{v}) + J_{*}(x, (1 - v)\varepsilon + v\mathfrak{v}),
$$
\n
$$
\frac{1}{\hbar(\frac{1}{2})} J^{*}(x, \frac{\varepsilon + \mathfrak{v}}{2}) \leq J^{*}(x, v\varepsilon + (1 - v)\mathfrak{v}) + J^{*}(x, (1 - v)\varepsilon + v\mathfrak{v}),
$$
\n(29)

and submitted for possible open access of the Creative Commons of the Creative and the contract of the contra \mathcal{A} $\mathbf{r} \cdot \mathbf{n}$ \mathbf{r} is \mathbf{r} is \mathbf{r} is \mathbf{r} is \mathbf{r} is \mathbf{r} is \mathbf{r} then \mathbf{r} is \math *Moreover, if* Ԓ *is -integrable over* Ω, *then* \mathbf{r} and \mathbf{r} T_{rad} \mathbf{r} and \mathbf{r} can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L} challenging, inequalities can be used to approximate the solution. Since \mathbb{R}^n challenging, inequalities can be used to approximate the solution. Since many inequalities can be used to approximate the solution. Since many inequalities can be used to approximate the solution. Since many inequalities c

 $\mathcal{A}^{\mathcal{A}}$ $\frac{1}{2}$

and
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$$
\frac{1}{\hbar(\frac{1}{2})} J_* \left(\frac{\sigma + i}{2}, y \right) \leq J_* (v_{\sigma} + (1 - v)i, y) + J_* ((1 - v)_{\sigma} + vi, y),
$$
\n
$$
\frac{1}{\hbar(\frac{1}{2})} J^* \left(\frac{\sigma + i}{2}, y \right) \leq J^* (v_{\sigma} + (1 - v)i, y) + J^* ((1 - v)_{\sigma} + vi, y).
$$
\n(30)

From (29) and (30) , we have $\sum_{i=1}^{n}$ $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ $\sum_{i=1}^{n}$, <u>Let *Z*₂, Zet *Q*²</sub>, *D*₂, Zet *Q*₂, *D*₂, Zet *D*_{2</u>} $\frac{1}{100}$ $\frac{1}{2}$ function $\frac{2}{3}$ $\frac{1}{2}$ function $\frac{2}{3}$ and $\frac{1}{2}$ $\frac{1}{2}$ we have $\sum_{i=1}^{\infty}$ and $\sum_{i=1}^{\infty}$, we have

From (29) and (30), we have
\n
$$
\frac{1}{\hbar(\frac{1}{2})} \left[J_{*}(x, \frac{\varepsilon + \mathfrak{v}}{2}), J_{*}(x, \frac{\varepsilon + \mathfrak{v}}{2}) \right]
$$
\n
$$
\leq_p \left[J_{*}(x, v\varepsilon + (1 - v)\mathfrak{v}), J_{*}(x, v\varepsilon + (1 - v)\mathfrak{v}) \right]
$$
\n
$$
+ \left[J_{*}(x, (1 - v)\varepsilon + v\mathfrak{v}), J_{*}(x, (1 - v)\varepsilon + v\mathfrak{v}) \right],
$$

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Abstract: In particular, the fractional forms of Hermite–Hadamard inequalities for the newly

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obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

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Montgomery, AL 36101, USA

Revised: 5 February 2024

 $x \mapsto \frac{1}{2} \int_{0}^{1} \frac{1}{\sqrt{2}} \, \mathrm{d}x \, \mathrm{d}x$

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x. https://doi.org/10.3390/xxxxx

Revised: 5 February 2024

Integral Inequalities via Coordinated

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conditions of the Creative Commons The result was mainly contributed to $\mathbf{1}$ T_{max} The result was mainly contributed to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865– T_{rad} inequality has several applications and a straightforward intrinsic geometric explanation. inequality has seen applications and a straightforward intrinsic geometric explanation. **Copyright:** © 2024 by the authors. $\frac{1}{2}$ submitted for possible open access $\frac{1}{2}$ submitted for possible open access $\frac{1}{2}$ t relationship to the theory of inequalities, convexity has advanced α tight relationship to the theory of inequalities, convexity has advanced quickly in recent α t relationship to the theory of inequalities, convexity has advanced α in recent α Version of Fractional Pachpatte-type restrictions of intervalse of intervalse functions of intervalse functions that can be seen as applications of intervalse functions of intervalse \mathbb{R}^n restrictions on endpoint functions of interval-valued functions that can be seen as applications of $rac{c}{c}$ $r_{\rm end}$ \mathcal{L} ϵ and ϵ \mathbf{c} and \mathbf{c} α and α convexity) over intervalse interval-valued codomain. We exploit the use of double Riemann– 29 Eroilor Boulevard, 500036 Brasov, Romania 29 Eroilor Boulevard, 500036 Brasov, Romania 2 Department of Mathematics and Computer Science, Alabama State University, $\overline{}$ **Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4 Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4 Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4 Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4 Inequalities via Coordinated ℏ-Convexity via Left and Right New Version of Fractional Pachpatte-type Integral**

right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also

Order Relation

and
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$$
\frac{\frac{1}{\hbar(\frac{1}{2})}\Big[J_{*}\Big(\frac{\sigma+i}{2},y\Big),J^{*}\Big(\frac{\sigma+i}{2},y\Big)\Big]}{\frac{\left[J_{*}\Big(v_{\mathcal{O}_{+}}+(1-v)i,y\Big),J^{*}\Big(v_{\mathcal{O}_{+}}+(1-v)i,y\Big)\right]}{\left[J_{*}\Big(v_{\mathcal{O}_{+}}+(1-v)i,y\Big),J^{*}\Big(v_{\mathcal{O}_{+}}+(1-v)i,y\Big)\right]}},
$$
\nIt follows that

The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–

 $1 - 1$ Financial Mathematics and Actuarial Science (FMAS)-Research Group, $\mathcal{L}(\mathcal{A})$

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

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challenging, inequalities can be used to approximate the solution. Since many inequalities

tight relationship to the theory of inequalities, convexity has advanced α

There are many uses for the convex sets and convex sets and convex $\mathcal{L}_{\mathcal{A}}$

There are many uses for the concepts of convex sets and convex functions in the

The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–

Fractal Fract. **2024**, *8*, x FOR PEER REVIEW 4 of 24

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challenging, in equalities can be used to approximate the solution. Since α

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The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known finding μ

Abstract: In particular, the fractional forms of Hermite–Hadamard inequalities for the newly

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There are many uses for the convex sets and convex sets and convex sets and convex \mathcal{A}

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

It follows that (https://creativecommons.org/license (https://creativecommons.org/license $\frac{1}{2}$ that $\frac{1}{2}$ 2 Department of Mathematics and Computer State University, $\frac{1}{2}$ It follows that $\sum_{i=1}^{n}$ \mathcal{L} Scheme, Badulazi **If follows that** It follows that It follows that It follows that **New Version of Fractional Pachpatte-type Integral** It follows that **New York 1 Fraction 2 GM New York School It follows that**

Liouville fractional integral to derive the major results of the research. We also examine the key

Abstract: In particular, the fractional forms of Hermite–Hadamard inequalities for the newly

challenging, in equalities can be used to approximate the solution. Since α

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New Version of Fractional Pachpatte-type Integral

Liouville fractional integral to derive the major results of the research. We also examine the key

$$
\frac{1}{\hbar\left(\frac{1}{2}\right)} J\!\!\left(x, \frac{\varepsilon + \mathfrak{v}}{2}\right) \leq_p J\!\!\left(x, v\varepsilon + (1 - v)\mathfrak{v}\right) + J\!\!\left(x, (1 - v)\varepsilon + v\mathfrak{v}\right),\tag{31}
$$

conditions of the Creative Common series of the Creative Commons of the Creati conditions of the Creative Common service Common Service Commons of the Creative Commons of the Creati inequality has seen and a straightforward intrinsic geometric geometric geometric explanation. The straightforward intrinsic geometric explanation. The straightforward intrinsic geometric explanation. The straightforward i inext has several applications and a straightforward intrinsic geometric explanation. The straightforward intrinsic geometric explanation. The straightforward intrinsic geometric explanation. The straightforward intrinsic \mathcal{L} and convexity and the theory of inequalities. realms of applied and pure sciences. Furthermore, because of its many applications and its many applications and α T_{max} are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex sets and convex sets and convex functions in the convex sets and convex sets and convex sets a \mathbf{I} absorptional forms of Hermite–Hadamard inequalities for the newly forms of \mathbf{I} α Correspondence: enwanting α α correspondence: enwantbox α $\mathcal{L} = \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L}$ α correspondence: enwanting α enwanting α enwanting α 4 Department of Mathematics, $\frac{1}{2}$ and $\frac{1}{2}$ \mathcal{A} and \mathcal{A} 4 Department of Mathematics, $\frac{1}{2}$ Department of Science, Jazan $\frac{1}{2}$ 4 and 3 Department of Science, $\frac{1}{2}$ and $\frac{1}{2}$ 4 Department of Mathematics, Faculty of Science, Jazan University, Jazan 4 anu. 2 Department of Mathematics and Computer Science, Alabama State University, Alabama State University, Alabama S **Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4 Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4** \mathbf{A} **Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4**

and
$$
\frac{1}{\hbar(\frac{1}{2})}\mathbb{J}\left(\frac{\sigma+i}{2},y\right) \leq_p \mathbb{J}(v_{\mathcal{O}} + (1-v)i,y) + \mathbb{J}(v_{\mathcal{O}} + (1-v)i,y).
$$
 (32)

and (32), we have Since $J(x,.)$ and $J(.,y)$ are both coordinated LR- \hbar -convex-IVMs; then, from (17), (31), $T(T, \cdot)$ and $T(T, \cdot)$ and $T(T, \cdot)$ are though $T(T, \cdot)$ $\frac{1}{2}$ and $\frac{1}{2}$, we nave ା *be an interval-valued mapping () and* Ԓ ∈ ℐℛሾℴ,ሿ*.* in equality has seen as several applications and a straightformation intrinsic geometric explanation. The result was mainly controlled to Hermite (1822–1901), $\frac{1}{2}$ $\frac{1}{2}$ and $\frac{1}{2}$, we first integration ା *be an interval-valued mapping () and* Ԓ ∈ ℐℛሾℴ,ሿ*.* \overline{q} in \overline{q} and a straightforward intrinsic geometric geometric geometric geometric explanation. $T(T_0, t)$ and $T_1(t, y)$ are boun coordinated L_{IV} - t -convex-1901), then and (22) , we have in the several applications and a straightforward intrinsic geometric geometric geometric geometric geometric geometric explanation. The straight of the straight straight of the straight straight straight straight straigh $T_{\text{max}}^{\text{max}}$ and $T_{\text{min}}^{\text{max}}$ are bout coordinated $T_{\text{min}}^{\text{max}}$ only to $T_{\text{min}}^{\text{max}}$ (17), $\frac{1}{2}$ and $\frac{1}{2}$, we have \tilde{p}_1 in the second applications and a straightforward integration intrinsic geometric geometric explanation. Since $J(x,.)$ and $J(.,y)$ are both coordinated LR- \hbar -convex-IVMs; then, from (17), (31), (92) , we first it $\frac{2}{3}$. publication under the terms and $\mathcal{F}_{\mathcal{F}}$ is one of the most well-known finding inequality is one of the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known f The Hausdorff $\lim_{y \to y} \frac{f(x, y)}{f(y, y)}$ are bounded than the convex 14 $\frac{f(x, y)}{f(x, y)}$ intervals $\frac{f(x, y)}{f(x, y)}$ ϵ ince $\pi(x)$ and $\pi(x)$ are beth coordinated lR is conveximated. ϵ and the theory of inequalities. Since $J(x,.)$ and $J(x, y)$ are both coordinated $LR-h$ -convex-IV Ms; then, from (17), (31), and (32), we have Since $J(x,.)$ and $J(.,y)$ are both coordinated LR- \hbar -convex-IVMs; then, from (17), (Since $J(x,.)$ and $J(x, y)$ are both coordinated LR-h-convex-IVMs; then, from (I7), (Since $J(x,.)$ and $J(x, y)$ are both coordinated LK-h-convex-TV Ms; then, from (17), (c Since $J(x,.)$ and $J(.,y)$ are both coordinated LR- \hbar -convex-IVMs; then, from (17), (31), challenging, including, inequalities can be used to approximate the solution. Since many inequalities α α in the used to approximate the solution to approximate the solution. Since α σ are many uses for the convex sets and convex sets and convex sets and convex sets and convex σ Since $J(x,.)$ and $J(x, y)$ are both Since $J(x,.)$ and $J(x, y)$ are both coordinated LK -n-convex-typus; the Since $J(x, \cdot)$ and $J(\cdot, y)$ are both coordinated LR- \hbar -convex-IVMs; then, from (17), (31), r^2 restrictions on the set of intervalse functions of r^2 intervalse functions of r^2 intervalse r^2 intervalse r^2 intervalse functions of r^2 intervalse r^2 intervalse functions of r^2 intervalse r^2 $\sum_{n=1}^{\infty}$ right coordinated \mathcal{L} -convexity, some new versions of \mathcal{L} integral integral integral integral integral in Since $J(x,.)$ and $J(.,y)$ are both coordin $rac{(0.2)}{N}$ we have Since $J(x, \lambda)$ and $J(x, y)$ are both coordinated LK -*n*-convex-1*V* Ms; then, from (17) and (32), we have $r_{\rm c}$ -convexity, some new versions of $r_{\rm c}$ Since $J(x, \cdot)$ and $J(x, y)$ are both coordin $\ddot{\alpha}$ of convex mapping proposed that are known as coordinated left and right $\ddot{\alpha}$ Since $J(x,.)$ and $J(.,y)$ are both coordinated LR- \hbar -convex-IVMs; then, from (17), (31), $\frac{1}{2}$ integral to derive the major results of the search. Since $J(x,.)$ and $J(.,y)$ are both coordinated LR- \hbar -convex-IVMs; then, from $rac{(0.2)}{v}$ we have defined convex mapping $f(x, t)$ and $f(x, t)$ are boundary proposed in a convex mapping $f(x)$ $\frac{\text{arcc}}{\text{arcc}}$ ($\frac{\text{arcc}}{\text{arcc}}$) over intervalse the use of double Riemann– $\pi(\alpha)$ and $\pi(\alpha)$ are both coordinated LR-E-convex-U/Ms; then from **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly $\sum_{i=1}^{n}$ Since $J(x,.)$ and $J(.,y)$ are both coording $\mathcal{L}(\mathcal{L}, \mathcal{L})$ 4.4 Department of Mathematics, \tilde{B} and \tilde{B} are \tilde{B} 2mce J Since $J(x,.)$ and $J(.,y)$ are both coordinated LR- \hbar -convex-IVMs; then \sqrt{D} 3.32 Department of Mathematics and Computer Science, Transilvania University of Bras \sim $\frac{d}{dx}$ (32), we have $\frac{1}{2}$ of $\frac{1}{2}$ **Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4 Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4** $1 - 1$ Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of \mathcal{R} $1 - 1$ Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of \mathcal{N}

can be directly derived from convex functions, there is a close relationship between

$$
\frac{1}{\beta\hbar\left(\frac{1}{2}\right)}J_x\left(\frac{\varepsilon+\mathfrak{v}}{2}\right) \leq_p \frac{\Gamma(\beta)}{(\mathfrak{v}-\varepsilon)^{\beta}}\left[\mathfrak{I}_{\varepsilon^+}^{\beta}J_x(\mathfrak{v})+\mathfrak{I}_{\mathfrak{v}^-}^{\beta}J_x(\varepsilon)\right] \leq_p \left[J_x(\varepsilon)+J_x(\mathfrak{v})\right] \int_0^1 \kappa^{\beta-1}[\hbar(\kappa)+\hbar(1-\kappa)]d\kappa \tag{33}
$$

khakamid. And the same state of the same s $\frac{1}{2}$ who first identified it $\frac{2}{3}$. The following is $\frac{2}{3}$. The following is stated: \mathbf{r} was the one who first identified it \mathbf{r} . The following is stated: T result was mainly contained to H and T $\frac{1}{2}$ who first identified it $\frac{2}{3}$. The following is $\frac{1}{2}$ $\frac{1}{2}$ who first identified it $\frac{2}{3}$. The following is how this inequality is stated: $\frac{1}{2}$ which first identified it $\frac{2}{3}$. The following is how this inequality is stated: $\frac{1}{2}$ was the one who first identified it $\frac{2}{3}$. The following is stated: $\frac{1}{2}$ which first identified it $\frac{2}{3}$. The following is the following is stated: publication under the terms and category of classical convex functions, according to Dragomir and Pearce $[1]$. This is the Dragomir and Pearce $[1]$. This is the Dragomir and Pearce [1]. This is the Dragomir and Pearce [1]. This is the Pearce [1]. This category of classical convex functions, according to D can be directed from convex functions, there is a close relations, there is a close relationship between α T_{max} can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L} T usdorff \mathcal{F} can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L} can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L} **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double these new outcomes. The same a these new outcomes. obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some obtained and classical exceptional exceptional cases are also discussed by taking some α obtained and classical exceptional exceptional cases are also discussed by taking some new also discussed by taking some α obtained **and** control exceptional exceptional cases are also discussed by taking some α obtained **and** contract exceptional exceptional cases are also discussed by taking some new and cases are also discussed by taking some α \mathbf{I} and \mathbf{I} and \mathbf{I} and \mathbf{I} and \mathbf{I} inequalities for the newly the \mathbf{I} absorptional forms of Hermite–Hadamard inequalities for the newly forms of \mathbf{I} 29 Eroilor Boulevard, 500036 Brasil Bras

defined class of convex mappings proposed that are known as coordinated that are known as coordinated left and right μ

1. Introduction

$$
\frac{1}{\alpha \hbar \left(\frac{1}{2}\right)} J_y\left(\frac{\sigma + i}{2}\right) \leq_p \frac{\Gamma(\alpha)}{\left(i - \sigma\right)^{\alpha}} \left[\mathfrak{I}_{\mathcal{O}^+}^{\alpha} J_y(i) + \mathfrak{I}_{i^-}^{\alpha} J_y(\sigma) \right] \leq_p \left[J_y(\sigma) + J_y(i) \right] \int_0^1 v^{\alpha - 1} \hbar(v) + \hbar (1 - v) dv \tag{34}
$$
\n
$$
\text{Since } J_x(w) = J(x, w), \text{ then (34) can be written as}
$$

Since $J_x(w) = J(x, w)$, then (34) can be written as $\text{Since } J_x(\omega) = J(x, \omega)$, then (34) can be written as $\text{Since } J_x(w) = J(x, w)$, then (34) can be written as Since $J_x(\omega) = J(x, \omega)$, then (34) can be written as Since $J_x(w) = J(x, w)$, then (34) can be written as Since $J_x(w) = J(x, w)$, then (34) can be written as challenging $\sum_{x} (w - \lambda) (x, w)$, then (λx) can be written as challenging $\sum_{x} (w_i - z_j(x, w_i))$ intervalse matter the solution. R iemann–Liouville fractional integral operator; P actional operator; Pachpatte-type inequalities in Since $J_{\mathcal{X}}(w) = J(x, w)$, then Since $J_{\mu}(w) = J(x, w)$, then (34) can be written as Since $J_{x}(w) = J(x, w)$, then (34) can be writter Since $J_x(w) - J(x, w)$, were Since $J_x(w) = J(x, w)$, then (34) can be written as results' numerical validations that examples are nontrivial. By taking the product of two left and $r(x) = \int_X (\omega y - \omega y) \arctan(\omega x) \arctan(\omega x)$ Since $J_x(w) = J(x, w)$, then (34) can be written as defined $J_X(\omega) = J_y(\omega, \omega)$, where (ω) can be known Since $J_x(w) = J(x, w)$, then (34) can be written as defined ι_j of ι_j ι_j ι_j ι_j and ι_j are known as ι_j -dinated left and ι_j $\det_{\mathbf{J}_X(\omega)} - \mathbf{J}_\mathbf{J}(\omega, \omega)$, then (ω, \mathbf{I}) can be written as

defined class of convex mappings proposed that are known as coordinated left and right \sim

$$
\frac{1}{\beta\hbar\left(\frac{1}{2}\right)}J\!\!\int\!\! \left(x,\frac{\varepsilon+\mathfrak{v}}{2}\right) \leq_p \frac{\Gamma(\beta)}{(\mathfrak{v}-\varepsilon)^{\beta}}\left[g_{\varepsilon^+}^{\alpha}J\!\!\int\!\! (x,\mathfrak{v}) + g_{\mathfrak{v}^-}^{\alpha}J\!\!\int\!\! (x,\varepsilon)\right] \leq_p \left[J\!\!\int\!\! (x,\varepsilon) + J\!\!\int\!\! (x,\mathfrak{v})\right] \int_0^1 \kappa^{\beta-1}[\hbar(\kappa) + \hbar(1-\kappa)]d\kappa. \tag{35}
$$
\nThat is,

 $\frac{1}{2}$ **Plact.** $\frac{1}{2}$ *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract That is, $\sum_{i=1}^{\infty}$ and following it is how the following is how this is stated: $\frac{1}{2}$ That is, $\frac{1}{2}$. The following it $\frac{1}{2}$. The following is stated: Γ has seen as several applications and a straightforward intrinsic geometric explanation. $T_{\rm max}$ ω Γ has seen as seen as seen as seen as straightforward intrinsic geometric explanation. $T(\alpha, \epsilon)$ T had is, $\frac{1}{\sqrt{2}}$ is one of the most well-known findings in the most well-known finding in Γ and determining Γ mathematical problem proble challenging, including to approximate the solution. Since $\mathcal{L}_{\mathcal{A}}$ \mathbb{R}^n years. When determining exact values for a mathematical problem pr $\lim_{\epsilon \to 0}$ Δ lenging, inequalities can be used to approximate the solution. Since Δ $t_{\rm i}$ relationship to the theory of inequalities, convexity has advanced $t_{\rm i}$ in recent $t_{\rm i}$ in \sum_i that is, convexity has advanced \sum_i Γ that is the theory of inequalities, convexity has advanced quickly in recent Γ years. When determining exact values for a mathematical problem proves to be $\sum_{i=1}^n$ that is, convexity has advanced quickly inequalities, convexity has advanced quickly in recent $\sum_{i=1}^n$ in recent $\sum_{$ Γ many uses Γ and convex Γ and convex Γ and convex Γ Γ many uses Γ many uses Γ and Γ convex Γ convex Γ convex Γ and convex $\$ T there are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex sets and convex functions in the convex functions in the convex functions in the convex functi r_{rel} and r_{rel} Γ matrix uses for the convex sets and convex Γ that is the convex functions in the convex funct $T_{\rm eff}$ are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex sets and convex sets and convex functions in the convex sets and convex functions in the convex r_{rel} **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double $R = \frac{1}{2}$ R ille fractional integral operator; Pachengal operator; Pachengal operator; Pachengalities in R **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double Γ and Γ intervalued functions of Γ restrictions on endpoint functions of interval-valued functions that can be seen as applications of obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some obtained **That is,** some new and classical exception \mathbf{r} are also discussed by taking some \mathbf{r} obtained **the intervention** \mathbf{R} more also discussed by taking some new and cases are also discussed by taking some \mathbf{R} Γ **hat is** obtained in \mathcal{L} more new and cases are also discussed by taking some new also discussed by taking some \mathcal{L} **obtained** $\overline{\text{C}}$ more new and cases are also discussed by taking some new and cases are also discussed by taking some $\overline{\text{C}}$ and $\overline{\text{C}}$ are also discussed by taking some $\overline{\text{C}}$ and $\overline{\text{C}}$ and $\overline{\text{C}}$ That is a more new and cases are also discussed by taking some new and cases are also discussed by taking some \mathbf{r}

$$
\frac{1}{\beta \hbar(\frac{1}{2})} J(x, \frac{\varepsilon + \mathfrak{v}}{2}) \leq p \frac{1}{(\mathfrak{v} - \varepsilon)^{\beta}} \Big[\int_{\varepsilon}^{\mathfrak{v}} (\mathfrak{v} - \kappa)^{\beta - 1} J(x, \kappa) dx + \int_{\varepsilon}^{\mathfrak{v}} (\kappa - \varepsilon)^{\beta - 1} J(x, \kappa) dx \Big]
$$
\n
$$
\leq p \left[J(x, \varepsilon) + J(x, \mathfrak{v}) \right] \int_{0}^{1} \kappa^{\beta - 1} [\hbar(\kappa) + \hbar(1 - \kappa)] dx. \tag{36}
$$

Multiplying double inequality (36) by $\frac{\overline{y}+\overline{y}}{\overline{y}-\overline{y}}$ and integrating with respect to x over $[\sigma, i]$, we have g double inequality (36) by
 $\frac{(i-x)^{\alpha-1}}{(i-\alpha)^{\alpha}}$ and in $\overline{}$ ble inequality (36) by $\frac{\overline{(1-\alpha)}^{\alpha}}{(1-\alpha)^{\alpha}}$ and integrating with respe equality (36) by $\frac{\overline{\mathcal{L}(-\alpha)}^{\alpha}}{\overline{\mathcal{L}(-\alpha)}^{\alpha}}$ and \overline{a} l integrating w Nuttiplying double inequality (36) by $\frac{1}{(1-\alpha)^{\alpha}}$ and integrating with respect to *x* over $[0, 1]$, we have *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $[0, 1]$, we have 이 사이트 STATE ST Multiplying double inequality (36) by $\frac{(i-x)^{\alpha-1}}{(i-x)^{\alpha}}$ Multiplying double inequality (36) by $\frac{(i-x)^{\alpha-1}}{(i-\sigma)^{\alpha}}$ and integrating with respect to *x* over inequality has several applications and a straightforward interior \mathbf{N} ultiplying double inequ $19³$ who first identified it is $1³$. integral matrice has several applications and a straightforward integral integral integral integral integral in The result was mainly credited to \mathbb{R}^2 , even though Hadamard (1822–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even $[U, Y]$, we have i ingly in a straightforward applications and a straightforward integration. T result was mainly contained the quality (00) ϵ $\frac{1}{2}$ Multiplying double inequal category of classical convex functions, according to Dragomir and Pearce [1]. This $\lbrack \sigma,\iota \rbrack$, we have M Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known f counter converged to Dragomir Convergence to Dragomir and Pearce Energy \ldots is one of the most well-known finding inequality is one of the most well-known finding in t Multiplying double inequality (36) by $\frac{(1-\lambda)}{(1-\lambda)^{\alpha}}$ and integrating with $\left[\begin{matrix}a & b\end{matrix}\right]$ we have $T_{\rm eff}$ and $T_{\rm eff}$ are dited to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1872–1901), even though Hadamard (1872–1901), even though Hadamard (1872–19 \mathbf{M} Hermite–Hadamard inequality is one of the most well-known findings in the most wellconvenience α convenience functions, α and α and α and α and α and α is α . \ldots is one of the most well-known finding inequality is one of the most well-known finding in t e inequality (36) by $\frac{(1-x)}{(1-x)^{\alpha}}$ and integrating with respect to x over $\left(\cdot\right)$ by a straightforward intrinsic geometric explanations and a straightforward intrinsic geometric explanation. Submitted for possible open access can be directly derived from convex functions, there is a close relationship between can be directly derived from convex functions, there is a close relationship between category of classical convex functions, according to Dragomir and Pearce [1]. This convexity and the theory of inequalities. T_{eff} is one of the most well-known finding inequality is one of the most well-known finding in the most well-known finding $\lbrack \sigma , \iota \rbrack$, we have α be directly derived from convex functions, there is a close relationship between α convexity and the theory of internal terms of $\mathbf{Multiplyn}$ category of classical convex functions, according to Dragomir and Pearce [1]. This conventions and the theory of the theory $(1-\mathcal{O})$ $\lbrack v,1],$ we have ultiplying double inequality (36) by $\frac{(1-x)^{n-1}}{n}$ and integrating with respect to x over can be directly derived from convex functions, there is a close relationship between $(1-\mathcal{O})$ c_n and c_n ultiplying double inequality (36) by $\frac{(1-x)^{n-1}}{n}$ and integrating with respect to x over challenging, including the solution $(1-\mathcal{O})$ ϵ have Multiplying double inequality (36) by $\frac{(-x)^{\alpha-1}}{2}$ Multiplying double inequality (36) by $\frac{(1-x)}{(1-\alpha)^{\alpha}}$ $\lbrack \sigma, \iota \rbrack$, we have tight relationship to the theory of inequalities, convexity has advanced α in recent α $\begin{bmatrix} a & i \end{bmatrix}$ we have $\left[\mathbf{v}\right]$ relatives, convexity has advanced quickly in \mathbf{v} in \mathbf{v} in \mathbf{v} in \mathbf{v} in \mathbf{v} in \mathbf{v} Multiplying double inequality (36) by $\frac{1-x}{x}$ $\frac{1}{\sqrt{2}}$ because of its many applied and pure sciences. Furthermore, because of its many applications and pure sciences. Multiplying double inequality (36) by $\frac{(-x)^{\alpha-1}}{x}$ and int realment and pure sciences. Furthermore, because of its many applications and $(i-\sigma)$ $\lbrack \sigma,1 \rbrack$, we have $[\alpha, \iota]$, we have realms of applied and pure sciences. Furthermore, because of its many applications and Multiplying double inequality (26) by $(i-x)^{\alpha-1}$ and integral

$$
\frac{1}{\beta(i-\alpha)^{\alpha}\hbar(\frac{1}{2})}\int_{\alpha}^{i} J(x,\frac{\varepsilon+\nu}{2})(i-x)^{\alpha-1}dx
$$
\n
$$
\leq p \frac{1}{(i-\alpha)^{\alpha}(v-\varepsilon)^{\beta}}\int_{\alpha}^{i} \int_{\varepsilon}^{v} (i-x)^{\alpha-1}(v-\kappa)^{\beta-1}J(x,\kappa)d\kappa dx + \int_{\alpha}^{i} \int_{\varepsilon}^{v} (i-x)^{\alpha-1}(v-\varepsilon)^{\beta-1}J(x,\kappa)d\kappa dx
$$
\n
$$
\leq p \frac{1}{(i-\alpha)^{\alpha}}\Big[\int_{\alpha}^{i} (i-x)^{\alpha-1}J(x,\varepsilon)dx + \int_{\alpha}^{i} (i-x)^{\alpha-1}J(x,v)dx\Big] \int_{0}^{1} \kappa^{\beta-1}[\hbar(\kappa)+\hbar(1-\kappa)]dx.
$$
\n(37)

 $\frac{H_{\text{E}}}{H_{\text{E}}}\left(\frac{H_{\text{E}}}{H_{\text{E}}}\right)$ and the main definition of $\frac{H_{\text{E}}}{H_{\text{E}}}\left(\frac{H_{\text{E}}}{H_{\text{E}}}\right)$ x over $[\sigma, i]$, we have $(i-\sigma)^{\alpha}$ (i) we have $(v, \cos v)$ $(i-\alpha)^{\alpha}$ and integrating when ng ababit mequanty able inequality (50) by $(i-\alpha)^{\alpha}$ and inequality $\left(i-\sigma\right)^{\alpha}$ and integrating with respect to ሾℴ, ሿ *are denoted by* ሾℴ,ሿ *and* ሾℴ,ሿ, *respectively*. A gain, multiplying double inequality (36) \mathcal{X} over $[\alpha,1]$, we have Again, multiplying double inequality (36) by $\frac{(x-\sigma)^{\alpha-1}}{(x-\sigma)^{\alpha}}$ and integrating with respect to *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *holds:*) *α*−1 (i− ain, multiplying double inequality (36) by $\frac{\sqrt{a^2 + b^2}}{(1 - b)^a}$ and integrating with respect to Δ gain multiplying double inequality (36) by $\left($ inguality has above has search applications and a straightforward interior α γ over \lbrack _{even} \lbrack _{even} though \lbrack $(9, 3)$. The following is the following it following it follows: Again, multiplying double inequality The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– λ sein multiplying double inequality (26) b $T₂$ ant, manipiying ababic inequality (bo). Λ and multiplying double inequality (26) by $(x-\theta)^{n-1}$ The result was mainly controlled to H^2 and H^2 and H^2 are H^2 and H^2 and H^2 are H^2 Δ axin multiplying double ineq category of classical convex functions, according to Dragomir and Pearce [1]. This T_{eff} is one of the most well-known findings in the most Again, mumplying worden flequally (50). $\int (x - \epsilon)^{\alpha}$ Again, multiplying double inequality (36) by $\frac{C_{1}}{2}$. γ over α il we have $T_{\rm eff}$ as mainly credited to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), Again, multiplying double inequality (36) by $\frac{\sqrt{2}}{(1-\lambda)^n}$ and integrating w $\frac{1}{\sqrt{2}}$ in the straightforward intrinsic geometric explanations and a straightforward intrinsic geometric explanation. $T_{\rm tot}$ as mainly credit was mainly credited to Hermite (1822–1901), even though Hadamard (1875–1901), even though Hadamard (1875–1901), even though Hadamard (1865–1901), even though Hadamard (1875–1901), even though Had γ over α if we have category of classical convex functions, according to Dragomir and Pearce [1]. This is the Dragomir and Pearce [T_{c} and T_{c} in the most well-known finding in the most α or category of convex functions, and α \mathcal{L} $\frac{1}{2}$ The most well-known finding in the most well-known findi Δ coinvex interprise. $T_{\rm e}$, $T_{\rm e}$ is one of the most most well-known findings in the most well-known finding α cain multiplying double inequality can be directly derived from convex functions, there is a close relationship between Δ cain multiplying double inequality. can be directly derived from convex functions, there is a close relationship between ϵ Again, multiplying double inequality (56) by $(x-\alpha)^{\alpha-1}$ Again, multiplying double inequality (36) by $\frac{1}{\sqrt{1-x}}$ $\mathcal{F}_{\mathcal{F}}$ is one of the most well-known finding inequality is one of the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known f Δ cain multiplying double inequality (26) by $(x-\alpha)^{\alpha-1}$ and integr can be directly derived from convex functions, the $\left(i-\alpha\right)^n$ and α $(x-\alpha)^{\alpha-1}$ Again, multiplying double inequality (56) by $\frac{1}{(1-\alpha)^{\alpha}}$ and integrating $\left(\gamma-\alpha\right)^{\alpha-1}$ Again, multiplying double inequality (36) by $\frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i}$ and integrating with resp x over $[\alpha, i]$, we have The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known finding in the most well-known fin

$$
\frac{1}{\beta(i-\sigma)^{\alpha}h(\frac{1}{2})}\int_{\sigma}^{i} J(x, \frac{\varepsilon+v}{2})(x-\sigma)^{\alpha-1}dx \n\leq p \frac{1}{(i-\sigma)^{\alpha}(v-\varepsilon)^{\beta}}\int_{\sigma}^{i} \int_{\varepsilon}^{v} (x-\sigma)^{\alpha-1}(v-\kappa)^{\beta-1}J(x,\kappa)d\kappa dx \n+\frac{1}{(i-\sigma)^{\alpha}(v-\varepsilon)^{\beta}}\int_{\sigma}^{i} \int_{\varepsilon}^{v} (x-\sigma)^{\alpha-1}(x-\varepsilon)^{\beta-1}J(x,\kappa)d\kappa dx \n\leq p \frac{1}{(i-\sigma)^{\alpha}}\left[\int_{\sigma}^{i} (x-\sigma)^{\alpha-1}J(x,\varepsilon)dx+\int_{\sigma}^{i} (x-\sigma)^{\alpha-1}J(x,\nu)dx\right]\int_{0}^{1} \kappa^{\beta-1}[\hbar(\kappa)+\hbar(1-\kappa)]dx.
$$
\n(38)

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Order Relation. *Fractal Fract.* **2024**, *8*,

Order Relation. *Fractal Fract.* **2024**, *8*,

1. Introduction

Montgomery, AL 36101, USA

Tareq Saeed 1*,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4**

Integration in the coordinated via Coordinated

Accepted: 6 February 2024

Khan, M.B.; Hakami, K.H. New

Integral Inequalities via Coordinated

Integral Inequalities via Coordinated

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convexity and the theory of inequalities.

Order Relation. *Fractal Fract.* **2024**, *8*,

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1. Introduction

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Order Relation. *Fractal Fract.* **2024**, *8*,

(ሾ∗, ∗ሿ,ሾ∗, ∗ሿ) = ሼ|∗ − ∗|, |∗ − ∗|ሽ. (7)

From (37), we have $T = \sqrt{2\pi}$ inequality has several applications and a straightforward intrinsic geometric explanation. $\ddot{}$ has several applications and a straightforward intrinsic geometric explanation. **Copyright:** © 2024 by the authors. $T = \frac{1}{2}$ is one of the most well-known finding inequality is one of the most well-known finding in the most well-known findin $T(\alpha)$ is one of the most well-known α $T(\mathcal{O})$ is one of the most well-known finding in the most well-k \mathcal{N} years. When determining exact values for a mathematical problem problem problem problem problem proves to be " ≤ " *"left and right" (or "LR" order, in short).* There are many uses for the convex sets and convex sets and convex \mathcal{A} and convex \mathcal{A} *for all* ሾ∗, ∗ሿ,ሾ∗, ∗ሿ ∈ ℝூ, *and it is a pseudo-order relation. The relation* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ *coincident to* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ *on* ℝூ *when it is* " ≤ "*. for all* ሾ∗, ∗ሿ,ሾ∗, ∗ሿ ∈ ℝூ, *and it is a pseudo-order relation. The relation* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ *for all* ሾ∗, ∗ሿ,ሾ∗, ∗ሿ ∈ ℝூ, *and it is a pseudo-order relation. The relation* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ There are matrices for the convex sets and convex sets and convex sets and convex \mathbb{R}^n From (37), we have $\text{F}(\mathbf{v})$, we have **Keywords: intervalue** mapping over convexity; doubled mapping over convexity; doubled and right α -Convexity; doubled and right α -Convexity; doubled and right α **Keywords:** $\frac{1}{2}$ obtained more new and cases are also discussed by the some restrictions are also discussed by the seen as applications of the seen as appl $\text{Low}(v)$ we have \mathcal{L} over intervalse intervalse convexity) over intervalse codomain. We exploit the use of double Riemann– From (37), we have \mathcal{L} iouville fractional to derive the major results of the major results of the key also examine the key also ex Δt are the major results of the major results of the major results of the key also examine t $\sum_{i=1}^{n}$ $c_{\alpha,\mu}$ over intervalse convexity. We exploit the use of double Riemann– $\frac{1}{2}$ (-,), we find $\frac{1}{2}$ 29 Eroilor Boulevard, 500036 Brasov, Romania $\overline{\mathbf{B}}$ From (37), we have From (37), we have **Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4** $\frac{1}{2}$ **IFOR** (27) we have Γ *From* (27) via baye $\frac{1}{2}$ **IFROM** (27) via have $\lim_{\epsilon \to 0} (37)$ we have **New Version of Fractional Pachpatte-type Integral** From (37) , we have **New Version of Fractional Pachpatte-type Integral** From (37), we have

category of classical convex functions, according to Dragomir and Pearce [1]. This

1. Introduction

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There are many uses for the convex sets and convex sets and convex sets and convex α

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category of classical convex functions, according to Dragomir and Pearce [1]. This

restrictions on endpoint functions of interval-valued functions that can be seen as applications of

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can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L}

realms of applied and pure sciences. Furthermore, because of its many applications and

realms of applied and pure sciences. Furthermore, because of its many applications and

Remark 1 ([47])**.** *(i) The relation* " ≤ " *is defined on* ℝூ *by*

$$
\frac{\Gamma(\alpha+1)}{2\hbar(\frac{1}{2})(i-\sigma)^{\alpha}} \left[\mathcal{J}^{\alpha}_{\mathbf{\sigma}} + \mathcal{J}(\mathfrak{i}, \frac{\varepsilon+\mathfrak{v}}{2}) \right]
$$
\n
$$
\leq p \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{(i-\sigma)^{\alpha}(\mathfrak{v}-\varepsilon)^{\beta}} \left[\mathcal{J}^{\alpha, \beta}_{\mathbf{\sigma}^{+}, \varepsilon^{+}} \mathcal{J}(\mathfrak{i}, \mathfrak{v}) + \mathcal{J}^{\alpha, \beta}_{i-\varepsilon^{+}} \mathcal{J}(\mathfrak{i}, \varepsilon) \right]
$$
\n
$$
\leq p \frac{\beta \Gamma(\alpha+1)}{(i-\sigma)^{\alpha}} \left[\mathcal{J}^{\alpha}_{\mathbf{\sigma}^{+}, \mathbf{\sigma}} \mathcal{J}(\mathfrak{i}, \varepsilon) + \mathcal{J}^{\alpha}_{\mathbf{\sigma}^{+}} \mathcal{J}(\mathfrak{i}, \mathfrak{v}) \right] \int_{0}^{1} \kappa^{\beta-1} [\hbar(\kappa) + \hbar(1-\kappa)] d\kappa.
$$
\nFrom (38), we have

The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most well-known fi

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

(ሾ∗, ∗ሿ,ሾ∗, ∗ሿ) = ሼ|∗ − ∗|, |∗ − ∗|ሽ. (7)

defined class of convex mappings proposed that are known as coordinated left and right \sim

defined class of convex mappings proposed that are known as coordinated left and right \sim

From (38), we have *b* and *is the gamma function.* 1963 was the one who first identified it (23) . The following is stated: From (38), we have $T_{\rm eff}$ mainly credited to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), ev The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– $T_{\rm eff}$ mainly credit was mainly credited to Hermite (1822–1901), even though Hadamard (1875–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadama **Copyright:** © 2024 by the authors. The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– $\sum_{i=1}^{n}$ $\mathbf{r} = \langle \mathbf{0} \mathbf{0} \rangle - 1$ Γ derived from convex functions, there is a close relationship between Γ ζ inequalities can be used to approximate the solution. Since ζ $t_{\rm c}$, convexity has advanced quickly inequalities, convexity has advanced quickly in recent qualities, convexity has advanced quality has advanced quality has advanced quality has advanced quality has advanced qualit realms of applied and pure sciences. Furthermore, pure sciences. Furthermore, because of its many applications and \overline{a} tight relationship to the theory of inequalities, convexity has advanced q $rac{1}{2}$ and pure sciences. Furthermore, because of its many applications and $binom{m}{k}$ $f^{(o)}$, ∗ o ∴ * ∗ o \mathcal{S}), we have \mathcal{S} **Remark 1** ([47])**.** *(i) The relation* " ≤ " *is defined on* ℝூ *by* **Keywords:** $\frac{1}{2}$ $\lim_{k \to \infty}$ intervalse matrix $\frac{1}{k}$ key, we have $\mathcal{L}(\mathcal{L}(\mathcal{L}))$ r_{tot} (50), we nave $\mathcal{L}(\mathbf{R})$ integral to derive the major results of the major results of the key also examine the key also examin $\frac{1}{2}$ numerical values are non-trivial. By taking the product of two left and product of two left and two left and $\frac{1}{2}$ \hat{z} 2 Department of Mathematics and Computer Science, Alabama State University, $3 \frac{1}{2}$ Department of Mathematics and Computer Science, Transilvania University of Bras α Montgomery, AL 36101, USA $\sum_{i=1}^{n}$ of Science, κ Science, κ and κ 80203, κ 80203, Saudi Arabia; tsalmalki κ ; tsalmalki κ \overline{N} $2D$ **Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4**

$$
\frac{\Gamma(\alpha+1)}{2\hbar(\frac{1}{2})(i-\sigma)^{\alpha}} \left[\mathcal{J}_{i}^{\alpha} \cdot \mathcal{J}(\sigma, \frac{\varepsilon+\mathbf{v}}{2}) \right] \n\leq p \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{(i-\sigma)^{\alpha}(\mathbf{v}-\varepsilon)^{\beta}} \left[\mathcal{J}_{i-\varepsilon}^{\alpha, \beta} \cdot \mathcal{J}(\sigma, \mathbf{v}) + \mathcal{J}_{i-\varepsilon}^{\alpha, \beta} \cdot \mathcal{J}(\sigma, \varepsilon) \right] \n\leq p \frac{\beta \Gamma(\alpha+1)}{(i-\sigma)^{\alpha}} \left[\mathcal{J}_{i}^{\alpha} \cdot \mathcal{J}(\sigma, \varepsilon) + \mathcal{J}_{i}^{\alpha} \cdot \mathcal{J}(\sigma, \mathbf{v}) \right] \int_{0}^{1} \kappa^{\beta-1} [\hbar(\kappa) + \hbar(1-\kappa)] d\kappa.
$$
\n(40)

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category of classical convex functions, according to Dragomir and Pearce [1]. This

1 milarly, since $J_y(z) = J(z, y)$, then, from (35), (41), and (42), we have *over* ሾℴ, ሿ, *then* $\mu_{\mu}(z) = \bar{J}(z, y)$, then, from (35), (41), and (42), we have *over* ሾℴ, ሿ, *then* **Similarly, since** Π (z) = $\Pi(z, y)$, then, from (35), (41), and (42), we have ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* **Su** $\Pi(z) = \Pi(z, y)$, then, from (35), (41), and (42), we have Similarly, since $J_y(z) = J(z, y)$, then, from (35), (41), and (42), we have *Then, interval Riemann–Liouville-type integrals of* Ԓ *are defined as* T result was mainly controlled to $\frac{1}{2}$ $\frac{1$ Similarly, since $J_y(z) = J(z, y)$, then, from (35), (41), and (42), we have inequality has seen applications and a straightforward intrinsic geometric explanation. Similarly, since $\mathcal{J}_{L}(z) = \mathcal{J}_{L}(z, y)$, then, from (35), (41), and (42), we have inequality has seen applications and a straightforward intrinsic geometric explanation. category of classical convex functions, according to D Similarly, since $J_y(z) = J(z, y)$, then, from (35), (41), and (42), we have Similarly, since $J_{y}(z) = J(z, y)$, then, from (35), (41), and (42), we have category of convex functions, according to Dragomir and Pearce (2π) . This decree $(1/42)$. This decree $(1/42)$ Similarly, since $J_y(z) = J(z, y)$, then, from (35), (41), and (42), we have category of classical convex functions, according to Dragomir and Pearce [1]. This Similarly, since $J_{\mu}(z) = J(z, y)$, then, from (35), (41), and (42), we have Similarly, since $J_{\mu}(z) = J(z, y)$, then, from (35), (41), and (42), we have can be directly derived from convex functions, there is a close relationship between can be directed from $\mathcal{L}_j(\neg \mathcal{F}_j)$, there is a convex functions, there is a close relationship between \mathcal{L}_j realms of applied and pure sciences. Furthermore, because of its many applications and bitimally, since $J_y(z) = J(z, y)$, then, from (55), (41), and (42), we in *coincident to* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ *on* ℝூ *when it is* " ≤ "*.* $J_{Jy}(z) = JJ(z,y)$, then, from (39), (41), and (42), we have Similarly, since $J_y(z) = J(z, y)$, then, from (35), (41), and (42), we have \mathcal{Y} $(x_α)$ $x₁ = x_α - x_α = x_α + x₁$ (*ta*) $x₁ = x₁$ (*ta*) $x₁ = x₁$ $\log J_y(z) = J(z, y)$, then, from (35), (41), and (42), we have y Similarly, since $J_{\mu}(z) = J(z, y)$, then, from (35), (41), and (42), we have Similarly, since $J_{\mu}(z) = J_{\mu}(z, y)$, then, from (35), (41), and (42), we have *coincident to* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ *on* ℝூ *when it is* " ≤ "*.* $T(x)$ is the concept of convex $T(x)$ and convex $T(x)$ (d) $T(x)$ and $T(x)$ and convex functions in the convex function $T(x)$ a commany, since $J_{y}(z) = J_{y}(z, y)$, then, homeosty, (41), and (42), we have Conclusive areas $\pi(z) = \pi(z, y)$ then from (25) (41) and (42) we have Similarly, since $J_y(z) = J(z, y)$, then, from (35), (41), and (42), we have *coincident to* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ *on* ℝூ *when it is* " ≤ "*.* $T(\alpha)$ is $T(\alpha)$ in the convex $T(\alpha)$ and (42) and (42) and (42) Similarly, since $J_y(z) = J(z, y)$, then, from (35), (41), and (42), we have Similarly, since $J_y(z) = J(z, y)$, then, from (35), (41), $r_{\rm eff}$ intervalse on endpoint functions that can be seen as applications that can be seen as applications of $r_{\rm eff}$ Similarly, since $J_{\nu}(z) = J(z, y)$, then, from (35), (41), and (42), we have Similarly, since $J_{Jy}(z) = JJ(z,y)$, then, from (35), (41), and (42), we have Similarly, since $J_y(z) = J(z, y)$, then, from (35), (41), and (42), we have $c_{\mathbf{y}}$ over intervalse codomain. We exploit the use of double Riemann– $\lim_{\epsilon \to 0} \lim_{\epsilon \to 0} \lim_{\epsilon \to 0} \frac{\pi(z, y)}{\epsilon - \pi(z, y)}$ then from (35) (41) and (42) we have defined class of convex mappings proposed that are known as coordinated left and right ℏ - $\lim_{\epsilon \to 0}$ $\lim_{\epsilon \to 0}$ $\pi(\epsilon) = \pi(\epsilon, \mu)$, then from (25) (41) and (42) we have Similarly, since $J_y(z) = J(z, y)$, then, from (35), (41), and (42), we have
 $J_y(z) = J(z, y)$, $\begin{bmatrix} 6 & (\alpha + i) \\ 1 & 1 \end{bmatrix}$ Similarly, since $J_y(z) = J(x, y)$, then, from (35), (41), and (42), we have 4 Department of Mathematics, Faculty of Science, Jazan University, Jazan 45142, Saudi Arabia; Sumariy, since $J_y(z)$

 $\frac{1}{\sqrt{2}}$ is a familiar familiar familiar factor $\frac{1}{\sqrt{2}}$ is a complete metric space.

challenging, inequalities can be used to approximate the solution. Since many inequalities

Keywords: interval-valued mappings over coordinates; left and right ℏ -Convexity; double

Liouville fractional to derive the major results of the major results of the results of the key also examine the key

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

restrictions on endpoint functions of interval-valued functions that can be seen as applications of

The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most well-known fi

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can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L}

defined class of convex mappings proposed that are known as coordinated that are known as coordinated left and right \sim

Remark 1 ([47])**.** *(i) The relation* " ≤ " *is defined on* ℝூ *by*

$$
\frac{\Gamma(\beta+1)}{2\hbar(\frac{1}{2})(\mathbf{v}-\varepsilon)^{\beta}} \left[\mathcal{J}_{\varepsilon^{+}}^{\beta} J\left(\frac{\mathcal{O}^{+i}}{2}, \mathbf{v}\right) \right]
$$
\n
$$
\leq p \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{(\mathbf{i}-\mathcal{O})^{\alpha}(\mathbf{v}-\varepsilon)^{\beta}} \left[\mathcal{J}_{\mathcal{O}^{+},\varepsilon^{+}}^{\alpha,\beta} J(\mathbf{i},\mathbf{v}) + \mathcal{J}_{\mathbf{i}-,\varepsilon^{+}}^{\alpha,\beta} J(\mathcal{O},\mathbf{v}) \right]
$$
\n
$$
\leq p \frac{\alpha \Gamma(\beta+1)}{(\mathbf{v}-\varepsilon)^{\beta}} \left[\mathcal{J}_{\varepsilon^{+}}^{\beta} J(\mathcal{O},\mathbf{v}) + \mathcal{J}_{\varepsilon^{+}}^{\beta} J(\mathbf{i},\mathbf{v}) \right].
$$
\n(41)

\nand

 $y = \frac{1}{2}$

Fractal Fract. **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract inequality has several applications and a straightforward intrinsic geometric explanation. inequality has several applications and a straightforward intrinsic geometric explanation. \mathbf{C} and \mathbf{C} and \mathbf{C} category of convex functions, and \overline{C} category of convex functions, and \overline{C} θ relationship to the theory of inequalities, convexity has advanced quickly in recent θ ersion of Francesco Integral

Expediance via Coordinated Via Coordinated Via Coordinated Via Coordinated Via Coordinated Via Coordinated Vi \mathbf{d} $r_{\rm end}$ and $r_{\rm end}$ results' numerical validations that examples are nontrivial. By taking the product of two left and

and
\n
$$
\frac{\Gamma(\beta+1)}{2\hbar(\frac{1}{2})(\mathfrak{v}-\varepsilon)^{\alpha}} \left[\mathfrak{I}_{\mathfrak{v}}^{\beta} - \mathfrak{I} \left(\frac{\mathfrak{O}+i}{2}, \varepsilon \right) \right]
$$
\n
$$
\leq p \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{(\mathfrak{i}-\mathfrak{O})^{\alpha}(\mathfrak{v}-\varepsilon)^{\beta}} \left[\mathfrak{I}_{\mathfrak{O}^{+},\mathfrak{v}}^{\alpha,\beta} - \mathfrak{I}(\mathfrak{i},\varepsilon) + \mathfrak{I}_{\mathfrak{i}^{-},\mathfrak{v}}^{\alpha,\beta} - \mathfrak{I}(\mathfrak{O}_{\ell},\varepsilon) \right]
$$
\n
$$
\leq p \frac{\alpha \Gamma(\beta+1)}{(\mathfrak{v}-\varepsilon)^{\beta}} \left[\mathfrak{I}_{\mathfrak{v}}^{\beta} - \mathfrak{I}(\mathfrak{O}_{\ell},\varepsilon) + \mathfrak{I}_{\mathfrak{v}}^{\beta} - \mathfrak{I}(\mathfrak{i},\varepsilon) \right].
$$
\n(42)

the inequalities (41) and (42). e inequalities (41) and (42).
Now, we have inequality (17)'s left portion. cond, third, and fourth inequalities of (27) will be the consequence of adding
ities (41) and (42) ఌ The second, third, and fourth inequalities of (27) will be the consequence of adding T_0 (42).
 T_2 $\frac{1}{2}$ $\frac{$ econa, thira, an
*** α , third, and fourth inequalities of (27) will be the consequence of adding α Ԓ∗() *both are Riemann integrable (-integrable) over* ሾℴ, ሿ. *Moreover, if* Ԓ *is -integrable* α is a integrative integrable (*27*) will be the consequence of adding Ԓ∗() *both are Riemann integrable (-integrable) over* ሾℴ, ሿ. *Moreover, if* Ԓ *is -integrable* **Theorem 2** (42). $\frac{1}{2}$ of $\frac{1}{2}$, $\frac{1}{2}$ and $\frac{1}{2}$ are consequence $\mathcal{L} = \mathcal{L} \left(\mathcal{L} \right)$, $\mathcal{L} = \mathcal{L} \left(\mathcal{L} \right)$, $\mathcal{L} = \mathcal{L} \left(\mathcal{L} \right)$, $\mathcal{L} = \mathcal{L} \left(\mathcal{L} \right)$ $\frac{1}{2}$ Mow, we have inequality (17)'s left portion. $\mathcal{L} = \mathcal{L} \mathcal$ The second, third, and fourth inequalities of (27) will be the consequence of adding α category of classical convex functions, according to α The second, third, and fourth inequalities of (27) will be the consequence of adding e second, third, and fourth inequalities of (27) will be the consequence of adding the mequanties (41) and (42) .
Now, we have inequality $(17)'s$ left portion. The second, third, and fourth inequalities of (27) will be the consequence of adding \mathcal{L} .
The result was mainly contributed to Hermite (1822–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadam $T₁$ The second, third, and fourth inequalities of (27) will be the consequence of add The second, third, and fourth inequalities of (27) will be the consequence of adding e second, third, and fourth inequalities of (27) will be the consequence of adding $T_{\rm eff}$ α fourth inequalities of (27) will be the consequence of adding α the directly derived from convex functions, there is a close relationship between \mathcal{L} The second, third, and fourth inequalities of (27) will be the consequence of adding
the inequalities (41) and (42) can be directly derived from convex functions, there is a close relationship between \mathbb{R}^2 Editor: Bruce Henry Received: 14 \mathbb{R}^2 The mequanties (41) and (42) $\frac{1}{2}$ Mow, we have inequality (17)'s left portion. $for **tion**$, ∗ $for **relation**$, ∗ T are many uses for the concepts of convex sets and convex T functions in the convex T $\frac{1}{\text{Now xwo hovo}}$ in the $\frac{1}{\text{Now xwo hovo}}$ for the convex functions in the convex function of the convex functions of the con Version of Fractional Pachpatte-type Integral Inequalities via Coordinated the inequalities (41) and (42)

 σ , we have inequality (17) s left portion. $a_z(parallelity (17) is left portion.$ Now, we have inequality (17)'s left portion. $=$ $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$ convexity and the theory of inequalities. The theory of inequalities is the theory of inequalities of inequalities \mathcal{L} convexity and the theory of inequalities. row, we have inequality (17) s left portion. r , we have inequality (r) is en portions. furthermore, $\frac{1}{2}$ and pure sciences. **Now, we have inequality (17)'s left portion.** $T = \frac{1}{2}$ are many uses for the convex sets and convex sets and convex sets and convex functions in the convex functions in the convex sets and convex functions in the convex functions in the convex functions in the con There are many uses for the convex sets and convex sets and convex sets and convex \mathbf{r} Order Relation. *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx

$$
\frac{1}{\hbar^2\left(\frac{1}{2}\right)} J\!\!\int \!\left(\frac{\sigma+i}{2}, \frac{\varepsilon+\mathfrak{v}}{2}\right) \leq_p \frac{\Gamma(\beta+1)}{\hbar\left(\frac{1}{2}\right)(\mathfrak{v}-\varepsilon)} \left[J\!\!\int_{\varepsilon+}^{\beta} J\!\!\int \!\left(\frac{\sigma+i}{2}, \mathfrak{v}\right) + J\!\!\int_{\mathfrak{v}}^{\beta} - J\!\!\int \!\left(\frac{\sigma+i}{2}, \varepsilon\right) \right] \tag{43}
$$

Definition 2 ([41])**.** *Let* Ԓ: Ω → ℝூ *and* Ԓ ∈ Ω*. The double fuzzy interval Rieman–-Liouville*published: and the terms and terms and terms and terms $\frac{1}{2}$ and $\frac{1}{2}$. The following is how this inequality is stated: $\frac{1}{2}$ and $\frac{1}{2}$. The following is how this inequality is $\frac{1}{2}$. T result was mainly credited to H and T and T convexity and the theory of inequalities. and $\overline{}$ can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L} α and β be used to approximate the solution. Since α is approximate the solution. Since α α be directly derived from convex functions, there is a close relationship between relationship between α Revised: 5 February 2024 Accepted: 6 February 2024 $t_{\rm t}$ relationship to the theory of inequalities, convexity has advanced quickly in recent α

and
\n
$$
\frac{1}{\hbar^2\left(\frac{1}{2}\right)} J\left(\frac{\sigma+i}{2}, \frac{\varepsilon+\mathfrak{v}}{2}\right) \leq_p \frac{\Gamma(\alpha+1)}{\hbar\left(\frac{1}{2}\right)(i-\sigma)} \left[g^{\alpha}_{\sigma} + J\left(i, \frac{\varepsilon+\mathfrak{v}}{2}\right) + g^{\alpha}_{i} - J\left(\sigma, \frac{\varepsilon+\mathfrak{v}}{2}\right) \right]
$$
\n(44)

 $\frac{1}{\sqrt{1-\frac{1}{n}}}\left\{ \frac{n(n+1)(n+1)}{n}, \frac{n(n+1)(n+1)}{n}, \frac{n(n+1)}{n}, \frac$ **The following inequality is created by adding the two inequalities (43) and (44):** $\frac{1}{2}$ are definitely to created by adding the two mediatrice $\left(\frac{1}{2}\right)$ he following inequality is created by adding the two inequalities (43) and (44): ted by adding the two inequalities (43) and (4 equalities (43) and (44) : *Moreover in a created by adding the two maging inequality is created by adding the two* ollowing inequality is created by adding the two inequalities (43) and (44): α α β j eng the two mequ wo inequalities (43) and $\lim_{t \to 0}$ and $\lim_{t \to 0}$ and $\lim_{t \to 0}$. \mathfrak{a} = $\mathfrak{c}(\mathfrak{m})$ and The following inequality is created by adding the two inequalities (43) and (44): (44) : The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– The following inequality is created by adding the two inequalities (43) and (44): following inequality is created by adding the two inequalities (43) and (44): publication under the terms and μ ahty is created by adding the two filequalities (45) and (44) . publication under the terms of $T_{\rm eff}$ result was mainly credited to $T_{\rm eff}$ (1822–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (18 quality is created by additing the two fitequalities $(\pm \nu)$ and $(\pm \pm)$. Submitted for possible open access public terms the terms and The following inequality is created by adding the two inequalities (43) and (44): σ and σ is the result was mainly compared to σ and σ σ The following inequality is created by adding the two inequalities (43) and (44): σ result was mainly credited to σ and the Hadamard (18) and (1872–1901), even though Hadamard (1865–1901), σ TL_{α} following inequality is exacted by the theory τ following inequality is created by adding the two filed antices (45) and (44).

$$
\frac{1}{\hbar^2\left(\frac{1}{2}\right)} J \left(\frac{\sigma+i}{2}, \frac{\varepsilon+\mathfrak{v}}{2} \right) \leq_p \frac{\Gamma(\alpha+1)}{\hbar\left(\frac{1}{2}\right)\left(i-\sigma\right)^\alpha} \left[\mathfrak{I}^{\alpha}_{\mathbf{\sigma^+}} J \left(i, \frac{\varepsilon+\mathfrak{v}}{2}\right) + \mathfrak{I}^{\alpha}_{i-} J \left(\sigma, \frac{\varepsilon+\mathfrak{v}}{2}\right) \right] \newline + \frac{\Gamma(\beta+1)}{\hbar\left(\frac{1}{2}\right)\left(\mathfrak{v}-\varepsilon\right)^\beta} \left[\mathfrak{I}^{\beta}_{\varepsilon^+} J \left(\frac{\sigma+i}{2}, \mathfrak{v}\right) + \mathfrak{I}^{\beta}_{\mathfrak{v}^-} J \left(\frac{\sigma+i}{2}, \varepsilon\right) \right].
$$

arly, since we obtain the set of IVM s $\, J \colon \Omega \to \mathbb{R}^+_I$, the inequality can be expressed . Similarly, since we obtain the set of *IVM*s $\bar{JI}:\Omega\to\mathbb{R}^+$, the inequality can be expressed $\frac{1}{\sqrt{2}}$ arly, since we obtain the set of IVM s $J\!:\!\Omega\to\mathbb{R}_I^+$, the inequality can be expressed . \overline{a} ince we obtain the set of $\overline{U}/M_{\mathbb{C}}$ $\overline{\mathbb{I}}$ **:** $\Omega \rightarrow \mathbb{R}^+$ the inequality can be expressed M_{max} is h_0 sot of $\mathit{IVM}_\mathcal{E}$ $\mathsf{\pi}\cdot\mathsf{O}\to\mathbb{R}^+$ the inequality can be expressed. *Moreover if the specifical* $\frac{1}{2}$, $\frac{1}{$ $T_{\rm eff}$: $T_{\rm eff}$: $T_{\rm eff}$ T arly, since we obtain the set of *IVM*s $\,J\!:\!\Omega\to\mathbb{R}_I^+$, the inequality can be expressed *(-integrable) over* Ω *if and only if* Ԓ∗(,) *and* Ԓ∗(,) *both are -integrable over* Ω. $T_{\rm eff}$: $T_{\rm eff}$ $T_{$ obtain the set of IVM s $J\!J:\Omega\to\mathbb{R}_I^+$, the inequality can be expressed he inequality can <mark>l</mark> \rm{ality} can be expressed nequality can b e expressed *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract Similarly, since we obtain the set of *IVMs* $J \colon \Omega \to \mathbb{R}_I^+$, the inequality can be expressed follows. *holds:* as follows:

Integral Inequalities via Coordinated

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The result was mainly controlled to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard

Riemann–Liouville fractional integral operator; Pachpatte-type inequalities

2 Department of Mathematics and Computer Science, Alabama State University, Montgomery, AL 36101, USA

Inequalities via Coordinated ℏ-Convexity via Left and Right

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

 $\mathcal{L}_{\mathcal{A}}$ integral to derive the major results of the major results of the key also examine the key also examin

results' numerical validations that examples are nontrivial. By taking the product of two left and

results' numerical validations that examples are nontrivial. By taking the product of two left and

Liouville fractional integral to derive the major results of the research. We also examine the key

Riemann–Liouville fractional integral operator; Pachpatte-type inequalities

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Inequalities via Coordinated ℏ-Convexity via Left and Right

Liouville fractional integral to derive the major results of the research. We also examine the key

category of classical convex functions, according to Dragomir and Pearce [1]. This

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Inequalities via Coordinated ℏ-Convexity via Left and Right

 $\mathcal{L}_{\mathcal{A}}$ integral to derive the major results of the major results of the key also examine the key also examin

" ≤ " *"left and right" (or "LR" order, in short).*

There are many uses for the convex sets and convex sets and convex sets and convex α

 R iemann–Liouville fractional integral operator; P actional operator; P actional operator; P

years. When determining exact values for a mathematical problem proves to be

Remark 1 ([47])**.** *(i) The relation* " ≤ " *is defined on* ℝூ *by*

Riemann–Liouville fractional integral operator; Pachpatte-type inequalities

The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–

$$
\frac{1}{\hbar^{2}(\frac{1}{2})} J\left(\frac{\sigma+i}{2}, \frac{\varepsilon+v}{2}\right)
$$
\n
$$
\leq p \frac{\Gamma(\alpha+1)}{\hbar(\frac{1}{2})(i-\sigma)^{\alpha}} \left[\mathcal{J}_{\sigma}^{\alpha} + J\left(i, \frac{\varepsilon+v}{2}\right) + \mathcal{J}_{i}^{\alpha} J\left(\sigma, \frac{\varepsilon+v}{2}\right) \right] + \frac{\Gamma(\beta+1)}{\hbar(\frac{1}{2})(v-\varepsilon)^{\beta}} \left[\mathcal{J}_{\varepsilon}^{\beta} + J\left(\frac{\sigma+i}{2}, v\right) + \mathcal{J}_{v}^{\beta} - J\left(\frac{\sigma+i}{2}, \varepsilon\right) \right].
$$
\n(45)

\nThe first inequality of (27) is this one.

years. When determining exact values for a mathematical problem proves to be

results' numerical validations that examples are nontrivial. By taking the product of two left and

There are many uses for the convex sets and convex sets and convex sets and convex \mathcal{A}

(ii) It can be easily seen that " ≤ " *looks like "left and right" on the real line* ℝ, *so we call*

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1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of Mathematics, Faculty

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

for all ሾ∗, ∗ሿ,ሾ∗, ∗ሿ ∈ ℝூ, *and it is a pseudo-order relation. The relation* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ

There are many uses for the convex sets and convex sets and convex sets and convex \mathcal{A}

3 Department of Mathematics and Computer Science, Transilvania University of Brasov,

category of classical convex functions, according to Dragomir and Pearce [1]. This

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the first inequality of (27) is this one. The first inequality of (27) is this one. List integral to (27) is this one. The first inequality of (27) is this one. $\sum_{i=1}^n$ Correspondence: $\sum_{i=1}^n$ Correspondence: The first inequality of (27) is this one.
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category of classical convex functions, according to D and P

for all ሾ∗, ∗ሿ,ሾ∗, ∗ሿ ∈ ℝூ, *and it is a pseudo-order relation. The relation* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ

1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of Mathematics, Faculty

There are many uses for the convex sets and convex sets and convex sets and convex \mathcal{A}

 R iemann–Liouville fractional integral operator; P actional operator; P actional operator; P

 R iemann–Liouville fractional integral operator; P actional operator; Pachpatte-type inequalities in

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Remark 1 ([47])**.** *(i) The relation* " ≤ " *is defined on* ℝூ *by*

(ii) It can be easily seen that " ≤ " *looks like "left and right" on the real line* ℝ, *so we call*

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

results' numerical validations that examples are nontrivial. By taking the product of two left and

The first mequality of (27) is this one.
Now, we have inequality $(17)'$ s right portion: 1963 we have negating (1) , $\frac{1}{3}$. The following it is stated: The result and $\frac{1}{2}$ (1822–1901), the set of $\frac{1}{2}$ Now, we have inequality $(17)'s$ right portion: 1963) we have inequality $\langle x \rangle$ state politicial $T_{\rm tot}$ was mainly considered to $T_{\rm tot}$ and $T_{\rm tot}$ the Hadamard (1822–1901), even though Hadamard (187 μ exists the next section). 1963 was the one who first identified it follows in 2.3 . The following is how this inequality is stated. The result we defined to (47) and (80) , the maximum Maximum Maximum Maximum Maximum Maximum Hadamard (187 $/$ 1963 , we have needed it (23) . Then for them. The result was mainly credit was mainly credit to $\frac{1}{2}$ and \frac 1963 was the one who first identified it follows in $\frac{2}{3}$. The following is $\frac{2}{3}$ The result was mainly credited to $\frac{1}{2}$ and $\frac{1$ 1963 , we have inequality $\left(1\right)$ stated inequality is $\frac{1}{2}$. The result was mainly credit was mainly control to $\frac{1}{2}$ and $\$ Now, we have inequality (17)'s right portion: convexity and the theory of inequalities. convexity and the theory of inequalities. Now, we have inequality $(17)'$ s right portion: Now, we have inequality (17)'s right portion: $R = \text{Now}, w$ R_{Cov} The mot integral of (2) is the one. $180w$, we have mequality (17) s right portion. $\frac{c_1}{c_2}$ is the use interval-valued codomain. We exploit the use of double $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of double Riemann– \mathcal{L} integral to derive the major results of the results of the key also examine the key al $\frac{N_{\text{DM}}}{\sqrt{17}}$ we have inequality $\frac{17}{2}$ with perform Now, we have inequality (17)'s right portion: Now, we have inequality (17)'s right portion: $\frac{1}{1}$ Department of Mathematics, Faculty of Science, Jazan 45142, Saudi Arabia; Ja Φ have inequality (17)'s right portion: $\frac{1}{\sqrt{2}}$ Department of Mathematics, $\frac{1}{\sqrt{2}}$ $(17)'$ eright portion. $\frac{1}{2}$ department of $\frac{1}{2}$ Now, we have inequality (17)'s right portion: $\frac{1}{2}$ Department of Mathematics, $\frac{1}{2}$ Now, we have inequality (17)'s right portion: INOW, we have mequality (17) s right portion: Now, we have inequality (17) stright portion: Now, we have mequality (17) s right portion: **Tareq Saeed 1***,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4**

years. When determining exact values for a mathematical problem proves to be

 R iemann–Liouville fractional integral operator; P actional operator; P actional operator; P

$$
\frac{\Gamma(\beta)}{(\mathfrak{v}-\varepsilon)^{\beta}} \Big[\mathfrak{J}_{\varepsilon^{+}}^{\beta} J(\sigma, \mathfrak{v}) + \mathfrak{J}_{\mathfrak{v}}^{\beta} J(\sigma, \varepsilon) \Big] \leq_{p} \left[J(\sigma, \varepsilon) + J(\sigma, \mathfrak{v}) \right] \times \int_{0}^{1} \kappa^{\beta-1} [\hbar(\kappa) + \hbar(1-\kappa)] d\kappa
$$
\n(46)

$$
\frac{\Gamma(\beta)}{(\mathfrak{v}-\varepsilon)^{\beta}} \Big[\mathfrak{J}_{\varepsilon^{+}}^{\beta} J(\mathfrak{i},\mathfrak{v}) + \mathfrak{J}_{\mathfrak{v}}^{\beta} J(\mathfrak{i},\varepsilon) \Big] \leq_{p} \big[J(\mathfrak{i},\varepsilon) + J(\mathfrak{i},\mathfrak{v}) \big] \times \int_{0}^{1} \kappa^{\beta-1} [\hbar(\kappa) + \hbar(1-\kappa)] d\kappa \tag{47}
$$

$$
\frac{\Gamma(\alpha)}{(i-\sigma)^{\alpha}} \left[\mathfrak{I}_{\mathcal{O}^{+}}^{\alpha} J(i,\varepsilon) + \mathfrak{I}_{i}^{\alpha} J(\sigma,\varepsilon) \right] \leq_{p} \left[J(\sigma,\varepsilon) + J(i,\varepsilon) \right] \times \int_{0}^{1} v^{\alpha-1} \hbar(v) + \hbar(1-v) dv \tag{48}
$$
\n
$$
\frac{\Gamma(\alpha)}{\alpha} \left[\mathfrak{I}_{\mathcal{O}^{+}}^{\alpha} J(i,\mathbf{n}) + \mathfrak{I}_{\mathcal{O}^{+}}^{\alpha} J(\sigma,\mathbf{n}) \right] \leq \left[J(\sigma,\mathbf{n}) + J(i,\mathbf{n}) \right] \times \int_{0}^{1} v^{\alpha-1} \hbar(v) + \hbar(1-v) dv \tag{49}
$$

$$
\frac{\Gamma(\alpha)}{(i-\sigma)^{\alpha}} \left[\jmath^{\alpha}_{\sigma^{+}} J(i, \mathfrak{v}) + \jmath^{\alpha}_{i-J} J(\sigma, \mathfrak{v}) \right] \leq_{p} \left[J(\sigma, \mathfrak{v}) + J(i, \mathfrak{v}) \right] \times \int_{0}^{1} v^{\alpha-1} \hbar(v) + \hbar(1-v) dv \tag{49}
$$
\nSumming inequalities (46), (47), (48), and (49), and then taking the multiplication of

Summing mequanties (40), (47), (46), and (49), and then taking the mumph. *The family of all -integrable of s over coordinates and -integrable functions over* συπίπτης πειραπικές (*φ*), (*φ*), (*φ*), απά (*φ*), απά τιστι ισ Summing inequalities (46), (47), (48), and (49), and then taking the multiplication of the resultant with $\alpha\beta$, we have Interval and fuzzy Riemann-type integrals are defined as follows for coordinated Interval and fuzzy Riemann-type integrals are defined as follows for coordinated as follows for coordinated as the resultant with $\alpha\beta$, we have T_{total} including credited to Hermite (1822–1901), even though Hadamard (1822–1901), even though Hadamard (1875–1901), even though Hadamard (1875–1901), even though Hadamard (1875–1901), even though Hadamard (1875–1901) Summing inequalities (46) , (47) , (48) , and (49) , and then taking the resultant with $\alpha\beta$, we have $\frac{1}{2}$ is the one who first identified it $\frac{1}{2}$. ା *be an interval-valued mapping () and* Ԓ ∈ ℐℛሾℴ,ሿ*.* T_{total} result was mainly conducted to θ and θ and ା *be an interval-valued mapping () and* Ԓ ∈ ℐℛሾℴ,ሿ*.* T_{total} results was mainly considered to ℓ and $\left(\frac{1}{2}\right)$, $\left(\frac{1}{2}\right)$, and then taking the mainly Summing inequalities (46), (47), (48), and (49), and then taking the multiplic
the resultant with α ^{*R*} we have $\frac{1}{2}$ is the one who first identified it $\frac{1}{2}$. the resultant with $\alpha\beta$, we have cc convex functions, and P \mathcal{L} functions, according to Dragomir and \mathcal{L} α convex functions, and α $\frac{1}{\sqrt{2}}$ α convex functions, according to α α convex functions, and α functions, and α years. When determining exact values for a mathematical problem proves to be $T_{\text{ref}}(x)$ for $\frac{1}{2}$ are many uses for the convex sets and convex $\frac{1}{2}$ for $\frac{1}{2}$ and $\frac{1}{2}$ realms of application and pure sciences. Furthermore, because of its many applications and pure sciences. Order Relation. *Fractal Fract.* **2024**, *8*, Summing inequalities (46), (47), (48), and (49), and then taking the multiplication of p, we have Summing inequalities (46) , (47) , (48) , and (49) , and then taking the multiplication of Version of Fractional Pachpatte-type the resultant with $\alpha\beta$, we have t summing in ng mequanties (46 \arg inequalities (40), (47), (40), and $(i - \sigma)^{n}$ [° σ] σ [° σ] σ [° σ] σ [° σ] σ] σ [° σ] σ [σ] σ] σ [σ] σ] σ] σ [σ] σ] σ] σ [σ] σ] σ] σ [Summing inequalities (46), (47), (48), and (49), and then taking the multiple obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some Summing inequalities (46) , (47) , (48) , and (49) , and then taking the multiplication o r_{minming} inequalities (40), (47), (46), and (49), and then taking the multiplication of ing inequalities (46), (47), (48), and (49), and then taking the multiplication of $\overline{}$ Summing inequalities (46), (47), (48), and (49), and then taking the multiplication of α convexity (ν) over α and ν) over α $d = \frac{d}{dt} \int_{-\infty}^{\infty} \frac{d}{dt} \int$ α_c convexity (α_c) over α_c intervalse α_c $d = \frac{d}{dt} \int_{-\infty}^{\infty} d\theta \cos \theta \cos \theta$ α -convexity (α -codomain. We exploit the use of double Riemann– d_{de} defined convex mapping are known as convex mapping μ and μ an α convenience α and α and α is of double Riemann–

$$
\frac{\beta \Gamma(\alpha+1)}{(i-\sigma)^{\alpha}} \left[\mathcal{J}^{\alpha}_{\mathcal{O}^{+}} J(j,\varepsilon) + \mathcal{J}^{\alpha}_{i} J(\sigma,\varepsilon) + \mathcal{J}^{\alpha}_{\mathcal{O}^{+}} J(j,\mathfrak{v}) + \mathcal{J}^{\alpha}_{i-} J(\sigma,\mathfrak{v}) \right] \n+ \frac{\alpha \Gamma(\beta+1)}{(\mathfrak{v}-\varepsilon)^{\beta}} \left[\mathcal{J}^{\beta}_{\varepsilon} + J(\sigma,\mathfrak{v}) + \mathcal{J}^{\beta}_{\mathfrak{v}} - J(\sigma,\varepsilon) + \mathcal{J}^{\beta}_{\varepsilon} + J(i,\mathfrak{v}) + \mathcal{J}^{\beta}_{\mathfrak{v}} - J(i,\varepsilon) \right] \n\leq_p \left[J(\sigma,\varepsilon) + J(\sigma,\mathfrak{v}) + J(i,\varepsilon) + J(i,\mathfrak{v}) \right] \times \int_0^1 \kappa^{\beta-1} [\hbar(\kappa) + \hbar(1-\kappa)] d\kappa \int_0^1 \upsilon^{\alpha-1} \hbar(\upsilon) + \hbar(1-\upsilon) d\upsilon.
$$
\nThis is the final inequality of (27) and the conclusion has been established. \square

 $\mathbf{1}$, $\mathbf{1}$ \mathbf{I} , \mathbf{I} ℴ ϵ final inequality of (27) an \mathbf{C} **This is the final inequality of (27) and the conclusion has be** This is the final inequality of (27) and the conclusion has been established. \square $\frac{1}{\sqrt{1}}$. This convex functions, according to Dragomir and Pearce $\frac{1}{\sqrt{1}}$. Dragomir and Pearce $\frac{1}{\sqrt{1}}$ \mathbf{r} category of \mathbf{r} , according to Dragomir and Pearce \mathbf{r} . This pear This is the final inequality of (27) and the concl This is the final inequality of (27) and the conclusion has been established. y_1 determining exact values for a mathematical problem problem problem problem proves to be problem proves to be proved by y_1 This is the final inequality of (27) and the conclusion has been established. \Box y_{1} and determining exact values for a mathematical problem problem problem problem problem problem proves to be proved problem proves to be problem proved by y_{2} \mathbf{y} e final inequality of (27) and the conclusion has been established. $□$ \mathcal{L}_1 and determining exact values for a mathematical problem problem problem problem problem problem proves to be proved by \mathcal{L}_1 This is the final inequality of (27) and the conclusion has been established. \square $\mathcal{L}_{\mathcal{A}}$ This is the final inequality of (27) and the conclusion has been established. \square $T \rightarrow \infty$ $T \rightarrow \infty$ $T \sim$ there are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex sets and convex functions in the convex sets and convex sets and convex sets and convex sets T , there are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex sets and convex \mathcal{L} $T_{\rm eff}$ are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex sets and convex sets and convex functions in the convex sets and convex sets and convex sets an \mathbf{H} This is the final inequality of (27) and the conclusion has been established. [This is the final inequality of (27) and the conclusion has been established. \Box **This is the final inequality of (27) and the conclusion has been established.** \Box

 $\frac{1}{2}$ ($\frac{1}{2}$, $\frac{1}{$ *The family of all -integrable of s over coordinates over coordinates is denoted by* \overline{a} , \overline{a} *The family of all -integrable of s over coordinates over coordinates is denoted by* \overline{a} , \overline{b} , \overline{c} , \overline{c} , \overline{c} *The family of all -integrable of s over coordinates over coordinates is denoted by* ఌ ℴ *The family of all -integrable of s over coordinates over coordinates is denoted by* **Example 3.** We assume the IVMs π : [0, 2] \times [0, 2] $\rightarrow \mathbb{R}^+$, define Ԓ(,) = ሾԒ∗(,), Ԓ∗(,)ሿ *for all* (,) ∈Ω= ሾℴ, ሿ × ሾ, ሿ *. Then,* Ԓ *is double integrable* **Example 3.** We assume the IVMs π : [0, 2] \times [0, 2] $\rightarrow \mathbb{R}^+$ defined by Ԓ(,) = ሾԒ∗(,), Ԓ∗(,)ሿ *for all* (,) ∈Ω= ሾℴ, ሿ × ሾ, ሿ *. Then,* Ԓ *is double integrable The family of all -integrable of s over coordinates and -integrable functions over* **Example 5.** We assume *The family of all -integrable of s over coordinates and -integrable functions over* **Example 3.** We assume the IVMs Л **ample 3.** We assume the IV $^7Ms \text{ J} : [0, 2] \times$ **3.** We assume the IVMs $\mathbb{J}:[0, 2] \times [0, 2] \rightarrow \mathbb{R}_{I}^{+}$ defined e assume the IVMs $JJ: [0, 2]$ $\vert \times [0, 2] \rightarrow \mathbb{R}^{+}_{I}$ **Example 3.** We assume the IVMs $JJ : [0, 2] \times [0, 2] \rightarrow \mathbb{R}^+_I$ defined by \mathbf{r} , then \mathbf{r} **Example 3.** We assume the IVMs π : [0, 2] \times [0, 2] $\rightarrow \mathbb{R}^+$ defined by Ԓ(,). **Example 3.** We assume the IVMs JJ : $[0, 2] \times [0, 2] \rightarrow \mathbb{R}^+_I$ defined by $T_{\rm eff}$ result was mainly credited to $T_{\rm eff}$ Hadamard (1822–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Had E_{V} ² inne the IVMs, $\overline{\Pi}:[0,2]\times [0,2]\to \mathbb{R}^+$ defined by The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– Example 3 W **Example 3.** We assume the IVMs J : $[0, 2] \times [0, 2] \rightarrow \mathbb{R}^+_l$ defined by Ex $\mathbf{F}_{\mathbf{v}}$ $\frac{1}{\sqrt{1-\frac{1$ years. When determining exact values for a mathematical problem proves to be **Example 3.** We assume the IV Ms $JJ: [0, 2] \times [0, 2]$ " ≤ " *"left and right" (or "LR" order, in short).* years. When determining exact values for a mathematical problem proves to be **Example 3.** We assume the IVMs $\mathbf{J} : [0, 2] \times [0, 2] \rightarrow \mathbb{R}_I^+$ defined can be directly derived from convex functions, there is a close relationship between **Example 3** We assume the IVMs π : $[0, 2] \times [0, 2] \rightarrow \mathbb{R}^+$ defined by can be directly derived from convex functions, there is a close relationship between **Example 3.** We assume the IVMs π : $[0, 2] \times [0, 2] \rightarrow \mathbb{R}^+$ defined by can be directly derived from convex functions, there is a close relationship between e assume the IVMs, $\pi \cdot [0, 2] \times [0, 2] \rightarrow \mathbb{R}^+$ defined by can be directly derived from convex functions, there is a close relationship between $\mathbf{E}_{\mathbf{v}}$ challenging, inequalities can be used to approximate the solution. Since many inequalities **Example 5.** We assume the TV NIS J : $[0, 2] \times [0, 2] \rightarrow \mathbb{R}$ *uefined by* $\begin{bmatrix} S & \Pi \cdot [0, 2] \times [0, 2] \end{bmatrix} \rightarrow \mathbb{R}^+$ defined by can be directly derived from convex functions, there is a close relationship between \mathcal{L} R evende 2 $M_{2.021}$ Accepted: 6 February 2024 challenging, inequalities can be used to approximate the solution. Since many inequalities is $J: [0, 2] \times [0, 2] \rightarrow \mathbb{R}$ a close relationship between y **Example 3.** We assume the IVMs $JJ : [0, 2] \times [0, 2] \rightarrow \mathbb{R}^+_I$ defined by **Example 3.** We assume the IVMs $\mathbf{J} : [0, 2] \times [0, 2]$ **Example 3.** We assume the IVMs $\mathbf{J} : [0, 2] \times [0, 2] \rightarrow \mathbb{R}_I^+$ defined by tight relationship to the theory of inequalities, convexity has advanced α *for all* ሾ∗, ∗ሿ,ሾ∗, ∗ሿ ∈ ℝூ, *and it is a pseudo-order relation. The relation* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ *compound the IVMs* $J_J : [0, 2] \times [0, 2] \rightarrow \mathbb{R}^+_I$ defined **Example 3.** We assume the IVMs $\mathbf{J} : [0, 2] \times [0, 2] \rightarrow \mathbb{R}_I^+$ defined by tight relationship to the theory of inequalities, convexity has advanced α *for all* ሾ∗, ∗ሿ,ሾ∗, ∗ሿ ∈ ℝூ, *and it is a pseudo-order relation. The relation* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ *We assume the IVMs* $\mathbf{J} : [0, 2] \times [0, 2] \rightarrow \mathbb{R}_{I}^{+}$ defined by **Example 3.** We assume the IVMs $JJ : [0, 2] \times [0, 2] \rightarrow \mathbb{R}^+_I$ defined by **Example 3.** We assume the IVMs $JJ : [0, 2] \times [0, 2] \rightarrow \mathbb{R}^+_I$ defined by *for all* ሾ∗, ∗ሿ,ሾ∗, ∗ሿ ∈ ℝூ, *and it is a pseudo-order relation. The relation* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ **Example 3.** We assume the IVMs $JJ : [0, 2] \times [0, 2] \rightarrow \mathbb{R}^+_I$ defined by $\frac{1}{2}$ relationship to the theory of inequality has advanced $\frac{1}{2}$ in recent $\frac{1}{2}$ in recent $\frac{1}{2}$ **Remark 1** ([47])**.** *(i) The relation* " ≤ " *is defined on* ℝூ *by* $\mathbf{y} \cdot \mathbf{y} = \mathbf{y} \cdot \mathbf{y} + \mathbf{y} \cdot \math$ **Remark 1** ([47])**.** *(i) The relation* " ≤ " *is defined on* ℝூ *by* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ if and only if ∗ ≤∗, [∗] ≤ ∗, (6) **1. Introduction Remark 1** (i) The relation \mathbf{r} is defined on \mathbf{r} is defined on \mathbf{r} is defined on \mathbf{r} **1. Intervention only 1.** The $\int_{a}^{b} \left[e^{b} \right]$ × $\int_{a}^{b} \left[e^{b} \right]$ × $\int_{a}^{b} \left[a \right]$ *notation by* **Remark 1** ([47])**.** *(i) The relation* " ≤ " *is defined on* ℝூ *by* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ if and only if ∗ ≤ ∗, [∗] ≤ ∗, (6) **1. Introduction Example 3.** We assume the **Example 3.** We assume the TV Ms J : $[0, 2] \times [0, 2] \rightarrow \mathbb{R}$ alguned by Version of Frample **Example 3.** We assume the IVMs π : [0, 2] \times [0, 2] -**Example 3.** We assume the IVMs π : [0, 2] \times [0, 2] $\rightarrow \mathbb{R}^+$; **Example 3.** We assume the IVMs π : [0, 2] \times [0, 2] $\rightarrow \mathbb{R}^+$ defined **Example 3.** We assume the IVMs π : [0, 2] \times [0, 2] $\rightarrow \mathbb{R}^+$; defined by

$$
J(x,y) = [(2-\sqrt{x})(2-\sqrt{y}), 2(2-\sqrt{x})(2-\sqrt{y})], \qquad (51)
$$

then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_*(x, y)$, $J^*(x, y)$ are coordinate LR- \hbar - $\frac{1}{2}$ converse and the concave functions. Hence $\overline{R}(x, y)$ is the coordinated I R-th-converse IVN *Moreover, if* Ԓ *is -integrable over* Ω, *then* $\frac{1}{\sqrt{2}}$ *over and the concerns functions* Hence $\frac{1}{\pi}(x, y)$ is the coordinated I B the courses IVI convex and \hbar -concave functions. Hence, $\stackrel{\sim}{{\rm J}}\!\! (x,y)$ is \hbar -coordinated LR- \hbar -convex IV N ሾℴ, ሿ *are denoted by* ሾℴ,ሿ *and* ሾℴ,ሿ, *respectively*. ሾℴ, ሿ *are denoted by* ሾℴ,ሿ *and* ሾℴ,ሿ, *respectively*. \mathcal{L} then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_{\ast}(x)$ $\overline{\mathcal{L}}(x, y)$ convex and \overline{h} -concave functions. Hence, $\overline{\mathcal{L}}(x, y)$ is \overline{h} -coordinated LR- \overline{h} -convex IVM. then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_*(x, y)$, $J^*(x, y)$ then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $\text{J}_{\ast}(x, y)$, $\text{J}^{\ast}(x, y)$ are coordinate then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_*(x, y)$, $J^*(x, y)$ are coordinate LR- \hbar *over* ሾℴ, ሿ, *then* ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* $\frac{1}{\alpha}$ ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* $\frac{1}{\sqrt{2}}$
 CONTAGE AND CONGREGE LIMITIONS Hance $\overline{\Pi}(x, u)$ is *b* coordinated I *P b* contract IVM ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* **Theorem 2** (*Concern Eunctions* Hanse $\overline{I}(x, y)$ is *b* coordinated I P *b* course IVM *nu n*−concave functions. Tience, sj(x, y) is n−coorumuteu Eix-n−convex 1 v ivi. convex and h-concave functions. Hence, $\overline{J}(x,y)$ is h-coordinated LR-h-convex IVM. then, for each $\gamma \in [0, 1]$, we have. Endpoint functions category of classical convex functions, according to Dragomir and Pearce [1]. This $\sum_{i=1}^{n}$ then for each $\alpha \in [0, 1]$ and have Endmoint functions π (x then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_{\ast}(x, y)$, J^{\ast} then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $\mathcal{J}_*(x, y)$, $\mathcal{J}^*(x, y)$ are coording to γ . then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_{*}(x, y)$, $J^{*}(x, y)$ are coordinate LR- \hbar m ave functions. Hence, $J(x, y)$ is h-coordinated EK-h-convex TV IV. **Copyright:** © 2024 by the authors. convex and h-concave functions. Hence, $\tilde{J}(x,y)$ is h-coordinated LR-h-convex IVM. then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_{\ast}(x, y)$, $J^{\ast}(x, y)$ are coordinate LR- \hbar -
convex and \hbar -concave functions. Hence, $\tilde{J}(x, y)$ is \hbar -coordinated LR- \hbar -convex IVM. T result, $T(x, y)$ is n-coordinated $L(x-t)$ -convex to ivi. **Copyright:** © 2024 by the authors. convex and h-concave functions. Hence, $\ddot{J}(x,y)$ is h-coordinated LR-h-con The Hausdorff – Pompeiu distance between intervals $J_{\frac{1}{2}}(n, \frac{1}{2})$, ∗ $J_{\frac{1}{2}}(n, \frac{1}{2})$ $\frac{N}{2}$, $\frac{N}{2}$ (ሾ∗, ∗ሿ,ሾ∗, ∗ሿ) = ሼ|∗ − ∗|, |∗ − ∗|ሽ. (7) then for each $\alpha \in [0, 1]$ and bega. Endmoint functions $\mathbb{I}^-(x, y)$ then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_{\ast}(x, y)$, $J^*(x, y)$ are coordinate LR-h- $\mathbf{r} \cdot \mathbf{r}$, $\mathbf{$ then for each $\alpha \in [0, 1]$ and bega. Endmoint functions π (x u) $\pi^*(x, u)$ are coord then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_{\ast}(x, y)$, $J^{\ast}(x, y)$ are coordinate LR-h- \overline{t} then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_*(x, y)$, $J^*(x, y)$ are coordinate LR-hthen, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_*(x, y)$, $J^*(x, y)$ are coordinate LR- \hbar i convex and n-concave fanctions. Tience, $j(x, y)$ is n-cool *(iii) It can be easily seen that and right and right and right and right and right* and right and r convex and \overrightarrow{h} -concave functions. Hence, $\overrightarrow{J}(x,y)$ is \overrightarrow{h} -coordinated LR- \overrightarrow{h} -convex IVM. α the theory of inequalities, convexity has advanced quickly indicated α then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_{*}(x, y)$, $J^{*}(x, y)$ convex and h-concave functions. Hence, $\overline{J}(x,y)$ is h-coordinated LR-h-convex IVM. $\sum_{i=1}^{n}$ to the theory of inequalities, convexity has advanced quickly in recent $\sum_{i=1}^{n}$ in recen then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $\text{J}_{*}(x, y)$, $\text{J}^{*}(x, y)$ are coordinate LR- \hbar convex and \hbar -concave functions. Hence, $\overline{J}(x,y)$ is \hbar -coordinated LR- \hbar -convex IVM. then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_{\ast}(x, y)$, $J_{\ast}(x, y)$ are coordinate LR-h then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $\text{J}_{*}(x, y)$, $\text{J}^{*}(x, y)$ are coordinate LR- \hbar convex and \hbar -concave functions. Hence, \tilde{J} (x, y) is \hbar -coordinated LR- \hbar -convex IVM. then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $\mathbb{J}_*(x, y)$, $\mathbb{J}^*(x, y)$ are coordinate LR- \hbar then, for each $\gamma \in [0, 1]$, we have. Endpoint functions $J_{*}(x, y)$, $J^{*}(x, y)$ are coordinate LR- \hbar t_{c} relationship to the theory of intervention c in c is advanced c in recent c course and powers functions. Hence, $\tilde{p}(x, y)$ is poordinated I P convex and h-concave functions. Hence, $\stackrel{\sim}{J}(x,y)$ is h-coordinated LR-h-convex IVM convex and \hbar -concave functions. Hence, $\stackrel{\sim}{J}(x,y)$ is \hbar -coordinated LR- \hbar -convex IVM. to the time the term of interference $f_{\text{J}}(x, y)$ is a coordinated E in convexity $f(x)$ course and programs functions. Hence, $\tilde{\mathbf{u}}(x, y)$ is programsted I.B. from any IVM convex and \hbar -concave functions. Hence, $\widetilde{J}(x,y)$ is \hbar -coordinated LR- \hbar -convex IVM. convex and n-concave functions. Hence, JJ(x, y) is n-coordinated LK-n-convex IV M.

$$
J\left(\frac{\sigma+i}{2}, \frac{\varepsilon+v}{2}\right) = [1, 2],
$$
\n
$$
\frac{\Gamma(\alpha+1)}{4(i-\sigma)^{\alpha}} \left[J_{\sigma,+}^{\alpha} J\left(i, \frac{\varepsilon+v}{2}\right) + J_{i}^{\alpha} J\left(\sigma, \frac{\varepsilon+v}{2}\right) \right] + \frac{\Gamma(\beta+1)}{4(v-\varepsilon)^{\beta}} \left[J_{\varepsilon}^{\beta} J\left(\frac{\sigma+i}{2}, v\right) + J_{v}^{\beta} J\left(\frac{\sigma+i}{2}, \varepsilon\right) \right]
$$
\n
$$
= 2 - \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{8} \pi \cdot [1, 2]
$$
\n
$$
\frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{4(i-\sigma)^{\alpha}(v-\varepsilon)^{\beta}} \left[J_{\sigma^+, \varepsilon^+}^{\alpha, \beta} J\left(i, v\right) + J_{\sigma^+, v}^{\alpha, \beta} J\left(i, \varepsilon\right) + J_{i-, \varepsilon^+}^{\alpha, \beta} J\left(\sigma, v\right) + J_{i-, v}^{\alpha, \beta} J\left(\sigma, \varepsilon\right) \right]
$$
\n
$$
= \frac{33}{8} - \sqrt{2} - \frac{\sqrt{2}}{2} \pi + \frac{\pi}{8} + \frac{\pi}{32} \cdot [1, 2]
$$
\n
$$
\frac{\Gamma(\alpha+1)}{8(i-\sigma)^{\alpha}} \left[J_{\sigma^+}^{\alpha} J\left(i, \varepsilon\right) + J_{\sigma^+}^{\alpha} J\left(i, v\right) + J_{i}^{\alpha} J\left(\sigma, \varepsilon\right) + J_{i}^{\alpha} J\left(\sigma, v\right) \right]
$$
\n
$$
+ \frac{\Gamma(\beta+1)}{8(\varepsilon - \varepsilon)^{\beta}} \left[J_{\varepsilon^+}^{\beta} J\left(i, v\right) + J_{\varepsilon^+}^{\beta} J\left(i, v\right) + J_{v}^{\beta} J\left(\sigma, \varepsilon\right) + J_{v}^{\beta} J\left(i, \varepsilon\right) \right]
$$
\n
$$
= \frac{34\sqrt{2} + (\sqrt{2} - 4)\pi - 24}{8\sqrt{2}} \cdot [1, 2]
$$

 $\frac{1}{1000}$ $\frac{1}{1}$ $\frac{1}{1}$ \overline{a} *type integrals* ℐℴశ,ఌశ $\mathbb{R}^n \times \mathbb{R}$ $H = m$ \mathbf{H} matrix \mathbf{F} \mathcal{L}_{H} and \mathcal{L}_{H} H_{eff} and \tilde{H}_{eff} and \tilde{H}_{eff} and the fuzzon of the fuzzy \tilde{H}_{eff} *The family of all -integrable of s over coordinates over coordinates is denoted by The family of all -integrable of s over coordinates over coordinates is denoted by The family of all -integrable of s over coordinates over coordinates is denoted by* Ω. *over* ሾℴ, ሿ, *then over* ሾℴ, ሿ, *then* $T\sum_{k=1}^{n}$ (*L₂*), $\sum_{k=1}^{n}$ *L₂*, $\sum_{k=1}^{n}$ *b*_× $\sum_{k=1}^{n}$ *d*_∗(*)*, $\sum_{k=1}^{n}$ *for all 3 f* $\frac{1}{2}$ *nut* is $\frac{1}{2}$ $T_{\rm eff}$: $T_{\rm eff}$, $T_{\rm eff}$ \sim $T_{\rm eff}$ ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* $The *left* is a constant, where α is the constant.$ Interval and function \mathcal{L} are defined as follows for coordinated as follows for c Ԓ(,). *That is*

$$
[1,2] \leq_{p} 2 - \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{8}\pi \cdot [1,2] \leq_{p} \frac{33}{8} - \sqrt{2} - \frac{\sqrt{2}}{2}\pi + \frac{\pi}{8} + \frac{\pi^{2}}{32} \cdot [1,2] \leq_{p} \frac{34\sqrt{2} + (\sqrt{2}-4)\pi - 24}{8\sqrt{2}} \cdot [1,2] \leq_{p} \left(\frac{9}{2} - 2\sqrt{2}\right)
$$

$$
\cdot [1,2]
$$

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Article

Article

years. When determining exact values for a mathematical problem proves to be

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tight relationship to the theory of inequalities, convexity has advanced α

can be directly derived from convex functions, there is a close relationship between

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Inequalities via Coordinated ℏ-Convexity via Left and Right

can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L}_c

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can be directly derived from convex functions, there is a close relationship between

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tight relationship to the theory of inequalities, convexity has advanced α

can be directly derived from convex functions, there is a close relationship between

convexity and the theory of inequalities.

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New Version of Fractional Pachpatte-type Integral

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New Version of Fractional Pachpatte-type Integral

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restrictions on endpoint functions of interval-valued functions that can be seen as applications of

Hence, Theorem 6 has been verified. including the straightforward intrinsic geometric explanation \mathcal{I}_{int} can be directly derived from convex functions, there is a close relationship between convexity and the theory of inequalities. Accepted: 6 February 2024 $\frac{1}{2}$ and $\frac{1}{2}$ a $\sum_{i=1}^{n}$ and determine $\sum_{i=1}^{n}$ mathematical problem problem problem problem problem problem proves to be been problem proble $\mathcal{L}_{\text{wave}}$ and $\mathcal{L}_{\text{wave}}$ and $\mathcal{L}_{\text{wave}}$ and $\mathcal{L}_{\text{wave}}$ problem problem problem problem problem proves to be a mathematical problem problem problem problem proves to be a mathematical problem problem problem p \mathcal{L}_{max} and \mathcal{L}_{max} and \mathcal{L}_{max} problem problem problem problem problem problem problem proves to be been proved by a mathematical problem proves to be a mathematical problem problem problem problem prob y_{max} and determining exact values for a mathematical problem problem problem problem problem problem proves to be y_{max} There are many uses for the convex sets and convex sets and convex sets and convex \mathcal{L} these new outcomes. these new outcomes. Hence, Theorem 6 has been verified. \mathcal{L}^{max} \ldots eze \ldots 4 Department of Mathematics, \tilde{B} and \tilde{B} 4 Department of Mathematics, \tilde{B} and \tilde{B} are straighted at \tilde{B} Hence, I heorem 6 has been verified. 29 Eroilor Boulevard, 500036 Brasov, Romania Hence, Ineorem 6 has been verified. 29 Eroilor Boulevard, 500036 Brasov, Romania Hence, I neorem o has been verified. \overline{A} Montgomery, AL 36101, USA Ω and Ω and Ω mathematical vertical University of Brasilvania University of Brasil \mathbf{M} 1 Department of Mathematics and Computer Science, Transieurs, 2000 Brasil \overline{M} \overline{M} \overline{M} 1 department of Mathematics and Computer Science, Transilvania University of Brasil \mathbb{R} al 36101, use \mathbb{R} 3.7 Theorem of Mas been beingthem. \overline{M} \overline{M} \overline{M} \overline{M} \overline{M} α , Theorem of Ms been betyched. of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa nence, i neorem o nus been verifieu. of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa Ω define, Theorem of Mas been being of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa Hence, Ineorem o nus been veryieu. of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa Ω department of Ω and Ω and Ω and Ω such that Ω of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa Γ ence, Theorem o hus been veryieu. of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa Hence, Theorem 6 has been verified. of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa $\overline{\mathcal{S}}$ abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki $\overline{\mathcal{S}}$ **Inequalities via Coordinated ℏ-Convexity via Left and Right Hence**, Theo. **New Version of Fractional Pachpatte-type Integral** Hence, I heorem 6 has been verified.

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results' numerical validations that examples are nontrivial. By taking the product of two left and

restrictions on endpoint functions of interval-valued functions that can be seen as applications of

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New Version of Fractional Pachpatte-type Integral

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Liouville fractional integral to derive the major results of the research. We also examine the key results' numerical validations that examples are nontrivial. By taking the product of two left and

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can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L}_c

Abstract: In particular, the fractional forms of Hermite–Hadamard inequalities for the newly

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Liouville fractional to derive the major results of the major results of the results of the key also examine the key

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Abstract: In particular, the fractional forms of Hermite–Hadamard inequalities for the newly

Tareq Saeed 1*,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4**

defined class of convex mappings proposed that are known as coordinated that are known as coordinated left and right μ

Remark 3. If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (27), as a $result, there will be inequality (see [43]):$ $result, there will be inequality (see [43]):$ $result, there will be inequality (see [43]):$ inequality has several applications and a straightforward intrinsic geometric explanation. inequality has several applications and a straightforward intrinsic geometric explanation. **category of convex functions, and** $\alpha = \alpha$ and $\beta = \alpha$, and $n(v) = v$, $n(\alpha) = \alpha$, **Remark 3.** If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (27), as a inequality has several applications and a straightforward intrinsic geometric explanation. convexity and the theory of inequalities. R indi R , I one assumes that α **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **Remark 3.** If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (27), as a **Rightly** $\lim_{n \to \infty} \frac{\partial u}{\partial n} \frac{\partial u}{\partial n} = \lim_{n \to \infty} \frac{\partial u}{\partial n} \frac{\partial u}{\partial n} = 1$, $\lim_{n \to \infty} \frac{\partial v}{\partial n} = 0$, $\lim_{n \to \infty} \frac{\partial v}{\partial n} = 0$, $\lim_{n \to \infty} \frac{\partial v}{\partial n} = 0$ result, there will be inequality (see [43]): **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **Remark 3.** If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (27), as a these new outcomes. The second se **Remark 3.** If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, the results' numerical validations that examples are nontrivial. By taking the product of two left and **Remark 5.** If one assumes that $\alpha = 1$ and $p = 1$, and $h(y) = v$, $h(x) = \kappa$, then from results' numerical validations that examples are nontrivial. By taking the product of two left and **Remark 5.** If one assumes that $\alpha = 1$ and $\rho = 1$, and $h(\nu) = \nu$, $h(\kappa) = \kappa$, then from (27), as a α are known as coordinated that are known as coordinated left and right α **Remark 3.** If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (27), as a d computed can be mapping poor α result, there will be inequality (see [43]): ϵ is the use of double ϵ intervalse codomain. We exploit the use of double Riemann– **Remark 3.** If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (27), as a recult there will be inequality (see [A31). \mathcal{L} khakamid ajazanu.
Saaraa 1980 - 1980 - 1980 - 1980 - 1980 - 1980 - 1980 - 1980 - 1980 - 1980 - 1980 - 1980 - 1980 - 1980 - 1980 29 Eroilor Boulevard, 500036 Brasov, Romania **Remark 3.** If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then khakamid.com **Remark 3.** If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(\upsilon) = \upsilon$, $\hbar(\kappa) = \kappa$, then from (2/), as result, there will be inequality (see [43]): **Remark 3.** If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (27), a 4 Department of \mathcal{B} Department of Science, Jazan University, Ja \mathcal{B} 4.4 Department of Mathematics, $\tilde{B}_{\rm eff}$ of Science, Jazan 45142 4 Department of \mathcal{B} Department of Science, Jazan University, Jazan \mathcal{B} 4.4 Department of Mathematics, Γ athematics, Γ andi Γ \mathbf{R} Montgomery, AL 36101, USA N chiart θ . 2 Department of Mathematics and Computer State University, Alabama State University, Alab **Nemal 3.** $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ **D**epartment of Mathematics and Computer State University, \mathbf{A} $M_{\rm H}$ \sim $M_{\rm H}$ \overline{P} department of Mathematics and Computer State University, $\frac{1}{2}$ **Remark 3.** If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (27), as a $\sum_{i=1}^n$ of \mathcal{L} science, \mathcal{L} and \mathcal{L} are \mathcal{L} and \mathcal{L} and \mathcal{L} are \mathcal{L} ; the same \mathcal{L} and \mathcal{L} are \mathcal{L} and \mathcal{L} are \mathcal{L} and \mathcal{L} are \mathcal{L} ; the same \mathcal{L} and $\mathcal{L$ $\overline{2}$ department of Mathematics and Computer State University, $\overline{2}$ T_{c} T_{c} **The Samultical Kemark 3.** If one assumes that $\alpha = 1$ and result, there will be inequality (see [43]): **Order Relation Remark 3.** If one assumes that $\alpha = 1$ and $\beta = 1$, and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (27), as a result, there will be inequality (see [43]):

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$$
J\left(\frac{\sigma+i}{2}, \frac{\varepsilon+v}{2}\right)
$$
\n
$$
\leq p \frac{1}{2} \left[\frac{1}{i-\sigma} \int_{\sigma}^{i} J\left(x, \frac{\varepsilon+v}{2}\right) dx + \frac{1}{v-\varepsilon} \int_{\varepsilon}^{v} J\left(\frac{\sigma+i}{2}, y\right) dy \right] \leq p \frac{1}{(i-\sigma)(v-\varepsilon)} \int_{\sigma}^{i} \int_{\varepsilon}^{v} J\left(x, y\right) dy dx
$$
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$$
\leq p \frac{1}{4(i-\sigma)} \left[\int_{\sigma}^{i} J\left(x, \varepsilon\right) dx + \int_{\sigma}^{i} J\left(x, v\right) dx \right] + \frac{1}{4(v-\varepsilon)} \left[\int_{\varepsilon}^{v} J\left(\sigma, y\right) dy + \int_{\varepsilon}^{v} J\left(i, y\right) dy \right]
$$
\n
$$
\leq p \frac{J\left(\sigma, \varepsilon\right) + J\left(i, \varepsilon\right) + J\left(\sigma, v\right) + J\left(i, v\right)}{4}.
$$
\n(52)

Fractal Fract. **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract convex, then from (27), as a result, there will be inequality (see [42]): *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract If one assumes that $\alpha = 1$ and $\beta = 1$, $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and J is coordinated left-LR- \hbar convex, then from (27), as a result, there will be inequa[lity](#page-22-27) (see [42]): convexity and the theory of inequalities. The theory of inequalities \mathbf{I} ι if of ι category of classical convex functions, according to Dragomir and Pearce [1]. This category of classical convex functions, according to Dragomir and Pearce [1]. This category of classical convex functions, according to Dragomir and Pearce [1]. This is the Dragomir and Pearce [1]. This is the convex, their from (27) as a result, there will be inequality (see 12). α convex, then from (27), as a result, there will be inequality (see [42]): convex, then $_{1}$ from $_{2}$, the a relation with be directly determined from convex $_{1}$ challenging in $\alpha = \text{unit}$ α y_1 is a mathematical problem problem proves to be a matter of $f(x) = x$ and π is assuminated late I D to $\epsilon_{\text{channel. The use of the solution of the solution of the solution of the solution of ϵ many integration. Since ϵ is the solution of ϵ and ϵ is the solution of ϵ and ϵ is the solution of ϵ is t$ f from $\langle 2r \rangle$, there is a close relationship between f the f the f . $\lim_{\alpha \to 0} p = 1$, $n(\alpha) = 0$, $n(\alpha) = \alpha$ and sj is coordinated the ER n If one assumes that $\alpha = 1$ and $\beta = 1$, $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and J is coordinated left-LR- \hbar - $\frac{2}{3}$ chemistry $convex$, then from (27), as a result, there will be inequality (see [42]): challenging, inequalities can be used to approximate the solution. Since many inequalities $\frac{1}{2}$ challenging, inequalities can be used to approximate the solution. Since many inequalities challenging, inequalities can be used to approximate the solution. Since \mathcal{L} challenging, inequalities can be used to approximate the solution. Since \mathcal{L} challenging, inequalities can be used to approximate the solution. Since many inequalities challenging, inequalities can be used to approximate the solution. Since many inequalities \mathcal{L} If one assumes that $\alpha = 1$ and $\beta = 1$, $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and J is coordinated left-LR- \hbar -
convex then from (27) as a result, there will be inequality (see [421). convex, then from (27), as a result, there will be inequality (see [42]): If one assumes that $\alpha=1$ If one assumes that $\alpha = 1$ and $\beta = 1$, $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and J is coordinated left-LR- \hbar convex, then from (27), as a result, there will be inequality (see [42]): **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly \ldots enew, men_from $\left(\frac{2R}{\lambda}\right)$ If one assumes that $\alpha = 1$ and $\beta = 1$, $convex,$ $unex$ $\frac{1}{4}$ one assumes of $\frac{1}{4}$ and $\frac{1}{4}$ $\frac{1}{4}$ are $\frac{1}{4}$ $\frac{1}{4}$ μ_{200} compared, that $x = 1$ and $\theta =$ $\frac{1}{2}$ department of Mathematics, $\frac{1}{2}$ $\frac{1}{2}$, $\frac{1}{2}$ $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$; $convex$, then from (27), as a result, there will be inequality (see [42]): If one assumes that $\alpha = 1$ and $\beta = 1$, $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and J is coord $\frac{1}{2}$ eroilor Bras $\frac{1}{2}$ $\frac{1}{2}$ are $\frac{1}{2}$ being and its (computer Section). $\frac{1}{2}$ eroilor Bras $\frac{1}{2}$

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J\left(\frac{\sigma+i}{2}, \frac{\varepsilon+v}{2}\right)
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\supseteq \frac{1}{2}\left[\frac{1}{i-\sigma}\int_{\sigma}^{i} J\left(x, \frac{\varepsilon+v}{2}\right)dx + \frac{1}{v-\varepsilon}\int_{\varepsilon}^{v} J\left(\frac{\sigma+i}{2}, y\right)dy\right] \supseteq \frac{1}{(i-\sigma)(v-\varepsilon)}\int_{\sigma}^{i} \int_{\varepsilon}^{v} J\left(x, y\right)dydx
$$
\n
$$
\supseteq \frac{1}{4(i-\sigma)}\left[\int_{\sigma}^{i} J\left(x, \varepsilon\right)dx + \int_{\sigma}^{i} J\left(x, v\right)dx\right] + \frac{1}{4(v-\varepsilon)}\left[\int_{\varepsilon}^{v} J\left(\sigma, y\right)dy + \int_{\varepsilon}^{v} J\left(i, y\right)dy\right]
$$
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$$
\supseteq \frac{J\left(\sigma, \varepsilon\right) + J\left(i, \varepsilon\right) + J\left(\sigma, v\right) + J\left(i, v\right)}{4}.
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about the upcoming inequality (see $[46]$): *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract about the upcoming inequality (see [46]): If $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $J_{*}(x, y) \neq J^{*}(x, y)$, then from (27), we succeed in bringing If $\hslash(v) = v$, $\hslash(\kappa) = \kappa$ and $J_*(x, y) \neq J^*(x, y)$, then from (27), we $if \ n(v) = v, \ n(x) = k \ un \ ilx$
It is a fact that $\text{Tr}(S)$. $T_{\rm eff}$ is one of the most well-known findings inequality is one of the most well-known findings in the most well-known find If $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $J_{*}(x, y) \neq J^{*}(x, y)$, then from (27), we succeed in bringing If $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $J_{\ast}(x, y) \neq J^*(x, y)$, then from (27), we succeed in bringing $i_n(x) = k$ and $j_{\frac{1}{2}}(x, y) \neq j_{\frac{1}{2}}(x, y)$, and form (z, t) , we sacceed in bringing. m_1 and m_2 (see 10), even though μ about the upcoming inequality (see [46]): If $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $\hbar(\kappa) \neq \hbar^*(\kappa \nu)$ then from (27), we succeed in brinoing $\frac{1}{2}$ ζ functions, according to Dragomir and ζ . If $\hslash(v) = v$, $\hslash(\kappa) = \kappa$ and $J_{*}(x, y) \neq J^{*}(x, y)$, then from (27), we succeed in bringing ζ functions, according to Dragomir and ζ . (ሾ∗, ∗ሿ,ሾ∗, ∗ሿ) = ሼ|∗ − ∗|, |∗ − ∗|ሽ. (7) If $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $J_{*}(x, y) \neq J^{*}(x, y)$, then from (27), we succeed in bringing ϵ functions, according to Dragomir and P (ሾ∗, ∗ሿ,ሾ∗, ∗ሿ) = ሼ|∗ − ∗|, |∗ − ∗|ሽ. (7) \mathcal{C} category of convex functions, according to Dragomir and Pearce \mathcal{C} convexity and the theory of inequalities. The theory of inequalities of inequalities of \mathcal{E} If $\hslash(v) = v$, $\hslash(\kappa) = \kappa$ and $J_{*}(x,y) \neq J^{*}(x,y)$, then from (27), we succeed in bringing about the upcoming inequality (see [46]): α is the theory of inequalities. conduct the uncoming inequality (see $[16]$). convexity and the theory of inequalities. $[46]$: can be directed from convex functions, there is a convex function \mathcal{E}^* δ in the theory of integration of δ If $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $\bar{\mu}(x, y) \neq \bar{\mu}^*(x, y)$, then from (27), we succeed in bringing about the upcoming inequality (see $[46]$): \overline{a} and the theory of inequalities. $y(x, y) = y$, $h(x) = k$ and $y_*(x, y) \neq y(x, y)$, then from tight relationship to the theory of inequalities, convexity has advanced q If $h(v) = v$, $h(\kappa) = \kappa$ and $J_{*}(x, y) \neq J^{*}(x, y)$, then from (27), we succeed in bringing If $n(v) = v$, $n(x) = \kappa$ and $J_*(x, y) \neq J_0^*(x, y)$, then from (27), we succ tight relationship to the theory of inequalities, convexity has advanced α If $h(v) = v$, $h(\kappa) = \kappa$ and $J_{*}(x, y) \neq J^{*}(x, y)$, then from (27), we succeed in bringing If $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $J_{*}(x, y) \neq J^{*}(x, y)$, then from (27), we succeed in bringing $y = v$, $n(\kappa) = \kappa$ and $J_*(x, y) \neq J_1^*(x, y)$, then from (27), we succeed in bringing really of a pure sciences. Furthermore, because of its many applications and its many appli $t_j(n(v) = v, n(x) = \kappa$ and $t_{j,k}(x, y) \neq t_j(x, y)$, then from (27), $r(x, k(x)) = \frac{1}{2\pi} \int_{0}^{\infty} \frac{1$ tight relationship to the theory of inequality $\langle \alpha, \beta \rangle \neq 0$ and $\langle \alpha, \beta \rangle$ in recent $\langle \alpha, \beta \rangle$ really of a pure sciences. Furthermore, because of its many applications and its many appli $t_j(n(v) = v, n(\kappa) = \kappa$ and $t_{j,k}(\lambda, y) \neq t_j(\lambda, y)$, then from (27), we such really of applied and pure sciences. Furthermore, because of its many applications and its $t_j(n(\sigma) = \sigma, n(\kappa) = \kappa n \mu \sigma \sigma_{\kappa}(\lambda, y) + \sigma \sigma_{\kappa}(\lambda, y)$, men from (27), we succed in $r(k\omega)$ or $k\omega$, and $\pi(\omega)$, furthermore, because of its manufactorizations and its many applications and its many tight relationship to the theory of $\{x, y, \pi\}$ π $\{y, \pi\}$ in recent $\{f(x)\}$ is advanced quickly in recent $\{f(x)\}$ really of a pure sciences. Furthermore, because of its many applications and its many appli If $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $J_*(x, y) \neq J^*(x, y)$, then from (27), we succeed in bringing \mathcal{A} $y₁$ about the apec $\mathcal{H}_1(\mathcal{C}) = \mathcal{C}, \mathcal{R}(\mathcal{K}) = \mathcal{K}$ and $\mathcal{J}_{\mathcal{K}}(\mathcal{K}, \mathcal{Y}) \neq \mathcal{J}$ \mathcal{L} integral operator; Pachpathe-type integrals in the set of \mathcal{L} about the upcoming inequality (see [46]): these new outcomes. $\frac{t}{t}$ these new outcomes. obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some $r(x) = v$, $h(x) = \kappa$ and $f_{\frac{1}{2}}(x, y) \neq f_{\frac{1}{2}}(x, y)$, in obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some $r = k$ and $J_{\ast}(x, y) \neq J_{\ast}(x, y)$, then from (2)), we succed in bringing obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some $r_{\ast}(x,y) \neq x_j(x,y)$, inter from (27), we succeed in bringing

$$
Pnets(304, 8, 125)
$$
\nHence, Theorem 6 has been verified.
\n**Remark 3.** If one assumes that a = 1 and β = 1, and h(v) = v, h(x) = k; then from (27), as a
\nresult, there will be inequality (see [43]):
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$$
\iint \left(\frac{p+1}{2}, \frac{t+u}{2}\right) dx + \frac{1}{u-1} \int_{t}^{u} g\left(\frac{p+1}{2}, y\right) dy\right] \leq_{F} \frac{1}{\left(-\frac{1}{v}\right)\left(\frac{u-1}{v}\right)} \int_{v}^{1} f\left(\frac{y}{2}, \frac{y}{2}\right) dy dx
$$
\n(52)
\n
$$
\leq_{F} \frac{1}{2\left(-\frac{1}{v}\int_{v}^{1} g\left(x, \frac{t+u}{2}\right) dx + \frac{1}{u-1} \int_{v}^{u} g\left(\frac{p+1}{2}, y\right) dy\right] \leq_{F} \frac{1}{\left(-\frac{1}{v}\right)\left(\frac{v-1}{v}\right)} \int_{v}^{1} f\left(\frac{y}{2}, \frac{y}{2}\right) dy dx
$$
\n(52)
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$$
\leq_{F} \frac{1}{4\left(-\frac{1}{v}\right)} \int_{v}^{1} \int_{v}^{1} g(x, \frac{1}{v} dx + \frac{1}{v}\int_{v}^{1} g\left(\frac{y+1}{v}\right) dy\right) \leq_{F} \frac{f\left(\frac{1}{v}\right)}{1-\left(-\frac{1}{v}\right)\left(\frac{y}{v}\right)} \int_{v}^{1} g\left(\frac{y}{2}, \frac{y}{2}\right) dy
$$
\n
$$
\leq_{F} \frac{f\left(\frac{y}{2}, \frac{1}{v}\right)}{1-\left(-\frac{1}{v}\right)\left(\frac{y}{2}, \frac{1}{v}\right)} \leq_{F} \frac{f\left(\frac{y+1}{v}\right) \cdot \frac{1}{2} \cdot f\left(\frac{y}{2}, \frac{1}{v}\right)}{1-\left(-\frac{1}{v}\right)} \int_{v}^{1} g\left(\frac{y}{2}, \frac{1}{v}\right) dy
$$
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$$
= \frac{1}{2} \left[\frac{1}{1-v}
$$

Fractal Fract. **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract the upcoming inequality (see [43]): ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* $\lim_{x \to 0} \frac{1}{x^{(1)}y^{(2)}y^{(3)}y^{(4)}}$ (x, y, y, y, men by $\frac{1}{x^{(2)}}$), we enceed the energing needs ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* If $f(x) = y$, $f(x) = x$ and \overline{u} $(x, y) \neq \overline{u}^*(x, y)$ then by (27) zine succeed in brinoing about (x, y, y) If $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $J_*(x, y) \neq J^*(x, y)$, then by (27), we succeed in bringing about $\frac{1}{2}$ and $\frac{1}{2}$ are defined as follows for coordinated as follows for coordina *where i and <i> is the gamma function. is the gamma function.* If $\hslash(v) = v$, $\hslash(\kappa) = \kappa$ and $J_{*}(x, y) \neq J^{*}(x, y)$, then by (27), we succe $T_{\rm c}$ coredited to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even tho $\frac{1}{\sqrt{2}}$ If $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $J_*(x, y) \neq J^*(x, y)$, then by (27), we succeed in bringing about If $h(v) = v$, $h(\kappa) = \kappa$ and $J_{\kappa}(x, y) \neq J^*(x, y)$, then by (2/), we succeed in bringing about
the uncoming inequality (see [43]). $T_{\rm eff}$ mainly credited to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), ev Submitted for possible open access $\ddot{}$ convex functions, according to Dragomir and Pearce $\ddot{}$ \ldots of convex functions, acceptance α the upcoming inequality (see [43]): If $k(x)$ If $h(v) = v$, $h(\kappa) = \kappa$ and $J_{\kappa}(x, y) \neq J^*(x, y)$, then by (27), we so \ldots of convex \ldots $T(h(v) = v, h(\kappa) = \kappa$ and $J_*(x, y) \neq J_*(x, y)$, then by (2/), we succeed in bringing about If $n(v) = v$, $n(\kappa) = \kappa$ and $\mathsf{J}_{\ast}(x, y) \neq \mathsf{J}_{\ast}(x, y)$, then by (27), we succeed in bringing about $\mathbf{H}=\kappa$ and $J_{\ast}(x,y)\neq J^{\ast}(x,y)$, then by (2/), we succeed in bringing about $\mathbf{r}(k)$ derived functions, there is a convex functions $\mathbf{r}(k)$ and $\mathbf{r}(k)$ between relationship between $\mathbf{r}(k)$ ι *y* ι _i(ι) $=$ ι _{*y*} c_{ℓ} functions, there is a close relationship between c_{ℓ} from convex functions, there is a close relationship between c_{ℓ} $\frac{1}{2}$ is the theory of inequalities. μ apcoming inequality (see $[$ +0]). ι ι can be directly derived from convex functions, there is a convex functions of \mathbf{r} and \mathbf{r} between relationship between \mathbf{r} $c_1 u(v) = v, u(\kappa) = \kappa u \kappa u$ c_1 derived from convex functions, there is a convex functions functions, there is a close relationship between c_1 $c_1 + c_2 = c_1 + c_2 = c_2$ the apcoming inequality (see [+0]). c_1 if $c(t) = c$, $c(t) = \kappa$ and $c(t) = c$, $c(t)$ \mathbf{y} and determining exact values for a mathematical problem prob i_j in $(v) = v$, $u(x) = \kappa$ and $j_{\kappa}(x, y) \neq j_{\kappa}(x, y)$, then years. When determining exact values for a mathematical problem proves to be $\alpha = \min_{\mathbf{y}} \mathbf{y}(\mathbf{x}, \mathbf{y}) + \mathbf{y}(\mathbf{x}, \mathbf{y})$, then by (27), we succeed in bringing about $(\text{sec } \left[\pm \omega \right]),$ Received: 14 November 2023 $\mathcal{L}_{\mathcal{F}}$ $\mathcal{F}_{\mathcal{F}}$ $\langle \cdot, \cdot \rangle$ determining exact values for a mathematical problem probl $(x, y) \neq J$ (x, y), then by (27), we succeed the bringing about Received: 14 November 2023 I_j $F_k(v) = v_j$, $F_k(v)$

$$
\mathcal{J}\left(\frac{\sigma+i}{2}, \frac{\varepsilon+v}{2}\right)
$$
\n
$$
\leq p \frac{1}{2} \left[\frac{1}{i-\sigma} \int_{\sigma}^{i} J\left(x, \frac{\varepsilon+v}{2}\right) dx + \frac{1}{\sigma-\varepsilon} \int_{\varepsilon}^{v} J\left(\frac{\sigma+i}{2}, y\right) dy \right] \leq p \frac{1}{(i-\sigma)(\sigma-\varepsilon)} \int_{\sigma}^{i} \int_{\varepsilon}^{v} J\left(x, y\right) dy dx
$$
\n
$$
\leq p \frac{1}{4(i-\sigma)} \left[\int_{\sigma}^{i} J\left(x, \varepsilon\right) dx + \int_{\sigma}^{i} J\left(x, v\right) dx \right] + \frac{1}{4(\sigma-\varepsilon)} \left[\int_{\varepsilon}^{v} J\left(\sigma, y\right) dy + \int_{\varepsilon}^{v} J\left(i, y\right) dy \right]
$$
\n
$$
\leq p \frac{J\left(\sigma, \varepsilon\right) + J\left(i, \varepsilon\right) + J\left(i, \varepsilon\right) + J\left(i, \varepsilon\right)}{4}
$$
\n
$$
(55)
$$

If π is coordinated LR-h-coursex with $f(x) = x$, $f(x) = x$ and π $(x, y) = \pi^*(x, y)$ from (28), we succeed in bringing about the upcoming classical inequality: If If is coordinated LK-h-convex with $h(v) = v$, $h(\kappa) = \kappa$ and $\iint_{\kappa} (x, y) = \iint_{\kappa} (x, y)$, then K-h-convex with h
∙ ex with $\hbar(v) = v$, $\hbar(\kappa) =$ LK -h-convex with $h(v) = v$, $h(\kappa) = \kappa$ and $J_{*}(x, y) = J$ $\mathfrak{m}(v) = v, \mathfrak{n}(\kappa)$ $= v, \hbar(\kappa) = \kappa$ and $J_*(x)$ with $h(v) = v$, $h(\kappa) = \kappa$ and $J_{\ast}(x, y) = J_{\cdot}(x, y)$, then $h(\kappa) = \kappa$ and $J_{\ast}(\kappa)$ $, and J_*(x,y) = J^*(x,y)$ $(v, h(\kappa)) = \kappa$ and $J_{\kappa}(x, y) = J_{\kappa}(x, y)$, then $d \iint_{*}(x, y) = \iint_{\mathbb{T}}(x, y)$ If $\mathcal J$ is coordinated LR-h-convex with $\hslash(v) = v$, $\hslash(\kappa) = \kappa$ and $\mathcal J_*(x,y) = \mathcal J^*(x,y)$, then \sim \sim \sim \sim \sim \sim \sim \sim \sim

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convexity and the theory of inequalities. The theory of inequalities of inequalities of inequalities.

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Riemann–Liouville fractional integral operator; Pachpatte-type inequalities

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these new outcomes.

2 Department of Mathematics and Computer Science, Alabama State University,

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Tareq Saeed 1*,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4**

Academic Editor: Bruce Henry

$$
\mathcal{I}\left(\frac{\partial^{+i}}{2},\frac{\varepsilon+v}{2}\right)
$$
\n
$$
\leq \frac{\Gamma(\alpha+1)}{4(i-\delta)^{\alpha}} \left[\mathcal{J}_{\mathcal{O}^{+}}^{\alpha} \mathcal{J}\left(i,\frac{\varepsilon+v}{2}\right) + \mathcal{J}_{i}^{\alpha} \mathcal{J}\left(\sigma,\frac{\varepsilon+v}{2}\right) \right] + \frac{\Gamma(\beta+1)}{4(\upsilon-\varepsilon)^{\beta}} \left[\mathcal{J}_{\varepsilon^{+}}^{\beta} \mathcal{J}\left(\frac{\partial^{+i}}{2},\upsilon\right) + \mathcal{J}_{\upsilon}^{\beta} \mathcal{J}\left(\frac{\partial^{+i}}{2},\varepsilon\right) \right]
$$
\n
$$
\leq \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{4(i-\delta)^{\alpha}(\upsilon-\varepsilon)^{\beta}} \left[\mathcal{J}_{\mathcal{O}^{+},\varepsilon^{+}}^{\alpha,\beta} \mathcal{J}\left(i,\upsilon\right) + \mathcal{J}_{\mathcal{O}^{+},\upsilon}^{\alpha,\beta} - \mathcal{J}\left(i,\varepsilon\right) + \mathcal{J}_{i-\varepsilon^{+}}^{\alpha,\beta} \mathcal{J}\left(\sigma,\upsilon\right) + \mathcal{J}_{i-\varepsilon^{+}}^{\alpha,\beta} \mathcal{J}\left(\sigma,\varepsilon\right) \right]
$$
\n
$$
\leq \frac{\Gamma(\alpha+1)}{8(i-\delta)^{\alpha}} \left[\mathcal{J}_{\mathcal{O}^{+}}^{\alpha} \mathcal{J}\left(i,\varepsilon\right) \mathcal{J}\mathcal{J}_{\mathcal{O}^{+}}^{\alpha} \mathcal{J}\left(i,\upsilon\right) + \mathcal{J}_{i}^{\alpha} \mathcal{J}\left(\sigma,\varepsilon\right) + \mathcal{J}_{i}^{\alpha} \mathcal{J}\left(\sigma,\upsilon\right) \right] \right].
$$
\n
$$
+ \frac{\Gamma(\beta+1)}{8(\upsilon-\varepsilon)^{\beta}} \left[\mathcal{J}_{\varepsilon^{+}}^{\beta} \mathcal{J}\left(\sigma,\upsilon\right) + \mathcal{J}_{\upsilon}^{\beta} \mathcal{J}\left(\sigma,\varepsilon\right) + \mathcal{J}_{\varepsilon^{+}}^{\beta} \mathcal{J}\left(i,\upsilon\right) + \mathcal{
$$

convexity and the theory of inequalities.

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convexity and the theory of inequalities.

Fractal Fract. **2024**, *8*, x FOR PEER REVIEW 4 of 24

(ii) It can be easily seen that " ≤ " *looks like "left and right" on the real line* ℝ, *so we call*

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–

 $\frac{1}{\sqrt{2}}$ is a familiar familiar fact that (ℝ $\frac{1}{\sqrt{2}}$ is a complete metric space.)

 \mathcal{N}_{max} and \mathcal{N}_{max} and \mathcal{N}_{max} and \mathcal{N}_{max}

Keywords: interval-valued mappings over coordinates; left and right ℏ -Convexity; double

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challenging, including, including, including, including to approximate the solution. Since many inequalities o

category of classical convex functions, according to Dragomir and Pearce [1]. This

challenging, including, including, including, including to approximate the solution. Since many inequalities o

Riemann–Liouville fractional integral operator; Pachpatte-type inequalities

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years. When determining exact values for a mathematical problem proves to be

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Fractal Fract. **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *The near emerges as the gent given being merceing emerges* the product of two coordinated LR-ħ-convex IVMs. These inequalities ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* Ԓ∗() *both are Riemann integrable (-integrable) over* ሾℴ, ሿ. *Moreover, if* Ԓ *is -integrable The next choicemes, we are going to plan bory interesting emerities that were because the product of two coordinated LR-h-convex IVMs. These inequalities are known as Pa* ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* Ԓ∗() *both are Riemann integrable (-integrable) over* ሾℴ, ሿ. *Moreover, if* Ԓ *is -integrable Theorem 3* (*Letting 22*) *I. <i>Letting and being climentic that* will be a common been been as pachaatte's ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* **Theorem 2** ([42])**.** *Let* Ԓ:ሾℴ, ሿ ⊂ℝ→ℝூ *be an , given by* Ԓ() = ሾԒ∗(), Ԓ∗()ሿ *for all* In the hext outcomes, we are going to find very interesting outcomes that will be obtained over
the product of two coordinated LR-h-convex IVMs. These inequalities are known as Pachpatte's
inequalities. $-$
In the next outcomes, we are going to find very interesting outcomes that will be obtained over T_{eff} *μπιτιες*, T_{rel} *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *inequalities. Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *The quarriers.* (ሾ∗, ∗ሿ,ሾ∗, ∗ሿ) = ሼ|∗ − ∗|, |∗ − ∗|ሽ. (7) In the next outcomes, we are going to find very interesting outcomes that will be comparison and the theory of inequalities. m equalities. in the next outcomes, we are going to final very interesting outcomes that will be $The quantities,$ \leq $\frac{1}{4}$. challenging, incorporation contains can be used to the solution. can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L} th the heat outcomes, we are going to find only interesting outcomes that will be obtained occurred the product of two coordinated LR- \hbar -convex IVMs. These inequalities are known as Pachpatte's m equatives. μ an bedirectly derived from convex functions, there is a close relationship between μ and μ between μ between μ and μ between μ and μ α be directly derived from convex functions, there is a close relationship between α close relationship between α $\overline{\mathfrak{p}}$ the product of two coordinated LR-h-convex IVMs. These inequalities are known as Pachpatte's
inequalities \ldots qualities can be used to approximate the solution. Since \ldots \dot{m} are manualities realms of applied and pure sciences. Furthermore, because of its many applications and \mathbf{F} right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also $inomialize$ \mathbf{r} i riequalities. results that examples are non-trivial. By taking the product of two left and product of two left and two \mathbf{r} $requations$. Liouville fractional integral to derive the major results of the research. We also examine the key α -converging converging converging codomain. We exploit the use of α $\mathcal{L}_{\text{reco}}$ 2 Department of Mathematics and Computer Science, Alabama State University, \overline{D}

 $\frac{1}{\sqrt{2}}$ is a familiar familiar fact that (ℝ $\frac{1}{\sqrt{2}}$ is a complete metric space.)

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

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obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

 $\overline{}$ is a familiar familiar fact that (ℝ $\overline{}$

let \hbar_1 , \hbar_2 : i, $\hbar_1, \hbar_2 : [0, 1] \rightarrow \mathbb{R}^+$. If J l, $\left[\begin{array}{cc} and & \jmath(x,y) \end{array}\right]$ and $\jmath(x,y)$ r the following inequa Γ *heore* $[1] \rightarrow \mathbb{R}^+$. If) \mathbb{R}^+ . If $J \times g \in \mathfrak{TD}_{\Omega}$, then , () න Ԓ∗() qualities hold: P *t* \mathcal{J} *l*, q : $\Omega \to \mathbb{R}$ α , then the follo \overline{a} n the following inequalities h ℴ tt (x, y) ∈ Ω .
pld: *be two coordina* ng inequalities . ℴ = ቈ() න Ԓ∗() **Theorem** 7. Let Π , $q: \Omega \to \mathbb{R}^+$ be two coordinated LR- \hbar -convex IVMs $\Pi(x, y) = [\Pi(x, y), \Pi^*(x, y)]$ and $q(x, y) = [q(x, y), q^*(x, y)]$ for all (**Theorem** 7. Let Π , $q: \Omega \to \mathbb{R}^+$ be two coordinated LR- \hbar -convex IVMs on Ω , given by $\Pi(x, y) = [\Pi(x, y), \Pi^*(x, y)]$ and $q(x, y) = [q(x, y), q^*(x, y)]$ for all $(x, y) \in \Omega$ and \int_{0}^{1} \int_{0}^{1} \int_{0}^{1} \int_{0}^{1} \int_{0}^{1} \int_{0}^{1} et ∏, σ : Ω \rightarrow \mathbb{R}^+ *be two coordinated LR-fi-convex IVMs on* Ω , given by $(x, y, y, z) = \int_{\mathcal{X}} f(x, y) dx$ and $g(x, y) = \int_{\mathcal{X}} (x, y) dx$, $g^*(x, y)$ for all $(x, y) \in \Omega$ and $\rightarrow \mathbb{R}^+$ *If* $\pi \times \rightarrow$ $α⁺$ *he two coordinated LR-ħ*-convex *IVMs on Ω, given by* \int_{1}^{1} concentration integrable (*x*₁*u*) or *i f*_{*i*} *over i j*_{*s*}*com i*_{*s*}*ni o*_{*i*} *o*_{*i*} *o*^{*i*} *f*_{*n*} *<i>n*^{*i*} *(x*₁*u*) *i f*_{*n*} *<i>n*^{*i*} *(x*₁*u*) *ic Q and* $\mathbf{a} \in \mathfrak{TD}_{\infty}$, then the **THEOREM 2.** ELEV, $f: \Sigma x \to \mathbb{R}$ be two coordinated EX-*N*-Concex TV MS on
 $\mathbb{I}(x, y) = [\mathbb{I}^-(x, y), \mathbb{I}^*(x, y)]$ and $g(x, y) = [g(x, y), g^*(x, y)]$ for all (x, y) $\mathcal{L}_{\mathcal{J}_{\mathcal{K}}(x,y)} = \left[\mathcal{I}_{\mathcal{K}}(x,y), \mathcal{I}_{\mathcal{J}}(x,y)\right]$ and $\mathcal{J}(x,y) = \left[\mathcal{I}_{\mathcal{K}}(x,y), \mathcal{I}_{\mathcal{K}}(x,y)\right]$ for an (x, y)
let $\mathcal{F}_{\mathcal{K}}$, $\mathcal{F}_{\mathcal{K}}$ on $\mathcal{I}_{\mathcal{K}}$ and $\mathcal{F}_{\mathcal{K}}$ and the following in μ_{ν} μ_{0} , μ_{2} \cdot $\lbrack v, r \rbrack$ \cdots $\lbrack v, r \rbrack$ **THEOREM 2.** Extrappendix $T^* \leq T^* \leq T^*$ is considered by the contentrated by the contentrated by T^* (\mathbf{y} ii) $T^* \leq T$ and \mathbf{y} $T^* \leq T$ $\Gamma_{\mathcal{O}_k}(x, y) = \Gamma_{\mathcal{O}_k}(x, y)$, $\Gamma_{\mathcal{O}_l}(x, y)$ is and $\Gamma_{\mathcal{O}_k}(x, y) = \Gamma_{\mathcal{O}_k}(x, y)$ for an $(x, y) \in \mathbb{R}^2$ and $\Gamma_{\mathcal{O}_l}(x, y) = \Gamma_{\mathcal{O}_k}(x, y)$. μ *i* μ ₁, μ ₂, μ ₃, μ ₁, μ ₃, μ ₂, μ ₂, μ ₂, μ ₂, μ ₁, μ ₂, μ ₂ $T_{\text{L}}(x, y) = \int \frac{1}{\pi^2} \int \frac{1}{x} \exp\left[\frac{1}{2} \int \frac{1}{x} \int \frac{1}{y} \int \frac{1}{y} \exp\left[\frac{1}{2} \int \frac{1}{y} \int$ $\begin{bmatrix} \mathcal{F}_{\mathbf{y}}(x,y), & \mathcal{F}_{\mathbf{y}}(x,y) \end{bmatrix}$ and $\begin{bmatrix} \mathcal{F}_{\mathbf{y}}(x,y), & \mathcal{F}_{\mathbf{y}}(x,y) \end{bmatrix}$ for an $(x,y) \in \mathbb{R}^n$ and \mathbb{R}^+ if $\Pi \times \mathfrak{g} \in \mathfrak{F}$. Then the following inequalities hold: $\mathcal{L}_{\mathcal{F}}[f] \rightarrow \mathbb{R}$, $\mathcal{L}_{\mathcal{F}}[f] \wedge \mathcal{L}_{\mathcal{F}}[f]$ and the following inequalities now. T_1 (\mathbb{F}_1 be two coolumnical EX-R convex TV MS on Ω , given by \mathbb{F}_1 or \mathbb{F}_1 for all $(\mathbb{F}_1) \subset \Omega$ and α, *α*) and $f(x, y) = [f_*(x, y), f(x, y)]$ for an $(x, y) \in \mathbb{R}^2$ and $f(x, y) = f(x, y)$ is an *i* $f(x, y) = f(x)$ *and if a* $f(x, y) = f(x)$ *f b*_{*y*} \rightarrow *b*₂, *o*_{*n*}, *inter int following incquaring nom.* Interval and fuzzy Riemann-type integrals are defined as follows for coordinated Let \hbar_1 , $\hbar_2\,:\, [0,\,1]\to\mathbb{R}^+$. If $\text{J}\hskip-2pt\downarrow \times$ $\jmath\in \mathfrak{L}\Omega_\Omega$, then the following inequalities hold: Fractal Interval and fuzzy Riemann-type integrals are defined as follows for coordinated **L** neorei Let \hbar_1 , $\hbar_2: [0,1]\to\mathbb{R}^+$. If $\text{J} \times g\in \mathfrak{LO}_{\Omega'}$ then the following inequalities hold: Fractal Fract Interval and fuzzy Riemann-type integrals are defined as follows for coordinated $Let \cup,$ $[T] \to \mathbb{R}^+$. If $J \times \gamma \in \mathfrak{LO}_{\Omega'}$ then the following inequalities hold: **FROMALE FRACTAL FRACTAL FRACTAL FRACTAL FRACTAL FRACTAL FRACTAL SECONDOM** Interval and fuzzy Riemann-type integrals are defined as follows for coordinated $\iota \rightarrow \mathbb{K}_I$ $\text{Im } \mathcal{F}_1 \times \mathcal{F}_2 \subseteq \text{Im } \Omega$, then the following inequalities hold: F_{ref} F_{ref} F_{ref} or two coolumnations in the concentration of $\frac{1}{2}$, given by $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 &$ **Theorem 7.** *Let* J , $j : \Omega \to \mathbb{R}^+_I$ *be two coordinated LR-h-convex IVMs on* Ω , given *b*
 $J(\alpha, \alpha) = \Pi(\alpha, \alpha) = \Pi^*(\alpha, \alpha) = \Pi^*(\$ ℐ ష ఈԒ() = ^ଵ ௰(ఈ)) −(ఈିଵԒ() ^௬ (<), (9) : Ω → R + *I be two coordinated LR-*ℏ*-convex IVMs on* Ω*, given by* $J\!J}(x,y) = [J_*(x,y), J^*(x,y)]$ and $J(x,y) =$ $J(x,y) = [J_*(x,y), J^*(x,y)]$ and $g(x,y) = [J_*(x,y), J^*$ $J(x,y) = [J_*(x,y), J^*(x,y)]$ and $J(x,y) = [J_*(x,y), J^*(x,y)]$ for I_{α} *t* Π_{α} : $\Omega \setminus \mathbb{P}^+$ be type coordinated IR α convex IVMs of $g(x, y) = [g_*(x, y), g^*(x, y)]$ for all $(x, y) \in \Omega$ and $\Omega \rightarrow \mathbb{P}^+$ be type coordinated LR \hbar conver *IVMs* on Ω given by $J_*(x,y), J^*(x,y)$ for all $(x,y) \in \Omega$ and
a following inaqualities hold: *<u>Photometricial Riversity IVMs</u> on O are defined as* $g^*(x, y)$ for all $(x, y) \in \Omega$ and *let* \hbar_1 , \hbar_2 : $[0, 1] \rightarrow \mathbb{R}^+$. If **Theorem 7.** Let $J, \gamma : \Omega \to \mathbb{R}^+_1$ be two coordinated LR-h-convex IVMs Let \hbar_1 , $\hbar_2:[0,\,1]\to\mathbb{R}^+$. If $\rm J\ltimes j\in\mathfrak{D}_{\Omega'}$ then the following inequalities hold: \mathbf{D} **d** \mathbf{D} and \mathbf{D} $\rightarrow \mathbb{R}^+$ be two coordinated LR-b-convex IVMs on \mathbf{D} aiven by $T = \left[\frac{\pi (x, y)}{\pi^*(x, y)} \right]$ and $g(x, y) = \left[a (x, y) \right] e^{x}(x, y)$ for a $g\in \mathfrak{TD}_{\Omega'}$ then the following inequalities hold: **categories convergences c** and Area **c** and Area **c** $\text{if}(x,y) = |J_{\ast}(x,y), J^{\ast}(x,y)|$ a Let \hbar_1 , \hbar_2 : $[0, 1] \rightarrow \mathbb{R}^+$. If $J \times g$ **Theorem 7.** Let \mathbf{J} , $\mathbf{g} : \Omega \to \mathbb{R}^+_l$ be tu Let $h_1, h_2 : [0, 1] \to \mathbb{R}^+$. If $J_1 \times J_2 \in \mathfrak{D}_0$, then the following inequality **catability** of **c** convex **c** convex functions, and \mathbb{R}^+ are \mathbb{R}^+ be two co $J(x,y) = [J(x,y), J^*(x,y)]$ and $g(x,y)$. Let $\hbar_1, \hbar_2 : [0, 1] \to \mathbb{R}^+$. If $\Lambda \times \gamma \in \mathfrak{TD}_{\Omega}$, though category of \mathcal{L} functions, according to Dragomir and Pearce \mathcal{L} . $\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n}$ $T_1(x,y) = [J_x(x,y),J_y(x)]$ can be resulted to $T_1(x,y) = [J_x(x,y)]$ $\alpha_1, \alpha_2, \alpha_1, \alpha_2, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_7, \alpha_8,$ **Theorem 7.** Let J_i , $\text{j}: \Omega \to \mathbb{R}^+_i$ be two coordinated I Let $h_1, h_2 : [0, 1] \to \mathbb{R}^+$. If $J_1 \times J_2 \in \mathfrak{SD}_{\Omega}$, then the following inequalities hold: **Drem 7.** Let $J, \, j: \Omega \to \mathbb{R}^+_1$ be two coordinated LR- \hbar -control Let $h_1, h_2 : [0, 1] \to \mathbb{R}^+$. If $J \times g \in \mathfrak{SD}_0$, then the following inequalities hold: **COMB** CONVERGIOUT Theorem 7. Let Π , $q: \Omega \to \mathbb{R}^+$ be two coordinated LR- \hbar -convex 1 $\bar{f}(x,y) = [f(x,y), f(x,y)]$ and $g(x,y) = [f(x,y), f(x,y)]$ for Let $\hbar_1, \hbar_2 : [0, 1] \to \mathbb{R}^+$. If $J \times \gamma \in \mathfrak{TD}_{\Omega}$, then the following inequalities category of convex functions, and \mathbb{R}^+ functions, and P $\sum_{i=1}^{n} \frac{1}{i!} \sum_{i=1}^{n} \frac{1}{i!} \sum_{i$ $T_1(x,y) = [T_1(x,y), T_2(x,y)]$ can $T_1(x,y) = [T_2(x,y)]$ and $T_2(x,y) = [T_1(x,y)]$ for n_1, n_2, \ldots, n_r . The state is $j \wedge j \wedge j \wedge \cdots \wedge j_r$ then the following inequalities now. $\mathfrak{I}, \ \overline{\jmath}: \Omega \to \mathbb{R}^+_I$ be two coordinated LR- \hbar -convex IVMs on Ω \overline{B} $\rightarrow \mathbb{R}^+$. If $J_1 \times J_2 \in \mathfrak{SD}_{\mathcal{O}}$, then the following inequalities hold: convexity and the theory of inequalities. $\overline{\Pi}(x, u) = \overline{\Pi}(x, u) \overline{\Pi}^*(x, u)$ and $g(x, u) = \overline{\Pi}(x, u)$ $\begin{bmatrix} \cos \theta & \sin \theta \\ \cos \theta & \sin \theta \end{bmatrix}$ (0, 1]. $\Rightarrow \mathbb{R}^+$ If $\mathbb{I} \times \mathfrak{a} \in \mathfrak{I}$), then the inequality has seen applications and a straightforward integrations and a straightforward intrinsic geometric explanation. tt Л, \jmath : $\Omega\to\mathbb{R}^+_I$ be two coordinated LR-ħ-convex IVMs on Ω , χ $I_{\sigma}(\alpha, \beta) = \left[\begin{matrix} \mathfrak{I}_{\mathfrak{F}}(\alpha, \beta), & \mathfrak{I} \end{matrix}\right] (\alpha, \beta)$ and $\int_{\alpha, \beta}(\alpha, \beta) = \left[\begin{matrix} \mathfrak{I}_{\mathfrak{F}}(\alpha, \beta), & \mathfrak{I} \end{matrix}\right]$
let \mathfrak{h}_{α} \mathfrak{h}_{α} \cdot [0, 1] $\rightarrow \mathbb{R}^+$ If $\pi \times \mathfrak{a} \in \mathcal{K}$), then the fo **Theorem 7.** Let $\mathcal{J}, \mathcal{J}: \Omega \to \mathbb{R}^+_1$ be two coordinated LR-h-convex I $J(x,y) = [J_*(x,y), J^*(x,y)]$ and $g(x,y) = [J_*(x,y), J^*(x,y)]$ for let $h_1, h_2 : [0, 1] \to \mathbb{R}^+$. If $J_1 \times J_2 \in \mathfrak{TD}_{\Omega}$, then the following inequalities **Theorem** 7. Let J_j , $j : \Omega \to \mathbb{R}^+_1$ be two coordinated LR-h-convex $c(x,y) = [J_*(x,y), J_*(x,y)]$ and $g(x,y) = [J_*(x,y), J_*(x,y)]$ for let $h_1,h_2: [0,\ 1]\rightarrow \mathbb{R}^+$. If $\mathbb{J}\times\mathbb{J}\in \mathfrak{U}_{\Omega}$, then the following inequalitie \hat{r}^+_I be two coordinated LR-ħ-convex IVMs on Ω , given by $\mathcal{I}_{1*}(\mathcal{X},\mathcal{Y})$, $\mathcal{I}_{1}(\mathcal{X},\mathcal{Y})$ is a complete metric $\mathcal{I}_{1*}(\mathcal{X},\mathcal{Y})$, $\mathcal{I}_{1}(\mathcal{X},\mathcal{Y})$ for an integral $\mathcal{I}_{1}(\mathcal{X},\mathcal{Y})$, $\mathcal{I}_{2*}(\mathcal{X},\mathcal{Y})$, $\mathcal{I}_{2*}(\mathcal{X},\mathcal{Y})$, $\mathcal{I}_{2*}(\mathcal{$ **Theorem** 7. Let J_l , $j : \Omega \to \mathbb{R}^+_l$ be two coordinated LR-h-convex IVMs on $c(x,y) = [J_*(x,y), J^*(x,y)]$ and $g(x,y) = [J_*(x,y), J^*(x,y)]$ for all (x,y) let $h_1,h_2: [0,\ 1]\to \mathbb{R}^+$. If $\mathsf{J}]\times g\in \mathfrak{U}_{\Omega}$, then the following inequalities hold: coordinated LR-ħ-convex IVMs on Ω , given by $\begin{bmatrix} \langle x,y \rangle \end{bmatrix}$ and $\begin{bmatrix} \langle x,y \rangle \end{bmatrix}$ is a complete metric space. Then the following inequalities hold: **n** 7. Let JJ, $g: \Omega \to \mathbb{R}^+_I$ be two coordinated LR-h-convex IVMs on Ω , given by $c(x,y) = [J_*(x,y), J^*(x,y)]$ and $g(x,y) = [J_*(x,y), J^*(x,y)]$ for all $(x,y) \in \Omega$ and $i_1, i_2: [0, 1] \to \mathbb{R}^+$. If $J \times g \in \mathfrak{TD}_{\Omega}$, then the following inequalities hold: **Copyright:** © 2024 by the authors. **m** 7. Let J , $j: \Omega \to \mathbb{R}^+_I$ be two coordinated LR-h-convex IVMs on Ω , given by $c(x,y) = [J_*(x,y), J^*(x,y)]$ and $g(x,y) = [J_*(x,y), J^*(x,y)]$ for all $(x,y) \in \Omega$ and $h_1,h_2\,:\, [0,\,1]\to\mathbb{R}^+\,.$ If $\,$ J \times $\,j\in\mathfrak{U}_\Omega,$ then the following inequalities hold: Λ - \hbar -convex IVMs on Ω , given by $\sum_{i=1}^{\infty} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{j$ t JJ, $\texttt{g}:\Omega\to \mathbb{R}_{I}^{+}$ be two coordinated LR-h-convex IVMs on Ω , given by $c_j[f] = [f]_*(x,y)$, $c_j[f] = [f]_*(x,y)$, $f^*(x,y)$ for all $(x,y) \in \Omega$ and let $h_1, h_2 : [0, 1] \to \mathbb{R}^+$. If $J] \times \overline{\jmath} \in \mathfrak{LO}_{\Omega}$, then the following inequalities hold: **can be directed from the direction from convex** I of \mathbb{R}^+ between convex Let $\bar{b}_1, \bar{b}_2, \bar{b}_1, \bar{b}_2 \in \mathbb{R}^+$ If $\bar{B} \times \bar{a} \in \mathcal{F}$. Then the most well-known finding in the most well-known f $c = 1$ convex functions, according to $\frac{1}{2}$ **theorem** 7. Let Π , $q: \Omega \to \mathbb{R}^+$ be two coording $\overline{u}(x,y) = \overline{u}(x,y)$, $\overline{u}(x,y)$ and $\overline{u}(x,y) = \overline{u}(x,y)$ let \hbar_1, \hbar_2 : $[0, 1] \rightarrow \mathbb{R}^+$. If $\pi \times \tau \in \mathfrak{D}_Q$, then the can be directly derived from convex functions, there is a close relationship between $J(x,y) = [J(x,y), J^*(x,y)]$ and $J(x,y) =$ let \hbar_1 , \hbar_2 : $[0, 1] \rightarrow \mathbb{R}^+$. If $\pi \times \eta \in \mathfrak{TD}_{\Omega}$, then th **Keywords: Theorem 7.** Let \mathcal{J}_1 , $\mathcal{J}_2 : \Omega \to \mathbb{R}^+_1$ be two coordinated **Theorem 7.** Let J_J , $g: \Omega \to \mathbb{R}_I^+$ be two coordinated LR- \hbar -convex IVMs on Ω ,
 $J_J(x,y) = [J_*(x,y), J^*(x,y)]$ and $g(x,y) = [J_*(x,y), J^*(x,y)]$ for all (x,y) $\overline{J}(x,y) = [\overline{J}(x,y), \overline{J}(x,y)]$ **obtained Theorem 7.** Let Π , $q: \Omega \to \mathbb{R}^+$ be two cases $\pi(x, y) = \pi(x, y)$, $\pi^*(x, y)$ and $\pi(x, y)$ obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some **Theorem 7.** Let $J, \, J: \Omega \to \mathbb{R}^+_I$ be two coordinated LR- \hbar -convex $\pi(x, y) = [\pi(x, y), \pi^*(x, y)]$ and $g(x, y) = [\pi(x, y), \pi^*(x, y)]$ for obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some restrictions on the set $\int_{I} \int_{I} x \cdot \Delta I \rightarrow \infty$ or two coordinated functions that can be seen as applications of $\int (x, y)$ results that examples are non-trivial. By taking the product of the product of the product of two left and the product of the produ $\pi(x, \theta) = \pi(x, \theta) \pi(x, \theta)$ $\mathcal{L}(\mathcal{M}, \mathcal{M}) = \mathcal{M}(\mathcal{M}, \mathcal{M})$ and cases are also discussed by taking some $\mathcal{M}(\mathcal{M}, \mathcal{M})$ $r_{1}, r_{2} \neq r_{1} \neq r_{2} \neq r_{1} \neq r_{2} \ne$ **Theorem 7.** Let \mathcal{J}_t , $\mathcal{J}_t : \Omega \to \mathbb{R}^+_t$ be two coordinated LF $J(x,y) = [J_*(x,y), J^*(x,y)]$ and $g(x,y) = [J_*(x,y),$ let $h_1, h_2 : [0, 1] \to \mathbb{R}^+$. If $J_1 \times J_2 \in \mathfrak{D}$ _O, then the followin **Theorem 7.** Let $J, \gamma : \Omega \to \mathbb{R}^+_I$ be two coordinated L $J(x,y) = |J_*(x,y), J^*(x,y)|$ and $g(x,y) = |J_*(x,y)|$ Let $h_1,h_2\,:\, [0,\,1]\to\mathbb{R}^+$. If $J\mathrm{J}\times g\in \mathfrak{LO}_{\Omega'}$ then the follown **Theorem 7.** Let $J, \gamma : \Omega \to \mathbb{R}^+_1$ be two coordinated LR-h-conversion $J(x,y) = |J_{*}(x,y), J^{*}(x,y)|$ and $g(x,y) = |J_{*}(x,y), J^{*}(x,y)|$ let \hbar_1 , $\hbar_2: [0, 1] \to \mathbb{R}^+$. If $\text{J} \times g \in \mathfrak{LO}_{\Omega}$, then the following inequal **Theorem 7.** Let Π , $q: \Omega \to \mathbb{R}^+_1$ be two coordinated LR- \hbar -convex IVMs on Ω , β $J(x,y) = |J_x(x,y), J^*(x,y)|$ and $g(x,y) = |J_x(x,y), J^*(x,y)|$ for all $(x,y) \in$ let $h_1, h_2 : [0, 1] \to \mathbb{R}^+$. If $J \times \gamma \in \mathfrak{TD}_\Omega$, then the following inequalities hold: **Theorem 7.** Let $J, \gamma : \Omega \to \mathbb{R}^+_I$ be two coordinated LR- \hbar -convex IVMs on Ω , $J(x,y) = |J_*(x,y), J^*(x,y)|$ and $g(x,y) = |J_*(x,y), J^*(x,y)|$ for all (x,y) let \hbar_1 , $\hbar_2: [0,1] \to \mathbb{R}^+$. If $\text{J} \times g \in \mathfrak{LO}_{\Omega}$, then the following inequalities hold: **Theorem 7.** Let $J, \gamma : \Omega \to \mathbb{R}^+_I$ be two coordinated LR- \hbar -convex IVMs on Ω , given by $J(x,y) = |J_*(x,y), J^*(x,y)|$ and $g(x,y) = |J_*(x,y), J^*(x,y)|$ for all $(x,y) \in \Omega$ and let \hbar_1 , $\hbar_2: [0,1] \to \mathbb{R}^+$. If $\text{J} \times g \in \mathfrak{LO}_{\Omega'}$ then the following inequalities hold: The example \vec{r} derive the major results of the results of the key $\pi(x, t) = \pi(x, t)$. $\pi^*(x, t)$ and $\pi^*(x, t)$ $I_{\text{det}}(x, y) = \begin{bmatrix} -3(x, y), & -3(x, y) \\ 2(x, y), & -3(x, y) \end{bmatrix}$ obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some **Theorem 7** Let \mathbb{F}_{q} \mathbb{F}_{q} of \mathbb{F}_{q} \mathbb{F}_{q} $\overline{\mathbf{d}}(x, y) = [\overline{\mathbf{d}}(x, y), \overline{\mathbf{d}}^*(x, y)]$ and $\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ \mathcal{L} integral to derive the major results of the major results of the key also examine the $\mathbf{I}(\gamma, \mu) = \begin{bmatrix} \mathbf{I} & \mathbf{I} & \mathbf{I} \\ \mathbf{I} & \mathbf{I} & \mathbf{I} \end{bmatrix}$ **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly $\frac{1}{2}$ Department of $\frac{1}{2}$ $\det\, h_1$, $h_2\;:\,[0,\;1]\to\mathbb{R}$

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\frac{\Gamma(\alpha)\Gamma(\beta)}{(i-\sigma)^{\alpha}(\mathfrak{v}-\varepsilon)^{\beta}} \left[\mathfrak{J}^{\alpha,\beta}_{\sigma^+,\varepsilon^+} J[(\mathfrak{v},\mathfrak{v}) \times \mathfrak{J}(\mathfrak{v},\mathfrak{v}) + \mathfrak{J}^{\alpha,\beta}_{\sigma^+,\mathfrak{v}} - J[(\mathfrak{v},\mathfrak{v}) \times \mathfrak{J}(\mathfrak{v},\mathfrak{v}) + \mathfrak{J}^{\alpha,\beta}_{i-\rho} - J[(\sigma,\varepsilon) \times \mathfrak{J}(\sigma,\varepsilon) \times \mathfrak{J}(\sigma,\varepsilon)] \right] \n\leq_{p} \mathcal{M}(\sigma,\mathfrak{v},\varepsilon,\mathfrak{v}) \int_{0}^{1} v^{\alpha-1} \kappa^{\beta-1} [\hbar_1(1-v) \hbar_2(1-v) \hbar_1(1-\kappa) \hbar_2(1-\kappa) \n+ \hbar_1(1-v) \hbar_2(1-v) \hbar_1(\kappa) \hbar_2(\kappa) + \hbar_1(v) \hbar_2(v) \hbar_1(1-\kappa) \hbar_2(1-\kappa) \n+ \hbar_1(v) \hbar_2(v) \hbar_1(\kappa) \hbar_2(\kappa)] dv dx \n+ P(\sigma,\mathfrak{v},\varepsilon,\mathfrak{v}) \int_{0}^{1} v^{\alpha-1} \kappa^{\beta-1} [\hbar_1(v) \hbar_2(1-v) \hbar_1(1-\kappa) \hbar_2(1-\kappa) + \hbar_1(1-\kappa) \hbar_2(1-\kappa) \nv) \hbar_2(v) \hbar_1(1-\kappa) \hbar_2(1-\kappa) + \hbar_1(v) \hbar_2(1-v) \hbar_1(\kappa) \hbar_2(\kappa) + \hbar_1(1-\nu) \hbar_2(v) \hbar_1(1-\kappa) \hbar_2(v) \hbar_1(v) \hbar_2(v) \
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If Π and a are both coordinated \hbar -concave IVMs on Ω , then the inequality about \mathcal{A} expressed as follows: $\frac{1}{2}$ or $\frac{1}{2}$ order $\frac{1}{2}$ order $\frac{1}{2}$ order Ω both coordinated \hbar -concave IVMs on Ω , then the inequality above can be μ , *are defined by the one is*, then the inequality move can be d ħ-concave IVMs on Ω, then the inequality above can be $\frac{1}{2}$ or $\frac{1}{2}$, then the inequality accession to If Π and a gre hoth coordinated \hbar -concause IVMs on Ω then the inequ $\mathcal{L}_{\mathcal{A}}$ (*Let* $\mathcal{L}_{\mathcal{A}}$ *and* $\mathcal{L}_{\mathcal{A}}$ *and* $\mathcal{L}_{\mathcal{A}}$ *interval Riemann*– D_{S} \mathcal{L} and \mathcal{L} and and the same of the contract of continuous contract of the same international and the contract of the second second second contract of the second α and α , β Ω. g are
.. *If J* and *J* are both coordinated *h*-concave IVMs on Ω, then the inequality above can b expressed as follows: $\mathit{53}$ $\mathit{54}$ $\mathit{73}$ $\mathit{84}$ $\mathit{85}$ $\mathit{167}$ $\mathit{168}$ $\mathit{168}$ $\mathit{169}$ $\mathit{16$ $\int f \, f$ and $\int f$ are both coordi \mathfrak{r} *The family of all -integrable of s over coordinates and -integrable functions over* ℴ *The family of all -integrable of s over coordinates and -integrable functions over* م
TVMs oiجrdinated ħ-concave IVMs ru *The family of all -integrable of s over coordinates and -integrable functions over* If J and J are both coordinated \hbar -concave IVMs on Ω , then the inequality above can be expressed as follows: $$ If J and J are both coordinated \hbar -concave IVMs on Ω , then the inequality μ_{2} whose who first identified it is how that μ_{2} ା *be an interval-valued mapping () and* Ԓ ∈ ℐℛሾℴ,ሿ*.* ା *be an interval-valued mapping () and* Ԓ ∈ ℐℛሾℴ,ሿ*.* The result was mainly contributed to conclude the result of the result in increment 1963 who first identified it 233 . The following is the following inequality is stated. ା *be an interval-valued mapping () and* Ԓ ∈ ℐℛሾℴ,ሿ*.* inequality has several applications and a straightforward intrinsic geometric explanation. If JJJ and JJJ are both coordinated \hbar -concave IV Ms on Ω If JJ and *3* are both coordinated h-concave IVMs on Ω, then th
expressed as follows: If J and *g* are both coordinated h-concave IVMs on Ω , then the inequality above can be
expressed as follows: category of classical convex functions, according to Dragomir and Pearce [1]. This

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\frac{\Gamma(\alpha)\Gamma(\beta)}{(\mathbf{i}-\boldsymbol{\sigma})^{\alpha}(\mathbf{b}-\boldsymbol{\varepsilon})^{\beta}}\left[\mathcal{J}_{\boldsymbol{\sigma}}^{a,\beta}+\mathcal{J}(\mathbf{i},\mathbf{b})\times\mathcal{J}(\mathbf{i},\mathbf{b})+\mathcal{J}_{\boldsymbol{\sigma}^{+},\mathbf{b}}^{a,\beta}-\mathcal{J}(\mathbf{i},\boldsymbol{\varepsilon})\times\mathcal{J}(\mathbf{i},\boldsymbol{\varepsilon})\right] \n+\frac{\Gamma(\alpha)\Gamma(\beta)}{(\mathbf{i}-\boldsymbol{\sigma})^{\beta}(\mathbf{b}-\boldsymbol{\varepsilon})^{\beta}}\left[\mathcal{J}_{\mathbf{i}-,\boldsymbol{\varepsilon}}^{a,\beta}+\mathcal{J}(\boldsymbol{\sigma},\mathbf{b})\times\mathcal{J}(\boldsymbol{\sigma},\mathbf{b})+\mathcal{J}_{\mathbf{i}-,\mathbf{b}}^{a,\beta}-\mathcal{J}(\boldsymbol{\sigma},\boldsymbol{\varepsilon})\times\mathcal{J}(\boldsymbol{\sigma},\boldsymbol{\varepsilon})\right] \n\geq_{p} \mathcal{M}(\boldsymbol{\sigma},\mathbf{i},\boldsymbol{\varepsilon},\mathbf{b})\int_{0}^{1} v^{\alpha-1}\kappa^{\beta-1}[\hbar_{1}(1-v)\hbar_{2}(1-v)\hbar_{1}(1-\kappa)\hbar_{2}(1-\kappa) \n+\hbar_{1}(1-v)\hbar_{2}(1-v)\hbar_{1}(\kappa)\hbar_{2}(\kappa)+\hbar_{1}(v)\hbar_{2}(v)\hbar_{1}(1-\kappa)\hbar_{2}(1-\kappa) \n+\hbar_{1}(v)\hbar_{2}(v)\hbar_{1}(\kappa)\hbar_{2}(v)[\hbar_{1}(1-\kappa)\hbar_{2}(1-v)\hbar_{1}(1-\kappa)\hbar_{2}(1-\kappa)+\hbar_{1}(1-\nu)\hbar_{2}(1-\kappa)\hbar_{2}(1-\kappa) \nv)\hbar_{2}(v)\hbar_{1}(1-\kappa)\hbar_{2}(1-\kappa)+\hbar_{1}(v)\hbar_{2}(1-v)\hbar_{1}(\kappa)\hbar_{2}(\kappa)+\hbar_{1}(1-\nu)\hbar_{2}(1-\kappa) \n+\hbar_{1}(1-v)\hbar_{2}(1-\kappa)\hbar_{1}(1-\kappa)\hbar_{2}(1-\kappa) \
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Published: date

Tareq Saeed 1*,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4**

Published: date

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 $\frac{1}{\sqrt{2\pi}}\left(\frac{1}{\sqrt{2\pi}}\right)^{1/2}$ is a complete metric space.

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 $\left(\begin{array}{c} 0 & 0 \\ 0 & 0 \end{array} \right)$

ା *be an interval-valued mapping () and* Ԓ ∈ ℐℛሾℴ,ሿ*.*

where where we have the set of th 1963 where where 123 . The following it 25 . The following is 123 . The following is stated: 1963 who first identified it 233 . The following is how this integrated: $\sum_{i=1}^{n}$ where $\sum_{i=1}^{n}$ *Then, integrals of an area integrals of a reduced as* α *are defined as* α *are defined as* α *are defined as* α \emph{where} η_{where} τ *tuhere* τ τ the result was mainly contained to τ $where$ \mathcal{L} the result was mainly contained to \mathcal{L} τ *tuhere* θ \overline{z} The result was mainly credited to Γ σ i unteral applications and a straightforward intrinsic geometric explanation. i i_{in} intrinsic geometric geometric geometric explanations and a straightforward intrinsic geometric explanation. i unteral applications and a straightforward intrinsic geometric explanation. i i_{where} inhere category of classical convex functions, according to Dragomir and Pearce [1]. This inequality has several applications and a straightforward intrinsic geometric explanation. \overline{z} τ convex functions, according to τ τ uhere \mathcal{L} convex functions, according Dragomir and \mathcal{L} . This Dragomir and Pearce \mathcal{L} \blacksquare \mathcal{L} functions, and \mathcal{L} and \mathcal{L} . This Dragomir and Pearce \mathcal{L} ω_{m} α functions, according to Dragomir and P ϵ functions, according Dragomir and ϵ $\frac{1}{\sqrt{2}}$ ϵ and the theory of ϵ T_{m} ω T_{eff} is one of the most well-known finding inequality is one of the most well-known finding in the most well-known finding ϵ uthe ϵ T_{eff} challenging, inequalities can be used to approximate the solution. Since many inequalities ω derived from convex functions, there is a close relationship between ω close relationship between ω challenging, inequalities can be used to approximate the solution. Since many inequalities ω derived from convex functions, there is a close relationship between ω can convex ω $where$ $where$ ϵ $\mathcal{L}_{\mathcal{A}}$ r_{m} ω where ω ω there are many uses for the convex sets and convex sets and convex sets and convex ω functions in the convex functions in the convex sets and convex functions in the convex sets and convex functions in the convex μ where μ \mathcal{R} interval operator; \mathcal{R} integral operator; \mathcal{R} is the set of \mathcal{R} in the set of \mathcal where k where α intervalses; left and right α -Convexity; double-coordinates; and right α -Convexity; double-coordinates; double-coordinates; double-coordinates; double-coordinates; double-coordinates; double-coordina **Keywords: intervalse mappings over coordinates; left and right** α **-Convexity; doublet and right** α **-Convexity; Keywords:** $\mathbf{v}_i = \mathbf{v}_i$ where restrictions on endpoint functions of intervalse functions of α ω *restrictions on endpoint functions of intervalsed* functions that can be seen as applications of ω ω there intervalse functions of intervalse functions that can be seen as applications that can be seen as applications of ω ω restrictions of intervalse functions of intervalse functions that can be seen as applications of intervalse functions of ω $r_{\rm{rel}}$ -convexity, some new versions of $r_{\rm{rel}}$ \mathcal{L} -convexity, some new versions of \mathcal{L} $r_{\rm w}$ numerical values are non-trivial. By taking the product of two left and product of two left and r_{circle} results on \mathcal{L} in that examples are non-trivial. By taking the product of two left and product of two left and \mathcal{L} $r_{\rm{m}}$ right convexity, some new versions of fractional integrals are also fractional integrals are also fractional in $r_{\rm w}$ $\mathcal I$ results' numerical validations that examples are nontrivial. By taking the product of two left and \mathbf{r} \mathcal{L}^{rel} \mathbf{r} intervalse intervalse convexity) over intervalse codomain. We exploit the use of double Riemann– defined class of convex mappings proposed that are known as coordinated that are known as coordinated left and right \mathcal{L} ω iter ϵ d convex mappings proposed that are known as coordinated that are known as coordinated left and right \overline{d} defined class of convex mappings proposed that are known as coordinated that are known as coordinated left and right μ -dimensional μ -dimensional μ ω ier intervalse intervalse convexity over intervalse codomain. We exploit the use of double Riemann– defined class of convex mappings proposed that are known as coordinated that are known as coordinated left and right μ -dimensional μ α -convexity) over intervalse convexity. We exploit use of α **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly ω mere ω department of ω arabia; ω ω arabia; ω ν Department of Mathematics, σ Science, σ Science, σ ω _{rter} ϵ 2 ν and 2 ν Z_{max} $10₁$ Financial Science (FMAS)-Research Group, $\frac{1}{2}$ $\overline{\mathbf{w}}$ **Target 1**, α **Explored 2, there** 2, α **Explored 2,** α

inequality has several applications and a straightforward intrinsic geometric explanation.

The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known finding μ

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1 Financial Mathematics and Actuarial Science (FMAS)-Research Group, Department of Mathematics, Faculty

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inequality has several applications and a straightforward intrinsic geometric explanation.

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realms of applied and pure sciences. Furthermore, because of its many applications and

2 Department of Mathematics and Computer Science, Alabama State University,

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of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; tsalmalki@kau.edu.sa

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obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

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convexity and the theory of inequalities.

Published: date

Tareq Saeed 1*,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4**

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\mathcal{M}(\sigma, \mathbf{i}, \varepsilon, \mathbf{v}) = \mathcal{J}(\sigma, \varepsilon) \times \mathcal{J}(\sigma, \varepsilon) + \mathcal{J}(\mathbf{i}, \varepsilon) \times \mathcal{J}(\mathbf{i}, \varepsilon) + \mathcal{J}(\sigma, \mathbf{v}) \times \mathcal{J}(\sigma, \mathbf{v}) + \mathcal{J}(\mathbf{i}, \mathbf{v}) \times \mathcal{J}(\mathbf{i}, \mathbf{v}),
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P(\sigma, \mathbf{i}, \varepsilon, \mathbf{v}) = \mathcal{J}(\sigma, \varepsilon) \times \mathcal{J}(\mathbf{i}, \varepsilon) + \mathcal{J}(\mathbf{i}, \varepsilon) \times \mathcal{J}(\sigma, \varepsilon) + \mathcal{J}(\sigma, \mathbf{v}) \times \mathcal{J}(\mathbf{i}, \mathbf{v}) + \mathcal{J}(\mathbf{i}, \mathbf{v}) \times \mathcal{J}(\sigma, \mathbf{v}),
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\mathcal{N}(\sigma, \mathbf{i}, \varepsilon, \mathbf{v}) = \mathcal{J}(\sigma, \varepsilon) \times \mathcal{J}(\sigma, \mathbf{v}) + \mathcal{J}(\mathbf{i}, \varepsilon) \times \mathcal{J}(\mathbf{i}, \mathbf{v}) + \mathcal{J}(\sigma, \mathbf{v}) \times \mathcal{J}(\sigma, \varepsilon) + \mathcal{J}(\mathbf{i}, \mathbf{v}) \times \mathcal{J}(\mathbf{i}, \varepsilon),
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Q(\sigma, \mathbf{i}, \varepsilon, \mathbf{v}) = \mathcal{J}(\sigma, \varepsilon) \times \mathcal{J}(\mathbf{i}, \mathbf{v}) + \mathcal{J}(\mathbf{i}, \varepsilon) \times \mathcal{J}(\sigma, \mathbf{v}) + \mathcal{J}(\sigma, \mathbf{v}) \times \mathcal{J}(\mathbf{i}, \varepsilon) + \mathcal{J}(\mathbf{i}, \mathbf{v}) \times \mathcal{J}(\sigma, \varepsilon),
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convexity and the theory of inequalities.

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realms of applied and pure sciences. Furthermore, because of its many applications and

convexity and the theory of inequalities.

 $M(x, i, s, n), P(x, i, s, n), M(x, i, s, n), and O(x, i, s, n), are defined as follows:$ $(\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O})))\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O})))\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O})))\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O})))\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O})))\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O}(\mathcal{O})))\mathcal{O}(\mathcal{O}(\mathcal{$), $\mathcal{N}(\alpha, \mathfrak{i}, \varepsilon, \mathfrak{v})$ and $\mathrm{Q}(\alpha, \mathfrak{i}, \varepsilon, \mathfrak{v})$ are defined as follows: \mathcal{W} $(0, 0, 0, 0)$ and $(0, 0, 0, 0)$ are active as follows *holds: holds: holds:* heu ho fonoù \sim and $\mathcal{M}(\sigma, \mathfrak{i}, \varepsilon, \mathfrak{v})$, $P(\sigma, \mathfrak{i}, \varepsilon, \mathfrak{v})$, $\mathcal{N}(\sigma, \mathfrak{i}, \varepsilon, \mathfrak{v})$ and $Q(\sigma, \mathfrak{i}, \varepsilon, \mathfrak{v})$ are defined as follows: and $\mathcal{M}(\sigma, \iota, \varepsilon, \mathfrak{v}),$ $F(\sigma, \iota, \varepsilon, \mathfrak{v})$, $\mathcal{N}(\sigma, \iota, \varepsilon, \mathfrak{v})$ and $\mathcal{Q}(\sigma, \iota, \varepsilon, \mathfrak{v})$ are aefined as for The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known finding μ and $\mathcal{N}(\sigma, \iota, \varepsilon, \mathfrak{v})$, $P(\sigma, \iota, \varepsilon, \mathfrak{v})$, $\mathcal{N}(\sigma, \iota, \varepsilon, \mathfrak{v})$ and $\mathcal{Q}(\sigma, \iota, \varepsilon, \mathfrak{v})$ are aetinea as follows: The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known finding μ and $\mathcal{M}(\sigma, \mathfrak{i}, \varepsilon, \mathfrak{v})$, $P(\sigma, \mathfrak{i}, \varepsilon, \mathfrak{v})$, $\mathcal{N}(\sigma, \mathfrak{i}, \varepsilon, \mathfrak{v})$ and $Q(\sigma, \mathfrak{i}, \varepsilon, \mathfrak{v})$ are defined as follows: $\mathbf{H} = \mathbf{H} \times \mathbf{H}$ is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding i $\mathbb{E}_{\mathcal{A}}$ is one of the most well-known findings inequality is one of the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding \mathcal{A} $\mathcal{F}_{\mathcal{F}}$ is one of the most well-known findings in the most well-known finding in the m and $\mathcal{M}(\rho, \mathfrak{i}, \varepsilon, \mathfrak{v})$, $P(\rho, \mathfrak{i}, \varepsilon, \mathfrak{v})$, $\mathcal{N}(\rho, \mathfrak{i}, \varepsilon, \mathfrak{v})$ and $Q(\rho, \mathfrak{i}, \varepsilon, \mathfrak{v})$ are defined as follows: and $\mathcal{M}(\alpha,i,\epsilon,\mathfrak{v})$, $P(\alpha,i,\epsilon)$ Integral Inequalities via Coordinated $\lim_{\epsilon \to 0} \int \frac{\Gamma(\epsilon)}{\Gamma(\epsilon)} \frac{\Gamma(\epsilon)}{\Gamma(\epsilon)} \frac{\Gamma(\epsilon)}{\Gamma(\epsilon)} \frac{\Gamma(\epsilon)}{\Gamma(\epsilon)}$ Riemann–Liouville fractional integral operator; Pachpatte-type inequalities Riemann–Liouville fractional integral operator; Pachpatte-type inequalities **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double and $\mathcal{N}(\rho, \iota, \varepsilon, \mathfrak{v})$, $P(\rho, \iota, \varepsilon, \mathfrak{v})$, $\mathcal{N}(\rho, \iota, \varepsilon)$ **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double and $\mathcal{W}(\mathcal{Y}, \mathfrak{t}, \varepsilon, \mathfrak{v})$, $\Gamma(\mathcal{Y}, \mathfrak{t}, \varepsilon, \mathfrak{v})$, $\mathcal{N}(\mathcal{Y}, \mathfrak{t}, \mathfrak{t})$ ana IVI (α , ι , ε , \mathfrak{v}), $P(\alpha,\iota,\varepsilon,\mathfrak{v})$, IV (α , ι , ε , \mathfrak{v}) and $Q(\alpha,\iota,\varepsilon,\mathfrak{v})$ are aefined **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double and JVt $(\alpha, t, \epsilon, \nu)$, P $(\alpha, t, \epsilon, \nu)$, JV $(\alpha, t, \epsilon, \nu)$ and $Q(\alpha, t, \epsilon, \nu)$ are aefine and $\mathcal{M}(\alpha, t, \varepsilon, \mathfrak{v})$, $P(\alpha, t, \varepsilon, \mathfrak{v})$, $\mathcal{N}(\alpha, t, \varepsilon, \mathfrak{v})$ and $Q(\alpha, t, \varepsilon, \mathfrak{v})$ are defined as follows: 이 사이트 STATE ST ana $\mathcal{M}(\alpha, \mathfrak{t}, \varepsilon, \mathfrak{c})$ restrictions on endpoint functions of interval-valued functions that can be seen as applications of unu $\mathcal{M}(\alpha, \mathfrak{t}, \mathfrak{e})$ $(\alpha, 1, \varepsilon, \mathfrak{v})$, N $(\alpha, 1, \varepsilon, \mathfrak{v})$ ana $\mathcal{M}(\sigma, t, \varepsilon, \mathfrak{v}),$ $F(\sigma, t, \varepsilon, \mathfrak{v}),$ $\mathcal{N}(\sigma, t, \varepsilon, \mathfrak{v})$ ana $\mathcal{Q}(\sigma, t, \varepsilon, \mathfrak{v})$ are aefined as follows: $\mathcal{L}(\mathcal{$ $r_1(\nu, \nu, \nu)$ numerical values are non-trivial. By taking the product of two left and product of two left and α convexity (\overline{C}) over \overline{C} intervalse of double \overline{C} $\mathcal{L}(\mathcal{O})$ is the sum of the results of the results of the results of the key also examine convexity (\mathcal{L} -convexity) over intervalse codomain. We exploit the use of double Riemann– $\mathcal{L}(\mathcal{O})$ is the set of the fractional to derive the major results of the responsibility of the set of the key also examine the key **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly and $\mathcal{M}(\alpha, \mathfrak{t}, \varepsilon, \mathfrak{v}), P(\alpha, \mathfrak{t}, \varepsilon, \mathfrak{v}), \mathcal{N}(\alpha, \mathfrak{t}, \varepsilon, \mathfrak{v})$ and $Q(\alpha, \mathfrak{t}, \varepsilon, \mathfrak{v})$ are defined 4 Department of Science, Jazan University, Jazan 45142, Saudi Arabia; Jazan 4 and $\mathcal{M}(\sigma, i, \varepsilon, \mathfrak{v})$, $P(\sigma, i, \varepsilon, \mathfrak{v})$, $\mathcal{N}(\sigma, i, \varepsilon, \mathfrak{v})$ and $Q(\sigma, i, \varepsilon)$ $\mathcal{L}(\mathcal{O}_I, \mathcal{O}_I, \mathcal{O}_I)$, $\mathcal{L}(\mathcal{O}_I, \mathcal{O}_I, \mathcal{O}_I)$, $\mathcal{L}(\mathcal{O}_I, \mathcal{O}_I, \mathcal{O}_I)$ and $M(a, i, \epsilon, n)$ $D(a)$ $\mathcal{S}(\mathcal{O}_f(\mathcal{A}, \mathcal{O}_f)) = (\mathcal{O}_f(\mathcal{A}, \mathcal{O}_f))$ and $\mathcal{S}(\mathcal{O}_f(\mathcal{A}, \mathcal{O}_f))$ on Brasilvania University of Br

 $\frac{1}{\sqrt{2}}$ is a familiar familiar fact that $\frac{1}{\sqrt{2}}$ is a complete metric space.

The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known

restrictions on endpoint functions of interval-valued functions that can be seen as applications of

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category of classical convex functions, according to Dragomir and Pearce [1]. This

$$
\mathcal{M}(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v}) = [\mathcal{M}_*(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v}), \mathcal{M}^*(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v})],
$$

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P(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v}) = [P_*(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v}), P^*(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v})],
$$

$$
\mathcal{N}(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v}) = [\mathcal{N}_*(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v}), \mathcal{N}^*(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v})],
$$

$$
Q(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v}) = [Q_*(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v}), Q^*(\sigma, \mathbf{i}, \varepsilon, \mathfrak{v})].
$$

 $\overline{}$ coordinated $\ddot{ }$ $\mathbb{E} \{ \mathbf{r} \mathbf{r}_1 \}$ and $\mathbb{E} \{ \mathbf{r} \mathbf{r}_2 \}$ d LR - \hbar ₂-convex *IVMs* of $\begin{bmatrix} n_1 \text{ and } n_2 \text{ and } n_3 \end{bmatrix}$ convex 17 mp on $\begin{bmatrix} v \\ v \end{bmatrix}$ $\begin{bmatrix} v \\ v \end{bmatrix}$ J $\frac{1}{2}$ *VMs* on $[\sigma, i] \times [\varepsilon, \mathfrak{v}],$ $\sum_{n=1}^{\infty}$ 7 $\left[\begin{matrix} 0 \\ 1 \end{matrix}\right]$ $\left[\begin{matrix} 0 \\ 1 \end{matrix}\right]$ $\left[\begin{array}{ccc} \cdots & \cdots & \cdots \end{array}\right]$ of Let Land e be two coordinated LR- \hbar , and LR- \hbar , convex IVMs on [e, i] \times [c, n] *our* Let *J* and
restively Then *o coordina* Ԓ∗() *both are Riemann integrable (-integrable) over* ሾℴ, ሿ. *Moreover, if* Ԓ *is -integrable* μ *LK-* ν and *LK* α ^{*b*} α α α α β α , β α , integrable *i*ntegrable *i*ntegrable *i*ntegrable *i*ntegrable *i*ntegrable *i*ntegrable *in*tegrable *in*tegrable *integrable in <u>Profiled</u>*, *Fig. Then i* ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and foof.* Let *f* and *y* be two coordinated *LK-n₁* and *LK-n₂*-convex *τν i*ns on $[δ,ι] × [ε, θ]$, respectively. Then, ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and j* be two coordinated LR - n_1 and LR - n_2 -convex *i v* wis on $[\delta, \iota] \times [\varepsilon, \nu]$, ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* $\mathcal{E}(\mathbf{a})$ *b*₁, and *b*₁, *h*₂-convex *iv i M*s on $[\mathcal{C}, \mathbf{u}] \times [\mathcal{E}, \mathbf{v}]$, ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* **Proof.** Let *J* and *j* be two coordinated *LR-* \hbar_1 and *LR-* \hbar_2 -convex *IVMs* on $[\sigma, i] \times [\varepsilon, \mathfrak{v}]$, respectively. Then, \mathbb{R}^n $\frac{1}{\sqrt{2}}$ can be directed from convex functions, there is a convex functions relationship $\frac{1}{\sqrt{2}}$ can be two convex functions $\frac{1}{\sqrt{2}}$ can be two convex functions of $\frac{1}{\sqrt{2}}$ can be two convex functions of $\frac{1}{2}$ convexity. **Proof** Let L and a be two coordinated I_{R-h} and I_{R-h} convoy in ϵ reconctively. Then convexity and the theory of inequalities. **Droof** Let H and a between coordinated ID k and ID k change the solution \mathbf{r} is a potential to approximate the solution. Since \mathbf{r} is many integration. $\frac{1}{\sqrt{2}}$ can be directed from convex functions, there is a close relationship between $\frac{1}{\sqrt{2}}$ **Droof** Let L and g be two coordinated LR \ddot{b} , and LR \ddot{b} , convex IVMs on $\begin{bmatrix} a & \text{i} \end{bmatrix} \times \begin{bmatrix} a & \text{j} \end{bmatrix}$ consider the space that $\int_C \mathcal{L}(\mathbf{r})^2 \mathbf{r} \cdot \mathbf{r}$ and $\mathcal{L}(\mathbf{r})^2$ convex mass in $[\mathbf{r}, \mathbf{r}]$, $[\mathbf{r}, \mathbf{r}]$ ϵ be directly derived from convex functions, there is a close relationship between ϵ **Proof** Let π and a be two coordinated LP \hbar , and LP \hbar , convex *WMs* on $\left[\begin{smallmatrix}a&i\end{smallmatrix}\right]\times\left[\begin{smallmatrix}c&i\end{smallmatrix}\right]$ $\frac{1}{\sqrt{2}}$ consider the solution. Since $\frac{1}{\sqrt{2}}$ is a solution. Since $\frac{1}{\sqrt{2}}$ \mathbf{r} derived from convex functions, there is a close relationship between relationship between relationship between \mathbf{r} ϵ between a mathematical LR ϵ and LR ϵ convey IVM e and problem problem proves to be proved proves to be proved proves to be proved pro $\sum_{i=1}^n \sum_{i=1}^n \sum_{j=1}^n \sum_{j$ can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L} **Received:** Let π and the two coordinated $ID \&$ and $ID \&$ convexity $N\Lambda$ on $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ respectively. Then, **Proof.** Let J and J be two coordinated LR - \hbar_1 and LR - \hbar_2 -convex IVMs on $[\sigma, i] \times$ c_{c} can be used to approximate the solution. 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Then, **Denot** Let π and cho two coordinated π is and π in the convex sets and π realment of \mathbf{r} and \mathbf{r} and \mathbf{r} and \mathbf{r} many applications and \mathbf{r} many applications and \mathbf{r} $t_{\rm ref}$ relatively of inequalities, convexity \sim \mathbf{T}_{max} are many uses for the concepts of contract of convex \mathbf{W}_{max} and \mathbf{W}_{max} in the convex \mathbf{W}_{max} in the convex \mathbf{W}_{max} realment of application \mathbb{R} and \mathbb{R} and \mathbb{R} many \mathbb{R} is many applications and \mathbb{R} and \mathbb{R} and \mathbb{R} is many applications and \mathbb{R} and \mathbb{R} and \mathbb{R} and \mathbb{R} is many applica t_{c} relatively. Then, convexity $\frac{d}{dt}$ **Droof** Let $\text{End}_{\mathcal{A}}$ be two coordinated CD_k and CD_k convex $\text{U/M}_{\mathcal{A}}$ on $\begin{bmatrix} \vdots \\ \vdots \end{bmatrix}$ realment of applied $\frac{1}{\sqrt{2}}$ because of $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$ because of $\frac{1}{\sqrt{2}}$ is more sciences. Then t_{ref} relationship to the theory of inequalities, convexity has advanced α T are are matriced the constant of the convex sets and convex T and convex T and convex T in the convex T be two coordinated E realment n_2 convex if m_3 on $[v, v] \wedge [v, v]$ **Droof** Lot There are many uses for the concentrated E_x v_1 and E_y convex functions $[v, v]$ respectively. First, **Proof.** Let *J* and *j* be two coordinated *LR-* \hbar_1 and *LR-* \hbar_2 -convex *IVMs* on $[\sigma, i] \times [\varepsilon, \mathfrak{v}]$, \mathbf{F} is the type integral operator; \mathbf{F} **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **Keywords: intervalse map Proof.** Let J and *j* be two coordinated LR- \hbar_1 and LR- \hbar_2 -convex IVMs on $[\sigma, i] \times [\varepsilon]$

$$
\begin{aligned}\n&\mathcal{J}(v_{\mathcal{O}} + (1 - v)i, \kappa \varepsilon + (1 - \kappa)v) \\
&\leq_{p} \hbar_{1}(v) \hbar_{1}(\kappa) \mathcal{J}(\sigma, \varepsilon) + \hbar_{1}(v) \hbar_{1}(1 - \kappa) \mathcal{J}(\sigma, v) + \hbar_{1}(1 - v) \hbar_{1}(\kappa) \mathcal{J}(i, \varepsilon) \\
&\quad + \hbar_{1}(1 - v) \hbar_{1}(1 - \kappa) \mathcal{J}(i, v), \\
&\mathcal{J}(v_{\mathcal{O}} + (1 - v)i, (1 - \kappa)\varepsilon + \kappa v) \\
&\leq_{p} \hbar_{1}(v) \hbar_{1}(1 - \kappa) \mathcal{J}(\sigma, \varepsilon) + \hbar_{1}(v) \hbar_{1}(\kappa) \mathcal{J}(\sigma, v) + \hbar_{1}(1 - v) \hbar_{1}(1 - \kappa) \mathcal{J}(i, \varepsilon) + \hbar_{1}(1 - v) \hbar_{1}(\kappa) \mathcal{J}(i, v), \\
&\mathcal{J}((1 - v)\sigma + vi, \kappa \varepsilon + (1 - \kappa)v) \\
&\leq_{p} \hbar_{1}(1 - v) \hbar_{1}(\kappa) \mathcal{J}(\sigma, \varepsilon) + \hbar_{1}(1 - v) \hbar_{1}(1 - \kappa) \mathcal{J}(\sigma, v) + \hbar_{1}(v) \hbar_{1}(\kappa) \mathcal{J}(i, \varepsilon) \\
&\quad + \hbar_{1}(v) \hbar_{1}(1 - \kappa) \mathcal{J}(i, v), \\
&\mathcal{J}((1 - v)\sigma + vi, (1 - \kappa)\varepsilon + \kappa v) \\
&\leq_{p} \hbar_{1}(1 - v) \hbar_{1}(1 - v) \mathcal{J}(\sigma, \varepsilon) + \hbar_{1}(1 - v) \hbar_{1}(\kappa) \mathcal{J}(\sigma, v) + \hbar_{1}(v) \hbar_{1}(1 - \kappa) \mathcal{J}(i, \varepsilon) \\
&\quad + \hbar_{1}(v) \hbar_{1}(\kappa) \mathcal{J}(\sigma, v) + \hbar_{1}(v) \hbar_{1}(1 - \kappa) \mathcal{J}(i, \varepsilon).\n\end{aligned}
$$

$$
\sum_{p} n_1(1-v) n_1(1-k) J J(\sigma, \varepsilon) + n_1(1-v) n_1(\varepsilon) J J(\sigma, \sigma) + n_1(v) n_1(1-k) J J(\varepsilon, \varepsilon)
$$

+ $\hbar_1(v) \hbar_1(\varepsilon) J J(\varepsilon, \sigma),$

Ω. and *The family of all -integrable of s over coordinates over coordinates is denoted by Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract **Definition 1** ([30,40])**.** *Let* Ԓ:ሾℴ, ሿ → ℝூ *The next* results of α are defined as α are defined as α are defined as α $\frac{1}{2}$ and $\frac{1}{2}$. The following is how this inequality is stated: $\frac{d}{dx}$ T_{rad} and T_{rad} T result was mainly credited to H and T $\frac{1}{\sqrt{2}}$ is a familiar familiar fact that ($\frac{1}{\sqrt{2}}$ is a complete metric space.) category of classical convex functions, according to D and T and inequality is one of the most well-known findings in can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L} challenging, inequalities can be used to approximate the solution. Since \mathbb{R}^n challenging, in the solution can be used to approximate the solution. Since \mathbb{R}^n *(ii) It can be easily seen that* " ≤ " *looks like "left and right" on the real line* ℝ, *so we call* θ relationship to the theory of inequalities, convexity has advanced quickly in recent θ c^{and} and and and $r_{\rm end}$ $t_{\rm max}$ relationship to the theory of inequalities, convexity has advanced α $r_{\rm end}$ $t_{\rm max}$ relationship to the theory of inequalities, convexity has advanced quickly in recent α

and
\n
$$
g(v_{\sigma} + (1 - v)i, \kappa \varepsilon + (1 - \kappa)v) \\
\leq_{p} \hbar_{2}(v) \hbar_{2}(\kappa) g(\sigma, \varepsilon) + \hbar_{2}(v) \hbar_{2}(1 - \kappa) g(\sigma, v) + \hbar_{2}(1 - v) \hbar_{2}(\kappa) g(i, \varepsilon) \\
+ \hbar_{2}(1 - v) \hbar_{2}(1 - \kappa) g(i, v), \\
g(v_{\sigma} + (1 - v)i, (1 - \kappa) \varepsilon + \kappa v) \\
\leq_{p} \hbar_{2}(v) \hbar_{2}(1 - \kappa) g(\sigma, \varepsilon) + \hbar_{2}(v) \hbar_{2}(\kappa) g(\sigma, v) + \hbar_{2}(1 - v) \hbar_{2}(1 - \kappa) g(i, \varepsilon) + \hbar_{2}(1 - v) \hbar_{2}(\kappa) g(i, v), \\
((1 - v)\sigma + v i, \kappa \varepsilon + (1 - \kappa)v) \\
\leq_{p} \hbar_{2}(1 - v) \hbar_{2}(\kappa) g(\sigma, \varepsilon) + \hbar_{2}(1 - v) \hbar_{2}(1 - \kappa) g(\sigma, v) + \hbar_{2}(v) \hbar_{2}(\kappa) g(i, \varepsilon) \\
+ \hbar_{2}(v) \hbar_{2}(1 - \kappa) g(i, v), \\
g((1 - v)\sigma + v i, (1 - \kappa) \varepsilon + \kappa v) \\
\leq_{p} \hbar_{2}(1 - v) \hbar_{2}(1 - \kappa) g(\sigma, \varepsilon) + \hbar_{2}(1 - v) \hbar_{2}(\kappa) g(\sigma, v) + \hbar_{2}(v) \hbar_{2}(1 - \kappa) g(i, \varepsilon) \\
+ \hbar_{2}(v) \hbar_{2}(\kappa) g(i, v),
$$

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convexity and the theory of inequalities. The theory of inequalities α

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Inequalities via Coordinated ℏ-Convexity via Left and Right

Inequalities via Coordinated ℏ-Convexity via Left and Right

tight relationship to the theory of inequalities, convexity has advanced α

convexity and the theory of inequalities. The theory of inequalities α

 1963 was the one who first identified it $\frac{2}{3}$. The following is the following is the following is stated:

The Hausdorff–Pompeiu distance between intervals στα παραγωγή στα παραγωγή στα παραγωγή στα παραγωγή στα παραγ
Στην σταθεία

Since *J* and *j* both are coordinated *LR-* \hbar_1 and *LR-* \hbar_2 -convex *IVMs* on $[\sigma, i] \times [\varepsilon, \mathfrak{v}]$, the state of the coordinated EX n_1 and EX n_2 convex TV Ms on $[\sigma, \tau] \times [\sigma, \sigma]$,
respectively, we have *Then, integrals of areas integrals of an areas integrals of areas integrals of areas integrals of a reduced as* $\frac{1}{2}$ Since *J* and *j* both are coordinated *LR-* \hbar_1 and *LR-* \hbar_2 :
respectively, we have Since *J* and *j* both are coordinated *LR-* \hbar_1 and *LR-* \hbar_2 -convex *IVMs* on $[\sigma, i] \times [\varepsilon, \mathfrak{v}]$, $\frac{1}{2}$ and $\frac{1}{2}$ be seen are coordinated for n_1 and obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some restrictions on the seen as σ on the seen as applications that can be seen as applications that can be seen as applications of σ C_{max} **T** and by the new coordinated ID k and ID k concern H/M_{max} (i) the r_{re} restrictions on the set of intervalse functions of intervalse functions of $\left[\mathbf{v},\mathbf{v}\right]$ t_{ref} out t_{ref} C^* results are nontrivial. That examples are non-trivial. By taking the product of two left and two le right coordinated \mathcal{L}_r are \mathcal{L}_r -convexity, some new version of \mathcal{L}_r in \mathcal{L}_r obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some Fince π and shoth are coordinated $\overline{I}D\overline{K}$ and $\overline{I}D\overline{K}$ reconceively, some new versions of \mathbb{R}^n -convenience \mathbb{R}^n in \mathbb{R}^n in \mathbb{R}^n obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some Fines π and shoth are seentimeted IPE and IPE convey IVM and ily faul right coordinated \mathbf{r} -convexiting \mathbf{r} -convexiting \mathbf{r} integrated integral inte obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some $Cinco \text{ and } c$ both are coordinated \overline{LP} is a \mathcal{L} is derive the major results of the major results of the key also examine the key also \mathbf{r} numerical values are non-trivial. By taking the product of two left and product of two left and two left and \mathbf{r} Since \mathbb{R} and a both are coordinated \mathbb{R} - \mathbb{R} , and \mathbb{R} - \mathbb{R} -convex IVMs on [a, i] \times [s, i Since J and *j* both are coordinated *LK-h*₁ and *LK* respectively, we have results' numerical validations that examples are nontrivial. By taking the product of two left and $\sum_{i=1}^{\infty}$ respectively we also examine the major results of the key also examine the key a results' numerical validations that examples are nontrivial. By taking the product of two left and Since *J* and *j* both are coordinated *LR-* \hbar_1 and *LR-* \hbar_2 -convex *IVMs* on $[\sigma, i] \times [\varepsilon, \sigma]$ respectively, we have

The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known finding μ

can be directly derived from convex functions, there is a close relationship between

category of classical convex functions, according to D and P

 1963 was the one who first identified it $\frac{2}{3}$. The following is the following is stated: **Definition 1** ([30,40])**.** *Let* Ԓ:ሾℴ, ሿ → ℝூ

can be directly derived from convex functions, there is a close relationship between

realms of applied and pure sciences. Furthermore, because of its many applications and

category of classical convex functions, according to Dragomir and Pearce [1]. This

category of classical convex functions, according to D and P

publication under the terms and

challenging, including, including, including, including to approximate the solution. Since many inequalities o

The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most well-known fi

The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known finding $\mathcal{L}_\mathcal{A}$

can be directly derived from convex functions, there is a close relationship between

New Version of Fractional Pachpatte-type Integral

convexity and the theory of inequalities. The theory of inequalities α

Inequalities via Coordinated ℏ-Convexity via Left and Right

$$
\begin{split}\nJ_{1}(v_{0} + (1-v)i, \kappa \varepsilon + (1-\kappa)v) \times \mathcal{J}(v_{0} + (1-v)i, \kappa \varepsilon + (1-\kappa)v) \\
+ J_{0}(v_{0} + (1-v)i, (1-\kappa)\varepsilon + \kappa v) \times \mathcal{J}(v_{0} + (1-v)i, (1-\kappa)\varepsilon + \kappa v) \\
+ J_{1}(1-v) \sigma + \nu i, \kappa \varepsilon + (1-\kappa)v) \times \mathcal{J}(1-v) \sigma + \nu i, \kappa \varepsilon + (1-\kappa)v) \\
+ J_{1}(1-v) \sigma + \nu i, (1-\kappa)\varepsilon + \kappa v) \times \mathcal{J}(1-v) \sigma + \nu i, (1-\kappa)\varepsilon + \kappa v) \\
\leq_{p} \mathcal{M}(\sigma, i, \varepsilon, v)[\hbar_{1}(1-v)\hbar_{2}(1-v)\hbar_{1}(1-\kappa)\hbar_{2}(1-\kappa) \\
+ \hbar_{1}(1-v)\hbar_{2}(1-v)\hbar_{1}(\kappa)\hbar_{2}(\kappa) + \hbar_{1}(v)\hbar_{2}(v)\hbar_{1}(1-\kappa)\hbar_{2}(1-\kappa) \\
+ \hbar_{1}(v)\hbar_{2}(v)\hbar_{1}(\kappa)\hbar_{2}(\kappa) \\
+ P(\sigma, i, \varepsilon, v)[\hbar_{1}(v)\hbar_{2}(1-v)\hbar_{1}(1-\kappa)\hbar_{2}(1-\kappa) \\
+ \hbar_{1}(1-v)\hbar_{2}(v)\hbar_{1}(1-\kappa)\hbar_{2}(1-\kappa) + \hbar_{1}(v)\hbar_{2}(1-v)\hbar_{1}(\kappa)\hbar_{2}(\kappa) \\
+ \hbar_{1}(1-v)\hbar_{2}(v)\hbar_{1}(\kappa)\hbar_{2}(\kappa) \\
+ \mathcal{N}(\sigma, i, \varepsilon, v)[\hbar_{1}(1-v)\hbar_{2}(1-v)\hbar_{1}(\kappa)\hbar_{2}(1-\kappa) \\
+ \hbar_{1}(1-v)\hbar_{2}(1-v)\hbar_{1}(1-\kappa)\hbar_{2}(\kappa) + \hbar_{1}(v)\hbar_{2}(v)\hbar_{1}(1-\kappa)\hbar_{2}(\kappa) \\
+ \hbar_{1}(v)\hbar_{2}(v)\hbar_{1}(1-\kappa)\hbar_{2}(1-\kappa) \\
+ \h
$$

Theorem 3 theorem 3 *Let* **Theorem 3** *Let* α *Let* α *Let* α *Let* α *****Let* α *Let* α Ԓ(,) = ሾԒ∗(,), Ԓ∗(,)ሿ *for all* (,) ∈Ω= ሾℴ, ሿ × ሾ, ሿ *. Then,* Ԓ *is double integrable* Taking the multiplication of the above fuzzy inclusion with $v^{\alpha-1} \kappa^{\beta-1}$ and then taking $\frac{1}{2}$ of the resultant over $\left[0, 1\right] \times \left[0, 1\right]$ with respect to $\left(v, \kappa\right)$ such that ሾℴ, ሿ *are denoted by* ሾℴ,ሿ *and* ሾℴ,ሿ, *respectively*. merchant with $\theta = \theta$ and then *The family of all -integrable of s over coordinates and -integrable functions over* $(0, 1] \times (0, 1]$ \times [0, 1] with respect to (*v*, (x) and $\lim_{k\to\infty}$ $\mathcal{L}[\mathcal{O}, 1] \wedge [\mathcal{O}, 1]$ what respect to (\mathcal{O}, n) such that ı $\frac{1}{2}$ where $\frac{1}{2}$ is $\frac{1}{2}$ spect to (v, κ) , such that the double integration of the resultant over $[0, 1] \times [0, 1]$ with respect to (v, κ) , such that The result was mainly cannot was mainly contained to the above fuz The result plus mainly cannot be above fuz the double integration of the resultant over $[0, 1] \times [0, 1]$ with respect to (v, κ) , such that inequality has several applications and a straightforward intrinsic geometric explanation. The result was mainly calculated to the above fuzzy inclusion with v^{α} - κ^{ρ} - and though category of classical convex functions, according to Dragomir and Pearce [1]. This inking the multiplication of the above fuzzy in category of classical convex functions, according to Dragomir and Pearce [1]. This Taking the multiplication of the above fuzzy inclusion with $v^{\alpha-1} \kappa^{\beta-1}$ and then to in taking the multiplication of the above fuzzy inclusion with $v = w$ and then taking T_{maxing} is T_{maxing} in the most well-known for the above ruzzy in category of convex functions, and α are resultant over [0, 1]. convexity and the theory of inequalities. The theory of inequalities of inequalities of inequalities. The theory of inequalities of inequalities of inequalities of inequalities. The theory of inequalities of inequalities o T_{max} the Hadamard inequality is one of the most well-known finding in the most well-known fi category of convex functions, and α convenient over $\left[\alpha, \alpha\right] \wedge \left[\alpha, \alpha\right]$. α ^{*k*}, α ^{*k*}, α _{*k*} and then taking the momentum of the above ruzzy inclusion with v^2 , α ^{*k*}, and then taking Taking the multiplication of the above fuzzy inclusion with $v^{\alpha-1} \kappa^{\beta-1}$ and then takin category of classical convex functions, $\mathcal{L}[\mathcal{O}, \mathcal{L}]$. This is the process $\mathcal{O}(\mathcal{O}, \mathcal{N})$, such that the double integration of the resultant over $[0, 1] \times [0, 1]$ with respect to (v, κ) , such that convexity and the theory of inequalities. The theory of inequalities of inequalities of inequalities. convexity and the theory of interesting the theory of \sim the double integration of the resultant over [0, 1] \times [0, 1] with respect to (v, κ), such that can be directly derived from convex functions, there is a close relationship between Taking the multiplication of the above fuzzy Taking the multiplication of the above fuzzy inclusion with $v^{\alpha-1}\kappa^{p-1}$ and then Taking the multiplication of the above fuzzy inclu laking the multiplication of the above fuzzy inclu results' numerical validations that examples are nontrivial. By taking the product of two left and Taking the multiplication of the above fuzzy inclusion with $v^{k-1}\kappa^{\rho-1}$ and then taki Liouville fractional to derive the major results of the major results of the results of the results of the key convexity (-ℏ-convexity) over interval-valued codomain. We exploit the use of double Riemann– Laking the mumphration of the above fuzzy inclusion v convexity) over intervalse intervalse convexity over intervalse codomain. We exploit the use of double Riemann– Laking the mumphration of the above fuzzy inclusion with $v = w$ and then taking **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly α can be converted to convex mapping that are the above ruzzy in α **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly defined convention of the above fuzzy method **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly α defined that are convenience that are known as convenience that are convenient and right are α -directed and right α * Correspondence: enwaeze@alasu.edu (E.R.N.); muhammad.bilal@unitbv.ro (M.B.K.) * Correspondence: enwaeze@alasu.edu (E.R.N.); muhammad.bilal@unitbv.ro (M.B.K.) * Correspondence: enwaeze@alasu.edu (E.R.N.); muhammad.bilal@unitbv.ro (M.B.K.) the double integration of the resultant over [0, 1] \times [0, 1] with respect to (*v*, κ), su

$$
\int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa \beta^{-1} J(v_{\sigma} + (1 - v)i, \kappa \epsilon + (1 - \kappa)v) \times J(v_{\sigma} + (1 - v)i, \kappa \epsilon + (1 - \kappa)v) dv dx \n+ \int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa \beta^{-1} J(v_{\sigma} + (1 - v)i, (1 - \kappa)\epsilon + \kappa v) \times J(v_{\sigma} + (1 - v)i, (1 - \kappa)\epsilon + \kappa v) du dx \n+ \int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa \beta^{-1} J((1 - v)_{\sigma} + vi, \kappa \epsilon + (1 - \kappa)v) \times J((1 - v)_{\sigma} + vi, \kappa \epsilon + (1 - \kappa)v) du dx \n+ \int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa \beta^{-1} J((1 - v)_{\sigma} + vi, (1 - \kappa)\epsilon + \kappa v) \times J((1 - v)_{\sigma} + vi, (1 - \kappa)\epsilon + \kappa v) du dx \n\leq p M(\sigma, i, \epsilon, v) \int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa \beta^{-1} [\hbar_{1}(1 - v) \hbar_{2}(1 - v) \hbar_{1}(1 - \kappa) \hbar_{2}(1 - \kappa) \n+ \hbar_{1}(1 - v) \hbar_{2}(1 - v) \hbar_{1}(\kappa) \hbar_{2}(\kappa) + \hbar_{1}(v) \hbar_{2}(v) \hbar_{1}(1 - \kappa) \hbar_{2}(1 - \kappa) \n+ \hbar_{1}(v) \hbar_{2}(v) \hbar_{1}(\kappa) \hbar_{2}(\kappa) du dx
$$
\n(59)
\n+ P(\sigma, i, \epsilon, v) \int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa \beta^{-1} [\hbar_{1}(v) \hbar_{2}(1 - v) \hbar_{1}(1 - \kappa) \hbar_{2}(1 - \kappa)
\n+ \hbar_{1}(1 - v) \hbar_{2}(v) \hbar_{1}(1 - \kappa) \hbar_{2}(1 - v) \hbar_{1}(1 - v) \hbar_{2}(1 - v) <

defined class of convex mappings proposed that are known as coordinated left and right \mathcal{L}

hand side of (59) , we have α , α \mathcal{L} have right-hand side of (59), we have $\frac{1}{2}$, (12), *o o e o o f side* *****of* (59), we h From the right-hand side of (59), we have *From the right-hand side of (59), we have* $\frac{d}{dt}$ From the right-hand side of (39) , we have **The result was made to Hermite in the Hadamard Succession (1937), we have** The result was mainly contained to (32) , we have Γ can be explored to Dragomir and σ Or (3) , we have Γ category of classical convex functions, and Γ $\frac{1}{2}$ functions, and $\frac{1}{2}$. The Dragomir and $\frac{1}{2}$. The Dragomir and Pearce $\frac{1}{2}$. From the right-hand side of (59), we have convexity and the theory of inequalities. $\frac{1}{2}$ convexity and the theory of $\left(\frac{1}{2}n\right)$ $\sum_{i=1}^{\infty}$ character the solution. Since $\sum_{i=1}^{\infty}$ From the right-hand side of (59), we have From the right-hand side of (59), we have

$$
\int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa^{\beta-1} J(v \sigma + (1-v)i, \kappa \varepsilon + (1-\kappa)v) \times \mathcal{J}(v \sigma + (1-v)i, \kappa \varepsilon + (1-\kappa)v) dv d\kappa \n+ \int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa^{\beta-1} J(v \sigma + (1-v)i, (1-\kappa)\varepsilon + \kappa v) \times \mathcal{J}(v \sigma + (1-v)i, (1-\kappa)\varepsilon +\n\kappa v) dv d\kappa \n+ \int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa^{\beta-1} JJ((1-v) \sigma + v i, \kappa \varepsilon + (1-\kappa)v) \times \mathcal{J}((1-v) \sigma + v i, \kappa \varepsilon + (1-\kappa)v) dv d\kappa \n+ \int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa^{\beta-1} JJ((1-v) \sigma + v i, (1-\kappa)\varepsilon + \kappa v) \times \mathcal{J}((1-v) \sigma + v i, (1-\kappa)\varepsilon + \kappa v) dv d\kappa \n= \frac{\Gamma(\alpha)\Gamma(\beta)}{(i-\sigma)^{\alpha}(\upsilon-\varepsilon)^{\beta}} \left[\mathcal{J}^{\alpha, \beta}_{\sigma^+, \varepsilon^+} JJ(i, \upsilon) \times \mathcal{J}(i, \upsilon) + \mathcal{J}^{\alpha, \beta}_{\sigma^+, \upsilon^-} J(i, \varepsilon) \times \mathcal{J}(i, \varepsilon) \right]
$$
\n(60)

New Version of Fractional Pachpatte-type Integral

Integral Inequalities via Coordinated

Combining (59) and (60), we have Combining (59) and (60), we have (32) and (00) , we have Combining (59) and (60), we have $Combining (50)$ and (60) , we have Combining (59) and (60), we have results that examples are non-trivial. By taking the product of two left and product of two left and two Combining (59) and (60), we have c_n intervalse intervalse convexity. We exploit the use of double Riemann– Combining (59) and (60), we have Combining (59) an Combining (59) and (60), we have $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $Combinom(50)$ and (60) , us 3.3 Department of Mathematics and Computer Science, Transilvania University of Bras \sim $Combining(59)$ and (60) we have \mathbf{S} department of Mathematics and Computer Science, Transilvania University of Bras \mathbf{S} \overline{O} by \overline{O} and \overline{O} and \overline{O} and \overline{O} and \overline{O} $3^{(2)}$ department of Mathematics and Computer Science, Transilvania University of Bras α Combining (59) and (60) , we have $\mathcal{O}(\sqrt{m})$ 3.9×10^{-10} Combining (59) and (60), we have

right coordinated $\overline{}$ -convexity, some new versions of fractional integral integral integral in

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results' numerical validations that examples are nontrivial. By taking the product of two left and

category of classical convex functions, according to D

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category of classical convex functions, according to Dragomir and Pearce [1]. This

right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also

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these new outcomes. The second control is a second control of the second control in the second control in the

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New Version of Fractional Pachpatte-type Integral

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results' numerical validations that examples are nontrivial. By taking the product of two left and

challenging, in equalities can be used to approximate the solution. Since α

realms of applied and pure sciences. Furthermore, because of its many applications and

 $\mathcal{L}_{\mathcal{A}}$ is one of the most well-known findings in the most well-known finding in the most

$$
\frac{\Gamma(\alpha)\Gamma(\beta)}{(\mathbf{i}-\sigma)^{\alpha}(\mathbf{v}-\varepsilon)^{\beta}} \left[\mathcal{J}_{\sigma^{+},\varepsilon}^{\alpha,\beta} J(\mathbf{i},\mathbf{v}) \times \mathcal{J}(\mathbf{i},\mathbf{v}) + \mathcal{J}_{\sigma^{+},\mathbf{v}}^{\alpha,\beta} - J(\mathbf{i},\varepsilon) \times \mathcal{J}(\mathbf{i},\varepsilon) \right]
$$
\n
$$
\leq_{p} \mathcal{M}(\sigma, \mathbf{i}, \varepsilon, \mathbf{v}) \int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa^{\beta-1} [\hbar_{1}(1-v) \hbar_{2}(1-v) \hbar_{1}(1-\kappa) \hbar_{2}(1-\kappa) + \hbar_{1}(1-v) \hbar_{2}(1-v) \hbar_{1}(\kappa) \hbar_{2}(\kappa) + \hbar_{1}(v) \hbar_{2}(v) \hbar_{1}(1-\kappa) \hbar_{2}(1-\kappa) + \hbar_{1}(1-v) \hbar_{2}(1-\kappa) + \hbar_{1}(v) \hbar_{2}(v) \hbar_{1}(\kappa) \hbar_{2}(\kappa) \right] d\omega \kappa
$$
\n
$$
+ P(\sigma, \mathbf{i}, \varepsilon, \mathbf{v}) \int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa^{\beta-1} [\hbar_{1}(v) \hbar_{2}(1-v) \hbar_{1}(1-\kappa) \hbar_{2}(1-\kappa) + \hbar_{1}(1-v) \hbar_{2}(1-\kappa) + \hbar_{1}(1-v) \hbar_{2}(1-\kappa) + \hbar_{1}(1-v) \hbar_{2}(1-v) \hbar_{1}(\kappa) \hbar_{2}(\kappa) + \hbar_{1}(1-v) \hbar_{2}(\kappa) \Big] d\omega \kappa
$$
\n
$$
+ \mathcal{N}(\sigma, \mathbf{i}, \varepsilon, \mathbf{v}) \int_{0}^{1} \int_{0}^{1} v^{\alpha-1} \kappa^{\beta-1} [\hbar_{1}(1-v) \hbar_{2}(1-v) \hbar_{1}(\kappa) \hbar_{2}(1-\kappa) + \hbar_{1}(1-v) \hbar_{2}(\kappa) \Big] d\omega \kappa
$$
\n

 $\frac{1}{\sqrt{2}}$ is a familiar familiar fact that $\frac{1}{\sqrt{2}}$ is a complete metric space.

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results' numerical validations that examples are nontrivial. By taking the product of two left and

category of classical convex functions, according to D

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results' numerical validations that examples are nontrivial. By taking the product of two left and

 \Box \Box Hence, the required result. \square *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract r_{center} are required research \Box T Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most well-known fi Hence, the required result. \Box It is a familiar fact that (ℝூ,) is a complete metric space. $\sum_{i=1}^{n}$ T is one of the most most well-known \mathcal{L} Hence, the required result. \square T_{rel} Hence, the required result. \Box realms of applied and pure sciences. Furthermore, because of its many applications and $|$ iii) *It can be easily seen that " so we call and right" on the real line* α *, so we call* α *, so we call* α r_{ref} are required result. \Box T are many uses for the convex sets and convex sets and convex α r_{f} and pure science of its many applications of its many applications and r_{f} $T_{\rm eff}$ are many uses for the convex sets and convex sets and convex sets and convex functions in the convex functions in the convex sets and convex functions in the convex functions in the convex functions in the convex realms of applications and pure sciences. Furthermore, because of its many applications and pure sciences. Furthermore, \Box There are many uses for the convex sets and convex sets and convex sets and convex sets and convex functions in the convex sets and convex sets and convex functions in the convex sets and convex functions in the convex se There are many uses for the concepts of convex sets and convex functions in the There are many uses for the convex sets and convex sets and convex sets and convex sets and convex \mathcal{L}_max Hence, the required result. \square \mathcal{L} Hence, the required result. \square **Citation:** Saeed, T.; Nwaeze, E.R.; **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double Honce the required result \Box $\frac{1}{\sqrt{2}}$ intervalse on endpoint functions that can be seen as applications that can be seen as applications of intervalse of intervalse $\frac{1}{\sqrt{2}}$ result r_{c} are negative new versions \Box r_{max} are numerical values at \Box Hence, the required result. \Box Hence, the required result. \square

Remark 4. If one assumes that \iint_S is coordinated left-LR-h-convex with $\hbar(v) = v$, $h(\kappa) = \kappa$ and $\kappa = 1$ and $\beta = 1$, then from (59), as a [re](#page-22-27)sult, there will be inequality (see [42]): $\frac{1}{2}$ was the one who first identified it $\frac{2}{3}$. The following is stated: $\frac{1}{\sqrt{2}}$ and $\hbar(\kappa) = \kappa$ and $\alpha = 1$ and $\beta = 1$, then from (59), as a result, there will be inequality (see [42]): **Remark 4.** If one assumes that π is coordinated left-LR- \hbar -convex with $\hbar(v) = v$, *Then, interval Riemann–Liouville-type integrals of* Ԓ *are defined as* $\hbar(r) = r$ and $r = 1$ and $\beta = 1$ then from (59) as a re- $\sum_{i=1}^N$ $\hbar(r) = r$ and $r = 1$ and $\beta = 1$ then from (59) as a result there will be Γ result was mainly contributed to Γ $\hbar(r) = r$ and $r = 1$ and $\beta = 1$ then from (59) as a result there will be inequality (see Γ result was mainly contributed to Hermite (1822–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865 is and $\alpha = 1$ and $\beta = 1$ then from (59) as a result there will be inequality (see [42]). The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– $\hbar(\kappa) = \kappa$ and $\alpha = 1$ and $\beta = 1$, then from (59), as a result, the Submitted for possible open access α be directly derived from convex functions, there is a close relationship between α **chalace example to a solution be used to approximate the solution of the solution**. Since $\frac{1}{2}$ $\hslash(\kappa)=\kappa$ and $\alpha=1$ and $\beta=1$, then from (59), as a result, there will be inequality (see [42]): **Remark 4.** If one assumes that \iint is coordinated left-LR-h-convex with $\hbar(v) = v$, $\frac{1}{2}$ (c) a mathematical problem problem problem proves to be a mathematical problem problem problem proves to be a mathematical problem $c(x) = k$ where $x = 1$ and $p = 1$, then from (co), we are κ and κ and κ and κ are a mathematical problem proves to be a κ are κ and κ and κ and κ and κ and κ and κ are κ and $c(x) = k$ be used to $x = 1$ be used to $y = 1$, even from (55)), we a recent fraction. Since we are inequality (56) $\mathbf{y} = \mathbf{y} \cdot \mathbf{y}$ and $\mathbf{y} = \mathbf{y} \cdot \mathbf{y}$ and $\mathbf{y} = \mathbf{y} \cdot \mathbf{y} \cdot \mathbf{y} = \mathbf{y} \cdot \mathbf{y} \cdot \mathbf$ $h(\kappa) = \kappa$ and $\alpha = 1$ and $\beta = 1$, then from (59), as a result, there will be inequality (see [42]): \sim Remark 4. $f(x) = x$ and $x = 1$ and $\beta = 1$ then from (59) as a result there will be inequality (see [42]). $\hbar(\kappa) = \kappa$ and $\kappa = 1$ and $\beta = 1$, then from (59), as a result, there will be inequality $\hbar(\kappa)=\kappa$ and $\alpha=1$ and $\beta=1$, then from (59), as a result, there will be inequality (see [42]):

$$
\frac{1}{(i-\sigma)(v-\epsilon)}\int_{\sigma}^{i} \int_{\epsilon}^{b} J(x,y) \times \jmath(x,y) dy dx
$$
\n
$$
\supseteq \frac{1}{9} \mathcal{M}(\sigma, i, \epsilon, \mathfrak{v}) + \frac{1}{18} [P(\sigma, i, \epsilon, \mathfrak{v}) + \mathcal{N}(\sigma, i, \epsilon, \mathfrak{v})] + \frac{1}{36} Q(\sigma, i, \epsilon, \mathfrak{v}).
$$
\n
$$
(61)
$$

If J is coordinated LR-h-convex with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and one assumes that $\alpha =$ 1 and $\beta = 1$, then from (59), as a result, there will be inequality (see [43]): $\frac{1}{2}$. (iii) $\frac{1}{2}$ \mathbf{L} \mathbf{v} $\frac{1}{2}$ was the one who first identified it $\frac{1}{2}$. 1 and $\beta = 1$, then from (59), as a result, there will be inequality (see [43]): If J is coordinated LR-h-convex with $h(v) = v$, $h(\kappa) = \kappa$ and one assumes that $\alpha =$
1 and $\beta = 1$ then from (59) as a result there will be inequality (see [43]). *The first (co), in a room, interval risk integrality (corportion)* If J is coordinated LR-h-convex with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and one assumes that $\alpha =$ 1 and $\beta = 1$ then from (59) as a result there will be inequality (see [43] $T_{\rm c}$ credited to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even thou $T_{\rm eff}$ and $T_{\rm eff}$ is Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even $\frac{1}{2}$ and $\frac{1}{2} - 1$ then from (59) as a result there will be inequality (see [43]). The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– ϵ in the from (59) as a result there will be inequality (see [43]). The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– 1 and $\beta = 1$, then from (59), as a result, there will be inequality (see [43] 1 and $\beta = 1$, then from (59), as a result, there will be inequality (see [43]): If J is coordinated LR-h-convex with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and one assumes that $\alpha =$ If J is coordinated LR-h-convex with $h(v) = v$, $h(\kappa) = \kappa$ and one assumes that $\alpha =$ challenging, inequalities can be used to approximate the solution. Since many inequalities \mathcal{L} challenging, inequalities can be used to approximate the solution. Since many inequalities \mathcal{L} challenging, inequalities can be used to approximate the solution. Since many inequalities \mathcal{L} challenging, inequalities can be used to approximate the solution. Since many inequalities \mathbf{r} challenging, inequalities can be used to approximate the solution. Since \mathcal{L} Received: 14 November 2023 challenging, including, inequalities can be used to approximate the solution. Since \mathcal{L} if if all eventually and pure sciences. For $n(v) = v$, $n(\kappa) = \kappa$ and one assumes that $\kappa =$ realms of applied and pure sciences. Furthermore, because of its many applications and 1 and $\beta = 1$, then from (59), as a result, there will be inequality (see [43]): ι ι JJ is cooruinuieu Lr

$$
\frac{1}{(i-\sigma)(v-\varepsilon)}\int_{\sigma}^{i}\int_{\varepsilon}^{v}J(x,y)\times\gamma(x,y)dydx
$$

\n
$$
\leq_{p}\frac{1}{9}\mathcal{M}(\sigma,i,\varepsilon,\mathfrak{v})+\frac{1}{18}[P(\sigma,i,\varepsilon,\mathfrak{v})+\mathcal{N}(\sigma,i,\varepsilon,\mathfrak{v})]+\frac{1}{36}Q(\sigma,i,\varepsilon,\mathfrak{v}).
$$
\n(62)

If $J_*(x, y) \neq J^*(x, y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ then, by (57), we succeed in bringing about
the uncoming inequality (see [461). the upcoming inequality (see [46]):
the upcoming inequality (see [46]): δ): ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract $\frac{1}{3}$ was the following inequality in $\frac{1}{3}$. $\alpha = \alpha$, *n*(κ) – κ inen, by (57), we succeed in bringing do the upcoming inequality (see [46]): If $\pi(x, y)$ in $\pi^*(y, y)$ and $\varepsilon(x)$ in $\varepsilon(x)$ and $\varepsilon(x)$ in the straightforward intervalse $\varepsilon(x)$ If $J_{*}(x, y) \neq J_{*}^{*}(x, y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ then, by (57), we succeed in bringing about
the uncoming inequality (see [A6]). *The next Riemannier integrals of area integrals of a reduce the definition of* α If $\pi(u, v)$ for $\pi^*(u, v)$ and $k(u)$ and $k(v)$ and the straightforward intrinsic geometric explanation. If $J_*(x,y) \neq J^*(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ then, by (57), we succeed in bringing about the upcoming inequality (see [46]): If $J_*(x,y) \neq J^*(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ then, by (57), we succeed in bringing about The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– The result was mainly credited to \mathcal{A} and \mathcal{A} are different though Hadamard (1875–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard If $J_*(x, y) \neq J^*(x, y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ then, by (57), we succeed in bringing about if $J_{\downarrow}(x,y) \neq J_{\downarrow}(x,y)$ and $h(v) = v$, $h(\kappa) = \kappa$ then, by (57), we succeed in bring: If $J_*(x,y) \neq J^*(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ then, by (57), we succeed in bringing about e (40)): T_{tot} is and convex sets and convex functions in the convex sets and convex functions in the convex functions in the convex sets and convex functions in $f(x)$

$$
\frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{4(i-\sigma)^{\alpha}(v-\varepsilon)^{\beta}} \left[g^{\alpha,\beta}_{\sigma^+,\varepsilon^+} Jj(i,\upsilon) \times J(i,\upsilon) + g^{\alpha,\beta}_{\sigma^+,\upsilon^-} Jj(i,\varepsilon) \times J(i,\varepsilon) \right] \n+ \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{4(i-\sigma)^{\alpha}(v-\varepsilon)^{\beta}} \left[g^{\alpha,\beta}_{i-\varepsilon^+} Jj(\sigma,\upsilon) \times J(\sigma,\upsilon) + g^{\alpha,\beta}_{i-\varepsilon^+} Jj(\sigma,\varepsilon) \times J(\sigma,\varepsilon) \right] \n\leq p \left(\frac{1}{2} - \frac{\alpha}{(\alpha+1)(\alpha+2)} \right) \left(\frac{1}{2} - \frac{\beta}{(\beta+1)(\beta+2)} \right) \mathcal{M}(\sigma,i,\varepsilon,\upsilon) \n+ \frac{\alpha}{(\alpha+1)(\alpha+2)} \left(\frac{1}{2} - \frac{\beta}{(\beta+1)(\beta+2)} \right) P(\sigma,i,\varepsilon,\upsilon) \n+ \left(\frac{1}{2} - \frac{\alpha}{(\alpha+1)(\alpha+2)} \right) \frac{\beta}{(\beta+1)(\beta+2)} \mathcal{N}(\sigma,i,\varepsilon,\upsilon) + \frac{\beta}{(\beta+1)(\beta+2)} \frac{\alpha}{(\alpha+1)(\alpha+2)} Q(\sigma,i,\varepsilon,\upsilon).
$$
\n(63)

 T_1 , $\bar{b}(x) = x$ and $\bar{b}(x, y) \perp \bar{b}(x, y)$, then by (57) we cuceed in by province about *v*, n(κ) = κ unu j_{*}(x, y) ≠ j] (x, y), then by (57), we succeed in bringing about
poquality (coo [431); *(beliamerical)* over Γ *i and i c i and <i>n i and i b*_{*i*} *d i and i c i b*_{*i*} *d j <i>n*</sup> *i d j d i d i d j d j d j d j d j d j d j d* \overline{a} then by (57*),* we su (57) , we succeed in bringing If $h(v) = v$, $h(\kappa) = \kappa$ and $J_{\mu}(x, y) \neq J^*(x, y)$, then by (57), we succeed in bringing about the upcoming inequality (see [\[43\]](#page-23-0)): *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract $\frac{1}{2}$ was the following inequality in $\frac{1}{2}$. If $h(v) = v$, $h(x) = \kappa$ and $J_{\ast}(x, y) \neq J^*(x, y)$, then by (57), we succeed in bringing about
the uncoming inequality (see [43]). the upcoming inequality (see [43]):
the upcoming inequality (see [43]): *Then, interval Riemann–Liouville-type integrals of* Ԓ *are defined as* If $E(y) = y$, $E(x) = x$ and \overline{A} $(x, y) \neq \overline{B}^*(x, y)$, then by (57) are cuceed in bringing about $T_{\rm c}$ coredited to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1875–1901), even though Hadamard (1872–1901), even though Hadamard (1875–1901), even though Hadamard (1875–1901), even tho $\frac{1}{2}$ corresponding to $\frac{1}{2}$ $T_{\rm c}$ case mainly credit to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1872–1901), even though Hadamard (1875–1901), even though Hadamard (1875–1901), \overline{S} result was mainly contained to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (18

$$
\frac{1}{(i-\sigma)(v-\epsilon)}\int_{\sigma}^{i}\int_{\epsilon}^{v}J(x,y)\times \jmath(x,y)dydx
$$

\n
$$
\leq_{p}\frac{1}{9}\mathcal{M}(\sigma,i,\epsilon,\mathfrak{v})+\frac{1}{18}[P(\sigma,i,\epsilon,\mathfrak{v})+\mathcal{N}(\sigma,i,\epsilon,\mathfrak{v})]+\frac{1}{36}Q(\sigma,i,\epsilon,\mathfrak{v}).
$$
\n(64)

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Published: date

Published: date

restrictions on endpoint functions of interval-valued functions that can be seen as applications of

 ϵ -convexity) over intervalse convexity) over intervalse codomain. We exploit the use of double Riemann–

obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

Riemann–Liouville fractional integral operator; Pachpatte-type inequalities

Published: date

1. Introduction

Published: date

Academic Editor: Bruce Henry

publication under the terms and

Published: date

Integral Inequalities via Coordinated

Version of Fractional Pachpatte-type

 1963 was the one who first identified it $\frac{2}{3}$. The following is the following is stated:

convexity and the theory of inequalities. The theory of inequalities α

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convexity and the theory of inequalities $\mathcal{L}_{\mathcal{A}}$

Academic Editor: Bruce Henry

The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known

 1963 was the one who first identified it $2\sqrt{2}$. The following is the following is the following is stated:

convexity and the theory of inequalities $\mathcal{L}_{\mathcal{A}}$

years. When determining exact values for a mathematical problem proves to be

Definition 1 ([30,40])**.** *Let* Ԓ:ሾℴ, ሿ → ℝூ

category of convex functions, according to D

If $J_{*}(x, y) = J^{*}(x, y)$ and $J_{*}(x, y) = J^{*}(x, y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (57), *we succeed in bringing about the upcoming classical inequality:*
We succeed in bringing about the upcoming classical inequality: $\frac{1}{2}$ was the following inequality in $\frac{1}{2}$. $\mathcal{Y}(y) = \mathcal{Y}(x, y)$ and $\mathcal{U}(y) = v$, $\mathcal{U}(x) = \kappa$, then from (*3*), incolassised inequality: *The emergence are defined we can also appearing emission inceptinity.* If $J_{*}(x, y) = J^{*}(x, y)$ and $g_{*}(x, y) = g^{*}(x, y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (57), T_{tot} and \dot{H}_{tot} credited to \dot{H}_{tot} and \dot{H}_{tot} $\lim_{x \to x} \lim_{y \to x} (x, y) = \lim_{x \to x} \lim_{y \to x} (x, y) = \lim_{x \to x} \lim_{y \to x}$ *The interval with the upcoming emission inequality.* **Definition 1** (*x, y*) = *J* (*x, y*) and $\iota(\iota)$ =
bout the uncoming classical inequality: *The appointing children and painting.* If $J_*(x, y) = J^*(x, y)$ and $J_*(x, y) = J^*(x, y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (57), T_{max} and $\frac{1}{2}$ a $\mathcal{L}(\mathcal{X}, \mathcal{Y}) = \mathcal{L}(\mathcal{X}, \mathcal{Y})$ and $\mathcal{L}(\mathcal{X}, \mathcal{Y}) = \mathcal{L}(\mathcal{X}, \mathcal{Y})$ and $h(\mathcal{X}) = \mathcal{X}, h(\mathcal{X}) = \mathcal{X}$ λ conservative-type integratively. T is one of the Hadamard inequality is one of the most well-known findings in the most well If $J_*(x, y) = J^*(x, y)$ and $J_*(x, y) = J^*(x, y)$ and $h(v) = v$, $h(\kappa) = \kappa$, then from (57), If $J_*(x,y) = J^*(x,y)$ and $J_*(x,y) = g^*(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (57), if $J_{\mu}(x,y) = J_{\mu}(x,y)$ and $\jmath_{\ast}(x,y) = \jmath_{\mu}(x,y)$ and $n(v) = v$, $n(\kappa) = \kappa$, then from years. When determining exact values for a mathematical problem proves to be) and $g_*(x,y) = g^*(x,y)$ and $n(v) = v$, $n(\kappa) = \kappa$, then from (57), ι ι ι ι ι ι ι If $J_*(x,y) = J^*(x,y)$ and $J_*(x,y) = J^*(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (57) *(ii) It can be easily seen that* " ≤ " *looks like "left and right" on the real line* ℝ, *so we call coinging work the aptenting emotion inequality. f* and $f_*(x, y) = f(x, y)$ and $i(x) = 0$, $i(x) = k$, then from (57), $y(x) = k$ the uncomino classical inequality: W^{\prime} who up coming cursulu when interacting. *f* **f g* = *f* (*x*, *y f and it is* (*x*) = *x*, *increfiem* (*x*), *f f* α , *n f i s f f o i s f o i n f o i s f o i i o i s f o i i o i i i o* we succeed in bringing about the upcoming classical inequality: and $g_*(x,y) = g^*(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (57), $y_j y_{\ast}(x, y) = y_j(x, y)$ and $y_{\ast}(x, y) = y(x, y)$ and $u(v) = v, u(x) = k$,
we succeed in bringing about the upcoming classical inequality: **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double $\lim_{y \to 0^+} \mathcal{L}(\mathcal{X}, y) = \mathcal{L}(\mathcal{X}, y)$ If $J_*(x,y) = J^*(x,y)$ and $J_*(x,y) = J^*(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, the If $J_{*}(x,y) = J^{*}(x,y)$ and $J_{*}(x,y) = g^{*}(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (57), α checked in oringing holds in apcoming $\mathcal{L}_{\mathcal{F}}(x, y) = \mathcal{L}_{\mathcal{F}}(x, y)$ and $\mathcal{L}_{\mathcal{F}}(x, y) = \mathcal{L}_{\mathcal{F}}(x, y)$ and $\mathcal{L}_{\mathcal{F}}(x, y) = \mathcal{L}_{\mathcal{F}}(x, y)$ and $\mathcal{L}_{\mathcal{F}}(x, y) = \mathcal{L}_{\mathcal{F}}(x, y)$ R_{R} and R_{R} interests the comparison integrals operator; P_{R} $F_{\mathcal{F}}(x, y) = \mathcal{F}_{\mathcal{F}}(x, y)$ and $\mathcal{F}_{\mathcal{F}}(x, y) = \mathcal{F}(x, y)$ and $\mathcal{F}_{\mathcal{F}}(x, y) = \mathcal{F}_{\mathcal{F}}(x, y) = \mathcal{F}_{\mathcal{F}}(x, y)$
The succeed in brinoino about the uncomino classical inequality: R_{R} and the fractional integrals integrals integrated operator; Package integrations we succeed in bringing about the upcoming classical inequality: $\mathcal{L}_{\mathcal{X}}(x, y) = \mathcal{L}_{\mathcal{X}}(x, y)$ and $\mathcal{L}_{\mathcal{X}}(x, y) = \mathcal{L}_{\mathcal{X}}(x, y)$ and $\mathcal{L}_{\mathcal{X}}(x, y) = \mathcal{L}_{\mathcal{X}}(x, y)$ and $\mathcal{L}_{\mathcal{X}}(x, y) = \mathcal{L}_{\mathcal{X}}(x, y)$ R are emergencies in terms into the decoming emergencies integrating. *Fractive and a fractal property the uncoming classical inequality:* we succeed in bringing about the upcoming classical inequality: $\int y J_{\mu}(x, y) = \int y J(x, y) d\mu$ and $y_{\mu}(x, y)$

convexity and the theory of inequalities $\mathcal{L}_{\mathcal{A}}$

Remark 1 ([47])**.** *(i) The relation* " ≤ " *is defined on* ℝூ *by*

category of convex functions, according to D

 $\mathbf{r} = \mathbf{r} + \mathbf$

Remark 1 ([47])**.** *(i) The relation* " ≤ " *is defined on* ℝூ *by*

tight relationship to the theory of inequalities, convexity has advanced α

Definition 1 ([30,40])**.** *Let* Ԓ:ሾℴ, ሿ → ℝூ

tight relationship to the theory of inequalities, convexity has advanced quickly in recent

convexity and the theory of inequalities. The theory of inequalities α

convexity and the theory of inequalities $\mathcal{L}_{\mathcal{A}}$

$$
\frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{4(i-\sigma)^{\alpha}(\upsilon-\varepsilon)^{\beta}} \left[\mathcal{J}^{\alpha,\beta}_{\sigma^+,\varepsilon^+} \mathcal{J}(i,\upsilon) \times \mathcal{J}(i,\upsilon) + \mathcal{J}^{\alpha,\beta}_{\sigma^+,\upsilon^-} \mathcal{J}(i,\varepsilon) \times \mathcal{J}(i,\varepsilon) \right] \n+ \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{4(i-\sigma)^{\alpha}(\upsilon-\varepsilon)^{\beta}} \left[+ \mathcal{J}^{\alpha,\beta}_{i-\varepsilon^+} \mathcal{J}(\sigma,\upsilon) \times \mathcal{J}(\sigma,\upsilon) + \mathcal{J}^{\alpha,\beta}_{i-\varepsilon^+} \mathcal{J}(\sigma,\varepsilon) \times \mathcal{J}(\sigma,\varepsilon) \right] \n\leq \left(\frac{1}{2} - \frac{\alpha}{(\alpha+1)(\alpha+2)} \right) \left(\frac{1}{2} - \frac{\beta}{(\beta+1)(\beta+2)} \right) \mathcal{M}(\sigma,i,\varepsilon,\upsilon) + \frac{\alpha}{(\alpha+1)(\alpha+2)} \left(\frac{1}{2} - \frac{\beta}{(\beta+1)(\beta+2)} \right) P(\sigma,i,\varepsilon,\upsilon) \n+ \left(\frac{\beta}{2} - \frac{\alpha}{(\alpha+1)(\alpha+2)} \right) \frac{\beta}{(\beta+1)(\beta+2)} \mathcal{N}(\sigma,i,\varepsilon,\upsilon) + \frac{\beta}{(\beta+1)(\beta+2)} \frac{\alpha}{(\alpha+1)(\alpha+2)} Q(\sigma,i,\varepsilon,\upsilon).
$$
\n(65)

o o o <i>o cct J, *j* let $h : [0, 1] \rightarrow \mathbb{R}^+$. If $J \times j \in \mathfrak{TD}_{\Omega}$, then the following inequalities hold: $J(x,y) = [J_*(x,y)/J_-(x,y)]$ for *J*_{*j*}, $g:\Omega\to\mathbb{R}_I^+$ $[J_{*}(x,y), J_{-}(x,y)]$ for an (x,y) **Theorem 8.** Let \mathbf{J} , $\mathbf{J} : \Omega \to \mathbb{R}^+_I$ be two coordinated LR- \hbar -convex IVMs on Ω , given by $J(x,y) = [J_*(x,y), J^*(x,y)]$ and $J(x,y) = [J_*(x,y), J^*(x,y)]$ for all $(x,y) \in \Omega$ and Let \mathcal{J}_J , $\mathcal{J} : \Omega \to \mathbb{R}_I^+$ be two coordinated LR- \hbar -convex IVMs on Ω , given by $\{f_{*}(x,y), J^{*}(x,y)]$ and $g(x,y) = [f_{*}(x,y), g^{*}(x,y)]$ for all $(x,y) \in \Omega$ and $\sigma:\Omega\rightarrow \mathbb{R}^+_I$ be two coordinated LR-ħ-convex IVMs on Ω , given by $[f^*(x, y)]$ and $\overline{g}(x, y) = [f^*(x, y), f^*(x, y)]$ for all $(x, y) \in \Omega$ and **two coordinated LR-ħ-convex IVMs on** Ω **, given by Theorem 8.** Let J_J , $j : \Omega \to \mathbb{R}_I^+$ be two coordinated LR-h-convex IVMs on Ω , given by $J(x,y) = [J_*(x,y), J^*(x,y)]$ and $g(x,y) = [J_*(x,y), J^*(x,y)]$ for all $(x,y) \in \Omega$ and ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* Γ . If $\overline{J} \times \overline{J} \in \mathfrak{D}_{\Omega}$, then the following inequalities hold: ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* $T_1 \in \mathfrak{D}_{\Omega}$, then the following inequalities hold: ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* let $\hbar : [0, 1] \to \mathbb{R}^+$. If $\pi \times \pi \in \mathfrak{D}_{\Omega}$, then the following inequalities hold: ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* $y \times y \in \mathfrak{D}_{\Omega}$, then the following thequatities hold: $J\!J}(x,y) = [J_*(x,y), J^*(x,y)]$ and $J(x,y) =$ $J(x,y) = [J_*(x,y), J^*(x,y)]$ and $g(x,y) = [J_*(x,y), J^*$ $J(x,y) = [J_*(x,y), J^*(x,y)]$ and $J(x,y) = [J_*(x,y), J^*(x,y)]$ for I_{α} *t* π_{α} : $\Omega \setminus \mathbb{P}^+$ be type coordinated IR α convex IVMs of $g(x, y) = [g_*(x, y), g^*(x, y)]$ for all $(x, y) \in \Omega$ and
the the following inequalities hold. $\Omega \rightarrow \mathbb{P}^+$ be type coordinated LR \hbar conver *IVMs* on Ω given by $J_*(x, y), J^*(x, y)$ for all $(x, y) \in \Omega$ and
eximp inoqualities hold. *<u>Photometricial Riversity IVMs</u> on O are defined as* $g^*(x, y)$ for all $(x, y) \in \Omega$ and ω let $\hbar : [0, 1] \to \mathbb{R}^+$. If $J \times \mathcal{J} \in \mathfrak{TD}_{\Omega}$, then the following inequalities hold: **Theorem 8.** Let J_J , $j : \Omega \to \mathbb{R}^+_I$ be two coordinated LR- \hbar -convex IVMs on Ω , given by $T(t) = [\Pi(x, t), \Pi^*(x, t)]$ and $g(x, t) = [g(x, t), g^*(x, t)]$ category of classical convex functions, according to Dragomir and Pearce [1]. This **inequality has set in a straightforward integral and a straightforward integral integral integral integral integral in a straightforward integral integral integral in the straightforward integral integral integral in the** $J(x,y) = [J_*(x,y), J_*(x,y)]$ and $J(x,y) = [J_*(x,y), J_*(x,y)]$ **incomediately has several assumediate** integral and a straightforward integration. Integration, $\Omega \to \mathbb{R}^+$ be two coordinated LF $\Pi(x, y) = \left[\Pi(x, y), \Pi^*(x, y) \right]$ and $g(x, y) = \left[g(x, y) \right]$ **m** 8. Let Π , $\tau : \Omega \to \mathbb{R}^+$ be two coordinated LR- \hbar -convex. $\left\{ \left(\frac{y}{x}, \frac{y}{y}, \frac{y}{y}, \frac{y}{y}, \frac{y}{y}, \frac{y}{y}\right) \right\}$ for an $\left(\frac{x}{y}, \frac{y}{y} \right)$ $\in \mathbb{Z}^2$ and \mathbb{Z}^2 *Then, interval Riemann–Liouville-type integrals of* Ԓ *are defined as* **incomediately has subset in a straightforward integration and a straightforward integration**. Let \mathbb{R} a straightforward integration. **Theorem 8.** Let J_J , $g : \Omega \to \mathbb{R}_I^+$ be two coordinated LR- \hbar -convex IVMs or $J(x, u) = [J(x, u), J^*(x, u)]$ and $g(x, u) = [g(x, u), g^*(x, u)]$ for all (x, u) Let $h:[0,1] \to \mathbb{R}^+$. If $\pi \times \pi \in \mathfrak{D}_{\mathcal{O}}$, then the following inequalities hold: \cdots intrinsic geometric straightforward intrinsic geometric explanation. $I_0(x,y) = [J_*(x,y), J_0(x,y)]$ and $J(x,y) = [J_*(x,y)]$
 $I_0(x,y) = [J_*(x,y)]$ The Hermite–Hadamard inequality is one of the most well-known finding in the most well-known find category of $\left[\mathbf{Q}_{\mathbf{S}}\left(\mathbf{A},\mathbf{y}\right),\mathbf{y}\right]$, and Pearce $\left[\mathbf{Q}_{\mathbf{S}}\left(\mathbf{A},\mathbf{y}\right)\right]$. This is equal to $\mathbf{Q}_{\mathbf{S}}$. $\begin{bmatrix} \partial_1(x,y) & -\partial_2(x,y), & \partial_3(x,y) \end{bmatrix}$ and $\begin{bmatrix} f(x,y) & -\partial_3(x,y), & \partial_3(x,y) \end{bmatrix}$
let $\hbar : [0, 1] \rightarrow \mathbb{R}^+$. If $J \times \mathcal{J} \in \mathfrak{TD}_{\Omega}$, then the following inequalities **Theorem 8.** Let $J, \, j: \Omega \to \mathbb{R}^+_1$ be two coordinated LR- \hbar -con $\text{C}(\bar{x}, \bar{y}) = |J_{\bar{x}}(x, y), J^*(x, y)|$ and $g(x, y) = |J_{\bar{x}}(x, y), J^*(x, y)|$ $\Omega \rightarrow \mathbb{R}_I^+$ be two coordinated LR-ħ-convex IVMs on Ω , given by $\left[\begin{matrix} \lambda, y), & \mathsf{J} \end{matrix}\right]$ (λ, y) is a complete metric $\left[\begin{matrix} \lambda, y \\ \mathsf{J} \end{matrix}\right]$ is a complete metric space. $T¹$ and $T¹$ and $T¹$ and $T¹$ The Hermite–Hadamard inequality is one of the most well-known finding in the most construction of $\mathcal{L}_{\mathcal{U}}(\mathbf{x},\mathbf{y}) = [y_{\mathbf{x}}(\mathbf{x},\mathbf{y}), y_{\mathbf{x}}]$ convex functions, and $\mathcal{L}_{\mathcal{U}}(\mathbf{x},\mathbf{y})$. The $\mathcal{L}_{\mathcal{U}}(\mathbf{x},\mathbf{y})$ for \mathbf{x} , \mathbf{y} , $\mathbf{$ let $h : [0, 1] \to \mathbb{R}^+$. If $J \times g \in \mathfrak{D}_{\Omega}$, then the following inequalities hold: **teorem 8.** Let \mathfrak{J} , $g:\Omega\to\mathbb{R}^+_1$ be two coordinated LR-ħ-convex IVMs on Ω , given i $c_n\mathrm{E} f(x,y) = \mathrm{E} f(x,y), \mathrm{E} f(x,y)$ and $g(x,y) = \mathrm{E} f(x,y), \mathrm{E} f(x,y)$ for all $(x,y) \in \Omega$ and i ted LR-ħ-convex IVMs on Ω , given by and $J(x,y) = [J_*(x,y), J_*(x,y)]$ for an $(x,y) \in \Omega$ and
 $\mathcal{L} \Omega$ then the following inequalities hold: **Theorem 8.** Let J_J , $j : \Omega \to \mathbb{R}^+_I$ be two coordinated LR- \hbar -convex IVMs of $\pi(x, y) =$ category of convex functions, according to $\frac{1}{2}$. **Conserved Figure 1** and I at $\Omega \rightarrow \mathbb{R}^+$ be two coordinated I R-5-**Theorem 8.** Let $J_J: \Omega \to \mathbb{R}_I^+$ be two coordinated LR- \hbar -convex IVMs on Ω , given by $J(\chi, \chi) = [J(\chi, \chi)]$ and $J(\chi, \chi) = [J(\chi, \chi)]$ for all $(\chi, \chi) \in \Omega$ and Let \hbar if $[0, 1] \rightarrow \mathbb{R}^+$ if $\pi \times \pi \in \mathcal{F}$. Then the following inequality category of classical convex functions, according to $\frac{1}{2}$. This is $\frac{1}{2}$ $[As(\omega, y) \rightarrow \mathbb{R}^+, \text{ If } \Pi \times \gamma \in \mathfrak{D}_{\infty}$, then the following inequalities hold: **Theorem 8.** Let $J_J: \Omega \to \mathbb{R}^+_I$ be two coordinated LR- \hbar -convex IVMs on Ω , given by $J(\chi, \chi) = [\Pi(\chi, \chi)] \Pi^*(\chi, \chi)]$ and $g(\chi, \chi) = [g(\chi, \chi)] \Pi^*(\chi, \chi)]$ for all $(\chi, \chi) \in \Omega$ and category of classical convex functions, according to $\frac{1}{2}$. This is $\frac{1}{2}$ (ሾ∗, ∗ሿ,ሾ∗, ∗ሿ) = ሼ|∗ − ∗|, |∗ − ∗|ሽ. (7) The Hermite–Hadamard inequality is one of the most well-known findings in the $\overline{\Pi}(x, y) = [\overline{\Pi}(x, y), \overline{\Pi}^*(x, y)]$ and a category of convex functions, according to Dragomir and Pearce $\frac{1}{2}$. **Theorem 8** Let $\overline{\Pi}$ \overline{q} : $\overline{Q} \rightarrow \mathbb{R}^+$ be two coordinated LR-5-convex IVMs on \overline{Q} orige **Theorem 8.** Let $J, g: \Omega \to \mathbb{R}^+_I$ be two coordinated LR- \hbar -convex IVMs on Ω , given by $J(x, y) = [\Pi(x, y) \Pi^*(x, y)]$ and $g(x, y) = [\Pi(x, y) \Pi^*(x, y)]$ for all $(x, y) \in \Omega$ and Let \hbar if $[0, 1] \rightarrow \mathbb{R}^+$ if $\pi \times \pi \in \mathcal{F}$ is then the following inequalities hold: category of classical convex functions, according to Dragomir and Pearce [1]. This is the Dragomir and Pearce [1]. This is the $(x, y, y, y, \dots, y, y)$ for (x, y, y, \dots, y, y) \in \Box \Box \Box **can be directed for a** $\Omega \to \mathbb{R}^+$ be two coordinated I R-h-convex IVMs on O given by $u = [\Pi (x, u), \Pi^*(x, u)]$ and $g(x, u) =$ $T_1 \rightarrow \mathbb{R}^+$ If $\pi \times \pi \in \mathcal{F}$ is then the following inequalities holdlet $n : [0, 1] \rightarrow \mathbb{R}^+$. If $J \times J \in \mathfrak{2} \mathfrak{D}_{\Omega}$, then the following mequalities hold: years. When determining exact values for a mathematical problem proves to be **challenging** can be used to approximate the solution. Let JJ , $J: \Omega \rightarrow \mathbb{R}$ where I $c(x,y) = [J_*(x,y), J_*(x,y)]$ and $g(x,y)$ **Theorem 8.** Let $\iint_R f(x) dx \to \iint_R f(x) dx$ $\iint_X (x, y) = [J]_*(x, y), J^*(x, y)]$ and $\iint_X (x, y) = [J^*(x, y)]$ **Theorem 8** \overline{A} of \overline{B} as \overline{C} . \overline{D} the two coordinated $\pi(x, y) = \pi(x, y)$ $\pi^*(x, y)$ and $\pi(x, y) = \pi(x, y)$

$$
\frac{1}{2\alpha\beta h_1^2(\frac{1}{2})h_2^2(\frac{1}{2})} J\left(\frac{\sigma+i}{2}, \frac{\varepsilon+v}{2}\right) \times J\left(\frac{\sigma+i}{2}, \frac{\varepsilon+v}{2}\right)
$$
\n
$$
\leq p \frac{\Gamma(\alpha)\Gamma(\beta)}{2(i-\sigma)^6} \left[\frac{\sigma_0^2}{2} + \frac{\beta}{2} J(i, v) \times J(i, v) + \frac{\sigma_0^2}{2} J(i, \varepsilon) \times J(i, \varepsilon) \right]
$$
\n
$$
+ \frac{\Gamma(\alpha)\Gamma(\beta)}{2(i-\sigma)^8} \left[\frac{\sigma_0^2}{2} + \frac{\beta}{2} J(i, \sigma) \times J(\sigma, v) + \frac{\beta}{4} J(i, \varepsilon) - J(\sigma, \varepsilon) \times J(\sigma, \varepsilon) \right]
$$
\n
$$
+ M(\sigma, i, \varepsilon, v) \int_0^1 v^{\alpha-1} \kappa^{\beta-1} [\tilde{h}_1(v) \tilde{h}_1(\kappa) [\tilde{h}_2(v) \tilde{h}_2(1-\kappa) + \tilde{h}_2(1-v) \tilde{h}_2(\kappa) + \tilde{h}_2(1-v) \tilde{h}_2(\kappa) \right]
$$
\n
$$
+ \tilde{h}_1(v) \tilde{h}_1(1-\kappa) [\tilde{h}_2(v) \tilde{h}_2(\kappa) + \tilde{h}_2(1-v) \tilde{h}_2(1-\kappa) + \tilde{h}_2(v) \tilde{h}_2(\kappa) \right]
$$
\n
$$
+ \tilde{h}_2(v) \int_0^1 v^{\alpha-1} \kappa^{\beta-1} [\tilde{h}_1(v) \tilde{h}_1(\kappa) [\tilde{h}_2(1-v) \tilde{h}_2(1-\kappa) + \tilde{h}_2(v) \tilde{h}_2(\kappa) + \tilde{h}_2(v) \tilde{h}_2(\kappa) + \tilde{h}_2(v) \tilde{h}_2(\kappa) \right]
$$
\n
$$
+ \tilde{h}_2(v) \tilde{h}_2(\kappa) [\tilde{d} u \, dx
$$
\n
$$
+ \mathcal{N}(\sigma, i, \varepsilon, v) \int_0^1 v^{\alpha-1} \kappa^
$$

ℐℴశ,ష ఈ,ఉ Ԓ(,) = ^ଵ expressed as follows: ງ botn are coorainate n-concave *τν i*vis on **x***1, then the ineq*
llows: Ω beth and coordinate \hbar concerns IUM_2 and Ω , then the *inconstitute from equality g both are coorainate n-concave IV I*Ns on **x1**, then the inequality above can
lows: **Definition 2** ([41])**.** *Let* Ԓ: Ω → ℝூ *and* Ԓ ∈ Ω*. The double fuzzy interval Rieman–-Liouville*e coorainate n-concave *IV I*NS on **s 1**, then the inequality above can be Ω *ncave IV I*NS on **si**, then the inequality above can be μ_1 , μ_2 (x c), μ_2 (x c), μ_2 (x), μ_2 (x), μ_2 (x), μ_2 (x), μ_1 (x), μ_2 (x), μ_2 (x), μ_2 (x), μ_2 (x), μ_2 (x), μ_2 (x), μ_1), μ_2 (x), μ_2 (x), μ_2 (x), μ_2 (**Definition 2** ([41])**.** *Let* Ԓ: Ω → ℝூ *and* Ԓ ∈ Ω*. The double fuzzy interval Rieman–-Liouville-M*_{*M*} *M*^{*M*} *M*^{*m*} *<i>M*^{*i*} *M*^{*n*} *<i>M <i>M*^{*i*} *<i>M <i><i>n <i>m <i><i>m <i>m <i><i>n***** *<i>m <i>m* *<i><i>n***** *<i>m***** *<i>m <i><i>n***** *<i>m***** *<i>m <i><i>n**<i>m***** *<i>m***** *<i>m <i><i>n**<i>m**<i><i>n (-integrable) over* Ω *if and only if* Ԓ∗(,) *and* Ԓ∗(,)*both are -integrable over* Ω. *Moreover, if concurse if the one all, then the in* h_1 **b**_−*concazy* IVMs on ∩ then the inequality ghozy can be *M* concurs *i V MO on* **2***_{<i>i*}, *then the inequality* U, Γαλλαβασία του προσταλείου του στο προσταλείου του στο στο προσταλείου προσταλείου προσταλείου του διαφορείου y sy and y boin are coordinate n-concube *xviss on \x, then the inequality above can be*
pressed as follows: \overline{r} (,), \overline{r} *o for all 0.* \overline{r} is double *integrable integrable integra The family of all -integrable of s over coordinates and -integrable functions over* **The denoted by and and the continuity** *The family of all -integrable of s over coordinates and -integrable functions over T J* and *J* both are coordinate n-concute TV ivis *The family of all -integrable of s over coordinates and -integrable functions over* ria γ boin are coorainate n-concave *τν ivis on x z, ι*
e follower *The family of all -integrable of s over coordinates and -integrable functions over* $\pi_1(\nu)\pi_1(1-\kappa)\pi_2(1-\nu)\pi_2(1-\kappa)+\pi_2(\nu)\pi_2(\kappa)+\pi_2(\nu)\pi_2(1-\kappa)]$ and g both are coordinate h-concave IVMs on Ω , then the inequality above can be
expressed as follows: *The family of all -integrable of s over coordinates and -integrable functions over*

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convexity and the theory of inequalities.

The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–

can be directly derived from convex functions, there is a close relationship between

realms of applied and pure sciences. Furthermore, because of its many applications and

Keywords: interval-valued mappings over coordinates; left and right ℏ -Convexity; double

category of classical convex functions, according to D and P

convexity and the theory of inequalities.

convexity and the theory of inequalities.

Liouville fractional integral to derive the major results of the research. We also examine the key

The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–

The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–

convexity and the theory of inequalities.

$$
\frac{1}{2\alpha\beta h_{1}^{2}(\frac{1}{2})h_{2}^{2}(\frac{1}{2})}J\left(\frac{\sigma+i}{2},\frac{\varepsilon+v}{2}\right)\times J\left(\frac{\sigma+i}{2},\frac{\varepsilon+v}{2}\right) \n\geq p \frac{\Gamma(\alpha)\Gamma(\beta)}{2(i-\sigma)^{n}(\upsilon-\varepsilon)^{\beta}}\left[\frac{\sigma_{\upsilon}\beta}{2(\upsilon+\varepsilon+1)}(i,\upsilon)\times\frac{\sigma(i,\upsilon)}{\sigma(\upsilon+\varepsilon)}+\frac{\sigma_{\upsilon}\beta}{2(\upsilon-\varepsilon)^{\beta}}\left[\frac{\sigma_{\upsilon}\beta}{2(\upsilon+\varepsilon+1)}(i\sigma,\upsilon)\times\frac{\sigma(\upsilon,\upsilon)}{\sigma(\upsilon+\varepsilon)}+\frac{\sigma_{\upsilon}\beta}{2(\upsilon-\varepsilon)^{\beta}}\left[\frac{\sigma_{\upsilon}\beta}{2(\upsilon+\varepsilon+1)}(i\sigma,\upsilon)\times\frac{\sigma(\upsilon,\upsilon)}{\sigma(\upsilon+\varepsilon)}+\frac{\sigma_{\upsilon}\beta}{2(\upsilon+\varepsilon)}\left[\frac{\sigma(\upsilon+\varepsilon)}{\sigma(\upsilon+\varepsilon)}\right]\right] \n+\mathcal{M}(\upsilon,i,\varepsilon,\upsilon)\int_{0}^{1} \upsilon^{\alpha-1}\kappa^{\beta-1}[\tilde{h}_{1}(\upsilon)\tilde{h}_{1}(\kappa)[\tilde{h}_{2}(\upsilon)\tilde{h}_{2}(\upsilon-\kappa)+\tilde{h}_{2}(1-\upsilon)\tilde{h}_{2}(\kappa) \n+\tilde{h}_{2}(1-\upsilon)\tilde{h}_{2}(\upsilon)]d\upsilon d\kappa \n+P(\upsilon,i,\varepsilon,\upsilon)\int_{0}^{1} \upsilon^{\alpha-1}\kappa^{\beta-1}[\tilde{h}_{1}(\upsilon)\tilde{h}_{1}(\upsilon)[\tilde{h}_{2}(1-\upsilon)\tilde{h}_{2}(1-\kappa)+\tilde{h}_{2}(\upsilon)\tilde{h}_{2}(\kappa)+\tilde{h}_{2}(\upsilon)\tilde{h}_{2}(\upsilon)]d\upsilon d\kappa \n+P(\upsilon,i,\varepsilon,\upsilon)\int_{0}^{1} \upsilon^{\alpha-1}\kappa^{\beta-1}[\tilde{h}_{1}(\upsilon)\tilde{h}_{1}(\upsilon)[\tilde{h}_{2}(1-\upsilon)\tilde{
$$

category of classical convex functions, according to D and P

restrictions on endpoint functions of interval-valued functions that can be seen as applications of

where
$$
M(\sigma, \mathbf{i}, \varepsilon, \mathbf{v})
$$
, $P(\sigma, \mathbf{i}, \varepsilon, \mathbf{v})$, $N(\sigma, \mathbf{i}, \varepsilon, \mathbf{v})$, and $Q(\sigma, \mathbf{i}, \varepsilon, \mathbf{v})$ are given in Theorem 7.

oof Since $\bar{\Pi}$ a · Ω → \mathbb{R}^+ is two *LR-h-convex IVMs* then from inequality (17) we have **Truot.** Since f_j , $j: \Omega \to \mathbb{R}$ is two *ER-h*-convex *TV INS*, then from hiequality (17), we have **Proof.** Since JJ , $J : \Omega \rightarrow \mathbb{R}^+_I$ is two **Proof.** Since JJ , $J : \Omega \rightarrow \mathbb{R}^+_I$ is two LR- \hbar -convex I $\det J$, \jmath : $\Omega \rightarrow \mathbb{R}_I^+$ is two *LR-fi-*convex *IVM*s, the \mathbb{R}^+_I is two *LR-fi*-convex *IVM*s, then from inequality **Proof.** Since JJ , $J : \Omega \to \mathbb{R}^+_I$ is two LR- \hbar -convex IVMs, then from inequality (17), we have 1963) was the $\int y_j f(x^2) dx$ in the following integration in the following inequality is stated. **Proof.** Since JJ , $J : \Omega \to \mathbb{R}_I^+$ is two LR- \hbar -convex IVMs, then from inequality (17), we have **Proof.** Since $J, \jmath: \Omega \to \mathbb{R}^+_1$ is two LR- \hbar -convex IVMs, then from inequality (1

The family of all -integrable of s over coordinates and -integrable functions over

The family of all -integrable of s over coordinates and -integrable functions over

$$
\cfrac{}{\sqrt{3}}\left(\frac{\sigma+\mathfrak{i}}{2},\frac{\varepsilon+\mathfrak{v}}{2}\right)\times\cfrac{}{\sqrt{3}}\left(\frac{\sigma+\mathfrak{i}}{2},\frac{\varepsilon+\mathfrak{v}}{2}\right)
$$

The family of all -integrable of s over coordinates and -integrable functions over

convexity and the theory of inequalities.

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 $\frac{1}{2}$ was the one who first identified it $\frac{1}{2}$ and $\frac{1}{2}$ is the following is the following is stated:

Received: 14 November 2023

can be directly derived from convex functions, there is a close relationship between

Integral Inequalities via Coordinated

 $\frac{1}{2}$ was the one who first identified it first identified it follows: $\frac{1}{2}$

category of classical convex functions, according to Dragomir and Pearce [1]. This product \mathcal{I}

category of classical convex functions, according to D and P

challenging, including, including, including, including, including, including, including, including, including

challenging, in equalities can be used to approximate the solution. Since α

 \mathcal{L}_{max} and \mathcal{L}_{max} and \mathcal{L}_{max} and \mathcal{L}_{max} problem problem problem problem proves to be

tight relationship to the theory of inequalities, convexity has advanced α in recent α

tight relationship to the theory of inequalities, convexity has advanced α

convexity and the theory of inequalities. The theory of inequalities α

The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the most well-known fi

The Hermite–Hadamard inequality is one of the most well-known findings in the most well-known findings in the most well-known finding $\mathcal{L}_\mathcal{A}$

 T ne Hausdorff $\mathcal{P}_\mathcal{A}$, $\mathcal{P}_\mathcal{A}$

these new outcomes. The second control of the second control of the second control of the second control of th

khakami@jazanu.edu.sa

(ii) It can be easily seen that " ≤ " *looks like "left and right" on the real line* ℝ, *so we call*

 $f₁₉$ of 24

 $r_{\text{19 of 24}}$

 19 of 24

$$
\frac{\text{Fradal Fnet. 2024, 8, 125}}{7} = J\left(\frac{v_{\mathcal{O}} + (1-v)_{1}}{2} + \frac{(1-v)_{\mathcal{O}} + vt}{2} \frac{v_{\mathcal{O}} + (1-v)_{2}}{2} + \frac{(1-v)_{2} + vt}{2} \right)
$$
\n
$$
= J\left(\frac{v_{\mathcal{O}} + (1-v)_{1}}{2} + \frac{(1-v)_{\mathcal{O}} + vt}{2} \frac{v_{\mathcal{O}} + (1-v)_{1}}{2} + \frac{(1-v)_{2} + vt}{2} \right)
$$
\n
$$
\leq_{p} h_{1}^{2} \left(\frac{1}{2}\right) h_{2}^{2} \left(\frac{1}{2}\right) \times \left[\frac{1}{2} \left(\frac{1}{2} \sqrt{1 - v}\right) \left(\frac{1 - v}{2} \sqrt{1 - v}\right) + \frac{1}{2} \left(\frac{1 - v}{2} \sqrt{1 - v}\right) + \frac{1}{
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DEFINITION 2 (EXECUTE: Let *CAL*) and *Let EXECUTE and Liouville-* $\begin{bmatrix} 0 & 1 \end{bmatrix}$ *. Let* $\begin{bmatrix} 0 & 1 \end{bmatrix}$ the double integration of the resultant over $[0, 1] \times [0, 1]$ with counter the function of the function $\frac{d}{dx}$ $\frac{H}{\text{m}}$ is the main parameter of the modulum form $[0, 1] \times [0, 1]$ with moment to μ is the house are ababie integration of the resultant over $\left[0, 1\right] \wedge \left[0, 1\right]$ with ς
Takir \mathcal{L}_{H} is the main definition of \mathcal{L}_{L} by \mathcal{L}_{L} and \mathcal{L}_{L} is the sequence on the sequence of \mathcal{L}_{H} τ ation of the above fuzzy inclusion with $v^{\alpha-1}\kappa^{\beta-1}$ and then taking י− ז−י
ion o *The resultant over* $\begin{bmatrix} 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \end{bmatrix}$ with respect to $\begin{bmatrix} v & \kappa \end{bmatrix}$ we have $\frac{1}{2}$ re fuzzy inclusion with zy inclusion with $v^{\alpha-1} \kappa^{\beta-1}$ and then taking $^{-1}$ and then taking T_{ref} of T_{ref} of T_{ref} over T_{ref} over *Moreover, if* Ԓ *is -integrable over* Ω, *then* er $[0, 1] \times [0, 1]$ with $[1] \times [0, 1]$ with respect to (v, κ) , we have o (v, κ) , we have Taking the multiplication of the above fuzzy inclusion with $ν^{α−1}κ^{β−1}$ and then taking the double integration of the resultant over $[0, \, 1] \times [0, \, 1]$ with respect to $(v, \, \kappa)$, we have Taking the several indications of the share forward inelevation settle ϵ The result was mainly contribution of the model to Hermite (1822–1901), even though Hadamard (1872–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though Hadamard (1865–1901), even though H $\frac{1}{2}$ and $\frac{1}{2}$ was the one who first identified it is the following inequality is stated. Taking the multiplication of the above fuzzy inclusion with the double integration of the resultant over $[0, 1] \times [0, 1]$ with re \mathbb{E} begration of the resultant over $[0, 1] \times [0, 1]$ with respect to (v, κ) , we have Taking the multiplication of the above fuzzy inclusion with $v^{\alpha-1}\kappa^{\beta-1}$ and then the double integration of the resultant over $[0, 1] \times [0, 1]$ with respect to (v, κ) , we h

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\int_0^1 \int_0^1 v^{\alpha-1} \kappa^{\beta-1} J \left(\frac{\sigma + i}{2}, \frac{\varepsilon + \mathfrak{v}}{2} \right) \times \jmath \left(\frac{\sigma + i}{2}, \frac{\varepsilon + \mathfrak{v}}{2} \right) dv d\kappa
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≤*^p* ℏ¹ 2 1 ℏ2 2 1 1963) was the one who first identified it [2,3]. The following is how this inequality is stated: 1963) was the one who first identified it [2,3]. The following is how this inequality is stated: The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– Submitted for possible open access inequality has several applications and a straightforward intrinsic geometric explanation. restrictions on endpoint functions of interval-valued functions that can be seen as applications of **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly khakami@jazanu.edu.sa * Correspondence: enwaeze@alasu.edu (E.R.N.); muhammad.bilal@unitbv.ro (M.B.K.) 29 Eroilor Boulevard, 500036 Brasov, Romania 4 Department of Mathematics, Faculty of Science, Jazan University, Jazan 45142, Saudi Arabia; 4 Department of Mathematics, Faculty of Science, Jazan University, Jazan 45142, Saudi Arabia; 3 Department of Mathematics and Computer Science, Transilvania University of Brasov, 2 2 × R 1 0 R 1 0 *υ α*−1 *κ β*−1 *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality* (*υ Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality* + (1 − *υ*)i, *κε* + (1 − *κ*)v) × ℐℴశ ^ఈ Ԓ() = ^ଵ ௰(ఈ)) −(ఈିଵԒ() ^௬ ^ℴ (>ℴ), (8) ℐ ష ఈԒ() = ^ଵ ௰(ఈ)) −(ఈିଵԒ() ^௬ (<), (9) *where* >0 *and is the gamma function.* Interval and fuzzy Riemann-type integrals are defined as follows for coordinated Ԓ(,). **Theorem 2** ([42])**.** *Let* Ԓ:ሾℴ, ሿ ⊂ℝ→ℝூ *be an , given by* Ԓ() = ሾԒ∗(), Ԓ∗()ሿ *for all* ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* Ԓ∗() *both are Riemann integrable (-integrable) over* ሾℴ, ሿ. *Moreover, if* Ԓ *is -integrable over* ሾℴ, ሿ, *then* () න Ԓ() = ቈ() න Ԓ∗() , () න Ԓ∗() *The family of all -integrable of s over coordinates and -integrable functions over* ሾℴ, ሿ *are denoted by* ሾℴ,ሿ *and* ሾℴ,ሿ, *respectively*. **Theorem 3** ([31])**.** *Let* Ԓ: Ω = ሾℴ, ሿ × ሾ, ሿ ⊂ ℝଶ → ℝூ *be an on coordinates, given by (-integrable) over* Ω *if and only if* Ԓ∗(,) *and* Ԓ∗(,) *both are -integrable over* Ω. *Moreover, if* Ԓ *is -integrable over* Ω, *then* (,)Ԓ () ఌ (,)Ԓ () () = ℴ ఌ ^ℴ . (11) (*υ Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract 1963) was the one who first identified it [2,3]. The following is how this inequality is stated: **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. 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Then,* Ԓ *is double integrable (-integrable) over* Ω *if and only if* Ԓ∗(,) *and* Ԓ∗(,) *both are -integrable over* Ω. *Moreover, if* Ԓ *is -integrable over* Ω, *then* (,)Ԓ () (,)Ԓ () () = ^ℴ . (11) ((1 − *υ*) *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– 1963) was the one who first identified it [2,3]. The following is how this inequality is stated: **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality holds:* publication under the terms and Attribution (CC BY) license (https://creativecommons.org/license + *υ*i, *κε* + (1 − *κ*)v) + *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract category of classical convex functions, according to Dragomir and Pearce [1]. This inequality has several applications and a straightforward intrinsic geometric explanation. The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– 1963) was the one who first identified it [2,3]. The following is how this inequality is stated: **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality* (*υ Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– 1963) was the one who first identified it [2,3]. The following is how this inequality is stated: **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. Then, the following double inequality* + (1 − *υ*)i,(1 − *κ*)*ε* + *κ*v) × **Definition 1** ([30,40])**.** *Let* Ԓ:ሾℴ, ሿ → ℝூ ା *be an interval-valued mapping () and* Ԓ ∈ ℐℛሾℴ,ሿ*. 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Submitted for possible open access conditions of the Creative Commons Attribution (CC BY) license + (1 − *υ*)i,(1 − *κ*)*ε* + *κ*v) + *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract The Hermite–Hadamard inequality is one of the most well-known findings in the category of classical convex functions, according to Dragomir and Pearce [1]. This inequality has several applications and a straightforward intrinsic geometric explanation. The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– 1963) was the one who first identified it [2,3]. The following is how this inequality is stated: **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. 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Then, the following double inequality holds:* **Copyright:** © 2024 by the authors. publication under the terms and conditions of the Creative Commons (https://creativecommons.org/license + *υ*i,(1 − *κ*)*ε* + *κ*v) *dυdκ* +ℏ¹ 2 1 2 ℏ2 2 1 2 M(*Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract The Hermite–Hadamard inequality is one of the most well-known findings in the category of classical convex functions, according to Dragomir and Pearce [1]. This inequality has several applications and a straightforward intrinsic geometric explanation. The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– 1963) was the one who first identified it [2,3]. The following is how this inequality is stated: **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. 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Since many inequalities can be directly derived from convex functions, there is a close relationship between The Hermite–Hadamard inequality is one of the most well-known findings in the category of classical convex functions, according to Dragomir and Pearce [1]. This inequality has several applications and a straightforward intrinsic geometric explanation. The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865– 1963) was the one who first identified it [2,3]. The following is how this inequality is stated: **Theorem 1.** *Assume that the convex mapping* Ԓ:ሾℴ, ሿ → ℜ*. 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Then, the following double inequality* , i,*ε*, v) × R 1 0 R 1 0 *υ α*−1 *κ β*−1 ℏ1(*υ*)ℏ1(*κ*)[ℏ2(¹ − *^υ*)ℏ2(*κ*) + ℏ2(*υ*)ℏ2(¹ − *^κ*) + ℏ2(*υ*)ℏ2(*κ*)] +ℏ1(*υ*)ℏ1(¹ − *^κ*)[ℏ2(¹ − *^υ*)ℏ2(¹ − *^κ*) + ℏ2(*υ*)ℏ2(*κ*) + ℏ2(*υ*)ℏ2(¹ − *^κ*)] +ℏ1(¹ − *^υ*)ℏ1(*κ*)[ℏ2(*υ*)ℏ2(*κ*) + ℏ2(¹ − *^υ*)ℏ2(¹ − *^κ*) + ℏ2(¹ − *^υ*)ℏ2(*κ*)] +ℏ1(*υ*)ℏ1(¹ − *^κ*)[ℏ2(*υ*)ℏ2(¹ − *^κ*) + ℏ2(¹ − *^υ*)ℏ2(¹ − *^κ*) + ℏ2(¹ − *^υ*)ℏ2(*κ*)] *dυdκ*, **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly defined class of convex mappings proposed that are known as coordinated left and right ℏ convexity (-ℏ-convexity) over interval-valued codomain. We exploit the use of double Riemann– Liouville fractional integral to derive the major results of the research. We also examine the key results' numerical validations that examples are nontrivial. By taking the product of two left and right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some restrictions on endpoint functions of interval-valued functions that can be seen as applications of **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double Riemann–Liouville fractional integral operator; Pachpatte-type inequalities There are many uses for the concepts of convex sets and convex functions in the realms of applied and pure sciences. Furthermore, because of its many applications and tight relationship to the theory of inequalities, convexity has advanced quickly in recent years. When determining exact values for a mathematical problem proves to be challenging, inequalities can be used to approximate the solution. 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The following is how this inequality is stated: inequality has several applications and a straightforward intrinsic geometric explanation. inequality has several applications and a straightforward intrinsic geometric explanation. inequality has several applications and a straightforward intrinsic geometric explanation. inequality has several applications and a straightforward intrinsic geometric explanation. **Copyright:** © 2024 by the authors. 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Inequalities via Coordinated ℏ-Convexity via Left and Right

There are many uses for the convex sets and convex sets and convex \mathcal{A}

 \sim Correspondence: enwaneze. Enwaneze \sim enwarespondence: \sim (M.B. \sim (M.B.); muhammad.bilal. \sim

Riemann–Liouville fractional integral operator; Pachpatte-type inequalities

Tareq Saeed 1*,* **Eze R. Nwaeze 2,****,* **Muhammad Bilal Khan 3,* and Khalil Hadi Hakami 4**

 $\frac{1}{\sqrt{2}}$ is a familiar familiar familiar fact that $\frac{1}{\sqrt{2}}$ is a complete metric space.

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The family of all -integrable of s over coordinates over coordinates is denoted by Ω. *The integrals of* μ *are defined as* μ are defined as μ are defined as μ are defined as μ including that several applications and a straightforward intrinsic geometric explanation. The straightforward intrinsic geometric explanation. The straightforward intrinsic geometric explanation. The straightforward intri \ldots of classical convex functions, according to D $\frac{1}{\sqrt{1}}$ \mathbf{r} $c_{\rm r}$ functions, according to Dragomir and P \mathbf{r} $\frac{1}{\sqrt{1-\frac{1$

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The result was mainly credited to Hermite (1822–1901), even though Hadamard (1865–

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1. Introduction

Khan, M.B.; Hakami, K.H. New York (1985)

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1. Introduction

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Remark 5. If one assumes that \overline{J} is coordinated left-LR- \hbar -convex with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and $\alpha = 1$ and $\beta = 1$, then from (66), as a result, there will be inequality (see [42]): *P Then, intervalue is the measurer integral convergences of* $\frac{1}{2}$ *.* $\hbar(\kappa) = \kappa$ and $\kappa = 1$ and $\beta = 1$, then from (66), as a result, there will be inequality (see [42]): Submitted for possible open access publication under the terms and $T_c(x) = x$ and $x = 1$ and $p = 1$, even from (00), as a result, there will be inequality (see $\frac{1}{12}$ $\frac{1}{2}$ in the second and a straightforward interior $\frac{1}{2}$ is considered intrinsical interior $\frac{1}{2}$. T_{F} results was mainly credit to the Hadamard (1805–1901), even the Hadamard (1865–1901), even the Hadamard (1865–1901), T_{F} $E(\lambda) = 1$ $\hbar(\kappa) = \kappa$ and $\kappa = 1$ and $\beta = 1$, then from (66), as a result, there will be inequality (see [42]): \sum $\hslash(\kappa)=\kappa$ and $\alpha=1$ and $\beta=1$, then from (66), as a result, there will be inequality (see [42]): **Kemark 5.** If one assumes that JJ is coordinated left-LR-h-convex with i *(i) is coordinated left-LR-h-convex with* $\hbar(v)$ *= v,* **nark 5.** If one assumes that JJ is coordinated left-LK-h-convex with $h(v) = v$, There are many uses for the concepts of convex sets and convex functions in the There are many uses for the concepts of convex sets and convex functions in the **realment** of a point is many applied and pure sciences. Furthermore, $\frac{1}{2}$ is many applications and $\frac{1}{2}$ is many applications and $\frac{1}{2}$ is $\frac{1}{2}$ in $\frac{1}{2}$ **Remark** 5. If the assumes that for the coordinated left-LK-h-convex with $h(v) = v$,
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4J\left(\frac{\sigma+i}{2},\frac{\varepsilon+v}{2}\right) \times J\left(\frac{\sigma+i}{2},\frac{\varepsilon+v}{2}\right)
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\supseteq \frac{1}{(i-\sigma)(v-\varepsilon)} \int_{\sigma}^{i} \int_{\varepsilon}^{v} J(x,y) \times J(x,y) dy dx + \frac{5}{36} \mathcal{M}(\sigma,i,\varepsilon,v)
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+ \frac{7}{36} [P(\sigma,i,\varepsilon,v) + \mathcal{N}(\sigma,i,\varepsilon,v)] + \frac{2}{9} \mathcal{Q}(\sigma,i,\varepsilon,v).
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1 and $\beta = 1$, then from (66), as a result, there will be inequality (see [43]): publication under the terms and T_{min} $\rho = T$, then from (00), as a result, there will be inequality (see [19]). $\frac{1}{2}$ in the concentration $\frac{1}{2}$ or $\frac{1}{2}$ integral integ $T(\cos \theta)$ is a result, while was consequently (see $[10]$), $\overrightarrow{1}$ $\overrightarrow{1}$ $\overrightarrow{0}$ $\overrightarrow{1}$ $\overrightarrow{0}$ 1 and $\beta = 1$, then from (66), as a result, there will be inequality (see [43]): $\frac{1}{\sqrt{2}}$ is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known findings in the most well-known finding in the most and $\beta = 1$ then from (66) as a result there $\mathcal{F}_{\mathcal{F}}$ is one of the most well-known findings in the most well-known findings in the most well-known findings in the most well-known finding in the most well-known finding in the most well-known finding in the mos ϵ chen from (66) as a result there will be inequality $\mathcal{F}_{\mathcal{F}}$ is one of the most well-known findings in the 1 and $\beta = 1$ then from (66) as a result there will be inequality (see [43]). 1 and $\beta = 1$, then from (66), as a result, there will be inequality (see [43]): If JJ is coordinated LR-ħ-convex with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and one assumes that $\alpha =$ *coordinated* LR -*n*-convex with $h(t) = v$, $h(k) = -1$, then from (66), so a recall the space will be inequality (c) *first from (60), we a recall, there was be inequally (bee [10]).* realms of application and pure sciences. Furthermore, $\frac{1}{2}$ of $\frac{1}{2}$ and $\frac{1}{2}$ and 1 and $\beta = 1$, then from (66), as a result, there will be inequality (see [43]): μ LK-n-convex whit $\mu(v) = v$, $\mu(x) = \kappa$ and one assume it is μ on μ and μ and μ *for all* ሾ∗, ∗ሿ,ሾ∗, ∗ሿ ∈ ℝூ, *and it is a pseudo-order relation. The relation* ሾ∗, ∗ሿ ≤ ሾ∗, ∗ሿ m α *x* w *in* $n(v) = v$, $n(x) = \kappa$ *und one ussumes that* m There are many uses the convex sets and convex functions in the convex functi is coordinated LR- \hbar -convex with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and one assumes that $\alpha =$ \mathbf{E} $c = v, n(x) = \kappa$ and one assumes that $\alpha = \kappa$ *incondity (cee [A2]*). *(ii) It can be easily seen that* " ≤ " *looks like "left and right" on the real line* ℝ, *so we call* $\mathbf{F}(\mathbf{F} : \mathbf{F} \cup \mathbf{F} \$ realment of a pure science σ and ρ and ρ is many applications and ρ is many applications and ρ **1. Introduction** T_{tot} are many uses for the convex sets and convex sets and convex sets and convex $\frac{d}{dt}$. 1 and $\beta = 1$, then from (66), as a result, there will be inequality (see [43]): and $\beta=1$, then from (66), as a result, there will be inequality (see [43]): If J is coordinated LR-h-convex with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and one assumes that $\alpha = 1$ and $\beta = 1$, then from $(f(\zeta))$ as a grouble there will be inequality (see [42]). x_1, y_2 is essentially convex (x_1, y_2) and $\beta = 1$, then from (66) as a recult there will be inequality (cee [A31). r_{corr} (see μ is mattern pure science of its many applications and μ *f* \overline{y} , \overline{y} is economical ER *n* concent with $\alpha(y) = c$, $\alpha(x) = a$ and $\beta = 1$, then from (66), as a result, there will be inequality (see [43]): real measures of a proprietor and pure sciences. Furthermore, $\frac{1}{2}$ 1 and $\beta = 1$, then from (66), as a resul 1 and $\beta = 1$, then from (66), as a result, there will be inequality (see [43]): If J is coordinated LR-h-convex with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and one assumes that $\alpha =$ **Keywords:** interval-valued mappings over coordinates; left and right ℏ -Convexity; double **Keywords:** intervalued mappings over coordinates; left and right α -Convexity; double-1 and $\beta = 1$, then from (66), as a result, there will be inequality (see [43]): η *s* coordinated EX-n-convex with $n(v) = v$, $n(x) = \kappa$ and one assume right coordinated ℏ -convexity, some new versions of fractional integral inequalities are also If \iint_S is coordinated EN-n-convex with $h(v) = v$, $h(\kappa) = \kappa$ and one assumes that α If If is coordinated EK-n-convex with $h(v) = v$, $h(k) =$ If J is coordinated LR-h-convex with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$ and one assumes that $\alpha =$ $r_{\rm{ref}}$ numberies are non-trivial. By taking the product of two left and product of two left and the product of two left and the product of two left and the product of two left and two left and two left and two left and 1 and $\beta = 1$, then from (66), as a result, there will be inequality (see [43]): 1 and $\beta = 1$, then from (66), as a result, there will be inequality (see [43]): results' numerical validations that examples are nontrivial. By taking the product of two left and Land $\beta = 1$ then from (66) as a result there will be inequality (see [43]). results' numerical validations that examples are nontrivial. By taking the product of two left and $\mu d \beta = 1$ then from (66) as a result there will be inequality (see [43]). results' numerical validations that examples are nontrivial. By taking the product of two left and $\mu_1(G)$ as a mouth them will be inequality (see $L(2)$). results' numerical validations that examples are nontrivial. By taking the product of two left and ω (66) as a result there will be inequality (see [43]). results' numerical validations that examples are nontrivial. By taking the product of two left and ϵ a result there will be inequality (cee [43]). results' numerical validations that examples are nontrivial. By taking the product of two left and 1 , then from (66), as a result, there will be inequality (see [43]): **Abstract:** In particular, the fractional forms of Hermite–Hadamard inequalities for the newly koorumued LK-n-cono

inequality has several applications and a straightforward intrinsic geometric explanation.

$$
4J\left(\frac{\sigma+i}{2},\frac{\varepsilon+v}{2}\right) \times \mathcal{J}\left(\frac{\sigma+i}{2},\frac{\varepsilon+v}{2}\right) \n\leq p \frac{1}{(i-\sigma)(v-\varepsilon)} \int_{\sigma}^{i} \int_{\varepsilon}^{v} J\left(\frac{x}{2},y\right) \times \mathcal{J}\left(\frac{x}{2},y\right) dy dx + \frac{5}{36} \mathcal{M}\left(\sigma,i,\varepsilon,v\right) \n+ \frac{7}{36} \left[P\left(\sigma,i,\varepsilon,v\right) + \mathcal{N}\left(\sigma,i,\varepsilon,v\right)\right] + \frac{2}{9} \mathcal{Q}\left(\sigma,i,\varepsilon,v\right).
$$
\n(69)

inequality has several applications and a straightforward intrinsic geometric explanation.

Liouville fractional to derive the major results of the major results of the key also examine t

left-LR-h-convex \sim \sim \sim \sim \sim \sim \sim If π is coordinated left-LR- \hbar -convex and π (x, y) $\neq \pi^*(x, y)$ with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, \mathbf{F}_t is positive that $\mathbf{F}_t(\mathbf{x}, \mathbf{y}) \neq \mathbf{F}_t(\mathbf{x}, \mathbf{y})$ with $\mathbf{F}_t(\mathbf{y}) = \mathbf{F}_t$, $\mathbf{F}_t(\mathbf{x}) = \mathbf{F}_t$, $\mathbf{F}_t(\mathbf{x}) = \mathbf{F}_t$ then from (66), we succeed in bringing about the upcoming inequality (see [41]): $\begin{array}{ccc} \circ & \circ & \circ & \circ & \circ \end{array}$ If J is coordinated left-LR-h-convex and $J_*(x,y) \neq J^*(x,y)$ with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, μ ^{*i*} be an intervalue of μ , μ _i μ _i σ , μ _i, σ , μ _i, σ , μ _i, σ *The critically mean the appearing inequality (ecc [11])* If J is coordinated left-LR- \hbar -convex and $J_*(x,y) \neq J^*(x,y)$ with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (66), we succeed in bringing about the upcoming inequality (see [41]): publication under the terms and inequality has several applications and a straightforward intrinsic geometric explanation. then from (66), we succeed in bringing about the upcoming inequality (see [41]): T_{rel} was mainly credit the appearing the funnity (186 $\frac{1}{2}$ 121). \overline{u} \overline{c} \overline{c} then from (66), we succeed in bringing about the upcoming inequality (see [41]): can be directly derived from convex functions, there is a close relations, there is a close relationship between $I = \langle \cdot, \cdot \rangle$ If π is coordinated left-LR-h-convex and $\pi(x, y) \neq \pi^*(x, y)$ with $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, $\mathcal{L}_{\mathcal{A}}$ is a matrix of a mathematical problem $T_{\rm eff}$ are many uses for the convex sets and convex sets and convex ϵ for $T_{\rm eff}$ (ϵ), and convex functions ϵ the convex functions ϵ real grounds of a pure succession of its many more in its many $T_{\rm eff}$ are many uses for the convex sets and convex sets and convex ϵ for ϵ and ϵ functions in the convex function then from (66), we succeed in bringing about the upcoming inequality (see [41]): $\sum_{i=1}^n$ *Fractional Fractional II* $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\frac{1}{2}$ m from (66), we succeed in bringing about the upcoming inequality (see [4], then from (66), we succeed in bringing about the upcoming inequality (see [41]): then from (66), we succeed in bringing about the upcoming inequality (see [41]): ded te_l extension and $J_*(x, y) \neq J$ (x, y) with $n(v) = v$, $n(x) = \kappa$, is coordinated left-I R-h-convex and $\overline{\Pi}(x, u) \neq \overline{\Pi}^*(x, u)$ with $\overline{b}(u) = u$, $\overline{b}(x) = v$ $r_{\text{ref}}(66)$ are succeed in bringing about the uncoming inequality (see [41]). obtained**.** Moreover, some new and classical exceptional cases are also discussed by taking some

$$
\begin{split}\n&\geq \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{4(i-\sigma)^{\alpha}(\upsilon-\varepsilon)^{\beta}} \begin{bmatrix}\n\frac{\partial^{\alpha+\frac{i}{2}}}{2}, \frac{\varepsilon+\upsilon}{2} \\
\frac{\partial^{\alpha+\beta}}{\partial^{\alpha+\beta}}\Gamma(\mathbf{i},\upsilon) \times \mathcal{J}(\mathbf{i},\upsilon) + \frac{\partial^{\alpha+\beta}}{\partial^{\alpha+\beta}}\Gamma(\mathbf{i},\varepsilon) \times \mathcal{J}(\mathbf{i},\varepsilon) \\
+\frac{\partial^{\alpha+\beta}}{4(i-\sigma)^{\alpha}(\upsilon-\varepsilon)^{\beta}} \begin{bmatrix}\n\frac{\partial^{\alpha+\beta}}{2}, \mathcal{J}(\mathbf{i},\upsilon) \times \mathcal{J}(\mathbf{i},\upsilon) + \frac{\partial^{\alpha+\beta}}{\partial^{\alpha+\beta}}\Gamma(\mathbf{i},\varepsilon) \times \mathcal{J}(\mathbf{i},\varepsilon) \\
+\frac{\partial^{\alpha+\beta}}{2(\alpha+1)(\alpha+2)} + \frac{\beta}{(\beta+1)(\beta+2)} \left(\frac{1}{2} - \frac{\alpha}{(\alpha+1)(\alpha+2)}\right) \mathcal{M}(\mathbf{0},\mathbf{i},\varepsilon,\upsilon) \\
+\frac{\partial^{\alpha+\beta}}{2} \left(\frac{1}{2} - \frac{\alpha}{(\alpha+1)(\alpha+2)}\right) + \frac{\alpha}{(\alpha+1)(\alpha+2)} \frac{\beta}{(\beta+1)(\beta+2)} \mathcal{N}(\mathbf{0},\mathbf{i},\varepsilon,\upsilon) \\
+\frac{\partial^{\alpha+\beta}}{2} \left(\frac{1}{2} - \frac{\beta}{(\beta+1)(\beta+2)}\right) + \frac{\alpha}{(\alpha+1)(\alpha+2)} \frac{\beta}{(\beta+1)(\beta+2)} \mathcal{N}(\mathbf{0},\mathbf{i},\varepsilon,\upsilon) \\
+\frac{\partial^{\alpha+\beta}}{2} - \frac{\alpha}{(\alpha+1)(\alpha+2)} \frac{\beta}{(\beta+1)(\beta+2)} \mathcal{Q}(\mathbf{0},\mathbf{i},\varepsilon,\upsilon).\n\end{bmatrix}\n\end{split}
$$
\n(70)

 $\mathit{nd}\,\, \hslash(v)=v,\,\, \hslash(\kappa)=\kappa$, then from (66), we succeed in bringing about the upcoming inequality (see $[46]$): $=$ κ , then from (66), we succeed in bringing If $J_*(x,y) \neq J^*(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (66), we succeed in bringing *The family of all -integrable of s over coordinates over coordinates is denoted by* **Theorem 3** ([31])**.** *Let* Ԓ: Ω = ሾℴ, ሿ × ሾ, ሿ ⊂ ℝଶ → ℝூ *be an on coordinates, given by over* ሾℴ, ሿ, *then* \mathcal{C} , $[46]$): \mathcal{L} *Let* \mathcal{L} *Let* \mathcal{L} , \mathcal{L} *be a c* \mathcal{L} *z s*_{*j*}. **Theorem 22** (*Let* ∑∞, *given b*, *g*^{α}), *d* **Theorem 2** ([42])**.** *Let* Ԓ:ሾℴ, ሿ ⊂ℝ→ℝூ *be an , given by* Ԓ() = ሾԒ∗(), Ԓ∗()ሿ *for all* If $J_{*}(x,y) \neq J^{*}(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (66), we succeed in bringing *J* g) unu n(v) – v, n(κ) – κ, inen jiom (60), we succeeu in oringing
iality (coo [46]) T_{rec} α α β are defined as α If $J_{*}(x,y) \neq J^{*}(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (66), we succeed in bringing publication under the terms and inequality has several applications and a straightforward intrinsic geometric explanation. If $J_*(x, y) \neq J^*(x, y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (66), we succeed in bringing
chart the uncoming inequality (cee [46]). If $J_*(x,y) \neq J^*(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (66), we succeed in bringing
shout the uncomine inequality (see [461). T_{temp} code 12 , μ about the upcoming inequality (see [46]): $\left(\frac{1}{\sigma}\right)$ can be directly derived from convex functions, there is a close relationship between If $J_*(x,y) \neq J^*(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (66), we succeed in bringing T and T is one of the most well-known finding in the most well-known $\frac{1}{N}$ If $\iint_A f(x,y) \neq \iint_A f(x,y)$ and $\hbar(v) = v$, $\hbar(\kappa) = \kappa$, then from (66), we succeed in bringing mequality (see [46]):
 $(x^{\pm i} + y^{\pm i})$ in equalities can be directly derived from convex functions, there is a close relationship between $(x^{\pm i} + y^{\pm i})$ \mathcal{P} are determined problem probl

$$
\begin{split}\n&\leq p \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{4(i-\sigma)^{\alpha}(\mathfrak{v}-\varepsilon)^{\beta}} \begin{bmatrix}\n\frac{\alpha^{\mu}\beta}{2}, \frac{\varepsilon+\mathfrak{v}}{2} \\
\frac{\alpha^{\mu}\beta}{2}, \frac{\varepsilon+\mathfrak{v}}{2}\n\end{bmatrix} \times g(i, \mathfrak{v}) + g^{\alpha, \beta}_{\sigma^+, \mathfrak{v}^-, \mathfrak{v}^+, \mathfrak{v}^+, \mathfrak{v}^-, \mathfrak{v}^+, \mathfrak{v}^+, \mathfrak{v}^-, \mathfrak{v}^+, \mathfrak{v}^+, \mathfrak{v}^+, \mathfrak{v}^+, \mathfrak{v}^+, \mathfrak{v}^+, \mathfrak{v}^+, \mathfrak{v}^+, \math
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can be directly derived from convex functions, there is a close relationship between

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realms of applied and pure sciences. Furthermore, because of its many applications and

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can be directly derived from convex functions, there is a close relationship between

Definition 1 ([30,40])**.** *Let* Ԓ:ሾℴ, ሿ → ℝூ

can be directly derived from convex functions, there is a close relationship between

can be directly derived from convex functions, there is a close relations, there is a close relationship between \mathcal{L}

convexity and the theory of inequalities.

category of classical convex functions, according to D

category of classical convex functions, according to Dragomir and Pearce [1]. This

$$
\leq \frac{4J\left(\frac{\sigma+i}{2}, \frac{\varepsilon+v}{2}\right) \times J\left(\frac{\sigma+i}{2}, \frac{\varepsilon+v}{2}\right)}{\frac{J(\alpha+1)\Gamma(\beta+1)}{4(i-\sigma)^{\alpha}(\upsilon-\varepsilon)^{\beta}}\left[\frac{J^{\alpha,\beta}_{\sigma^+, \beta} - J(i, \upsilon) + J^{\alpha,\beta}_{\sigma^+, \beta} - J(i, \varepsilon) \times J(i, \varepsilon)}{+J^{\alpha,\beta}_{i-\varepsilon} + J(\sigma, \upsilon) \times J(\sigma, \upsilon) + J^{\alpha,\beta}_{i-\varepsilon} - J(\sigma, \varepsilon) \times J(\sigma, \varepsilon)\right]} + \left[\frac{\alpha}{2(\alpha+1)(\alpha+2)} + \frac{\beta}{(\beta+1)(\beta+2)}\left(\frac{1}{2} - \frac{\alpha}{(\alpha+1)(\alpha+2)}\right)\right] \mathcal{M}(\sigma, i, \varepsilon, \upsilon) + \left[\frac{1}{2}\left(\frac{1}{2} - \frac{\alpha}{(\alpha+1)(\alpha+2)}\right) + \frac{\alpha}{(\alpha+1)(\alpha+2)}\frac{\beta}{(\beta+1)(\beta+2)}\right] P(\sigma, i, \varepsilon, \upsilon) + \left[\frac{1}{2}\left(\frac{1}{2} - \frac{\beta}{(\beta+1)(\beta+2)}\right) + \frac{\alpha}{(\alpha+1)(\alpha+2)}\frac{\beta}{(\beta+1)(\beta+2)}\right] \mathcal{N}(\sigma, i, \varepsilon, \upsilon) + \left[\frac{1}{4} - \frac{\alpha}{(\alpha+1)(\alpha+2)}\frac{\beta}{(\beta+1)(\beta+2)}\right] Q(\sigma, i, \varepsilon, \upsilon).
$$
\n(72)

4. Conclusions and Future Plans ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* Ԓ∗() *both are Riemann integrable (-integrable) over* ሾℴ, ሿ. *Moreover, if* Ԓ *is -integrable* ∈ ሾℴ, ሿ*. Then,* Ԓ *is Riemann integrable (-integrable) over* ሾℴ, ሿ *if and only if* Ԓ∗() *and* Ԓ∗() *both are Riemann integrable (-integrable) over* ሾℴ, ሿ. *Moreover, if* Ԓ *is -integrable* Hence Plans **Theorem 22** ($\frac{1}{2}$) $\frac{1}{2}$ $\frac{1}{$ *Fractal Fract.* **2024**, *8*, x. https://doi.org/10.3390/xxxxx www.mdpi.com/journal/fractalfract

 1963 was the one who first identified it $\frac{2}{3}$. The following is the following is stated:

convexity and the theory of inequalities. The theory of inequalities $\mathcal{L}_{\mathcal{A}}$

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 1963 was the one who first identified it $2\sqrt{2}$. The following is the following is the following is stated:

can be directly derived from convex functions, there is a close relationship between

convexity and the theory of inequalities.

Definition 1 ([30,40])**.** *Let* Ԓ:ሾℴ, ሿ → ℝூ

convexity and the theory of inequalities. The theory of inequalities $\mathcal{L}_{\mathcal{A}}$

To sum up, this study offers a new extension of interval-valued convexity. Through the use of fractional integral operators, various inequalities for *LR-fi*-convexity are produced and classical versions of integral inequalities are also acquired that can be considered as applications of this article's outcomes. Some very interesting examples are also given to The results of the main results. The results of this research paper could potentially have applications in various areas of mathematics, physics, and engineering. The extension of the proposed iterative method to systems of equations could be an interesting future research problem. In the future, we will try to explore these concepts in quantum calculus. lying interval-valued mapping. Many exceptional cases are discussed and new $\frac{1}{2}$ **d** $\frac{1}{2}$ $\frac{1}{2}$ **d** $\frac{1}{2}$ **Theorem 3** ([31])**.** *Let* Ԓ: Ω = ሾℴ, ሿ × ሾ, ሿ ⊂ ℝଶ → ℝூ *be an on coordinates, given by* **Theorem 3** ([31])**.** *Let* Ԓ: Ω = ሾℴ, ሿ × ሾ, ሿ ⊂ ℝଶ → ℝூ *be an on coordinates, given by* by applying interval-valued mapping. Many exceptional cases are discussed and new

The family of all -integrable of s over coordinates over coordinates is denoted by .contr writing—review and editing, M.B.K. and K.H.H.; visualization, T.S. and K.H.H.; supervision, T.S. and project administration, i.b. and E.R.IV. All authors have *The family of all -integrable of s over coordinates over coordinates is denoted by* Author Contributions: Conceptualization, M.B.K.; validation, E.R.N.; formal analysis, E.R.N.; investigation, M.B.K. and T.S.; resources, M.B.K. and T.S.; writing—original draft, M.B.K. and T.S.; *The family of all authors have read and across to the ministerates of C and C D M All authors have read and across to the ministerates* g, *M.*D.K. and K.H.H.; visualization, 1.5. and K.H.H.; supervision, 1.5. and
ation, T.S. and E.R.N. All authors have read and agreed to the published **The family of the manuscript.** \overline{a} over *complements of the manuscript.* $\frac{1}{\sqrt{2}}$ K.H.H.; project administration, T.S. and E.R.N. All authors have read and agreed to the published
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Data Availability Statement: No new data were created or analyzed in this study. Data sharing is le to this article.
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Conflicts of Interest: The authors claim to have no conflicts of interest. α order **defined** by: α order **by:** α *order* **defined** by: *o* α

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