



# Article Command Filter-Based Adaptive Neural Control for Nonstrict-Feedback Nonlinear Systems with Prescribed Performance

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Abstract: In this paper, a new command filter-based adaptive NN control strategy is developed to address the prescribed tracking performance issue for a class of nonstrict-feedback nonlinear systems. Compared with the existing performance functions, a new performance function, the fixed-time performance function, which does not depend on the accurate initial value of the error signal and has the ability of fixed-time convergence, is proposed for the first time. A radial basis function neural network is introduced to identify unknown nonlinear functions, and the characteristic of Gaussian basis functions is utilized to overcome the difficulties of the nonstrict-feedback structure. Moreover, in contrast to the traditional Backstepping technique, a command filter-based adaptive control algorithm is constructed, which solves the "explosion of complexity" problem and relaxes the assumption on the reference signal. Additionally, it is guaranteed that the tracking error falls within a prescribed small neighborhood by the designed performance functions in fixed time, and the closed-loop system is semi-globally uniformly ultimately bounded (SGUUB). The effectiveness of the proposed control scheme is verified by numerical simulation.

**Keywords:** nonstrict-feedback nonlinear systems; neural networks; prescribed performance; prescribed-time tracking control; command filter

## 1. Introduction

Nonlinear systems have received extensive attention since they were proposed because these systems can well model most actual systems [1,2]. Many useful techniques have been proposed to control nonlinear systems. Among these methods, the adaptive Backstepping method is a powerful and popular tool, and many meaningful control achievements have been reported for nonlinear systems in the framework of the traditional Backstepping method [3,4]. As is known, in the Backstepping control design, the repeated derivatives of the virtual control functions result in the "explosion of complexity" (EOC) problem. To solve the EOC problem, the dynamic surface control (DSC) strategy [5–7] and the command filter (CF) technique [8–12] are incorporated into the Backstepping method. Based on the DSC technique, the adaptive NN/FLS control problem is addressed for strict-feedback nonlinear fractional-order systems [6] and switched strict-feedback nonlinear systems (SFNSs) [7], respectively. However, the DSC technique reported in [5–7] does not take into account the filtering errors created by the virtual function passing through the filter. Therefore, it is hard to achieve high control performance for the considered systems. To overcome this limitation, the CF control method was proposed originally in [8] where the compensating mechanism is introduced to handle the filtering errors. Subsequently, the command-filtered Backstepping approach was extended to the single-input-single-output



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**Copyright:** © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). (SISO) SFNSS with constant parameters [9], and then the adaptive Backstepping control scheme based on CF was constructed to warrant that all signals are bounded. Recently, for switched SFNSs with hysteresis input [10], the adaptive finite-time neural network (NN) control strategy has been developed by exploiting the command-filtered Backstepping technique. However, it is worth noting that most of the above control schemes are only concerned with the nonlinear systems in affine form.

In fact, many control systems, e.g., helicopter systems [13], servo mechanisms [14], and aircraft systems [15], are inherently nonlinear, and their input variables may not be expressed in an affine form. Thus, it is essential and interesting to study the control issues of non-affine nonlinear systems. Nevertheless, most of the control algorithms proposed above are unavailable for non-affine systems. To overcome this restriction, much effort has been made and some meaningful results have been published for a kind of nonlinear system in non-affine form [16–18]. For instance, by combining multilayer NN with implicit function theory, both state and output feedback adaptive NN controllers are constructed for a type of SISO non-affine nonlinear system with zero dynamics [16]. For non-affine pure-feedback nonlinear multiagent systems with unmodeled dynamics and unknown dead zones [17], an adaptive event-triggered NN control scheme has been designed to warrant that the closed-loop system is bounded. However, it has to be emphasized that most of the obtained results are focused only on nonlinear systems with a strict-feedback structure.

In reality, many actual systems are modeled as nonstrict-feedback nonlinear systems (NSFNSs) with a relaxed system structure, such as uncertain robot systems [19] and aircraft sight control systems [20]. Nevertheless, the previously developed control schemes for strict-feedback systems are unavailable. It is common knowledge that under the framework of the Backstepping approach, a necessary condition is that there must exist a state feedback control law to stabilize the corresponding subsystem. That is, to ensure the existence of these control laws  $\alpha_i$  (i = 1, 2, ..., n), the designed controllers for each subsystem are dependent on the *i*-th system states only. If  $\alpha_i$  is related to all the system states  $x_1, x_2, \ldots, x_n$ , the algebraic loop problem will arise [21]. Therefore, how to control NSFNSs has attracted much attention, and then a great deal of effort has been made. Recently, to deal with the nonstrict-feedback structure, the variable separation method has been utilized under the assumption that the system function is less than or equal to a strictly increasing function [22,23]. Apparently, the assumption condition limits the application of the control algorithm reported in [22–24]. More recently, some effective adaptive fuzzy-logic system (FLS)/NN control schemes have been designed for uncertain NSFNSs [25-28], which relax the restriction on the system functions. For example, by combining the adaptive Backstepping and FLS techniques [25], the adaptive finite-time control issue of the NSFNSs with unknown dynamics is investigated. However, these reported control strategies [25–28] ensure that the tracking error (TE) converges to a small set of residuals, whose size is unknown.

To achieve high tracking performance, prescribed performance control (PPC) was first proposed in [29]. The PPC method guarantees both the transient (such as convergence rate, overshoot, etc.) and steady-state (such as control accuracy) performance of the TE for the controlled systems. Thus, much attention has been given and many favorable achievements have been published by utilizing the PPC and adaptive control techniques [29–38]. For instance, an adaptive optimal control method is proposed in [31] to address the path tracking control problem with prescribed performance for autonomous vehicles. It is significant to note that the initial value of the performance functions reported in [29–31,36–38] are dependent on the initial value of the system states and the reference signal. That is, the exact value of  $e_1(0)$  is required in advance. To overcome the restriction of the initial value, several types of performance functions have been constructed [32–34]. For example, Bu et al. propose a new performance function to handle the prescribed tracking performance functions reported in [29–34] ensure that the TE converges asymptotically to within a prescribed bound. To obtain better convergence performance, several new types of performance functions are designed in [35–38], such that the TE falls into a predetermined small neighborhood of zero in finite time. However, the convergence time of TE depends on initial values  $e_1(t)$  or system design parameters. Therefore, it is a rewarding and challenging problem regarding how to design a performance function without initial value constraints to ensure that tracking error converges within a predetermined time.

Motivated by the foregoing discussion, this study is the first attempt to address the fixed-time prescribed tracking performance control problem for a kind of NSFNS. Moreover, based on the PPC method, a new fixed-time performance function (FTPF) is constructed first, which is not constrained by initial values  $e_1(0)$ , and the convergence time can be predetermined according to the practical system requirements. And then, by utilizing the adaptive control technique, the radial basis function neural network (RBFNN) method, and CF approach, a novel CF-based adaptive NN control scheme is constructed to warrant the fixed-time PPC of the TE and handle the EOC problem. Furthermore, it is also proved that all the signals of the closed-loop system are SGUUB. Compared with existing works, the following contributions are worth being emphasized:

- (1) This paper focuses on a class of NSFNSs, which are more general than SFNSs reported in [3,4] and the traditional NSFNSs proposed in [25,28]. And thus, the developed control scheme has wider applicability. Moreover, for the nonstrict-feedback structure, differently from the variable separation method with the requirement that the system function is less than or equal to a strictly increasing function in [22,23], the property of the Gaussian basis function is utilized without any restriction on the unknown functions.
- (2) A novel FTPF with fixed-time boundedness is proposed for the first time. Compared with the existing PPC in [29,31,38] where the accurate  $e_1(0)$  has to be known in advance, the limitation is removed in this paper. Moreover, differently from the performance functions reported in [32,34] where the asymptotic convergence of the TE is warranted, the FTPF is developed, and the fixed-time prescribed performance of the TE is guaranteed. That is, the transient and steady-state performance of TE is guaranteed within a fixed time, and the convergence time can be designed according to the actual system requirements.
- (3) The proposed fixed-time PPC control strategy solves the EOC problem while eliminating the effects of filtering errors. In contrast to the traditional Backstepping technique with the restriction that the *n*-th derivatives of the reference signal are continuous [3,4], this paper tackles the EOC problem and relaxes the assumption where only the reference signal and its first-order derivative are continuous. Although the control strategies designed based on the DSC method [5–7] also deal with the EOC problem, these strategies ignore the influence of filtering errors and do not consider the PPC of the TE.

#### 2. Problem Description and Preliminaries

### 2.1. Problem Formulation

Consider the NSFNS as follows

$$\begin{cases} \dot{x}_i = g_i(X) + f_i(X), & i = 1, 2, \dots, n-1, \\ \dot{x}_n = g_n(X, u) + f_n(X), \\ y = x_1, \end{cases}$$
(1)

where  $X = [x_1, ..., x_n]^T$  denotes the system state vector; y and u stand for the system output and control input, respectively;  $g_i(X)$ ,  $g_n(X, u)$ , and  $f_i(X)$  are unknown smooth functions with  $g_i(\mathbf{0}) = 0$ ,  $g_n(\mathbf{0}, 0) = 0$ , and  $f_i(\mathbf{0}) = 0$ ; and  $\mathbf{0}$  is a vector whose elements are all zero.

**Assumption 1** ([39]). *Smooth functions*  $g_i(X)$  (i = 1, 2, ..., n - 1) and  $g_n(X, u)$  are continuously differentiable, such that

$$\frac{\partial g_i(X)}{\partial x_{i+1}} \ge b_i > 0, \quad \frac{\partial g_n(X,u)}{\partial u} \ge b_n > 0,$$

where  $b_i$  (i = 1, 2, ..., n) are positive constants.

**Remark 1.** Assumption 1 is a common assumption of nonstrict-feedback systems [39] to ensure that system (1) is controllable.

**Remark 2.** System (1) represents a class of general nonlinear systems. When  $g_i(X) = g_i(x_i)x_{i+1}$  and function  $f_i(X) \leq \phi_i(||X||)$  with  $\phi_i(0) = 0$ , system (1) becomes the NSFNSs reported in [23,24]. Unlike references [23,24], this paper has no such restriction on function  $f_i(X)$ . If functions  $g_i(X)$  and  $f_i(X)$  are both dependent on system state vector  $\bar{x}_i = [x_1, \ldots, x_i]^T$ , system (1) degrades into the well-known SFNSS.

**Definition 1** ((SGUUB) [40]). If there exists a compact set  $\Omega \in \mathbb{R}^n$ , for every  $x_0 \in \Omega$ , there are positive constant b and  $T = T(x_0, b) \ge 0$ , independent of  $t_0$ , such that  $||x(t)|| \le b$ ,  $\forall t \ge t_0 + T(x_0, b)$ ; then, the solutions of system (1) are SGUUB with ultimate bound b.

**Definition 2** (Fixed-Time SGUUB). *The solutions of system* (1) *are said to be Fixed-Time SGUUB if it is SGUUB with settling time*  $T > t_0$ *, independent on initial value*  $x_0$  *and constant b.* 

**Remark 3.** Compared with Definition 1 where T is dependent on the initial value  $x_0$  and ultimate bound b, the settling time T in Definition 2 is a prescribed constant and irrelevant to the value of  $x_0$  and b. According to Definitions 1 and 2, for every  $x_0 \in \Omega$ , if there exist constants b and  $T > t_0$ , such that  $\lim_{t \to T} ||x(t)|| \le b$ , then the solutions of system (1) are Fixed-Time SGUUB.

The objective of this paper is to construct a CF-based adaptive NN control scheme for NSFNSs (1), such that the TE eventually converges to the prescribed bounded by the designed FTPFs in fixed time, and the SGUUB stability of the considered system is warranted.

To achieve this purpose, in what follows, some preparatory work is provided.

**Assumption 2** ([10]). *Reference signal*  $y_d(t)$  *and its first derivative are continuous.* 

#### 2.2. Radial Basis Function Neural Networks

As is known, the more popular methods for dealing with unknown system functions are FLSs and NNs, which can be expressed as  $W^{*T}S(Z)$  with input vector  $Z \in \mathbb{R}^n \in \Omega$ , ideal weight vector  $W^* \in \mathbb{R}^q$ , node number q, and basis function vector  $S(Z) \in \mathbb{R}^q$ .

As reported in [41], RBFNN is utilized to identify unknown nonlinear function f(Z), and it yields

$$f(Z) = W^T S(Z) + \varepsilon(Z), \quad Z \in \Omega,$$
(2)

where  $Z \in \Omega \subset \mathbb{R}^n$  is the input vector;  $\varepsilon(Z)$  is the approximation error;  $W = [w_1, w_2, \dots, w_q]^T \in \mathbb{R}^q$  is the weight vector; and q is the number of RBFNN nodes. And ideal weight vector  $W^*$  can be described as

$$W^* = \arg\min_{W \in \mathbb{R}^q} \left\{ \sup_{Z \in \Omega} |f(Z) - W^T S(Z)| \right\},\$$

where  $S(Z) = [s_1(Z), s_2(Z), \dots, s_q(Z)]^T \in \mathbb{R}^q$  is the basis function vector. Throughout this paper, Gaussian function  $s_l(Z) = e^{-\frac{||Z-c_l||^2}{b_l^2}}$  with  $c_l = [c_{l1}, c_{l2}, \dots, c_{ln}]^T$  is chosen as the RBFNN basis function  $s_l(Z)$ ; for l = 1, 2, ..., q, i = 1, 2, ..., n,  $c_{li}$  and  $b_l$  are the centers and widths of Gaussian functions  $s_1(Z)$ .

**Lemma 1** ([42]). For any positive constants m, n and  $m \leq n$ , the basis function of NN satisfies

$$||S(Z_n)||^2 \le ||S(Z_m)||^2$$

where  $Z_n = [x_1, \ldots, x_n]^T$  and  $Z_m = [x_1, \ldots, x_m]^T$  are the input vectors of RBFNN.

**Proof.** For  $l = 1, 2, ..., q, i = 1, 2, ..., n, c_{li}$  and  $b_l$  are the centers and widths of Gaussian functions  $s_l(Z)$ . According to the Definition of the basis function vector and Gaussian function  $s_l(Z) = e^{-\frac{\|Z-c_l\|^2}{b_l^2}}$ , one has

$$||S(Z)||^{2} = \sum_{l=1}^{q} \left( \prod_{i=1}^{n} \exp\left[ -\frac{1}{b_{l}^{2}} (x_{i} - c_{li})^{2} \right] \right)^{2}$$
  
$$\leq \sum_{l=1}^{q} \left( \prod_{i=1}^{m} \exp\left[ -\frac{1}{b_{l}^{2}} (x_{i} - c_{li})^{2} \right] \right)^{2} = ||S(Z_{m})||^{2}.$$

**Remark 4.** According to the Proof in Lemma 1, it can be concluded that the characteristics of the *NN* basis function are related to the dimensionality of the input vector and that A is independent of the order of arrangement of state information. That is, m-dimensional vector elements can be arbitrarily selected from n-dimensional system states  $x_1, x_2, \ldots, x_n$ .

## 2.3. Performance Function

To obtain the good tracking performance, TE  $e_1(t) = y_1(t) - y_d(t)$  satisfies

$$-\underline{d}\rho(t) < e_1(t) < \overline{d}\rho(t),\tag{3}$$

where  $\rho(t)$  is the designed performance function and <u>d</u> and <u>d</u> are positive constants.

**Definition 3.** A smooth function  $\rho(t)$  is defined as an FTPF if it satisfies four properties: (1)  $\rho(t) > 0$ ; (2)  $\dot{\rho}(t) \leq 0$ ; (3)  $\rho(0)$  is independent on initial error  $e_1(0)$ ; and (4)  $\lim_{t \to T_f} \rho(t) = \rho_{T_f} > 0$  and  $\rho(t) = \rho_{T_f}$  for any  $t \ge T_f$  with  $\rho_{T_f}$  being an arbitrarily small constant and  $T_f$  representing the settling time.

From Definition 3, an FTPF can be designed as

$$\rho(t) = \begin{cases}
\coth(l_1 t + l_2)(1 - \frac{l_3 t}{l})^l + \rho_{T_f}, & t \in [0, T_f), \\
\rho_{T_f}, & t \in [T_f, +\infty),
\end{cases}$$
(4)

where  $l \ge n$  (*n* denotes the system order),  $\rho_{T_f} > 0$ ,  $l_1, l_2, l_3$  are constants, and  $T_f = \frac{l}{l_3}$ .

The relationship between TE  $e_1(t)$  and the performance function  $\rho(t)$  stated above is clearly shown in Figure 1.



**Figure 1.** Performance functions  $\rho(t)$  and tracking error  $e_1(t)$ .

**Remark 5.** It is evident that function (4) satisfies the four properties of Definition 3 mentioned above.

**Remark 6.** The existing PPC methods require that the accurate initial value of the TE be known in advance to set the initial value of the performance function to satisfy constrict condition (3) with t = 0. However, the requirement is difficult to achieve for some practical systems. When only the rough information of  $e_1(0)$  is known, the proposed FTPF has an advantage over the reported performance functions. That is, when t = 0, constrained condition (3) becomes an unconstrained one that  $\lim_{l_2 \to -\infty} -\underline{d}\rho(0) < e(0) < \lim_{l_2 \to \infty} \overline{d}\rho(0)$ .

**Remark 7.** Settling time  $T_f$  is dependent on constants l and  $l_3$ , which is specified according to the actual system requirements. It is very meaningful for practical models with high convergence time requirements, such as unmanned aerial vehicles and robotic arm systems. Constant  $\rho_{T_f}$  represents the prescribed accuracy of TE  $e_1(t)$  in advance, with the maximum allowable size of TE  $e_1(t)$  at the steady state. The value of parameter  $l_2$  determines the maximum overshoot of the TE based on the analysis of Remark 6. Consequently, a different tracking performance can be achieved by selecting the appropriate parameters of function  $\rho(t)$ . Likewise, when parameters  $l, l_1, l_2$ , and  $l_3$  are given, the performance function is predetermined.

To represent constrict condition (3) by an unconstrained form, the following state transformation, as reported in [7], is employed

$$e_1(t) = \rho(t) R(\bar{\zeta}_1),$$

where

$$R(\bar{\zeta}_1) = \frac{\bar{d}e^{\bar{\zeta}_1} - \underline{d}e^{-\bar{\zeta}_1}}{e^{\bar{\zeta}_1} + e^{-\bar{\zeta}_1}}$$
(5)

and  $\bar{\zeta}_1$  denotes the transformed error. According to (5), smooth and strictly increasing function  $R(\bar{\zeta}_1)$  satisfies that  $\lim_{\bar{\zeta}_1 \to -\infty} R(\bar{\zeta}_1) = -\underline{d}$  and  $\lim_{\bar{\zeta}_1 \to +\infty} R(\bar{\zeta}_1) = \overline{d}$ . The inverse transformation of (5) can be obtained

$$\bar{\zeta}_1 = R^{-1} \left( \frac{e_1}{\rho} \right) = \frac{1}{2} \ln \left( \frac{\frac{e_1}{\rho} + d}{\bar{d} - \frac{e_1}{\rho}} \right).$$

To deal with the zero equilibrium point inconformity problem in the state transformation, transformed error  $\bar{\zeta}_1$  can be rewritten as

$$\zeta_1 = \bar{\zeta}_1 - \frac{1}{2} \ln\left(\frac{\underline{d}}{\overline{d}}\right). \tag{6}$$

And then, the derivation of  $\zeta_1$  is

$$\dot{\zeta}_1 = \chi_1 \bigg( \dot{e}_1 - \frac{\dot{\rho}}{\rho} e_1 \bigg),$$

where

$$\chi_1 = \frac{1}{2\rho} \left( \frac{1}{\frac{l_1}{\rho} + \underline{d}} - \frac{1}{\frac{l_1}{\rho} - \overline{d}} \right) > 0,$$
  
$$\dot{\rho} = (l_1 - l_1 \coth^2(l_1 t + l_2))(1 - \frac{l_3 t}{l})^l - l_3 \coth(l_1 t + l_2)(1 - \frac{l_3 t}{l})^{l-1}$$

#### 3. Adaptive NN Controller Design

In this section, a CF-based adaptive NN control scheme is proposed by using the Backstepping technique to ensure the tracking property and prove the stability of the system.

The definition of the error variables is presented below.

Define error variables

$$\begin{cases} z_1 = \zeta_1, \\ z_i = x_i - x_{i,c}, \quad i = 2, 3, \dots, n, \end{cases}$$
(7)

where  $x_{i+1,c}(t)$  and  $\dot{x}_{i+1,c}(t)$  are the output of the CF. And as reported in [8], the CF is defined as

$$\begin{cases} \phi_{i,1} = \omega_n \phi_{i,2}, & i = 1, 2, \dots, n-1, \\ \dot{\phi}_{i,2} = -2\zeta \omega_n \phi_{i,2} - \omega_n (\phi_{i,1} - \alpha_i), \end{cases}$$
(8)

where  $x_{i+1,c}(t) = \phi_{i,1}$  and  $\dot{x}_{i+1,c}(t) = \omega_n \phi_{i,2}$  with  $\phi_{i,1}(0) = 0$  and  $\phi_{i,2}(0) = 0$ . The design parameters of filter (8) satisfy  $\omega_n > 0$  and  $\zeta \in (0, 1]$ .

**Remark 8.** From (8), it can be seen that the CF is a second-order filter, which is more complicated than the first-order low-pass filter in the DSC method reported in [5–7]. However, in the published control schemes based on the DSC method [5–7], filtering error  $x_{i+1,c} - \alpha_i$  is neglected, which makes the controller difficult to achieve accurate control performance. To achieve good control performance, the CF method is introduced into the Backstepping technique to solve the EOC problem, where the filtering error is eliminated by introducing the compensating signal.

**Remark 9.** Parameters of filter (8) are required to satisfy  $\omega_n > 0$  and  $\zeta \in (0, 1]$ . The output signals of the filter and control input are affected by the selection of parameter  $\omega_n$  and  $\zeta$ , which may lead to the peak phenomenon. Different filter performance and control performance can be achieved by selecting appropriate parameters  $\omega_n$  and  $\zeta$ . Thus, in practical applications, these parameters of the filter can be selected eclectically for achieving suitable control performance.

Next, we define compensated TE signal  $v_i$ 

$$\begin{cases}
\nu_1 = z_1 - \xi_1, \\
\nu_i = z_i - \xi_i, \quad i = 2, \dots, n - 1, \\
\nu_n = z_n,
\end{cases}$$
(9)

where  $\xi_i$  is the compensating signal to be devised.

In what follows, the designed adaptive controllers are presented as follows. The virtual controllers are designed as

$$\alpha_1 = -\eta_1 z_1 - \nu_1 \chi_1 + \dot{y}_d + \frac{\dot{\rho}}{\rho} e_1 - \hat{\theta}_1 a_1 \nu_1 \chi_1 S^T(Z_{12}) S(Z_{12})$$
(10)

and

$$\alpha_i = -k_i z_i - \nu_i - a_i \nu_i \hat{\theta}_i S^T(Z_{i2}) S(Z_{i2}) + \dot{x}_{i,c} - \chi_1(\nu_{i-1} - \xi_{i-1}), \ i = 2, \dots, n$$
(11)

The compensating signal laws are constructed as

$$\dot{\xi}_1 = \chi_1(-\eta_1\xi_1 + (x_{2,c} - \alpha_1) + \xi_2), \tag{12}$$

and

$$\tilde{\xi}_{i} = -k_{i}\xi_{i} + (x_{i+1,c} - \alpha_{i}) + \xi_{i+1} - \chi_{1}\xi_{i-1}, \ i = 2, 3, \dots, n-1 
\tilde{\xi}_{n} = -k_{n}\xi_{n} - \xi_{n-1},$$
(13)

where  $\eta_1 > 0$  and  $k_i > 0$  are constants;  $Z_{i2}$  is the input vector of RBFNN. Additionally, according to the calculation process, it is easy to find that when i > 2,  $\chi_1(\nu_{i-1} - \xi_{i-1})$  of controller (11) and  $\chi_1\xi_{i-1}$  of controller (13) will be independent of  $\chi_1$ . At the same time, the corresponding parameter learning laws are developed as

$$\hat{\theta}_{i} = \gamma_{i} a_{i} v_{i}^{2} S^{T}(Z_{i2}) S(Z_{i2}) - \sigma_{i} \hat{\theta}_{i}, \ i = 1, 2, \dots, n,$$
(14)

where  $\sigma_i > 0$  i = 1, 2, ..., n are the coefficients of the modification terms, and modification terms  $\sigma_i \hat{\theta}_i$  are used to increase the robustness of the closed-loop system;  $\hat{\theta}_i$  are the estimate of constants  $\theta_i^*$ . Throughout this paper, estimation errors  $\tilde{\theta}_i$  are defined as  $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$ .

#### 4. Stability Analysis

And the prescribed performance of the TE and the boundedness of the closed-loop system, are summarized as the theorem below. In what follows, the proof of the theorem is presented.

**Theorem 1.** Under Assumptions 1 and 2, system (1), controllers (10) and (11), parameter learning laws (14), and compensating signal laws (12) and (13), it is concluded that

- (1) the closed-loop system is SGUUB;
- (2) the fixed-time prescribed performance of the TE is guaranteed, i.e., Ineq. (3) holds.

**Proof.** In what follows, based on the Lyapunov stability theory, the CF method, and the proposed FTPF, an adaptive NN PPC strategy is designed to prove the conclusion of Theorem 1.

**Step 1.** According to (6), (7), and (9) and following from system description (1), the dynamic of compensated TE  $\nu_1$  is

$$\dot{\nu}_1 = \dot{\zeta}_1 - \dot{\xi}_1 = \chi_1 \left( x_2 + h_1(Z_{11}) - \dot{y}_d - \frac{\dot{\rho}}{\rho} e_1 - \chi_1^{-1} \dot{\xi}_1 \right), \tag{15}$$

where  $h_1(Z_{11}) = -x_2 + g_1(X) + f_1(X)$  is an unknown smooth nonlinear function.

With the help of RBFNN, from (2), function  $h_1(Z_{11})$  can be expressed as

$$h_1(Z_{11}) = W_1^{*T} S(Z_{11}) + \varepsilon_1(Z_{11}), \tag{16}$$

where  $Z_{11} = X = [x_1, ..., x_n]^T \in \Omega$  is the input vector of RBFNN with compact set  $\Omega$ . According to (15) and (16), the dynamic of  $\frac{1}{2}\nu_1^2$  results in

$$\dot{\nu}_1 \nu_1 = \nu_1 \chi_1 (x_2 + W_1^{*T} S(Z_{11}) + \varepsilon_1 (Z_{11}) - \dot{y}_d - \frac{\dot{\rho}}{\rho} e_1 - \chi_1^{-1} \dot{\xi}_1).$$
(17)

Based on Lemma 1 and Young's inequality  $xy \le \frac{x^2}{2a} + \frac{ay^2}{2}$  (a > 0 is a positive constant), we have

$$\chi_{1}\nu_{1}W_{1}^{*T}S(Z_{11}) \leq a_{1}\nu_{1}^{2}\chi_{1}^{2}\theta_{1}^{*}S^{T}(Z_{11})S(Z_{11}) + \frac{1}{4a_{1}}$$

$$\leq a_{1}\nu_{1}^{2}\chi_{1}^{2}\theta_{1}^{*}S^{T}(Z_{12})S(Z_{12}) + \frac{1}{4a_{1}},$$
(18)

and

$$\chi_1 \nu_1 \varepsilon_1(Z_{11}) \le \nu_1^2 \chi_1^2 + \frac{1}{4} \varepsilon_1^{*2},$$
(19)

where  $Z_{12}$  is only dependent on system state  $x_1$ ;  $\theta_1^* = ||W_1^*||^2$  is an unknown constant since  $W_1^*$  is a constant vector;  $\varepsilon_1^* > 0$  is the upper bound of approximation error  $\varepsilon_1(Z_{11})$ ; and  $a_1$  is a positive constant.

**Remark 10.** From (16), apparently, the NN input vector  $Z_{11}$  is dependent on all the state signals. According to the principle of the Backstepping technique and the Lyapunov stability theory, virtual functions  $\alpha_i$  are associated with all the system states, which result in the algebraic loop problem [21]. To avoid this issue, the NN input variable  $Z_{11}$  is replaced by  $Z_{12}$  based on Lemma 1, so that the designed virtual controllers can be designed and implemented directly.

Choose Lyapunov function as

$$V_1 = \frac{1}{2}\nu_1^2 + \frac{1}{2\gamma_1}\tilde{ heta}_1^2,$$

where  $\gamma_1 > 0$  is a constant.

In view of (17)–(19), the derivative of function  $V_1$  is

$$\frac{dV_1}{dt} = \nu_1 \dot{\nu}_1 - \frac{1}{\gamma_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1 
\leq \chi_1 \nu_1 \Big( (x_2 - x_{2,c}) + (x_{2,c} - \alpha_1) + \alpha_1 - \dot{y}_d + a_1 \hat{\theta}_1 \nu_1 \chi_1 S^T(Z_{12}) S(Z_{12}) + \nu_1 \chi_1 
- \frac{\dot{\rho}}{\rho} e_1 - \chi_1^{-1} \dot{\xi}_1 \Big) + \frac{1}{4} \varepsilon_1^{*2} + \frac{1}{4a_1} - \frac{1}{\gamma_1} \tilde{\theta} \Big( \dot{\hat{\theta}}_1 - \gamma_1 a_1 \nu_1^2 \chi_1^2 S^T(Z_{12}) S(Z_{12}) \Big).$$
(20)

Moreover, by utilizing the completion of the square, it is easy to conclude that

$$\frac{\sigma_1}{\gamma_1}\tilde{\theta}_1\hat{\theta}_1 = \frac{\sigma_1}{\gamma_1}\tilde{\theta}_1(\theta_1^* - \tilde{\theta}_1) = -\frac{\sigma_1}{\gamma_1}\tilde{\theta}_1^2 + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_1\theta_1^* 
\leqslant -\frac{\sigma_1}{2\gamma_1}\tilde{\theta}_1^2 + \frac{\sigma_1}{2\gamma_1}\theta_1^{*2}.$$
(21)

Substituting (10), (12), and (14) with i = 1 and (21) into (20) yields

$$\frac{dV_1}{dt} \le -k_1 \nu_1^2 - \frac{\sigma_1}{2\gamma_1} \tilde{\theta}_1^2 + \nu_1 \chi_1 \nu_2 + D_1,$$
(22)

where  $k_1 := \eta_1 \chi_1 > 0$  since  $\eta_1 > 0, \chi_1 > 0; D_1 = \frac{\sigma_1}{2\gamma_1} \theta_1^{*2} + \frac{1}{4} \varepsilon_1^{*2} + \frac{1}{4a_1}$ .

**Step** *i* ( $2 \le i \le n - 1$ ). Based on (1), (7), and (9), the dynamic of compensated TE  $v_i$  satisfies

$$\dot{\nu}_{i} = \dot{x}_{i} - \dot{\xi}_{i-1} 
= x_{i+1} + h_{i}(Z_{i1}) - \dot{x}_{i,c} - \dot{\xi}_{i},$$
(23)

where  $h_i(Z_{i1}) = -x_{i+1} + g_i(X) + f_i(X)$  is an unknown smooth function. In view of approximator (2), the unknown smooth function can be described as

$$h_i(Z_{i1}) = W_i^{*T} S(Z_{i1}) + \varepsilon_i(Z_{i1}), \quad Z_{i1} \in \Omega,$$
 (24)

where  $Z_{i1} = X = [x_1, x_2, ..., x_n]^T$  is the input vector of RBFNN.

Subsequently, from (23) and (24), the dynamic of  $\frac{1}{2}v_i^2$  is

$$\nu_{i}\dot{\nu}_{i} = \nu_{i}\left(x_{i+1} + W_{i}^{*T}S(Z_{i1}) + \varepsilon_{i}(Z_{i1}) - \dot{x}_{i,c} - \dot{\xi}_{i}\right)$$

$$\leq \nu_{i}\left(x_{i+1} + a_{i}\nu_{i}\theta_{i}^{*}S^{T}(Z_{i2})S(Z_{i2}) + \nu_{i} - \dot{x}_{i,c} - \dot{\xi}_{i}\right) + \frac{1}{4}\varepsilon_{i}^{*2} + \frac{1}{4a_{i}}$$
(25)

with NN input vector  $Z_{i2} = [x_1, x_2, ..., x_i]^T$ . The following two inequalities are used in the derivation of (25). Based on Young's inequality and the property of basis function  $S(\cdot)$ , we have

$$\nu_i W_i^{*T} S(Z_{i1}) \le a_i \nu_i^2 \theta_i^* S^T(Z_{i2}) S(Z_{i2}) + \frac{1}{4a_i},$$
(26)

and

$$\nu_i \varepsilon_i(Z_{i1}) \le \nu_i^2 + \frac{1}{4} \varepsilon_i^{*2},$$
(27)

where  $\theta_i^* = ||W_i^*||^2$  and  $\varepsilon_i^*$  is a positive constant with  $\varepsilon_i^* > |\varepsilon_i(Z_{i1})|$ . Consider the Lyapunov function

$$V_i = V_{i-1} + rac{1}{2}
u_i^2 + rac{1}{2\gamma_i} ilde{ heta}_i^2,$$

and its first time derivative is

$$\frac{dV_i}{dt} = \frac{dV_{i-1}}{dt} + \nu_i \dot{\nu}_i - \frac{1}{\gamma_i} \tilde{\theta}_i \dot{\hat{\theta}}_i, \qquad (28)$$

where  $\gamma_i > 0$  is a constant parameter.

According to (22), (25)–(28), it can be obtained that

$$\frac{dV_{i}}{dt} \leq -\sum_{j=1}^{i-1} k_{j} \nu_{j}^{2} - \sum_{j=1}^{i-1} \frac{\sigma_{j}}{2\gamma_{j}} \tilde{\theta}_{j}^{2} + \chi_{1} \nu_{i-1} \nu_{i} + D_{i-1} + \frac{1}{4} \varepsilon_{i}^{*2} + \frac{1}{4a_{i}} - \frac{1}{\gamma_{i}} \tilde{\theta}_{i} \hat{\theta}_{i} 
+ \nu_{i} \left( x_{i+1} + a_{i} \nu_{i} \theta_{i}^{*} S^{T}(Z_{i2}) S(Z_{i2}) + \nu_{i} - \dot{x}_{i,c} - \dot{\xi}_{i} \right) 
\leq -\sum_{j=1}^{i-1} k_{j} \nu_{j}^{2} - \sum_{j=1}^{i-1} \frac{\sigma_{j}}{2\gamma_{j}} \tilde{\theta}_{j}^{2} + D_{i-1} + \frac{1}{4} \varepsilon_{i}^{*2} + \frac{1}{4a_{i}} + \nu_{i} \left( (x_{i+1} - x_{i+1,c}) + (x_{i+1,c} - \alpha_{i}) + \alpha_{i} + \chi_{1} \nu_{i-1} + a_{i} \nu_{i} \hat{\theta}_{i} S^{T}(Z_{i2}) S(Z_{i2}) + \nu_{i} - \dot{x}_{i,c} - \dot{\xi}_{i} \right) 
+ \left( x_{i+1,c} - \alpha_{i} \right) + \alpha_{i} + \chi_{1} \nu_{i-1} + a_{i} \nu_{i} \hat{\theta}_{i} S^{T}(Z_{i2}) S(Z_{i2}) + \nu_{i} - \dot{x}_{i,c} - \dot{\xi}_{i} \right) 
- \frac{1}{\gamma_{i}} \tilde{\theta}_{i} \left( \dot{\theta}_{i} - \gamma_{i} a_{i} \nu_{i}^{2} S^{T}(Z_{i2}) S(Z_{i2}) \right),$$
(29)

where  $D_{i-1} = \sum_{j=1}^{i-1} (\frac{\sigma_j}{2\gamma_j} \theta_j^{*2} + \frac{1}{4} \varepsilon_j^{*2} + \frac{1}{4a_j}).$ 

Substituting (11), (13), and (14) into (29) leads to

$$\frac{dV_i}{dt} \le -\sum_{j=1}^{i} k_j z_j^2 - \sum_{j=1}^{i-1} \frac{\sigma_j}{2\gamma_j} \tilde{\theta}_j^2 + D_{i-1} + \frac{1}{4} \varepsilon_i^{*2} + \frac{1}{4a_i} + \frac{\sigma_i}{\gamma_i} \tilde{\theta}_i \hat{\theta}_i.$$
(30)

Similar to the calculation of (21), it yields

$$\frac{\sigma_i}{\gamma_i}\tilde{\theta}_i\hat{\theta}_i \leqslant -\frac{\sigma_i}{2\gamma_i}\tilde{\theta}_i^2 + \frac{\sigma_i}{2\gamma_i}{\theta_i^{*2}}.$$
(31)

Combining (30) and (31), we obtain

$$\frac{dV_i}{dt} \le -\sum_{j=1}^{i} k_j \nu_j^2 - \sum_{j=1}^{i} \frac{\sigma_j}{2\gamma_j} \tilde{\theta}_j^2 + D_i + \nu_i \nu_{i+1},$$
(32)

where  $D_i = \sum_{j=1}^i \left(\frac{\sigma_j}{2\gamma_j}\theta_j^{*2} + \frac{1}{4}\varepsilon_j^{*2} + \frac{1}{4a_j}\right).$ 

Step n. Consider

$$V_n = V_{n-1} + \frac{1}{2}\nu_n^2 + \frac{1}{2\gamma_n}\tilde{\theta}_n^2,$$

where  $\gamma_n$  and  $\theta_n^* = ||W_n^*||^2$  are positive constants.

Then, noticing (32) with i = n - 1 and combining (11) and (14) yields

$$\begin{aligned} \frac{dV_n}{dt} &\leq -\sum_{i=1}^n k_i \nu_i^2 - \sum_{i=1}^n \frac{\sigma_i}{2\gamma_i} \tilde{\theta}_i^2 + D_{n-1} + \frac{\sigma_n}{2\gamma_n} \theta_n^{*2} + \frac{1}{4} \varepsilon_n^{*2} + \frac{1}{4a_n} \\ &\leq -cV_n + D_n, \end{aligned}$$

where  $c = \min\{2k_i, \sigma_i : i = 1, ..., n\}$  and  $D_n = \sum_{i=1}^n \left(\frac{\sigma_i}{2\gamma_i}\theta_i^{*2} + \frac{1}{4}\varepsilon_i^{*2} + \frac{1}{4a_i}\right)$ . Apparently,  $D_n$  is bounded since  $\gamma_i, \sigma_i, \theta_i^*$ , and  $\varepsilon_i^*$  are constants.

According to the recursive process, Lyapunov function  $V_n$  satisfies  $\frac{dV_n}{dt} \leq 0$ . Thus, we can conclude that all the signals  $(v_i \text{ and } \tilde{\theta}_i, i = 1, ..., n)$  are uniformly ultimately bounded based on the Boundedness Theorem (e.g., Theorem 4.18 in [40]). At the same time, it is easy to obtain that  $\hat{\theta}_i$  are bounded since  $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$ . Choose the Lyapunov function of the compensating system as follows:

$$V_{\xi} = \frac{1}{2} \sum_{i}^{n} \xi_{i}^{2}$$

According to compensating law (13), the first derivative of Lyapunov function  $V_{\xi}$  is

$$\frac{dV_{\xi}}{dt} = \xi_1 \dot{\xi}_1 + \xi_2 \dot{\xi}_2 + \dots + \xi_n \dot{\xi}_n 
= -\eta_1 \chi_1 \xi_1^2 + \chi_1 \xi_1 (x_{2,c} - \alpha_1) + \chi_1 \xi_1 \xi_2 - k_2 \xi_2^2 + \xi_2 (x_{i+1,c} - \alpha_i) + \xi_2 \xi_3 
- \chi_1 \xi_1 \xi_2 - k_i \xi_i^2 + \xi_i (x_{i+1,c} - \alpha_i) + \xi_i \xi_{i+1} - \xi_i \xi_{i-1} + \dots - k_n \xi_n^2 - \xi_n \xi_{n-1}.$$
(33)

As cited in [8],  $|x_{i+1,c} - \alpha_i| \le \omega_i$  can be ensured with arbitrarily small positive constants  $\omega_i$  (i = 1, 2, ..., n). Therefore, with the help of the inequality that  $ab \le 1/2a^2 + 1/2b^2$ , Equation (33) can be rewritten as

$$\frac{dV_{\xi}}{dt} \le -\mathcal{K}V_{\xi} + 1/2\sum_{i=1}^{n-1}\varpi_i^2 \tag{34}$$

where  $\mathcal{K} = 2 \min\{l_1\chi_1 - 1/2, k_2 - 1/2, \dots, k_n - 1/2\}$ . Thus, according to the Boundedness Theorem [40], all the compensating signals are bounded. Thus, according to the boundedness of  $\xi_i$  and  $\nu_i$ , we can obtain that error signals  $z_i$  ( $i = 1, 2, \dots, n$ ) are bounded. As cited in [29,30], if transformed error  $\zeta_1$  is bounded, the prescribed performance of the TE is guaranteed. Thus, according to the boundedness of error signal  $z_1$  (i.e.,  $\zeta_1$ ), the fixed-time prescribed tracking performance is ensured. That is, TE  $e_1(t)$  enters the prescribed domain bounded by performance functions within fixed time and stay therein.

Consequently, the proof of Theorem 1 is completed.  $\Box$ 

### 5. Simulation

To verify the effectiveness of the proposed control algorithm, a simulation study for NSFNSs is performed.

**Example 1.** Consider the NSFNSs below

$$\begin{cases} \dot{x}_1 = g_1(X) + f_1(X), \\ \dot{x}_2 = g_2(X, u) + f_2(X), \\ y = x_1, \end{cases}$$
(35)

where  $X = [x_1, x_2]^T$ ; *u* and *y* denote the system state vector, input and output, respectively; system functions are chosen as  $g_1(X) = 4x_2 + \cos(x_1x_2)$ ,  $g_2(X, u) = 2u(\sin(x_1 + x_2) + 2)$ ,  $f_1(X) = -\cos(x_1^2x_2)$ , and  $f_2(X) = \cos(x_1x_2) + 1$ ; reference trajectory  $y_d(t)$  is chosen as  $y_d(t) = \cos(1.5t) + \cos(t)$ ; and performance function

$$\rho(t) = \begin{cases} \coth(0.5t + 0.5)(1 - \frac{4t}{4})^4 + \rho_{T_f}, & t \in [0, T_f], \\ \rho_{T_f}, & t \in [T_f, +\infty), \end{cases}$$

where  $\rho_{T_f} = 0.05$  and settling time  $T_f = 1$ .

In what follows, adaptive NN controllers based on CF are developed for system (35) to ensure the prescribed fixed-time performance of the TE. Similar to the designed controllers (10) and (11), the controllers for system (35) are formulated as

$$u = -k_2 z_2 - \nu_2 - \nu_2 a_2 \hat{\theta}_2 S^T(Z_{22}) S(Z_{22}) - \chi_1(\nu_1 - \xi_1) + \dot{x}_{2,c},$$
(36)

and

$$\alpha_1 = -\eta_1 z_1 - \nu_1 \chi_1 + \dot{y}_d + \frac{\dot{\rho}}{\rho} e_1 - a_1 \hat{\theta}_1 \nu_1 \chi_1 S^T(Z_{12}) S(Z_{12}), \tag{37}$$

where  $Z_{12} = x_1$  and  $Z_{22} = [x_1, x_2]^T$  are the NN inputs;  $\theta_1^*$  and  $\theta_2^*$  are the norm's squares of NN ideal weight vectors; and  $\hat{\theta}_1$  and  $\hat{\theta}_2$  are their estimates. At the same time, the dynamics of the compensating signals are

$$\dot{\xi}_1 = \chi_1(-\eta_1\xi_1 + (x_{2,c} - \alpha_1) + \xi_2),$$
(38)

$$\dot{\xi}_2 = -k_2\xi_2 - \chi_1\xi_1. \tag{39}$$

And the parameter learning laws are developed as

$$\hat{\theta}_1 = \gamma_1 a_1 \nu_1^2 \chi_1^2 S^T(Z_{12}) S(Z_{12}) - \sigma_1 \hat{\theta}_1, \tag{40}$$

$$\dot{\hat{\theta}}_2 = \gamma_2 a_2 \nu_2^2 S^T(Z_{22}) S(Z_{22}) - \sigma_2 \hat{\theta}_2.$$
(41)

The chosen simulation parameters are provided in Table 1, which include the initial conditions of system (35), the number of NN nodes, and the parameters of (36)–(41).

To verify the effectiveness of the proposed method, this paper compares the designed fixed-time prescribed tracking performance control scheme with the standard prescribed tracking performance control scheme by selecting the same design parameters and initial values.

| Initial<br>conditions     | $x_1(0)$           | 0.5 | Parameters            | Value |
|---------------------------|--------------------|-----|-----------------------|-------|
|                           | $x_2(0)$           | 1   | $\eta_1$              | 1     |
|                           | $\hat{	heta}_1(0)$ | 0   | <i>k</i> <sub>2</sub> | 200   |
|                           | $\hat{	heta}_2(0)$ | 0   | $\gamma_1$            | 10    |
| Node number               | q                  | 9   | $\gamma_2$            | 20    |
| Performance<br>function   | đ                  | 1   | $\sigma_1$            | 0.05  |
|                           | <u>d</u>           | 1   | $\sigma_2$            | 0.05  |
| Command filter parameters | $\omega_n$         | 100 | <i>a</i> <sub>1</sub> | 1     |
|                           | ζ                  | 0.5 | <i>a</i> <sub>2</sub> | 1     |

Table 1. Simulation Parameters.

The obtained simulation results are presented in Figures 2–7. From Figure 2, it can be seen that the system output tracks reference trajectory  $y_d(t)$ . To clearly display the prescribed tracking performance, the trajectories of the TE and the performance functions are provided in Figure 3, which indicates that the TE falls within the prescribed bounds by the performance functions in settling time  $T_f = 1$ . The small figure of Figure 3 further demonstrates that the tracking error signal converges the prescribed bounds of the performance functions within a fixed time. However, the trajectory of the tracking error–based standard PPC methodology converges asymptotically to a prescribed bounded neighborhood. Additionally, compared to the tracking error obtained without the PPC method, the transient and steady-state performances of the tracking error are guaranteed by using the proposed control algorithm simultaneously. Moreover, the trajectories of the control input u(t), the compensated TE signal, the compensating signal, and the norm's squares of the weight estimations are presented in Figures 4–7, respectively. It can be clearly seen that the trajectories of these estimates are stable in a bounded domain from Figures 4–7.

In summary, these simulation results suggest the fixed-time boundedness of the TE and the SGUUB of all the signals of the closed-loop system, which accords with Theorem 1. Furthermore, the obtained control performance verifies the effectiveness of the proposed control method.



**Figure 2.** Reference signal  $y_d(t)$  and output signal y(t).



**Figure 3.** Error signals  $e_1(t)$  and performance functions  $\rho(t)$ .



**Figure 4.** Control input u(t).



**Figure 5.** Compensated tracking error signal  $v_1(t)$ .



**Figure 6.** Compensating signal  $\xi_1(t)$ .



Figure 7. Estimations of norm's squares of NN weight vectors.

#### 6. Conclusions

This paper investigates the tracking control problem with prescribed performance for a class of NSFNSs. A new FTPF is proposed firstly, which does not need to determine the exact initial values in advance and whose convergence time is specified only according to the actual system requirements. Moreover, the approximator RBFNN is exploited to identify the unknown system functions, and the nonstrict-feedback structure is addressed by using the characteristics of NN basis functions. In this paper, the constructed CF-based adaptive NN PPC scheme relaxes the assumption on the reference signal and solves the EOC issue while eliminating the influence of filtering errors. Furthermore, it is proven that the closed-loop system is SGUUB and the fixed-time prescribed tracking performance is ensured. That is, the TE converges to the prescribed small neighborhood of origin within a fixed time.

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