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Article

Impressive Exact Solitons to the Space-Time Fractional Mathematical Physics Model via an Effective Method

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Abstract: A new class of truncated M-fractional exact soliton solutions for a mathematical physics model known as a truncated M-fractional (1+1)-dimensional nonlinear modified mixed-KdV model are achieved. We obtain these solutions by using a modified extended direct algebraic method. The obtained results consist of trigonometric, hyperbolic trigonometric and mixed functions. We also discuss the effect of fractional order derivative. To validate our results, we utilized the Mathematica software. Additionally, we depict some of the obtained kink, periodic, singular, and kink-singular wave solitons, using two and three dimensional graphs. The obtained results are useful in the fields of fluid dynamics, nonlinear optics, ocean engineering and others. Furthermore, these employed techniques are not only straightforward, but also highly effective when used to solve non-linear fractional partial differential equations (FPDEs).

Keywords: fractional modified mixed-KdV equation; modified extended direct algebraic method; exact soliton solutions

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1. Introduction

Fractional calculus have much importance in many fields of science and engineering like; fluid dynamics, plasma physics, optical fibers, chemistry, biology, economics etc. Naturally occurring phenomenon are represented in the form of fractional partial differential equations (FPDEs). Distinct techniques have been developed to the numerical and exact solutions of the FPDEs. For example, generalized exponential rational function method [1], extended trial equation technique [2], improved Fan sub-equation technique [3], modified extended tanh function technique [4], multi exp-function scheme [5], generalized double auxiliary equation technique [6], Bäcklund transformations [7], transformed rational function method [8], Darboux transformation [9] etc.

In our study, we utilize a straightforward and powerful method: modified extended direct algebraic method. This method has various applications. For example, optical soliton solutions to the nonlinear Schrödinger equation are obtained [10], kink wave and lump wave solutions for the fractional coupled Higgs system are achieved [11], bright and dark-singular combo soliton solutions for the Lakshmanan-Porseizian-Daniel model are gained [12], singular, dark and bright wave solutions for the Gerdjikov-Ivanov equation are gained [13], singular-periodic, rational, exponential and other solutions for the highly dispersive perturbed nonlinear Schrödinger equation are obtained [14], dark, bright, singular and other kinds of soliton solutions for the extended (2+1)-dimensional perturbed nonlinear Schrödinger equation are gained [15].

One of the mathematical physics model; (1+1)-dimensional modified mixed Korteweg-de Vries (mmKdV) equation. This model have much importance in nonlinear optics, fluid dynamics and other fields. This model is solved by different techniques earlier. For example; distinct kinds of topological and non-topological soliton solutions are gained by applying the extended rational sinh-cosh technique, extended rational sine-cosine technique and polynomial function technique [16], dark, bright and periodic wave solutions are obtained by utilizing the Homogenous balance technique [17] etc.

The objective of our study is to investigate some new types of M-fractional exact soliton solutions for the (1+1)-dimensional non-linear modified mixed-KdV model by employing the modified extended direct algebraic method and show the effect of truncated M-fractional derivative on the solutions.

This paper contains the various sections; Modified extended direct algebraic method is explained in Section 2, concerning model and it's mathematical analysis are explained in Section 3, exact soliton solutions are gained in Section 4, some gained results are shown graphically in Section 5, conclusion in Section 6.

2. Modified Extended Direct Algebraic Method

The fundamental steps of this technique are given as [11]:
Step 1: Considering a NLPDE:

$$G(f, f^2, f^2 f_x, f_{xx}, f_{xt}, \dots) = 0, \tag{1}$$

here $f = f(x,t)$ represents a wave profile. Consider a wave transformations:

$$f(x,t) = F(\xi), \ \ \xi = x + \lambda t. \tag{2}$$

Using Equation (2) into Equation (1), a NLODE is gained

$$Z(F, F^2 F', F'', \dots) = 0. \tag{3}$$

Step 2: Consider Equation (3) has solution given as:

$$F(\xi) = \sum_{s=-n}^{n} \alpha_s \psi^s(\xi) \tag{4}$$

here $\alpha_s (s = -n, \dots, 0, 1, 2, 3, \dots, n)$ are the unknowns and $\alpha_n \neq 0$. A new profile $\psi(\xi)$ satisfies the ODE:

$$\psi'(\xi) = \log(d)\Big(p + q\psi(\xi) + r\psi(\xi)^2\Big) \tag{5}$$

where p, q and r are the constants and $d \neq 0, 1$. Notice that Equation (5) has solutions for the following different cases:
Case 1: if $q^2 - 4pr < 0$ and $r \neq 0$, we have

$$\psi_1(\xi) = -\frac{q}{2r} + \frac{\sqrt{-(q^2-4pr)}\tan_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r}. \tag{6}$$

$$\psi_2(\xi) = -\frac{q}{2r} - \frac{\sqrt{-(q^2-4pr)}\cot_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r}. \tag{7}$$

$$\psi_3(\xi) = -\frac{q}{2r} + \frac{\sqrt{-(q^2-4pr)}(\tan_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\ \sec_B(\sqrt{-(q^2-4pr)}\xi)))}{2r}. \tag{8}$$

$$\psi_4(\xi) = -\frac{q}{2r} - \frac{\sqrt{-(q^2-4pr)}(\cot_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\ \csc_B(\sqrt{-(q^2-4pr)}\xi)))}{2r}. \tag{9}$$

$$\psi_5(\xi) = -\frac{q}{2r} + \frac{\sqrt{-(q^2-4pr)}(\tan_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi) - (\cot_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi)))}{2r}. \tag{10}$$

Case 2:

$$\psi_6(\xi) = -\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}\tanh_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r}. \tag{11}$$

$$\psi_7(\xi) = -\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}\coth_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r}. \quad (12)$$

$$\psi_8(\xi) = -\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\tanh_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\ sech_B(\sqrt{(q^2-4pr)}\xi)))}{2r}. \quad (13)$$

$$\psi_9(\xi) = -\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\coth_B(\sqrt{(q^2-4pr)}\xi) \pm (\sqrt{cf}\ csch_B(\sqrt{(q^2-4pr)}\xi)))}{2r}. \quad (14)$$

$$\psi_{10}(\xi) = -\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\tanh_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi) - (\coth_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi)))}{2r}. \quad (15)$$

Case 3: if $pr > 0$ and $q = 0$, we have

$$\psi_{11}(\xi) = \sqrt{\frac{p}{r}}\tan_B(\sqrt{pr}\xi). \quad (16)$$

$$\psi_{12}(\xi) = -\sqrt{\frac{p}{r}}\cot_B(\sqrt{pr}\xi). \quad (17)$$

$$\psi_{13}(\xi) = \sqrt{\frac{p}{r}}(\tan_B(2\sqrt{pr}\xi) \pm (\sqrt{cf}\sec_B(2\sqrt{pr}\xi))). \quad (18)$$

$$\psi_{14}(\xi) = -\sqrt{\frac{p}{r}}(\cot_B(2\sqrt{pr}\xi) \pm (\sqrt{cf}\csc_B(2\sqrt{pr}\xi))). \quad (19)$$

$$\psi_{15}(\xi) = \frac{1}{2}\sqrt{\frac{p}{r}}(\tan_B(\frac{1}{2}\sqrt{pr}\xi) - \cot_B(\frac{1}{2}\sqrt{pr}\xi)). \quad (20)$$

Case 4: if $pr < 0$ and $q = 0$, we have

$$\psi_{16}(\xi) = -\sqrt{-\frac{p}{r}}\tanh_B(\sqrt{-pr}\xi). \quad (21)$$

$$\psi_{17}(\xi) = -\sqrt{-\frac{p}{r}}\coth_B(\sqrt{-pr}\xi). \quad (22)$$

$$\psi_{18}(\xi) = -\sqrt{-\frac{p}{r}}(\tanh_B(2\sqrt{-pr}\xi) \pm (\iota\sqrt{cf}sech_B(2\sqrt{-pr}\xi))). \quad (23)$$

$$\psi_{19}(\xi) = -\sqrt{-\frac{p}{r}}(\coth_B(2\sqrt{-pr}\xi) \pm (\sqrt{cf}csch_B(2\sqrt{-pr}\xi))). \quad (24)$$

$$\psi_{20}(\xi) = -\frac{1}{2}\sqrt{-\frac{p}{r}}(\tanh_B(\frac{1}{2}\sqrt{-pr}\xi) + \coth_B(\frac{1}{2}\sqrt{-pr}\xi)). \quad (25)$$

Case 5: if $r = p$ and $q = 0$, we have

$$\psi_{21}(\xi) = \tan_B(p\xi). \quad (26)$$

$$\psi_{22}(\xi) = -\cot_B(p\xi). \quad (27)$$

$$\psi_{23}(\xi) = \tan_B(2p\xi) \pm (\sqrt{cf}\sec_B(2p\xi). \quad (28)$$

$$\psi_{24}(\xi) = -\cot_B(2p\xi) \pm (\sqrt{cf}\csc_B(2p\xi). \quad (29)$$

$$\psi_{25}(\xi) = \frac{1}{2}\tan_B(\frac{1}{2}p\xi) - \frac{1}{2}\cot_B(\frac{1}{2}p\xi). \quad (30)$$

Case 6: if $r = -p$ and $q = 0$, we have

$$\psi_{26}(\xi) = -\tanh_B(p\xi). \quad (31)$$

$$\psi_{27}(\xi) = -\coth_B(p\xi). \tag{32}$$

$$\psi_{28}(\xi) = -\tanh_B(2p\xi) \pm (\iota\sqrt{cf} sech_B(2p\xi). \tag{33}$$

$$\psi_{29}(\xi) = -\coth_B(2p\xi) \pm (\sqrt{cf} csch_B(2p\xi). \tag{34}$$

$$\psi_{30}(\xi) = -\frac{1}{2}\tanh_B(\frac{1}{2}p\xi) - \frac{1}{2}\coth_B(\frac{1}{2}p\xi). \tag{35}$$

Case 7: if $q^2 - 4pr = 0$, we have

$$\psi_{31}(\xi) = -2\frac{p(q\xi\log(d) + 2)}{q^2\xi\log(d)}. \tag{36}$$

Case 8: if $q = \rho$ $p = m\rho$ $(m \neq 0)$ and $r = 0$, we have

$$\psi_{32}(\xi) = d^{\rho\xi} - m. \tag{37}$$

Case 9: if $q = r = 0$, we have

$$\psi_{33}(\xi) = p\,\xi\log(d). \tag{38}$$

Case 10: if $p = q = 0$, we have

$$\psi_{34}(\xi) = -\frac{1}{r\,\xi\log(d)}. \tag{39}$$

Case 11: if $p = 0$, $q \neq 0$ and $r \neq 0$, we have

$$\psi_{35}(\xi) = -\frac{cq}{r(\cosh_B(q\xi) - \sinh_B(q\xi) + c)}. \tag{40}$$

$$\psi_{36}(\xi) = -\frac{q(\cosh_B(q\xi) + \sinh_B(q\xi))}{r(\cosh_B(q\xi) + \sinh_B(q\xi) + f)}. \tag{41}$$

Case 12: if $q = \rho$ $r = m\rho$ $(m \neq 0)$ and $p = 0$, we have

$$\psi_{37}(\xi) = \frac{cd^{\rho\xi}}{c - mfd^{\rho\xi}}. \tag{42}$$

where $c, f > 0$ and represent the deformation parameters.

Step 3: Begin this step by putting Equation (4) along Equation (5) into Equation (3). Next sum coefficients of each ψ^s term, then set the coefficients of same order equal to zero to gain a set of equations in α_s and λ. By manipulating this set of equations, results for the unknowns are found. Step 4: Inserting Equation (4), where the values of α_s and λ are now known into Equation (3) will yield solutions to Equation (1).

3. Model Description and Mathematical Analysis

Let us assume a truncated M-fractional (1+1)-dimensional nonlinear modified mixed-KdV model given as:

$$D^{\alpha,\Upsilon}_{M,t}u + (a\sqrt{u} + bu)D^{\alpha,\Upsilon}_{M,x}u + \tau D^{3\alpha,\Upsilon}_{M,3x}u = 0, \tag{43}$$

where

$$D^{\alpha,\Upsilon}_{M,x}u = \lim_{\tau\to 0}\frac{u\,(x\,E_\Upsilon(\tau x^{1-\alpha})) - u(x)}{\tau}, \quad 0 < \alpha \leq 1, \;\; \Upsilon \in (0,\infty) \tag{44}$$

where $E_\Upsilon(.)$ is a truncated Mittag-Leffler (TML) function mention in [18,19].

Here $u = u(x,t)$ is a wave function. Equation (1) represents a modified mixed Korteweg-de Vries (mmKdV) model which explains the flat-topped electron distribution having a stronger non-linearity that corresponds to the small-width and high-velocity of the wave [20–22]. To find the solutions of Equation (1), we suppose the following:

$$u = g^2; \qquad g = g(x,t). \tag{45}$$

By putting Equation (45) into Equation (43), yields

$$gD^{\alpha,\Upsilon}_{M,t}g + (ag^2 + bg^3)D^{\alpha,\Upsilon}_{M,x}g + \tau(gD^{3\alpha,\Upsilon}_{M,3x}g + 3D^{\alpha,\Upsilon}_{M,x}gD^{2\alpha,\Upsilon}_{M,2x}g) = 0. \tag{46}$$

Consider a wave transformation given as:

$$g = G(\xi); \qquad \xi = \theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha - \lambda t^\alpha). \tag{47}$$

here θ represents the wave-amplitude and λ denotes the wave velocity.

By using Equation (47) into Equation (46), we get

$$-6\lambda G^2 + 4aG^3 + 3bG^4 + 12\tau\theta^2(GG'' + (G')^2) = 0. \tag{48}$$

By applying the Homogenous balance approach into Equation (48), we gain the natural number $m = 1$. Now we will solve the Equation (48) with the help of two different techniques.

4. Exact Soliton Solutions

For $m = 1$, Equation (4) changes into

$$G(\xi) = \alpha_{-1}(\psi(\xi))^{-1} + \alpha_0 + \alpha_1\psi(\xi). \tag{49}$$

Putting Equation (49) along Equation (5) into Equation (48), we obtain the following sets for discussion.

Set 1:

$$\left\{\alpha_{-1} = -\frac{4ap}{5bq}, \alpha_0 = -\frac{4a}{5b}, \alpha_1 = -\frac{4ar}{5bq}, \lambda = -\frac{16a^2(q^2-4pr)}{75bq^2}, \tau = -\frac{4a^2}{75b\theta^2q^2\log^2(d)}\right\} \tag{50}$$

Case 1: if $q^2 - 4pr < 0$ and $r \neq 0$, we have

$$u(x,t) = \Big(-\frac{4a}{5b} - \frac{4ar}{5bq}\Big(-\frac{q}{2r} + \frac{\sqrt{-(q^2-4pr)}\tan_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r}\Big) - \frac{4ap}{5bq}\Big(-\frac{q}{2r} + \frac{\sqrt{-(q^2-4pr)}\tan_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r}\Big)^{-1}\Big)^2. \tag{51}$$

$$u(x,t) = \Big(-\frac{4a}{5b} - \frac{4ar}{5bq}\Big(-\frac{q}{2r} - \frac{\sqrt{-(q^2-4pr)}\cot_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r}\Big) - \frac{4ap}{5bq}\Big(-\frac{q}{2r} - \frac{\sqrt{-(q^2-4pr)}\cot_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r}\Big)^{-1}\Big)^2. \tag{52}$$

$$u(x,t) = \Big(-\frac{4a}{5b} - \frac{4ar}{5bq}\Big(-\frac{q}{2r} + \frac{\sqrt{-(q^2-4pr)}(\tan_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\ \sec_B(\sqrt{-(q^2-4pr)}\xi))}{2r}\Big) - \frac{4ap}{5bq}\Big(-\frac{q}{2r} + \frac{\sqrt{-(q^2-4pr)}(\tan_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\ \sec_B(\sqrt{-(q^2-4pr)}\xi)))}{2r}\Big)^{-1}\Big)^2. \tag{53}$$

$$u(x,t)=(-\frac{4a}{5b}-\frac{4ar}{5bq}(-\frac{q}{2r}-\frac{\sqrt{-(q^2-4pr)}(\cot_B(\sqrt{-(q^2-4pr)}\xi)\pm(\sqrt{cf}\ \csc_B(\sqrt{-(q^2-4pr)}\xi))}{2r})$$
$$-\frac{4ap}{5bq}(-\frac{q}{2r}-\frac{\sqrt{-(q^2-4pr)}(\cot_B(\sqrt{-(q^2-4pr)}\xi)\pm(\sqrt{cf}\ \csc_B(\sqrt{-(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (54)$$

$$u(x,t)=(-\frac{4a}{5b}-\frac{4ar}{5bq}(-\frac{q}{2r}+\frac{\sqrt{-(q^2-4pr)}(\tan_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi)-(\cot_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi))}{2r})$$
$$-\frac{4ap}{5bq}(-\frac{q}{2r}+\frac{\sqrt{-(q^2-4pr)}(\tan_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi)-(\cot_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (55)$$

where $\xi=\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2(q^2-4pr)}{75bq^2}t^\alpha)$.

Case 2:

$$u(x,t)=(-\frac{4a}{5b}-\frac{4ar}{5bq}(-\frac{q}{2r}-\frac{\sqrt{(q^2-4pr)}\tanh_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r})$$
$$-\frac{4ap}{5bq}(-\frac{q}{2r}-\frac{\sqrt{(q^2-4pr)}\tanh_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r})^{-1})^2. \quad (56)$$

$$u(x,t)=(-\frac{4a}{5b}-\frac{4ar}{5bq}(-\frac{q}{2r}-\frac{\sqrt{(q^2-4pr)}\cot_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r})$$
$$-\frac{4ap}{5bq}(-\frac{q}{2r}-\frac{\sqrt{(q^2-4pr)}\coth_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r})^{-1})^2. \quad (57)$$

$$u(x,t)=(-\frac{4a}{5b}-\frac{4ar}{5bq}(-\frac{q}{2r}-\frac{\sqrt{(q^2-4pr)}(\tanh_B(\sqrt{-(q^2-4pr)}\xi)\pm(\sqrt{cf}\ sech_B(\sqrt{(q^2-4pr)}\xi))}{2r})$$
$$-\frac{4ap}{5bq}(-\frac{q}{2r}-\frac{\sqrt{(q^2-4pr)}(\tanh_B(\sqrt{-(q^2-4pr)}\xi)\pm(\sqrt{cf}\ sech_B(\sqrt{(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (58)$$

$$u(x,t)=(-\frac{4a}{5b}-\frac{4ar}{5bq}(-\frac{q}{2r}-\frac{\sqrt{(q^2-4pr)}(\coth_B(\sqrt{(q^2-4pr)}\xi)\pm(\sqrt{cf}\ csch_B(\sqrt{(q^2-4pr)}\xi))}{2r})$$
$$-\frac{4ap}{5bq}(-\frac{q}{2r}-\frac{\sqrt{(q^2-4pr)}(\coth_B(\sqrt{(q^2-4pr)}\xi)\pm(\sqrt{cf}\ csch_B(\sqrt{(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (59)$$

$$u(x,t)=(-\frac{4a}{5b}-\frac{4ar}{5bq}(-\frac{q}{2r}-\frac{\sqrt{(q^2-4pr)}(\tanh_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi)-(\coth_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi))}{2r})$$
$$-\frac{4ap}{5bq}(-\frac{q}{2r}-\frac{\sqrt{(q^2-4pr)}(\tanh_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi)-(\coth_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (60)$$

where $\xi=\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2(q^2-4pr)}{75bq^2}t^\alpha)$.

Case 8:

$$u(x,t)=(-\frac{4a}{5b}-\frac{4am}{5b}(d^{\rho\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)}-m)^{-1})^2. \quad (61)$$

Case 11:

$$u(x,t)=(-\frac{4a}{5b}-\frac{4ar}{5bq}(-\frac{cq}{r(\cosh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))-\sinh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))+c)}))^2. \quad (62)$$

$$u(x,t)=(-\frac{4a}{5b}-\frac{4ar}{5bq}(-\frac{q(\cosh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))+\sinh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))}{r(\cosh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))+\sinh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))+f)}))^2. \quad (63)$$

Case 12:

$$u(x,t)=(-\frac{4a}{5b}-\frac{4am}{5b}(\frac{cd^{\rho\xi}}{c-mfd^{\rho\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)}}))^2. \quad (64)$$

Set 2:

$$\{\alpha_{-1}=\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)},\alpha_0=\frac{2\sqrt{a^2q^2(q^2-4pr)}-2a(q^2-4pr)}{5b(q^2-4pr)},\alpha_1=0,$$
$$\lambda=-\frac{16a^2}{75b},\tau=-\frac{4a^2}{75b\theta^2\log^2(d)(q^2-4pr)}\} \quad (65)$$

Case 1 :

$$u(x,t)=(\frac{2\sqrt{a^2q^2(q^2-4pr)}-2a(q^2-4pr)}{5b(q^2-4pr)}+\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}+\frac{\sqrt{-(q^2-4pr)}\tan_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r})^{-1})^2. \quad (66)$$

$$u(x,t)=(\frac{2\sqrt{a^2q^2(q^2-4pr)}-2a(q^2-4pr)}{5b(q^2-4pr)}+\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}-\frac{\sqrt{-(q^2-4pr)}\cot_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r})^{-1})^2. \quad (67)$$

$$u(x,t)=(\frac{2\sqrt{a^2q^2(q^2-4pr)}-2a(q^2-4pr)}{5b(q^2-4pr)}+\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}+\frac{\sqrt{-(q^2-4pr)}(\tan_B(\sqrt{-(q^2-4pr)}\xi)\pm(\sqrt{cf}\ \sec_B(\sqrt{-(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (68)$$

$$u(x,t)=(\frac{2\sqrt{a^2q^2(q^2-4pr)}-2a(q^2-4pr)}{5b(q^2-4pr)}+\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}-\frac{\sqrt{-(q^2-4pr)}(\cot_B(\sqrt{-(q^2-4pr)}\xi)\pm(\sqrt{cf}\ \csc_B(\sqrt{-(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (69)$$

$$u(x,t)=(\frac{2\sqrt{a^2q^2(q^2-4pr)}-2a(q^2-4pr)}{5b(q^2-4pr)}+\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}+\frac{\sqrt{-(q^2-4pr)}(\tan_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi)-(\cot_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (70)$$

where $\xi=\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)$.
Case 2 :

$$u(x,t)=(\frac{2\sqrt{a^2q^2(q^2-4pr)}-2a(q^2-4pr)}{5b(q^2-4pr)}+\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}-\frac{\sqrt{(q^2-4pr)}\tanh_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r})^{-1})^2. \quad (71)$$

$$u(x,t) = (\frac{2\sqrt{a^2q^2(q^2-4pr)}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}\coth_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r})^{-1})^2. \quad (72)$$

$$u(x,t) = (\frac{2\sqrt{a^2q^2(q^2-4pr)}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\tanh_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\, sech_B(\sqrt{(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (73)$$

$$u(x,t) = (\frac{2\sqrt{a^2q^2(q^2-4pr)}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\coth_B(\sqrt{(q^2-4pr)}\xi) \pm (\sqrt{cf}\, csch_B(\sqrt{(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (74)$$

$$u(x,t) = (\frac{2\sqrt{a^2q^2(q^2-4pr)}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\tanh_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi) - (\coth_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (75)$$

where $\xi = \theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)$.
Case 3:

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-4pr}}{5br}(\sqrt{\frac{p}{r}}\tan_B(\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (76)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-4pr}}{5br}(-\sqrt{\frac{p}{r}}\cot_B(\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (77)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-4pr}}{5br}(\sqrt{\frac{p}{r}}(\tan_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\sqrt{cf}\sec_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))))^{-1})^2. \quad (78)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-4pr}}{5br}(-\sqrt{\frac{p}{r}}(\cot_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\sqrt{cf}\csc_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))))^{-1})^2. \quad (79)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-4pr}}{5br}(\frac{1}{2}\sqrt{\frac{p}{r}}(\tan_B(\frac{1}{2}\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) - \cot_B(\frac{1}{2}\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^{-1})^2. \quad (80)$$

Case 4:

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-4pr}}{5br}(-\sqrt{-\frac{p}{r}}\tanh_B(\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (81)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-4pr}}{5br}(-\sqrt{-\frac{p}{r}}\coth_B(\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (82)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-4pr}}{5br}(-\sqrt{-\frac{p}{r}}(\tanh_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))\pm (\iota\sqrt{cf}sech_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))))^{-1})^2. \quad (83)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-4pr}}{5br}(-\sqrt{-\frac{p}{r}}(\coth_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))\pm (\sqrt{cf}csch_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))))^{-1})^2. \quad (84)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-4pr}}{5br}(-\frac{1}{2}\sqrt{-\frac{p}{r}}(\tanh_B(\frac{1}{2}\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))+ \coth_B(\frac{1}{2}\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^{-1})^2. \quad (85)$$

Case 5:

$$u(x,t) = (\frac{-2a}{5b} - \frac{4a\iota}{5b}(\tan_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (86)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{4a\iota}{5b}(-\cot_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (87)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{4a\iota}{5b}(\tan_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\sqrt{cf}\sec_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (88)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{4a\iota}{5b}(-\cot_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\sqrt{cf}\csc_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (89)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{4a\iota}{5b}(\frac{1}{2}\tan_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) - \frac{1}{2}\cot_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (90)$$

Case 6:

$$u(x,t) = (\frac{-2a}{5b} - \frac{4a}{5b}(-\tanh_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (91)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{4a}{5b}(-\coth_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (92)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{4a}{5b}(-\tanh_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\iota\sqrt{cf}sech_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (93)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{4a}{5b}(-\coth_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\sqrt{cf}csch_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (94)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{4a}{5b}(-\frac{1}{2}\tanh_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) - \frac{1}{2}\coth_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (95)$$

Case 8:

$$u(x,t) = (\frac{4am}{5b}(d^{\rho\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)} - m)^{-1})^2. \quad (96)$$

Set 3:

$$\{\alpha_{-1} = 0, \alpha_0 = \frac{2aq\sqrt{q^2-4pr} - 2a(q^2-4pr)}{5b(q^2-4pr)}, \alpha_1 = \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}, \lambda = -\frac{16a^2}{75b}, \tau = -\frac{4a^2}{75b\theta^2\log^2(d)(q^2-4pr)}\} \quad (97)$$

Case 1:

$$u(x,t) = (\frac{2aq\sqrt{q^2-4pr}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r}+ \frac{\sqrt{-(q^2-4pr)}\tan_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r}))^2. \quad (98)$$

$$u(x,t) = (\frac{2aq\sqrt{q^2-4pr}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r}- \frac{\sqrt{-(q^2-4pr)}\cot_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r}))^2. \quad (99)$$

$$u(x,t) = (\frac{2aq\sqrt{q^2-4pr}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r}+ \frac{\sqrt{-(q^2-4pr)}(\tan_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\ \sec_B(\sqrt{-(q^2-4pr)}\xi)))}{2r}))^2. \quad (100)$$

$$u(x,t) = (\frac{2aq\sqrt{q^2-4pr}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r}- \frac{\sqrt{-(q^2-4pr)}(\cot_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\ \csc_B(\sqrt{-(q^2-4pr)}\xi)))}{2r}))^2. \quad (101)$$

$$u(x,t) = (\frac{2aq\sqrt{q^2-4pr}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r}+ \frac{\sqrt{-(q^2-4pr)}(\tan_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi) - (\cot_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi)))}{2r}))^2. \quad (102)$$

where $\xi = \theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)$.
Case 2:

$$u(x,t) = (\frac{2aq\sqrt{q^2-4pr}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r}- \frac{\sqrt{(q^2-4pr)}\tanh_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r}))^2. \quad (103)$$

$$u(x,t) = (\frac{2aq\sqrt{q^2-4pr}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r}- \frac{\sqrt{(q^2-4pr)}\coth_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r}))^2. \quad (104)$$

$$u(x,t) = (\frac{2aq\sqrt{q^2-4pr}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r}- \frac{\sqrt{(q^2-4pr)}(\tanh_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\ sech_B(\sqrt{(q^2-4pr)}\xi)))}{2r}))^2. \quad (105)$$

$$u(x,t) = (\frac{2aq\sqrt{q^2-4pr}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\coth_B(\sqrt{(q^2-4pr)}\xi) \pm (\sqrt{cf}\, csch_B(\sqrt{(q^2-4pr)}\xi)))}{2r}))^2. \quad (106)$$

$$u(x,t) = (\frac{2aq\sqrt{q^2-4pr}-2a(q^2-4pr)}{5b(q^2-4pr)} + \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\tanh_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi) - (\coth_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi)))}{2r}))^2. \quad (107)$$

where $\xi = \theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)$.
Case 3:

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-pr}}{5pb}(\sqrt{\frac{p}{r}}\tan_B(\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (108)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-pr}}{5pb}(-\sqrt{\frac{p}{r}}\cot_B(\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (109)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-pr}}{5pb}(\sqrt{\frac{p}{r}}(\tan_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\sqrt{cf}\sec_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))))^2. \quad (110)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-pr}}{5pb}(-\sqrt{\frac{p}{r}}(\cot_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\sqrt{cf}\csc_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))))^2. \quad (111)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-pr}}{5pb}(\frac{1}{2}\sqrt{\frac{p}{r}}(\tan_B(\frac{1}{2}\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) - \cot_B(\frac{1}{2}\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))))^2. \quad (112)$$

Case 4:

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-pr}}{5pb}(-\sqrt{-\frac{p}{r}}\tanh_B(\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (113)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-pr}}{5pb}(-\sqrt{-\frac{p}{r}}\coth_B(\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (114)$$

$$u(x,t) = (\frac{-2a}{5b} - \frac{2a\sqrt{-pr}}{5pb}(-\sqrt{-\frac{p}{r}}(\tanh_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\iota\sqrt{cf} sech_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))))^2. \quad (115)$$

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a\sqrt{-pr}}{5pb}(-\sqrt{-\frac{p}{r}}(\coth_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))\pm(\sqrt{cf}csch_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))))^2. \quad (116)$$

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a\sqrt{-pr}}{5pb}-\frac{1}{2}\sqrt{-\frac{p}{r}}(\tanh_B(\frac{1}{2}\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))+\coth_B(\frac{1}{2}\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^2. \quad (117)$$

Case 5:

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a\iota}{5b}(\tan_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^2. \quad (118)$$

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a\iota}{5b}(-\cot_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^2. \quad (119)$$

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a\iota}{5b}(\tan_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))\pm(\sqrt{cf}\sec_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^2. \quad (120)$$

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a\iota}{5b}(-\cot_B(2p\xi)\pm(\sqrt{cf}\csc_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^2. \quad (121)$$

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a\iota}{5b}(\frac{1}{2}\tan_B(\frac{1}{2}p\xi)-\frac{1}{2}\cot_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^2. \quad (122)$$

Case 6:

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a}{5b}(-\tanh_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^2. \quad (123)$$

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a}{5b}(-\coth_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^2. \quad (124)$$

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a}{5b}(-\tanh_B(2p\xi)\pm(\iota\sqrt{cf}sech_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^2. \quad (125)$$

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a}{5b}(-\coth_B(2p\xi)\pm(\sqrt{cf}csch_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^2. \quad (126)$$

$$u(x,t)=(\frac{-2a}{5b}-\frac{2a}{5b}(-\frac{1}{2}\tanh_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))-\frac{1}{2}\coth_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^2. \quad (127)$$

Case 11:

$$u(x,t)=(\frac{4ar}{5bq}(-\frac{cq}{r(\cosh_B(q\xi)-\sinh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))+c)}))^2. \quad (128)$$

$$u(x,t)=(\frac{4ar}{5bq}(-\frac{q(\cosh_B(q\xi)+\sinh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))}{r(\cosh_B(q\xi)+\sinh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))+f)}))^2. \quad (129)$$

Case 12:

$$u(x,t)=(\frac{4am}{5b}(\frac{cd^{\rho\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)}}{c-mfd^{\rho\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)}}))^2. \quad (130)$$

Set 4:

$$\{\alpha_{-1}=-\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)},\alpha_0=-\frac{2\sqrt{a^2q^2(q^2-4pr)}+2a(q^2-4pr)}{5b(q^2-4pr)},\alpha_1=0,$$
$$\lambda=-\frac{16a^2}{75b},\tau=-\frac{4a^2}{75b\theta^2\log^2(d)(q^2-4pr)}\} \quad (131)$$

Case 1:

$$u(x,t)=(-\frac{2\sqrt{a^2q^2(q^2-4pr)}+2a(q^2-4pr)}{5b(q^2-4pr)}-\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}+$$
$$\frac{\sqrt{-(q^2-4pr)}\tan_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r})^{-1})^2. \quad (132)$$

$$u(x,t)=(-\frac{2\sqrt{a^2q^2(q^2-4pr)}+2a(q^2-4pr)}{5b(q^2-4pr)}-\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}-$$
$$\frac{\sqrt{-(q^2-4pr)}\cot_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r})^{-1})^2. \quad (133)$$

$$u(x,t)=(-\frac{2\sqrt{a^2q^2(q^2-4pr)}+2a(q^2-4pr)}{5b(q^2-4pr)}-\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}+$$
$$\frac{\sqrt{-(q^2-4pr)}(\tan_B(\sqrt{-(q^2-4pr)}\xi)\pm(\sqrt{cf}\ \sec_B(\sqrt{-(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (134)$$

$$u(x,t)=(-\frac{2\sqrt{a^2q^2(q^2-4pr)}+2a(q^2-4pr)}{5b(q^2-4pr)}-\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}-$$
$$\frac{\sqrt{-(q^2-4pr)}(\cot_B(\sqrt{-(q^2-4pr)}\xi)\pm(\sqrt{cf}\ \csc_B(\sqrt{-(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (135)$$

$$u(x,t)=(-\frac{2\sqrt{a^2q^2(q^2-4pr)}+2a(q^2-4pr)}{5b(q^2-4pr)}-\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}+$$
$$\frac{\sqrt{-(q^2-4pr)}(\tan_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi)-(\cot_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (136)$$

where $\xi=\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)$.
Case 2:

$$u(x,t)=(-\frac{2\sqrt{a^2q^2(q^2-4pr)}+2a(q^2-4pr)}{5b(q^2-4pr)}-\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}-$$
$$\frac{\sqrt{(q^2-4pr)}\tanh_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r})^{-1})^2. \quad (137)$$

$$u(x,t)=(-\frac{2\sqrt{a^2q^2(q^2-4pr)}+2a(q^2-4pr)}{5b(q^2-4pr)}-\frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r}-$$
$$\frac{\sqrt{(q^2-4pr)}\coth_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r})^{-1})^2. \quad (138)$$

$$u(x,t) = (-\frac{2\sqrt{a^2q^2(q^2-4pr)}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\tanh_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\, sech_B(\sqrt{(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (139)$$

$$u(x,t) = (-\frac{2\sqrt{a^2q^2(q^2-4pr)}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\coth_B(\sqrt{(q^2-4pr)}\xi) \pm (\sqrt{cf}\, csch_B(\sqrt{(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (140)$$

$$u(x,t) = (-\frac{2\sqrt{a^2q^2(q^2-4pr)}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4p\sqrt{a^2(q^2-4pr)}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\tanh_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi) - (\coth_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi)))}{2r})^{-1})^2. \quad (141)$$

where $\xi = \theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)$.

Case 3:

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-4pr}}{5br}(\sqrt{\frac{p}{r}}\tan_B(\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (142)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-4pr}}{5br}(-\sqrt{\frac{p}{r}}\cot_B(\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (143)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-4pr}}{5br}(\sqrt{\frac{p}{r}}(\tan_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\sqrt{cf}\sec_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))))^{-1})^2. \quad (144)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-4pr}}{5br}(-\sqrt{\frac{p}{r}}(\cot_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\sqrt{cf}\csc_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))))^{-1})^2. \quad (145)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-4pr}}{5br}(\frac{1}{2}\sqrt{\frac{p}{r}}(\tan_B(\frac{1}{2}\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) - \cot_B(\frac{1}{2}\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^{-1})^2. \quad (146)$$

Case 4 :

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-4pr}}{5br}(-\sqrt{-\frac{p}{r}}\tanh_B(\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (147)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-4pr}}{5br}(-\sqrt{-\frac{p}{r}}\coth_B(\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (148)$$

$$u(x,t)=(\frac{2a}{5b}+\frac{2a\sqrt{-4pr}}{5br}(-\sqrt{-\frac{p}{r}}(\tanh_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))\pm (\iota\sqrt{cf}sech_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^{-1})^2. \quad (149)$$

$$u(x,t)=(\frac{2a}{5b}+\frac{2a\sqrt{-4pr}}{5br}(-\sqrt{-\frac{p}{r}}(\coth_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))\pm (\sqrt{cf}csch_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^{-1})^2. \quad (150)$$

$$u(x,t)=(\frac{2a}{5b}+\frac{2a\sqrt{-4pr}}{5br}(-\frac{1}{2}\sqrt{-\frac{p}{r}}(\tanh_B(\frac{1}{2}\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))+ \coth_B(\frac{1}{2}\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))))^{-1})^2. \quad (151)$$

Case 5 :

$$u(x,t)=(\frac{2a}{5b}+\frac{4a\iota}{5b}(\tan_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (152)$$

$$u(x,t)=(\frac{2a}{5b}+\frac{4a\iota}{5b}(-\cot_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (153)$$

$$u(x,t)=(\frac{2a}{5b}+\frac{4a\iota}{5b}(\tan_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))\pm(\sqrt{cf}\sec_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (154)$$

$$u(x,t)=(\frac{2a}{5b}+\frac{4a\iota}{5b}(-\cot_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))\pm(\sqrt{cf}\csc_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (155)$$

$$u(x,t)=(\frac{2a}{5b}+\frac{4a\iota}{5b}(\frac{1}{2}\tan_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))-\frac{1}{2}\cot_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (156)$$

Case 6:

$$u(x,t)=(\frac{2a}{5b}+\frac{4a}{5b}(-\tanh_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (157)$$

$$u(x,t)=(\frac{2a}{5b}+\frac{4a}{5b}(-\coth_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (158)$$

$$u(x,t)=(\frac{2a}{5b}+\frac{4a}{5b}(-\tanh_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))\pm(\iota\sqrt{cf}sech_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (159)$$

$$u(x,t)=(\frac{2a}{5b}+\frac{4a}{5b}(-\coth_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))\pm(\sqrt{cf}csch_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (160)$$

$$u(x,t)=(\frac{2a}{5b}+\frac{4a}{5b}(-\frac{1}{2}\tanh_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha))-\frac{1}{2}\coth_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)))^{-1})^2. \quad (161)$$

Case 8:

$$u(x,t)=(-\frac{4am}{5b}(d^{\rho\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha+\frac{16a^2}{75b}t^\alpha)}-m)^{-1})^2. \quad (162)$$

Set 5:

$$\{\alpha_{-1}=0,\alpha_0=-\frac{2aq\sqrt{q^2-4pr}+2a(q^2-4pr)}{5b(q^2-4pr)},\alpha_1=-\frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)},\lambda=-\frac{16a^2}{75b}, \tau=-\frac{4a^2}{75b\theta^2\log^2(d)(q^2-4pr)}\} \quad (163)$$

Case 1 :

$$u(x,t) = (-\frac{2aq\sqrt{q^2-4pr}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r} + \frac{\sqrt{-(q^2-4pr)}\tan_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r}))^2. \quad (164)$$

$$u(x,t) = (-\frac{2aq\sqrt{q^2-4pr}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{-(q^2-4pr)}\cot_B(\frac{1}{2}\sqrt{-(q^2-4pr)}\xi)}{2r}))^2. \quad (165)$$

$$u(x,t) = (-\frac{2aq\sqrt{q^2-4pr}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r} + \frac{\sqrt{-(q^2-4pr)}(\tan_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\ \sec_B(\sqrt{-(q^2-4pr)}\xi)))}{2r}))^2. \quad (166)$$

$$u(x,t) = (-\frac{2aq\sqrt{q^2-4pr}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{-(q^2-4pr)}(\cot_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\ \csc_B(\sqrt{-(q^2-4pr)}\xi)))}{2r}))^2. \quad (167)$$

$$u(x,t) = (-\frac{2aq\sqrt{q^2-4pr}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r} + \frac{\sqrt{-(q^2-4pr)}(\tan_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi) - (\cot_B(\frac{1}{4}\sqrt{-(q^2-4pr)}\xi)))}{2r}))^2. \quad (168)$$

where $\xi = \theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)$.
Case 2:

$$u(x,t) = (-\frac{2aq\sqrt{q^2-4pr}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}\tanh_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r}))^2. \quad (169)$$

$$u(x,t) = (-\frac{2aq\sqrt{q^2-4pr}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}\coth_B(\frac{1}{2}\sqrt{(q^2-4pr)}\xi)}{2r}))^2. \quad (170)$$

$$u(x,t) = (-\frac{2aq\sqrt{q^2-4pr}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r} - \frac{\sqrt{(q^2-4pr)}(\tanh_B(\sqrt{-(q^2-4pr)}\xi) \pm (\sqrt{cf}\ sech_B(\sqrt{(q^2-4pr)}\xi)))}{2r}))^2. \quad (171)$$

$$u(x,t) = (-\frac{2aq\sqrt{q^2-4pr}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r}- \frac{\sqrt{(q^2-4pr)}(\coth_B(\sqrt{(q^2-4pr)}\xi) \pm (\sqrt{cf}\, csch_B(\sqrt{(q^2-4pr)}\xi)))}{2r}))^2. \quad (172)$$

$$u(x,t) = (-\frac{2aq\sqrt{q^2-4pr}+2a(q^2-4pr)}{5b(q^2-4pr)} - \frac{4ar\sqrt{q^2-4pr}}{5b(q^2-4pr)}(-\frac{q}{2r}- \frac{\sqrt{(q^2-4pr)}(\tanh_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi) - (\coth_B(\frac{1}{4}\sqrt{(q^2-4pr)}\xi)))}{2r}))^2. \quad (173)$$

where $\xi = \theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)$.
Case 3:

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-pr}}{5pb}(\sqrt{\frac{p}{r}}\tan_B(\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (174)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-pr}}{5pb}(-\sqrt{\frac{p}{r}}\cot_B(\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (175)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-pr}}{5pb}(\sqrt{\frac{p}{r}}(\tan_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))\pm (\sqrt{cf}\sec_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))))^2. \quad (176)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-pr}}{5pb}(-\sqrt{\frac{p}{r}}(\cot_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))\pm (\sqrt{cf}\csc_B(2\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))))^2. \quad (177)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-pr}}{5pb}(\frac{1}{2}\sqrt{\frac{p}{r}}(\tan_B(\frac{1}{2}\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))- \cot_B(\frac{1}{2}\sqrt{pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))))^2. \quad (178)$$

Case 4:

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-pr}}{5pb}(-\sqrt{-\frac{p}{r}}\tanh_B(\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (179)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-pr}}{5pb}(-\sqrt{-\frac{p}{r}}\coth_B(\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (180)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-pr}}{5pb}(-\sqrt{-\frac{p}{r}}(\tanh_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))\pm (\iota\sqrt{cf}sech_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))))^2. \quad (181)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-pr}}{5pb}(-\sqrt{-\frac{p}{r}}(\coth_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\sqrt{cf}csch_B(2\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))))^2. \quad (182)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\sqrt{-pr}}{5pb} - \frac{1}{2}\sqrt{-\frac{p}{r}}(\tanh_B(\frac{1}{2}\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) + \coth_B(\frac{1}{2}\sqrt{-pr}\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (183)$$

Case 5:

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\iota}{5b}(\tan_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (184)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\iota}{5b}(-\cot_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (185)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\iota}{5b}(\tan_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) \pm (\sqrt{cf}\sec_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (186)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\iota}{5b}(-\cot_B(2p\xi) \pm (\sqrt{cf}\csc_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (187)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a\iota}{5b}(\frac{1}{2}\tan_B(\frac{1}{2}p\xi) - \frac{1}{2}\cot_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (188)$$

Case 6:

$$u(x,t) = (\frac{2a}{5b} + \frac{2a}{5b}(-\tanh_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (189)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a}{5b}(-\coth_B(p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (190)$$

$$u(x,t) = (\frac{2a}{5b} + \frac{2a}{5b}(-\tanh_B(2p\xi) \pm (\iota\sqrt{cf}sech_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (191)$$

$$u(x,t) = (\frac{+2a}{5b} + \frac{2a}{5b}(-\coth_B(2p\xi) \pm (\sqrt{cf}csch_B(2p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (192)$$

$$u(x,t) = (\frac{+2a}{5b} + \frac{2a}{5b}(-\frac{1}{2}\tanh_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) - \frac{1}{2}\coth_B(\frac{1}{2}p\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha))))^2. \quad (193)$$

Case 11:

$$u(x,t) = (-\frac{4ar}{5bq}(-\frac{cq}{r(\cosh_B(q\xi) - \sinh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) + c)}))^2. \quad (194)$$

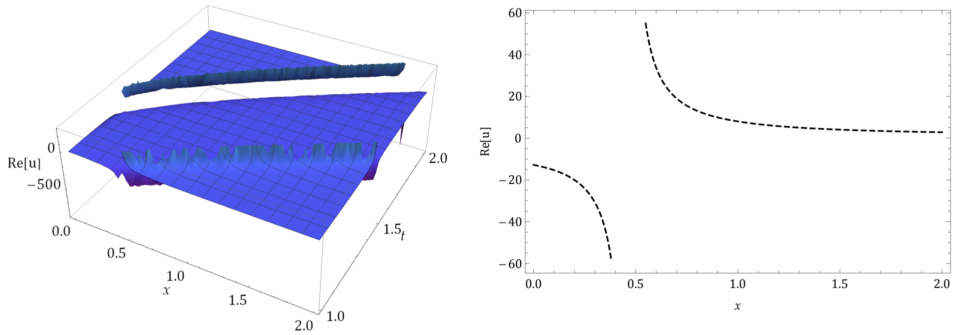
$$u(x,t) = (-\frac{4ar}{5bq}(-\frac{q(\cosh_B(q\xi) + \sinh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)))}{r(\cosh_B(q\xi) + \sinh_B(q\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)) + f)}))^2. \quad (195)$$

Case 12:

$$u(x,t) = (-\frac{4am}{5b}(\frac{cd^{\rho\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)}}{c - mfd^{\rho\theta\frac{\Gamma(1+\Upsilon)}{\alpha}(x^\alpha + \frac{16a^2}{75b}t^\alpha)}}))^2. \quad (196)$$

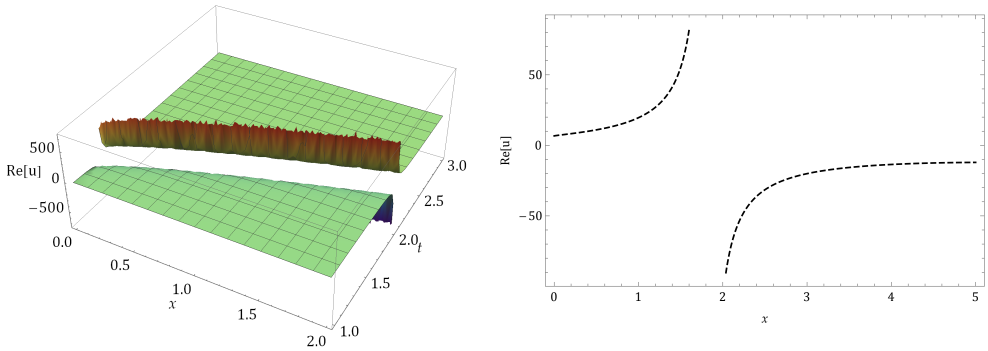
5. Physical Behavior of Solutions

In this portion, we presenta the behavior of our gained kink, periodic, singular, and kink-singular solitons solutions are shown in the following Figures. Figure 1 illustrations a kink-singular shape of $u(x,t)$ occur in Equation (51) in (a) 3-dim and (b) 2-dim graphs with $d = 3; a = 2.5; b = 0.4; \Upsilon = 1; r = 0.5; \theta = 0.9; p = 1; q = 1; \alpha = 0.8$. Figure 2 illustrations a kink-singular shape of $u(x,t)$ occur in Equation (52) in (a) 3-dim and (b) 2-dim graphs with $d = 3; a = 2.5; b = 0.4; \Upsilon = 1; r = 0.5; \theta = 0.3; p = 1; q = 1; \alpha = 0.8$. Figure 3 illustrations a kink shape of $u(x,t)$ occur in Equation (53) in (a) 3-dim and (b) 2-dim graphs with $d = 3; a = 2.5; b = 0.4; c = 1.3; \Upsilon = 1; r = 0.5; \theta = 0.3; p = 1; q = 1; f = 1.5; \alpha = 0.8$. (Figure 4) illustrations a kink shape of $u(x,t)$ occur in Equation (56) in (a) 3-dim and (b) 2-dim graphs with $d = 3; a = 2.5; b = 0.4; \Upsilon = 1; r = 0.5; \theta = 0.3; p = 1; q = 1; \alpha = 0.8$. Figure 5 illustrations a kink-singular shape of $u(x,t)$ occur in Equation (57) in (a) 3-dim and (b) 2-dim graphs with $d = 3; a = 0.5; b = 1.4; \Upsilon = 1; r = 0.5; \theta = 1; p = 1; q = 1; \alpha = 0.8$. Figure 6 illustrations a periodic shape of $u(x,t)$ occur in Equation (60) in (a) 3-dim and (b) 2-dim graphs with $d = 3; a = 5.1; b = 2.4; \Upsilon = 2; r = 0.5; \theta = 2.3; p = 1; q = 1; \alpha = 0.5$. Figure 7 illustrations a singular shapeof $u(x,t)$ occur in Equation (61) in (a) 3-dim and (b) 2-dim graphs with $d = 3; a = 2; b = 0.4; \Upsilon = 1; m = 1; \theta = 0.3; \alpha = 0.6$.



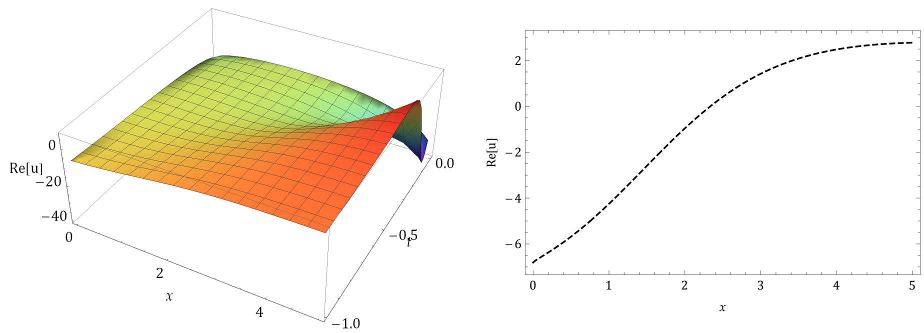
(**a**) (**b**)

Figure 1. Graph of $u(x,t)$ occur in Equation (51) in (**a**) 3-dim and (**b**) 2-dim graphs.



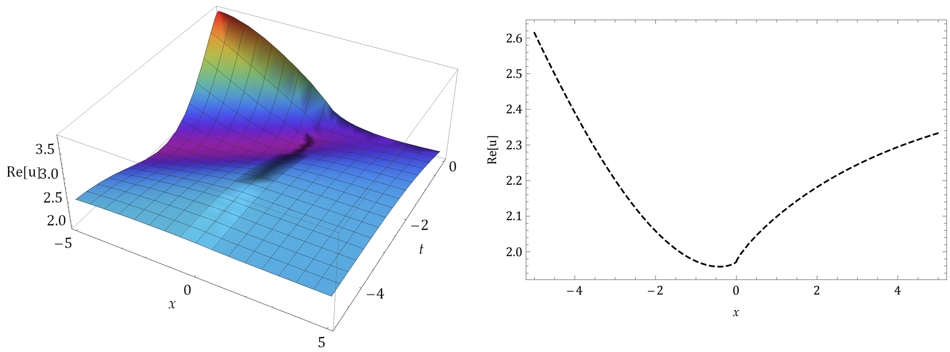
(**a**) (**b**)

Figure 2. Graph of $u(x,t)$ occur in Equation (52) in (**a**) 3-dim and (**b**) 2-dim graphs.



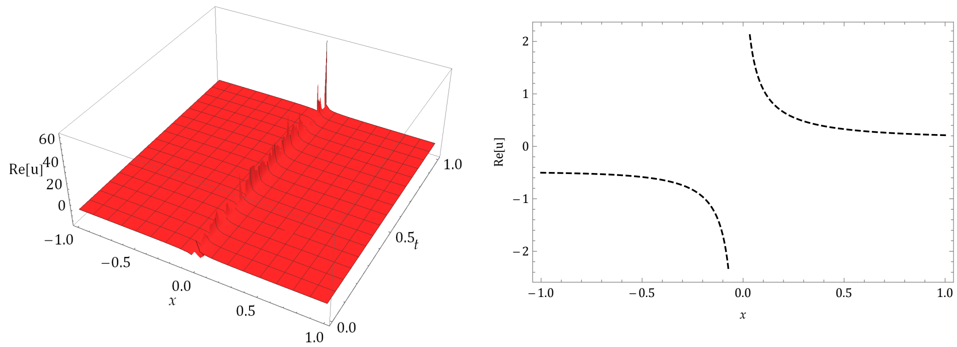
(**a**) (**b**)

Figure 3. Graph of $u(x,t)$ occur in Equation (53) in (**a**) 3-dim and (**b**) 2-dim graphs.



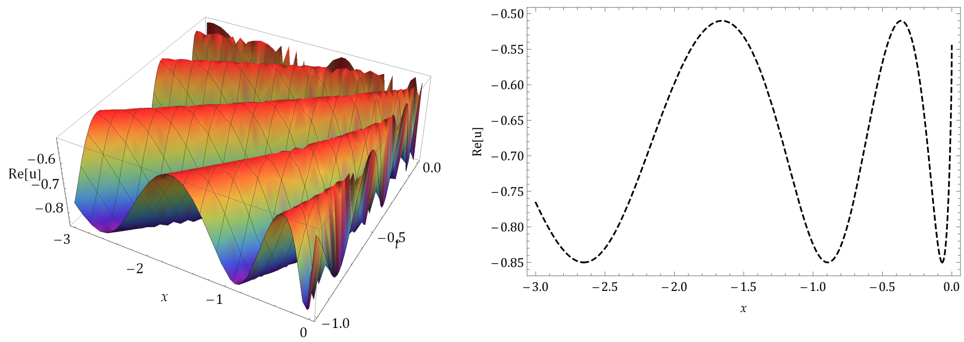
(**a**) (**b**)

Figure 4. Graph of $u(x,t)$ occur in Equation (56) in (**a**) 3-dim and (**b**) 2-dim graphs.



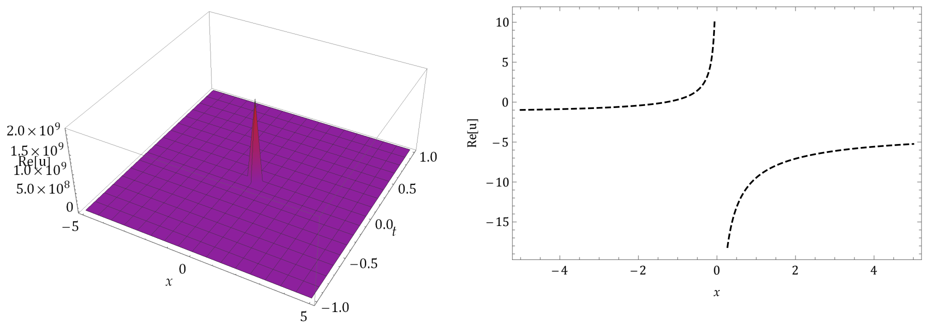
(**a**) (**b**)

Figure 5. Graph of $u(x,t)$ occur in Equation (57) in (**a**) 3-dim and (**b**) 2-dim graphs.



(**a**) (**b**)

Figure 6. Graph of $u(x,t)$ occur in Equation (60) in (**a**) 3-dim and (**b**) 2-dim graphs.



(**a**) (**b**)

Figure 7. Graph of $u(x,t)$ occur in Equation (61) in (**a**) 3-dim and (**b**) 2-dim graphs.

6. Conclusions

A new class of truncated M-fractional exact soliton solutions for a truncated M-fractional (1+1)-dimensional nonlinear modified mixed-KdV model are obtained successfully. For this aim; modified extended direct algebraic method is used. The obtained results including the kink, periodic, singular, and kink-singular soliton solutions. We also discuss the effect of fractional order derivative. To validate as well as to obtain our results, we utilized the Mathematica software. Additionally, we depict some of the obtained wave solitons, using two and three dimensional graphs. The obtained results are useful in the fields of fluid dynamics, nonlinear optics, ocean engineering and others. Furthermore, these employed techniques are not only straightforward, but also highly effective when used to solve non-linear fractional partial differential equations (FPDEs).

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