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Day of the Week Effect on the World Exchange Rates through Fractal Analysis

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Abstract: The foreign exchange rate market is one of the most liquid and efficient. In this study, we address the efficient analysis of this market by verifying the day-of-the-week effect with fractal analysis. The presence of fractality was evident in the return series of each day and when analyzing an upward trend and a downward trend. The econometric models showed that the day-of-the-week effect in the studied currencies did not align with previous studies. However, analyzing the Hurst exponent of each day revealed that there a weekday effect in the fractal dimension. Thirty main world currencies from all continents were analyzed, showing weekday effects according to their fractal behavior. These results show a form of market inefficiency, as the returns or price variations of each day for the analyzed currencies should have behaved similarly and tended towards random walks. This fractal day-of-the-week effect in world currencies allows us to generate investment strategies and to better complement or support buying and selling decisions on certain days.

Keywords: calendar anomalies; day-of-the-week effect; market indices; multifractal detrended fluctuation analysis

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1. Introduction

In an integrated global economy, growing trade relations and easier access to financial markets, currencies, and exchange rates are fundamental elements in the composition of economic results and investment performance, influencing trade flows, investment decisions, and general economic stability. The dynamic nature of exchange rates reflects the complex interactions between various economic, financial, and geopolitical factors, making them a topic of great interest to policymakers, businesses, and investors that impact both individuals and companies.

Currencies, or foreign exchange markets, are very deep financial markets, which means that they should be efficient. When currencies are traded on the market, supply and demand usually work without anomalies, with the exception of those currencies which, according to their exchange regime, may experience intervention (mainly from a central bank) or not be freely traded on the market. Economic fundamentals such as interest rates, inflation, and growth prospects play a crucial role in the supply and demand of currencies, and consequently in exchange rate movements. In addition, market sentiment, investor expectations, and geopolitical events can influence exchange rates in the short term, creating volatility and uncertainty in currency markets. Goodell et al. [1] have summarized several studies on investor emotions and market anomalies.

Financial markets are often considered efficient, with prices fully reflecting all available information, as suggested by the efficient market hypothesis (EMH). However, empirical evidence suggests that financial markets are not always efficient, and that there may be anomalies in the form of persistent patterns or behaviors that traditional financial theories cannot explain. These anomalies challenge the notion of market efficiency and have important implications for investors, portfolio managers, and financial policymakers. The exchange rate market has previously been studied with respect to its efficiency [2–10].

One way to show that markets are inefficient is by showing evidence of market anomalies, particularly evidence of asset prices consistently exhibiting patterns or behaviors that defy rational expectations or traditional economic theories. These anomalies can manifest in various ways, from predictable stock price patterns to abnormal trading behavior and asset pricing inefficiencies. One of the most well-known and studied types consists of calendar anomalies, particularly the day-of-week effect. Several works have studied this effect in different financial markets, equity markets [11–14], commodity markets [15–17], interest rates [18], US REITs [19], and even in the context of the effect of the COVID-19 pandemic on stock markets [20].

One innovative approach to addressing these anomalies is through fractal analysis, which explores self-similar patterns and structures that manifest across different time scales in financial data. Fractals offer a robust framework for capturing the complex dynamics of market prices, revealing hidden relationships and patterns that traditional methods might miss. Although fractals have previously been applied to the analysis of financial markets [21–32], only a few studies have applied fractals to determine the anomaly of the weekday effect.

Stosic et al. [33] applied multifractal detrended fluctuation analysis (MF-DFA) to the daily returns of market indices worldwide, finding that the returns on Mondays tend to be more persistent and to possess a higher degree of multifractality than returns on the other days. Plastun et al. [34] studied developed and emerging stock, FOREX, commodity, and cryptocurrency markets and found differences in persistence intensity depending on the day and market being analyzed. Sakalauskas et al. [35] analyzed the day-of-week effect in emerging financial markets, showing that the Hurst exponent differs depending on the day. For their part, Bolek et al. [36] analyzed the efficiency effect and the day-of-week effect for OMX exchange indices during the COVID-19 pandemic, finding different fractal behavior for different days.

The significance of studying the anomalies in the currency market cannot be understated. It is crucial to determine the level of efficiency in the market, the potential of investors to devise investment strategies, and the design of more effective instruments for risk hedging in currency portfolios. This knowledge also influences investment decisions, remittance settlement choices, and the commercial financial policies of companies involved in import and export.

Most articles on the fractal efficient market hypothesis test whether markets can be considered efficient by measuring the value of the Hurst exponent and testing whether it is different from 0.5 (the coefficient of a random walk process). Suppose that the market or financial asset under analysis has a coefficient above 0.5; in this case, we say that the asset has long-term persistence, while if it is below 0.5 then the asset is anti-persistent. This logic allows for a better understanding of market behavior and opens up the potential for building profitable trading strategies for financial assets. If $H > 0.5$, then investors can use momentum strategies to make investment decisions. If $H < 0.5$, then investors can instead use a mean-reversion strategy. Both of these approaches have shown potential profitability.

The same idea can be used to analyze pairs of financial assets. If the difference between asset prices has a coefficient of H < 0.5, then investors can develop buying and short-selling strategies for these assets based on a predicted reversion to the mean. If the prices diverge, then an investment strategy can instead be set up on the basis that the difference between the prices will revert to the mean. Ramos-Requena et al. [37] developed a pair trading strategy using these ideas and demonstrated that these strategies can be considered profitable. Other researchers, such as Guasoni et al. [38] and Garcin et al. [39], have also developed strategies that take into account the fractal nature of financial time series, which can allow for future price forecasting and help to position buying or short-selling signals for investors.

In the case of financial assets with multifractal characteristics, a single Hurst coefficient cannot explain the temporal dynamics. There are times when the coefficients change significantly. In this case, a logic similar to that of single-fractal markets can be employed. It is possible to use the idea that coefficients that deviate from 0.5 lead to trading strategies that can generate abnormal profits; in other words, it is possible to build strategies that bet on either mean reversion (anti-persistence) or even moments (persistence).

Dewandaru et al. [40] used this idea to construct an indicator of the efficiency of a financial asset as D = $1/2(|H(-q) - 0.5| + |H(q) - 0.5|)$. In this case, using a choice of q = 4, an efficiency indicator D = $1/2(|H(-4) - 0.5| + |H(4) - 0.5|)$ is applied, which measures the distance of H from the random walk benchmark of 0.5 in the case of small and large fluctuations, respectively. As D moves away from zero, there should be opportunities to build trading strategies. The authors found that their proposed strategy can generate significant profits compared to passive investment based on the market index. Here, we focus on testing how the fractal dimension changes over the days of the week. We leave the construction of specific strategies to further research, as it is a theme that demands several additional considerations.

The novelty of this study is that is addresses the day-of-the-week analysis using fractal theory for thirty currencies from around the world. We analyze the generalized Hurst exponent with a confidence interval in order to distinguish behaviors with statistical significance, thereby expanding the field of study of econophysics.

The rest of this paper consists of a Methodology section, where the foundations of the generalized Hurst exponent are explained, an Analysis of Results section, which provides an in-depth analysis of the series of variations of the studied exchange rates along with the obtained results, and a Conclusions section which discusses the results and highlights the main findings.

2. Materials and Methods

Multi-Fractal Detrended Fluctuation Analysis (MFDFA) allows the asymmetric multifractal characteristics of time series to be examined. This method was introduced by Peng et al. [41] as Detrended Fluctuation Analysis (DFA) and generalized by Kantelhardt et al. [42] as MFDFA. The method performs well even with highly nonstationary series, and is based on five steps.

For a time series x_i with a length of N observations, the method can be summarized as follows:

- **Step 1:** Construct the profile

$$X(i) = \sum_{t=1}^{i} (x_t - \overline{x}), i = 1, ..., N, \tag{1}$$

  where $\overline{x}$ represents the series average of time series x_i.
- **Step 2:** Divide the profiles $X(i)$ into $N_s = [N/s]$ non-overlapping windows of equal length s. Because the length of the series N is not necessarily a multiple of the time scale s, parts of the profile may remain at the end; thus, the same procedure is applied from the end of the series as well. The final result is $2N_s$ segments.
- **Step 3:** The trend $X^v(i)$ for each of the $2N_s$ segments is estimated using a linear regression as $X^v(i) = a_{X^v} + b_{X^v} \cdot i$. This process precedes the determination of the detrended variance, calculated as

$$F(v,s) = \frac{1}{s} \sum_{i=1}^{s} [X[(v-1)s + i] - X^v(i)]^2 \tag{2}$$

  for each segment v, $v = 1, \cdots, N_s$ and

$$F(v,s) = \frac{1}{s} \sum_{i=1}^{s} [X[N - (v - N_s)s + i] - X^v(i)]^2 \tag{3}$$

  for each segment v, $v = N_s + 1, \ldots, 2N_s$.

- **Step 4:** By averaging all segments, the qth order fluctuation function can be obtained for the different behaviors of trends in the time series x_t, as follows:

$$F_q^+(s) = \left(\frac{1}{M^+} \sum_{v=1}^{2N_s} \frac{sign(b_{X^v}) + 1}{2} [F(v,s)]^{q/2} \right)^{1/q} \tag{4}$$

$$F_q^-(s) = \left(\frac{1}{M^-} \sum_{v=1}^{2N_s} \frac{-[sign(b_{X^v}) - 1]}{2} [F(v,s)]^{q/2} \right)^{1/q} \tag{5}$$

when $q \neq 0$ and

$$F_0^+(s) = exp\left(\frac{1}{2M^+} \sum_{v=1}^{2N_s} \frac{sign(b_{X^v}) + 1}{2} [F(v,s)] \right) \tag{6}$$

$$F_0^-(s) = exp\left(\frac{1}{2M^-} \sum_{v=1}^{2N_s} \frac{-[sign(b_{X^v}) - 1]}{2} [F(v,s)] \right) \tag{7}$$

for $q = 0$. Here, $M^+ = \sum_{v=1}^{2N_s} \frac{sign(b_{X^v})+1}{2}$ and $M^- = \sum_{v=1}^{2N_s} \frac{-[sign(b_{X^v})-1]}{2}$ are the number of sub-time series with positive and negative trends. We assume that $b_{X^v} \neq 0$ for all $v = 1, ..., 2N_s$, such that $M^+ + M^- = 2N_s$.
The traditional MF-DFA is implemented by computing the average fluctuation function for $q \neq 0$ as

$$F_q(s) = \left(\frac{1}{2N_s} \sum_{v=1}^{2N_s} [F(v,s)]^{q/2} \right)^{1/q} \tag{8}$$

and for $q = 0$ as

$$F_q(s) = exp\left(\frac{1}{4N_s} \sum_{v=1}^{2N_s} ln[F(v,s)] \right). \tag{9}$$

- **Step 5:** The scaling behavior of the fluctuations is analyzed by observing the log–log plots of $F_q(s)$ versus s for each value of q. In the case where the two series are long-range cross-correlated, $F_q(s)$ will increase for large values of s as a power law.

$$F_q(s) \sim s^{H_x(q)} \tag{10}$$

$$F_q^+(s) \sim s^{H_x^+(q)} \tag{11}$$

$$F_q^-(s) \sim s^{H_x^-(q)} \tag{12}$$

The scaling exponent $H_x(q)$ is the slope of the log–log plots of $F_q(s)$ versus s using the ordinary least squares (OLS) method. In the case of $q = 2$, the scaling exponent $H_x(q)$ has similar properties and interpretations of the Hurst exponent. If $H_x(2) > 0.5$, then the series is persistent, meaning that a positive (negative) change in one price is more statistically likely to be followed by a positive (negative) value of the other price. In the case where $H_x(2) < 0.5$, the series is antipersistent, which means that a positive (negative) change in one price is more statistically likely to be followed by a negative (positive) change in the other price. For $H_x(2) = 0.5$, we have only short-range auto-correlations or no auto-correlations, indicating the presence of random walks.

To measure the multifractal degree, for every return series we calculate the ΔH_x according to Equation (13).

$$\Delta H_x = H_x^+(q_{min}) - H_x^-(q_{max}) \tag{13}$$

To measure the multifractal asymmetric degree, we can calculate the $DH_x(q)$ for every q according to Equation (14). In particular, it is important to analyze the multifractal asymmetric degree of the Hurst exponent, $DH_x(2)$:

$$DH_x(q) = H_x^+(q) - H_x^-(q). \tag{14}$$

Here, the greater the absolute value, the greater the asymmetric behavior. If $DH_x(q)$ is equal to or close to zero, then the auto-correlations are symmetric for different time series x_t trends. If the value of $DH_x(q)$ is positive, then the auto-correlation exponent is higher when the time series x_t has a positive trend than when it is negative. If it is negative, then the auto-correlation exponent is lower when the time series x_t has a positive trend than when it is negative.

If the scaling exponent $H_x(q)$ value depends on the value of q, then the auto-correlation is multifractal. For $q > 0$, the scaling behavior of large fluctuations is described by $H_x(q)$, $H_x^+(q)$, and $H_x^-(q)$, while for $q < 0$ these describe the scaling behavior of small fluctuations.

3. Results

Thirty worldwide currencies were selected for this study. The currencies are classified into six major currencies, seven European currencies, nine Asian currencies, two African currencies, and six American currencies. Table 1 presents the list of the currencies that we studied.

Table 1. List of currencies.

Major Currencies	Polish Zloty (PLN)	Thai Baht (THB)
Euro (EUR)	Hungarian Forint (HUF)	**African currencies**
British Pound Sterling (GBP)	Russian Ruble (RUB)	South African Rand (ZAR)
Japanese Yen (JPY)	**Asian currencies**	Moroccan Dirham (MAD)
Swiss Franc (CHF)	Korean Won (KRW)	**American currencies**
Australian Dollar (AUD)	Israeli New Shekel (ILS)	Mexican Peso (MXN)
Canadian Dollar (CAD)	Hong Kong Dollar (HKD)	Brazilian Real (BRL)
European currencies	Singapore Dollar (SGD)	Chilean Peso (CLP)
Swedish Krona (SEK)	New Taiwan Dollar (TWD)	Colombian Peso (COP)
Danish Krone (DKK)	Indonesian Rupiah (INR)	Peruvian Sol (PEN)
Norwegian Krone (NOK)	Indian Rupee (INR)	Argentine Peso (ARS)
Czech Koruna (CZK)	Malaysian Ringgit (MY)	

All the values of these currencies are expressed in relation to the US dollar. The analysis period established for the study was from 1 December 2013–29 December 2023, a total of 5240 days. All values were extracted from www.yahoo.com/currencies. Each currency's return or variation was calculated according to Equation (15). For each currency return time series, the days with abnormal returns ($abs(r_t)$ >10%) were eliminated. With the restriction of abnormal returns, 77 observations were eliminated out of 154,314 observations, or 0.05%. The highest percentage of one of the time series eliminated as abnormal was 0.31% in the case of RUB (16 observations out of 5110 total observations). Then, each return currency time series was split into five return time series corresponding to each day of the week. Thus, we obtained five return time series for each currency, with one for each day.

$$r_t = \ln(P_t / P_{t-1}) \tag{15}$$

When analyzing the descriptive statistics of each currency's daily return series (Table 2), it can be observed that of the major currencies and European currencies, all except JPY had their highest daily average on Mondays. For European currencies, the lowest return was on Wednesday or Friday, except for DKK. The most volatile currencies were AUD in the case of major currencies and RUB in the case of European currencies.

Table 2. Descriptive statistics for currency price returns by day.

	Return Average					Standard Deviation				
	Mon.	**Tue.**	**Wed.**	**Thu.**	**Fri.**	**Mon.**	**Tue.**	**Wed.**	**Thu.**	**Fri.**
EUR	0.04%	−0.01%	−0.01%	−0.01%	−0.01%	0.66%	0.62%	0.65%	0.66%	0.68%
GBP	0.06%	−0.02%	0.01%	−0.03%	−0.03%	0.67%	0.55%	0.57%	0.59%	0.61%
JPY	0.00%	0.00%	0.01%	0.01%	0.01%	0.65%	0.61%	0.69%	0.63%	0.66%
CHF	0.02%	−0.01%	−0.01%	−0.02%	−0.02%	0.62%	0.55%	0.66%	0.59%	0.71%
AUD	0.04%	−0.03%	0.00%	−0.01%	−0.01%	0.85%	0.73%	0.77%	0.80%	0.79%
CAD	0.03%	−0.01%	0.01%	−0.01%	−0.01%	0.58%	0.50%	0.56%	0.58%	0.56%
SEK	0.06%	−0.01%	−0.03%	−0.02%	−0.02%	0.78%	0.75%	0.75%	0.76%	0.76%
DKK	0.04%	−0.02%	−0.01%	−0.01%	−0.01%	0.59%	0.55%	0.54%	0.58%	0.62%
NOK	0.03%	0.01%	−0.01%	−0.03%	−0.03%	0.86%	0.83%	0.81%	0.86%	0.78%
CZK	0.02%	0.02%	−0.02%	−0.02%	−0.02%	0.72%	0.78%	0.69%	0.77%	0.78%
PLN	0.04%	0.00%	−0.03%	0.02%	0.02%	0.91%	0.91%	0.83%	0.90%	0.84%
HUF	0.04%	0.03%	−0.03%	0.01%	0.01%	0.92%	0.91%	0.87%	0.96%	0.87%
RUB	0.11%	0.04%	−0.01%	−0.05%	−0.05%	1.10%	1.14%	0.96%	1.17%	1.08%
KRW	−0.12%	0.15%	−0.02%	−0.01%	−0.01%	0.94%	0.97%	0.83%	0.85%	0.77%
ILS	−0.01%	0.01%	−0.01%	−0.02%	−0.02%	0.54%	0.53%	0.55%	0.48%	0.50%
HKD	0.01%	0.00%	0.00%	0.00%	0.00%	0.10%	0.03%	0.04%	0.03%	0.10%
SGD	0.01%	0.00%	−0.02%	−0.01%	−0.01%	0.35%	0.30%	0.31%	0.36%	0.33%
TWD	−0.09%	0.14%	−0.02%	−0.02%	−0.02%	0.86%	0.78%	0.60%	0.61%	0.67%
IDR	−0.09%	0.14%	−0.01%	0.01%	0.01%	0.88%	0.84%	0.68%	0.66%	0.68%
INR	−0.03%	0.10%	0.00%	−0.01%	−0.01%	0.51%	0.49%	0.50%	0.46%	0.48%
MYR	0.02%	0.04%	0.00%	−0.03%	−0.03%	0.60%	0.68%	0.56%	0.53%	0.55%
THB	−0.21%	0.22%	−0.01%	−0.01%	−0.01%	0.83%	0.78%	0.51%	0.50%	0.52%
ZAR	0.07%	0.02%	−0.04%	0.00%	0.00%	1.09%	1.03%	1.02%	1.12%	1.01%
MAD	0.17%	0.31%	0.05%	−0.08%	−0.08%	1.76%	1.42%	1.10%	1.08%	1.37%
MXN	0.05%	0.03%	0.01%	0.00%	0.00%	0.80%	0.72%	0.81%	0.81%	0.73%
BRL	−0.15%	0.24%	−0.02%	−0.02%	−0.02%	1.25%	1.17%	1.05%	1.05%	1.09%
CLP	−0.06%	0.22%	0.00%	−0.02%	−0.02%	1.21%	1.25%	1.06%	1.07%	1.05%
COP	−0.07%	0.18%	−0.01%	−0.08%	−0.08%	1.20%	1.26%	1.15%	1.18%	1.16%
PEN	−0.46%	0.47%	0.06%	−0.03%	−0.03%	1.32%	1.37%	1.10%	1.13%	1.18%
ARS	−0.07%	0.21%	0.04%	0.09%	0.09%	0.78%	0.86%	0.72%	0.74%	0.72%

In the case of Asian currencies, the highest return occurred on Tuesdays, except for HKD and SGD, for which they were on Mondays. On the other hand, the majority of currencies had the lowest profitability on Mondays. The low volatility of HKD is striking, while KRW has the highest volatility. The two African currencies had high returns on Mondays and Tuesdays, while the lowest returns were on Wednesdays in the case of ZAR and Thursdays and Fridays for MAD. Among the American currencies the highest returns were on Tuesdays, except for MXN, for which they were on Mondays. At the same time, all currencies except for MXN and COP had their lowest returns on Mondays. The currencies with the lowest volatility in the analyzed period were MXN and ARS.

One classic way of addressing the day-of-week effect is through the GARCH econometric model [43–48]. Traditionally, GARCH (p,q) models are applied according to Equations (16)–(18) to determine the day-of-week effect.

$$r_t = \sum_{i=1}^{5} a_i r_{t-i} + \sum_{j=1}^{5} b_j D_j + \epsilon_t \tag{16}$$

$$\epsilon_t = \mathcal{N}\left(0, \sigma_t^2\right) \tag{17}$$

$$\sigma_t^2 = \alpha_0 + \sum_{k=1}^{p} \alpha_k \sigma_{t-k}^2 + \sum_{l=1}^{q} \gamma_l \epsilon_{t-l}^2 \tag{18}$$

Here, D_1, D_2, D_3, D_4, and D_5 are dummy variables with values of 1 for Monday, Tuesday, Wednesday, Thursday, and Friday, respectively. In this econometric model, a day-of-the-week effect exists for day j when b_j is statistically significant. To determine the day-of-the-week effect using a GARCH model, all the combinations (p, q) for p and q equal to 1, 2, 3, 4, and 5 were adjusted for each currency, thereby maximizing the likelihood. From among the 35 different configurations available for each currency, the best model was selected based on the minimum Akaike Criterion Information (AIC).

Table 3 shows the dummy variables associated with each day's returns for the best model of each major currency. In general, there is no weekday effect, with the exceptions of a Tuesday effect for EUR (negative), a Monday effect for GBR (positive), and a Wednesday effect for JPY (positive). For the European currencies, there is only a weekday effect for SEK on Mondays (positive), DKK on Tuesdays (negative), and RUB on Tuesdays, Thursdays, and Fridays, with negative effects and Mondays showing a positive effect. Among the Asian currencies, TWD has day-of-the-week effects on all five trading days, while HKD, INR, and MYR have day-of-the-week effects on four of five trading days. KRW, IDR, and THB show a day-of-the-week effect on Mondays (negative) and Tuesdays (positive). ILS and SGD only show a very light day-of-the-week effect. Regarding the African currencies, only MAD shows a day-of-the-week effect on four days. Very homogeneous behavior can be observed in the case of the American currencies. All American currencies except ARS have a negative weekday effect on Fridays, while all except MXN have a positive weekday effect on Tuesdays. On Mondays there is a negative weekday effect for all currencies except for MXN, which has a positive effect, and ARS, with no effect. Finally, BRL, CLP, PEN, and ARS all have a positive weekday effect on Wednesdays.

Table 3. Day-of-the-week effect results obtained by the GARCH model.

	$\mathbf{b}_1$		$\mathbf{b}_2$		$\mathbf{b}_3$		$\mathbf{b}_4$		$\mathbf{b}_5$	
EUR	0.0002	−0.0004	*	−0.0001	0.0000	0.0001				
GBP	0.0005	**	−0.0001	−0.0001	−0.0003	0.0000				
JPY	0.0001	0.0001	0.0004	*	0.0001	0.0002				
CHF	−0.0001	−0.0001	0.0000	−0.0001	0.0000					
AUD	0.0003	−0.0002	−0.0001	0.0000	0.0001					
CAD	0.0002	−0.0002	0.0002	0.0000	−0.0002					
SEK	0.0007	**	−0.0004	−0.0003	0.0000	0.0002				
DKK	0.0002	−0.0003	*	0.0000	0.0000	0.0001				
NOK	0.0002	−0.0002	−0.0001	−0.0001	0.0002					
CZK	0.0002	−0.0001	−0.0002	0.0000	−0.0001					
PLN	0.0002	−0.0003	−0.0003	0.0003	0.0000					
HUF	0.0004	0.0001	−0.0002	0.0002	−0.0003					
RUB	0.0002	**	−0.0003	**	−0.0001	−0.0002	**	−0.0004	**	
KRW	−0.0005	**	0.0004	*	0.0000	−0.0003	0.0000			
ILS	−0.0002	*	0.0001	0.0001	−0.0002	−0.0001				
HKD	0.0000	**	0.0000	**	0.0000	**	0.0000	**	0.0000	
SGD	0.0001	0.0000	−0.0002	*	−0.0002	*	−0.0001			
TWD	0.0003	**	−0.0004	**	−0.0006	**	−0.0003	**	−0.0007	**
IDR	−0.0006	**	0.0008	**	0.0001	−0.0002	*	0.0000		
INR	0.0001	0.0006	**	0.0002	**	−0.0005	**	−0.0002	**	
MYR	0.0000	0.0001	**	−0.0003	**	0.0004	**	−0.0005	**	
THB	−0.0013	**	0.0003	**	0.0000	0.0000	−0.0001			
ZAR	0.0005	0.0004	−0.0004	−0.0001	0.0001					
MAD	−0.0006	*	0.0015	**	0.0007	**	−0.0002	−0.0028	**	
MXN	0.0004	*	0.0002	−0.0001	−0.0002	−0.0009	**			
BRL	−0.0009	**	0.0017	**	−0.0002	−0.0006	**	0.0001		
CLP	−0.0007	**	0.0017	**	0.0007	**	−0.0003	−0.0005	**	
COP	−0.0024	**	0.0004	*	0.0002	−0.0004	*	−0.0015	**	
PEN	−0.0067	**	0.0016	**	0.0024	**	0.0008	**	−0.0008	**
ARS	−0.0001	0.0016	**	0.0007	**	0.0008	**	0.0000		

Note: ** and * represent statistical significance at 1% and 5%, respectively.

To analyze the presence of autocorrelation, we applied the cross-correlation test model proposed by Podobnik et al. [49], with the difference that we applied it to determine the presence of auto-correlation. In this approach, the Q_{cc} statistic is defined according to Equations (19) and (20). The Q_{cc} statistic is distributed according to χ^2 with m degrees of freedom:

$$Q_{cc}(m) = N^2 \sum_{i=1}^{m} \frac{X_i^2}{N-i} \tag{19}$$

where

$$X_i = \frac{\sum_{k=i+1}^{N} r_t r_{t-i}}{\sum_{t=1}^{N} r_t^2}. \tag{20}$$

In Appendix A, Figures A7–A9 present the Q_{cc} for the series of returns for each day and for each of the currencies, making for a range of m [0, 100]. All series of returns for each day for all currencies reject the null hypothesis, indicating the existence of auto-correlation. The sole exception is JPY on Mondays for $m > 50$; also interesting is the disproportion of Mondays and Tuesdays for THB and PEN and of Mondays, Tuesdays, and Fridays for MAD. In light of this evidence around Q_{cc}, we have conditions for the possible existence of fractality.

As a second analysis, we used the test from [50,51] for whether the local regularity of a signal varies in time to determine whether there is multifractality in the series of returns for each day and each currency. The multifractal spectrum can effectively show the signal's distribution of scaling exponents [52]. If the time series exhibits the same regularity everywhere in time, then its multifractal spectrum will be narrow. On the contrary, time series with changing regularities over time imply multifractal behavior, which is reflected in a broad multifractal spectrum. The Hölder exponent, calculated using wavelet leaders, is used to determine the multifractal spectrum (see [53]). Figures A4–A6 present the multifractal spectrum as a function of the Hölder exponent. All series of returns for each day and each currency show a sufficient amplitude of the multifractal spectrum, which indicates that multifractality exists in the series. This finding does not imply a day-of-the-week effect, as the multifractal behavior could be the same for all days for a given currency. Nonetheless, it provides a basis for continuing with the multifractal analysis and determining whether there is different multifractal behavior on different days for any of the analyzed currencies.

To determine the degree of multifractality, we analyzed ΔH_x (Table 4). In the case of the major currencies, all days had a higher positive degree of multifractality. The only exceptions where multifractality did not exist were Tuesdays for EUR and Wednesdays for CAD. In the case of the European currencies, there was also a positive degree of multifractality, except on Wednesdays for CZK, for which there was no multifractality, and for the case of HUF. In the case of HUF, there was no multifractality on Tuesdays or Fridays, and on Wednesdays the degree of multifractality was negative ($H(5)$ was greater than $H(-5)$). Among the Asian currencies, the multifractality degree was positive. The only exceptions were KRW (Tuesdays), ILS (Mondays and Wednesdays), and TWD (Tuesdays). For the African currencies, there was a degree of positive multifractality on all days, with the only exception being ZAR on Wednesdays. Finally, for the American currencies, a large majority showed positive multifractality, with a few exceptions on certain days: MXN (Mondays and Wednesdays), BRL (Mondays, Tuesdays, and Wednesdays), and CLP (Thursdays).

Table 4. Multifractality degree of currency price returns by day.

	Monday		Tuesday		Wednesday		Thrusday		Friday	
EUR	0.317	**	−0.016	0.512	**	0.322	**	0.211	**	
GBP	0.336	**	0.175	**	0.292	**	0.149	**	0.296	**
JPY	0.188	**	0.205	**	0.237	**	0.123	**	0.228	**
CHF	0.223	**	0.160	**	0.440	**	0.109	**	0.221	**
AUD	0.360	**	0.274	**	0.018	0.197	**	0.525	**	
CAD	0.163	**	0.282	**	0.114	**	0.125	**	0.303	**
SEK	0.227	**	0.056	*	0.224	**	0.246	**	0.064	*
DKK	0.308	**	0.237	**	0.311	**	0.133	**	0.180	**
NOK	0.489	**	0.121	**	0.159	**	0.128	**	0.193	**
CZK	0.395	**	0.129	**	0.016	0.236	**	0.078	*	
PLN	0.061	*	0.108	**	0.185	**	0.326	**	0.194	**
HUF	0.313	**	0.014	−0.071	*	0.331	**	−0.030		
RUB	0.433	**	0.430	**	0.127	*	0.355	**	0.521	**
KRW	0.225	**	−0.022	0.342	**	0.293	**	0.429	**	
ILS	0.056	0.310	**	0.049	0.202	**	0.372	**		
HKD	0.714	**	0.593	**	0.692	**	0.245	**	0.452	**
SGD	0.309	**	0.276	**	0.147	**	0.246	**	0.332	**
TWD	0.264	**	−0.002	0.538	**	0.234	**	0.373	**	
IDR	0.406	**	0.165	**	0.725	**	0.454	**	0.812	**
INR	0.499	**	0.264	**	0.225	**	0.114	**	0.212	**
MYR	0.309	**	0.839	**	0.557	**	0.560	**	0.471	**
THB	0.217	**	0.119	**	0.158	**	0.096	**	0.497	**
ZAR	0.187	**	0.131	**	−0.005	0.195	**	0.152	**	
MAD	0.161	**	0.210	**	0.532	**	0.614	**	0.179	**
MXN	−0.043	0.220	**	−0.050	0.433	**	0.203	**		
BRL	0.047	0.016	0.038	0.295	**	0.193	**			
CLP	0.212	**	0.192	**	0.084	*	0.027	0.313	**	
COP	0.804	**	0.533	**	0.597	**	0.473	**	0.524	**
PEN	0.398	**	0.232	**	0.603	**	0.314	**	0.476	**
ARS	0.535	**	0.248	**	0.624	**	0.760	**	0.215	**

Note: ** and * represent statistical significance at 1% and 5%, respectively.

Next, the Hurst exponent was analyzed along with its confidence interval to determine the day-of-week effect for each of the studied currencies. Here, $H_x(q)$ is obtained from Equation (10), and the confidence intervals are obtained from these regressions. To determine the confidence interval, 95% probabilities were used. This analysis allows us to distinguish whether the behaviors of the daily returns were persistent, anti-persistent, or random walks. In the case of the major currencies (Figure 1), for EUR, only Thursday returns showed random walk behavior and only Tuesday returns showed persistence, while the returns from the other days showed antipersistence. For GBP, a single daily return (Wednesdays) showed random walk behavior, and only one day (Tuesdays) showed persistence; the returns from the other days (Mondays, Thursdays, and Fridays) showed antipersistence. For JPY, returns on Thursdays showed random walk behavior, while the returns on Mondays and Tuesdays showed persistence, with Mondays being more persistent than Tuesdays. The returns on Wednesdays and Fridays showed antipersistence. CHF was the only major currency to show random walk behavior for three daily returns (Mondays, Tuesdays, and Fridays). In the case of AUD, only the returns on Tuesdays showed random walk behavior. In contrast, the returns on Mondays and Wednesdays showed persistence, with the returns on Wednesdays being more persistent than those on Mondays. The returns on Thursdays and Fridays showed antipersistence, with Fridays being the most antipersistent. Only the CAD returns on Wednesdays showed a random walk behavior; the returns on Mondays and Thursdays showed persistence, while those on Tuesdays and Fridays showed antipersistence.

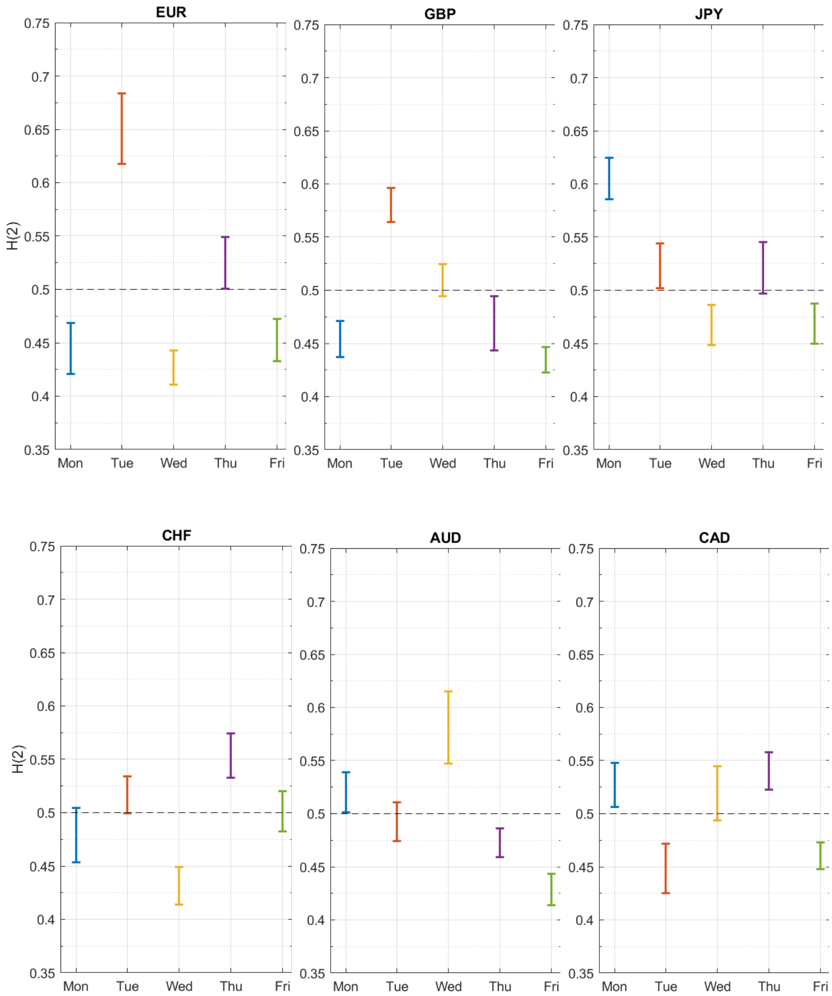

Figure 1. Hurst exponent ($H_x(2)$) and 95% confidence interval on each day for major currencies.

For the European currencies (Figure 2), SEK returns on Tuesdays and Wednesdays showed random walk behavior, while those on Thursdays and Fridays showed persistent behavior and those on Mondays showed antipersistent behavior. DKK did not have any days with random walk behavior; on Mondays, Wednesdays, and Fridays the returns showed antipersistence, while on Tuesdays and Thursdays the behavior was persistent. For NOK, returns on Tuesdays and Fridays showed random walk behavior during the study period, returns on Wednesdays and Thursdays showed persistence, and those on Mondays showed antipersistence. For CZK, only the returns on Thursdays showed random walk behavior, while the returns on Tuesdays, Wednesdays, and Fridays showed persistence, and those on Monday showed antipersistence. For PLN, Monday and Wednesday returns showed random walk behavior, Tuesdays and Fridays showed persistence, and Thursdays showed antipersistence. For HUF, the returns on Fridays showed random walk behavior, while those on Tuesdays and Wednesdays showed persistence and those on Mondays and Thursdays showed antipersistence. Finally, the RUB returns showed random walk behavior only on Tuesdays, showing persistence on Wednesdays, Thursdays, and Fridays and antipersistence only on Mondays.

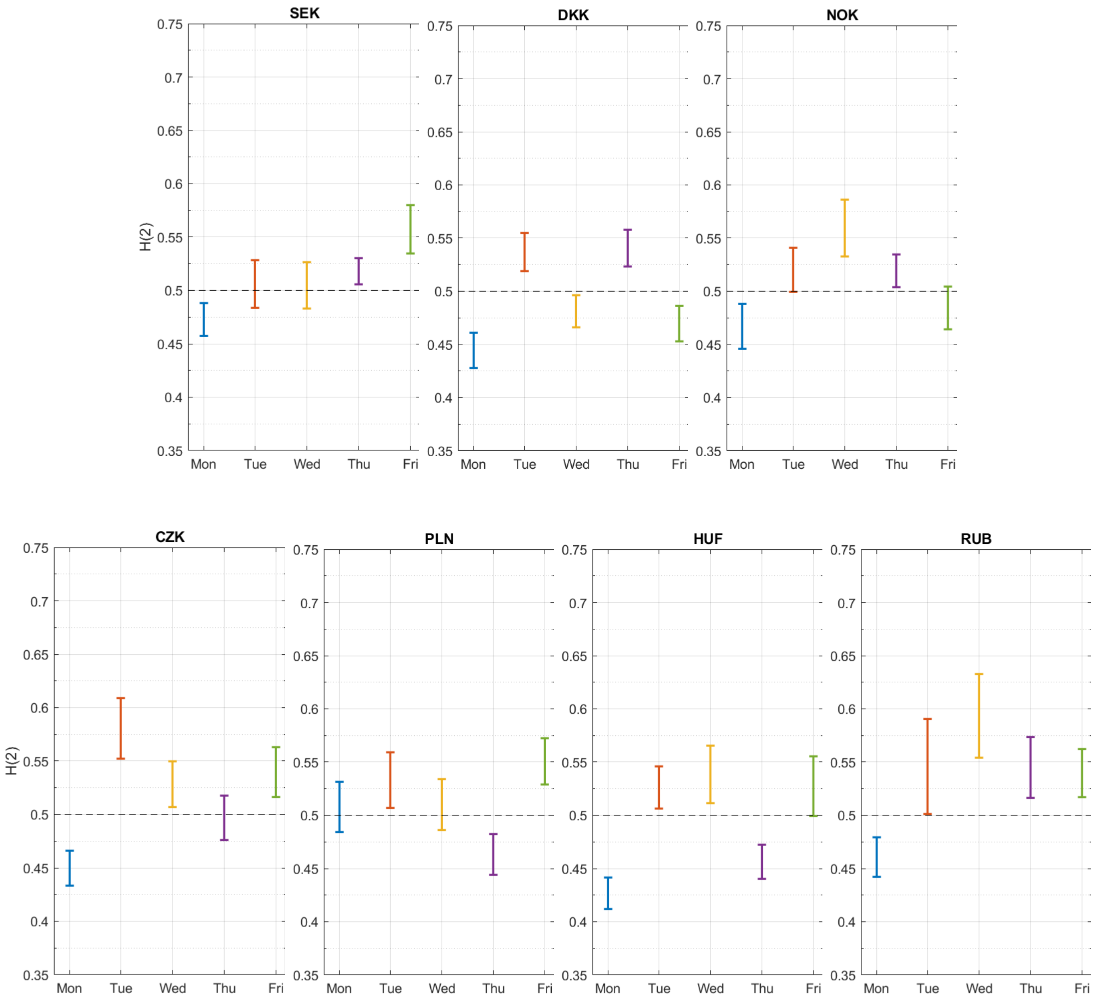

Figure 2. Hurst exponent ($H_x(2)$) and 95% confidence interval on each day for European currencies.

Regarding the Asian currencies (Figure 3), KRW had three daily returns with persistence (Tuesdays, Wednesdays, and Thursdays), with Tuesday being the most persistent, while antipersistent behavior was apparent on Mondays and Fridays. ILS returns showed random walk behavior on Wednesdays and Thursdays, with antipersistent behavior on Tuesdays and Fridays and persistent behavior only on Mondays. In the case of HKD, the returns on Mondays, Thursdays, and Fridays showed persistent behavior, while those on Tuesdays and Wednesdays showed antipersistence. For SGD, the returns on Wednesdays and Thursdays showed random walk behavior, while those on Tuesdays were antipersistent and those on Thursdays were persistent. The TWD returns showed two days of high persistence (Mondays and Tuesdays), while on Fridays they showed random walk behavior and on Wednesdays and Thursdays they showed antipersistent behavior. IDR and INR had no random walk days at all, and showed persistent behavior on four days. For the MYR returns, random walk behavior was present on Wednesdays and Fridays, while Mondays and Thursdays showed persistent behavior and Tuesdays showed antipersistent behavior. For THB, the returns only showed random walk behavior on Wednesdays, with the returns on Mondays, Tuesdays, and Thursdays showing persistence and those on Fridays showing antipersistence. Of the two African currencies analyzed in this study (Figure 3, bottom right), only the returns of ZAR on Mondays, Tuesdays, and Wednesdays showed random walk behavior. MAD had three days with persistent returns (Mondays, Tuesdays, and Fridays), while ZAR only showed this behavior on Fridays. ZAR and MAD returns both had antipersistent behavior on Wednesdays and Thursdays.

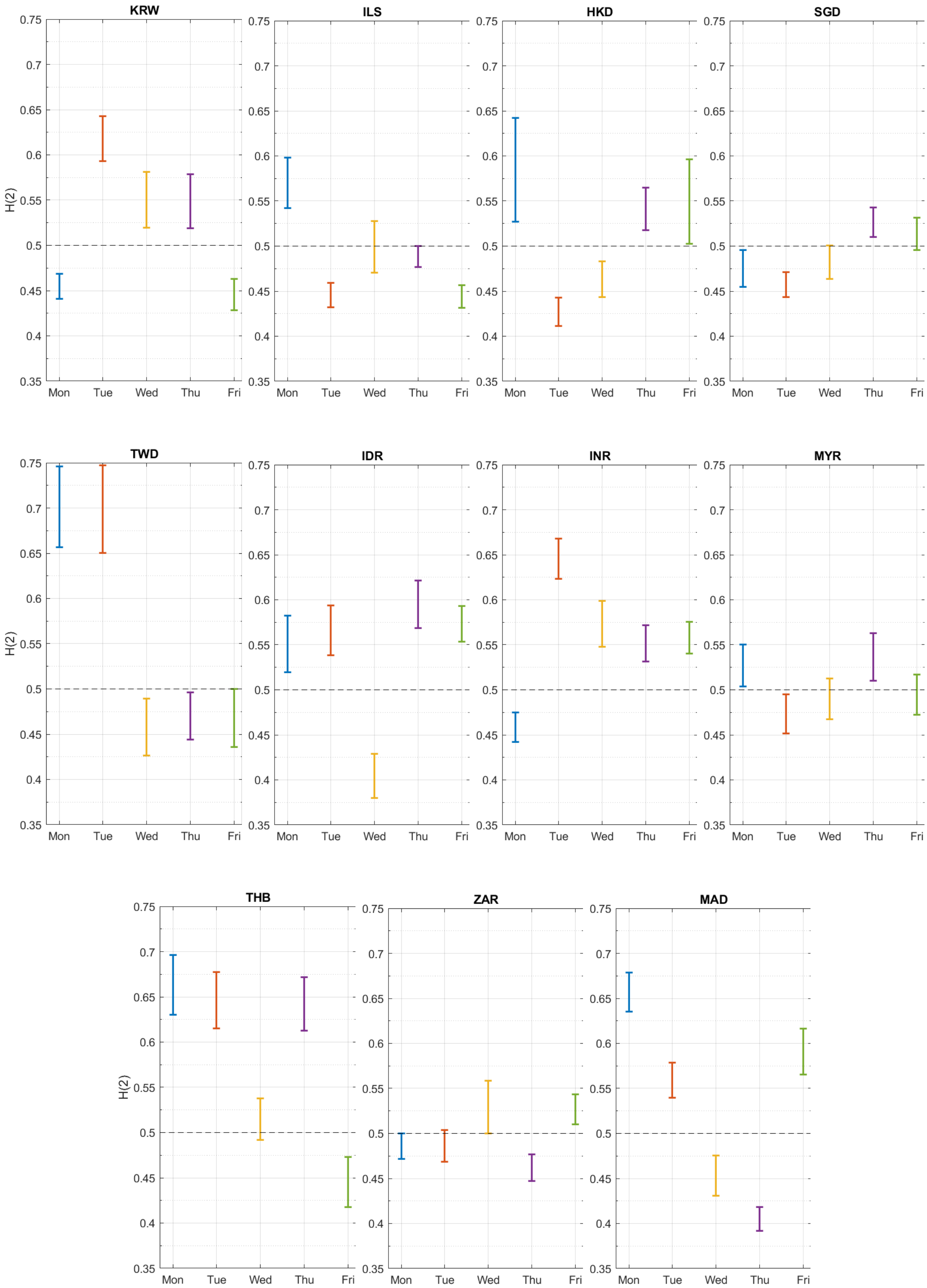

Figure 3. Hurst exponent ($H_x(2)$) and 95% confidence interval on each day for Asian and African currencies.

Among the American currencies (Figure 4), MXN daily returns showed persistent behavior on Mondays, Tuesdays, Wednesdays, and Fridays and antipersistent behavior on Thursdays. For BRL, the returns on Mondays, Tuesdays, Wednesdays, and Thursdays showed persistent behavior, with Tuesdays and Wednesdays being the most persistent. In contrast, the returns on Fridays behaved as a random walk. The behavior is similar in the case of MXN, with four days of returns showing persistence (Mondays, Tuesdays, Wednesdays, and Fridays) and Thursdays showing antipersistence. COP and PEN also had four persistent daily returns. In the case of PEN, persistence was present on Mondays, Tuesdays, Thursdays, and Fridays, while the returns for Wednesdays showed random walk behavior. In the case of COP, the days with persistent behavior were Mondays, Wednesdays, Thursdays, and Fridays, with Mondays and Wednesdays having higher persistence, while the returns for Tuesdays showed random walk behavior. The case of CLP was more extreme than that of other currencies; all the daily returns showed persistence, with the returns on Tuesdays and Mondays having the highest persistence. Finally, in the case of ARS returns, four days showed persistence (Tuesdays, Wednesdays, Thursdays, and Fridays), while the returns on Mondays showed antipersistence.

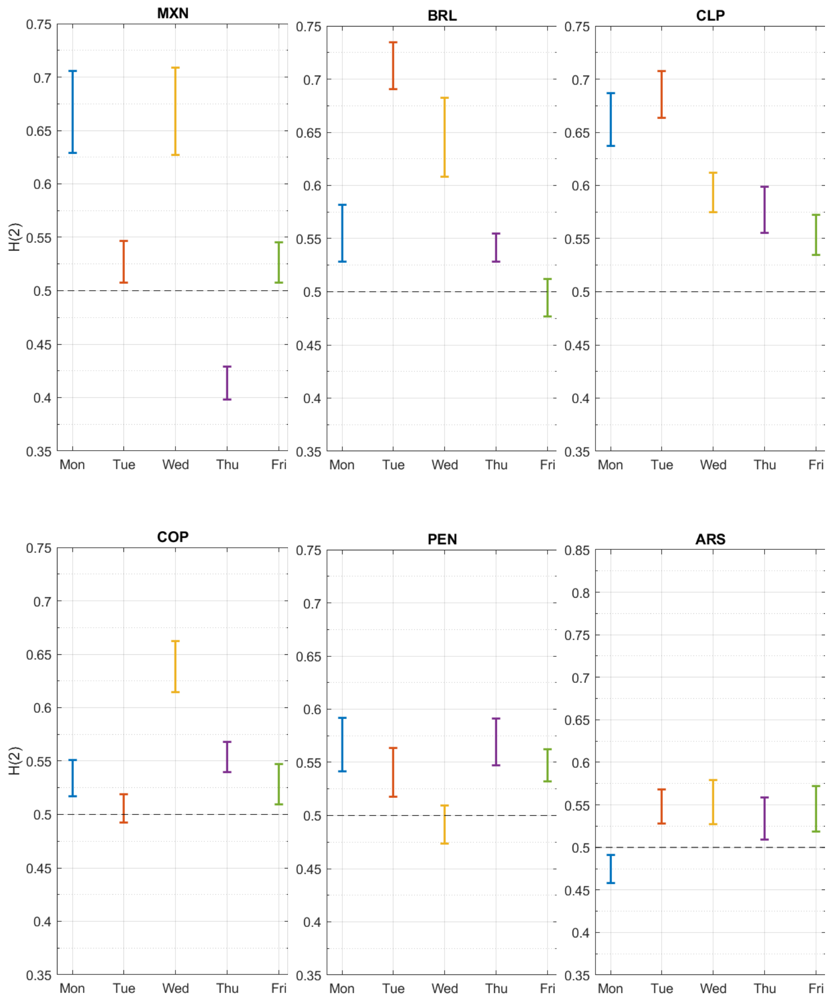

Figure 4. Hurst exponent ($H_x(2)$) and 95% confidence interval on each day for American currencies.

By separating the analysis of the generalized exponent according to the trends of the different currencies with respect to the US Dollar, it is possible to observe asymmetry among the major currencies (Figure 5). In particular, there is asymmetry on three days of the week for GBP (Wednesdays, Thursdays, Fridays), JPY (Tuesdays, Wednesdays, Fridays), and AUD (Mondays, Wednesdays, Fridays), while in the cases of CAD and CHF there is asymmetry on two days. For EUR, only one day (Tuesdays) presents asymmetry, while three days under an upward trend and two days under a downward trend are random walks. Based on the asymmetry analysis, the currency that showed the most days of random walk behavior was CAD, with three days under an upward trend and four days under a downward trend. All major currencies had a day-of-the-week effect under the upward and downward trends.

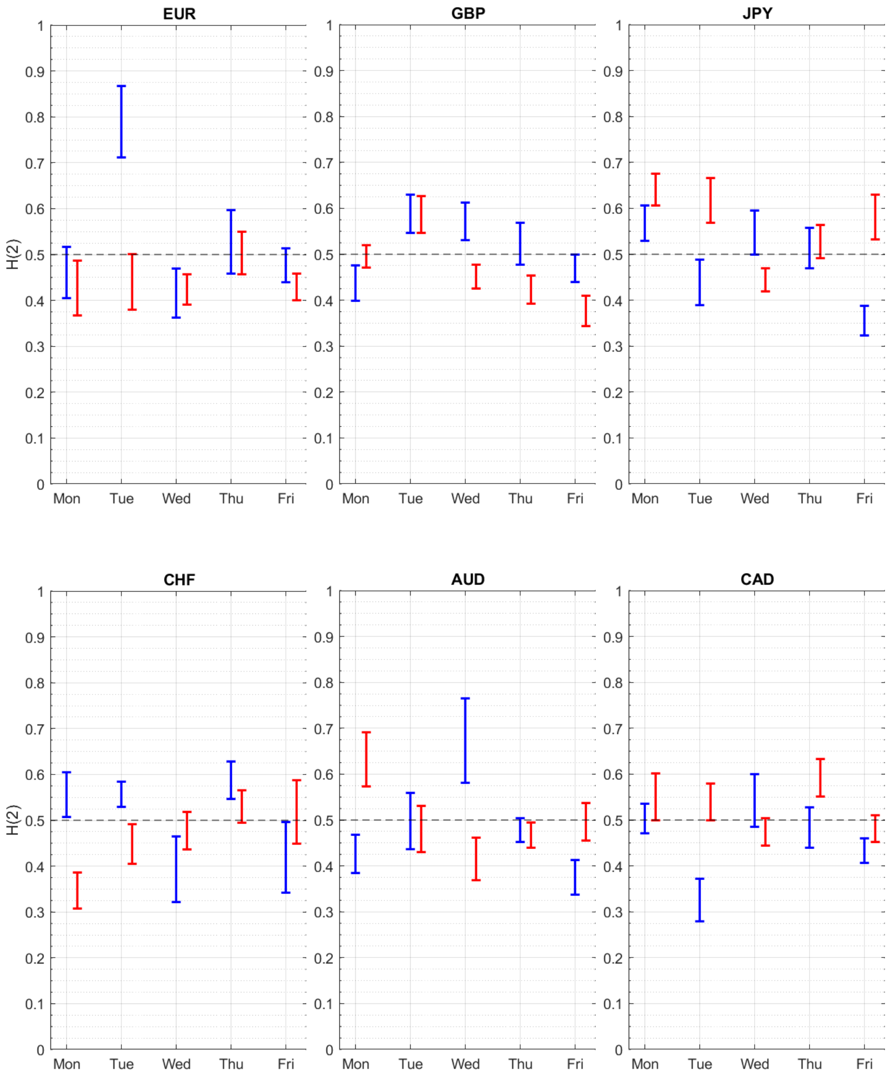

Figure 5. Hurst exponent ($H_x(2)$) and 95% confidence interval on each day for upward and downward trends for major currencies. Note: in blue, the $H_x(2)$ under upward trend; in red, under downward trend.

In the trend analysis, all studied European currencies presented a day-of-the-week effect (Figure 6). DKK and RUB had asymmetry on three days of the week, while SEK, NOK, and PLN had asymmetry on two days. NOK showed random walk behavior on four days under an upward trend and three days under a downward trend, while HUB showed random walk behavior for three days under an upward trend and four days under a downward trend. PLN and RUB had days of persistence, antipersistence, and random walk behavior for both trends. There was no antipersistence under the upward trend for SEK or NOK, and no persistence under the downward trend for SEK, DKK, CZK, or HUB.

The Asian currencies behave similarly to the major and European currencies, presenting a weekday effect under both trends for all currencies (Figure 7). In particular, there is asymmetry on four days of the week in the case of MYR (all days except Tuesdays) and on three days of the week for KRW, HKD, and IDR. For ILS currency, there was no asymmetry. In the cases of KRW and MYR, four days under a downward trend were random walks and only one day was under an upward trend. HKD only had one day that behaved as a random walk, and was in an upward trend on Wednesdays. KRW, TWD, INR, and THB had no under a downtrend with antipersistence.

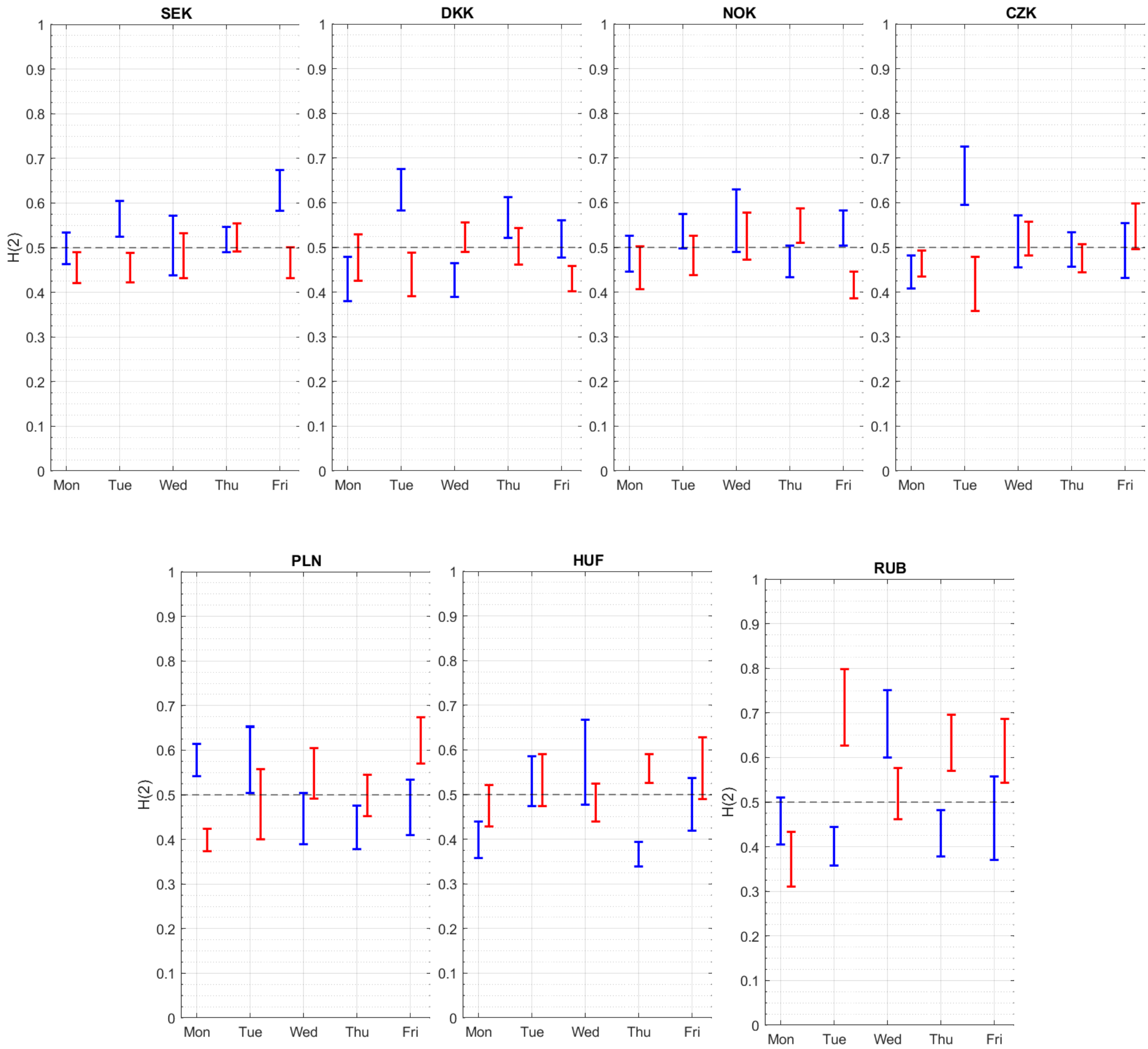

Figure 6. Hurst exponent ($H_x(2)$) and 95% confidence interval on each day for upward and downward trends for European currencies. Note: in blue, the $H_x(2)$ under upward trend; in red, under downward trend.

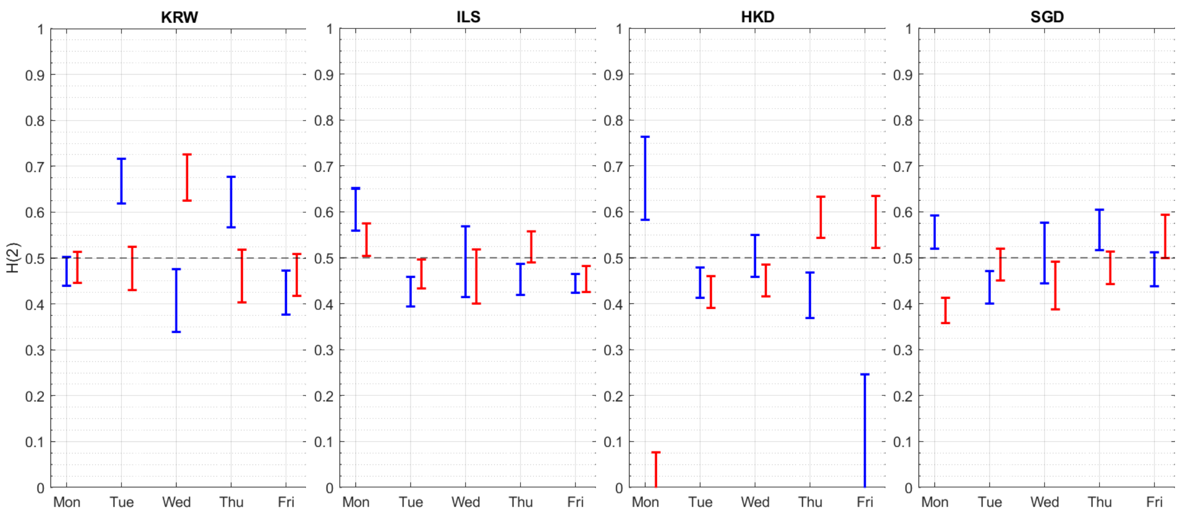

Figure 7. *Cont.*

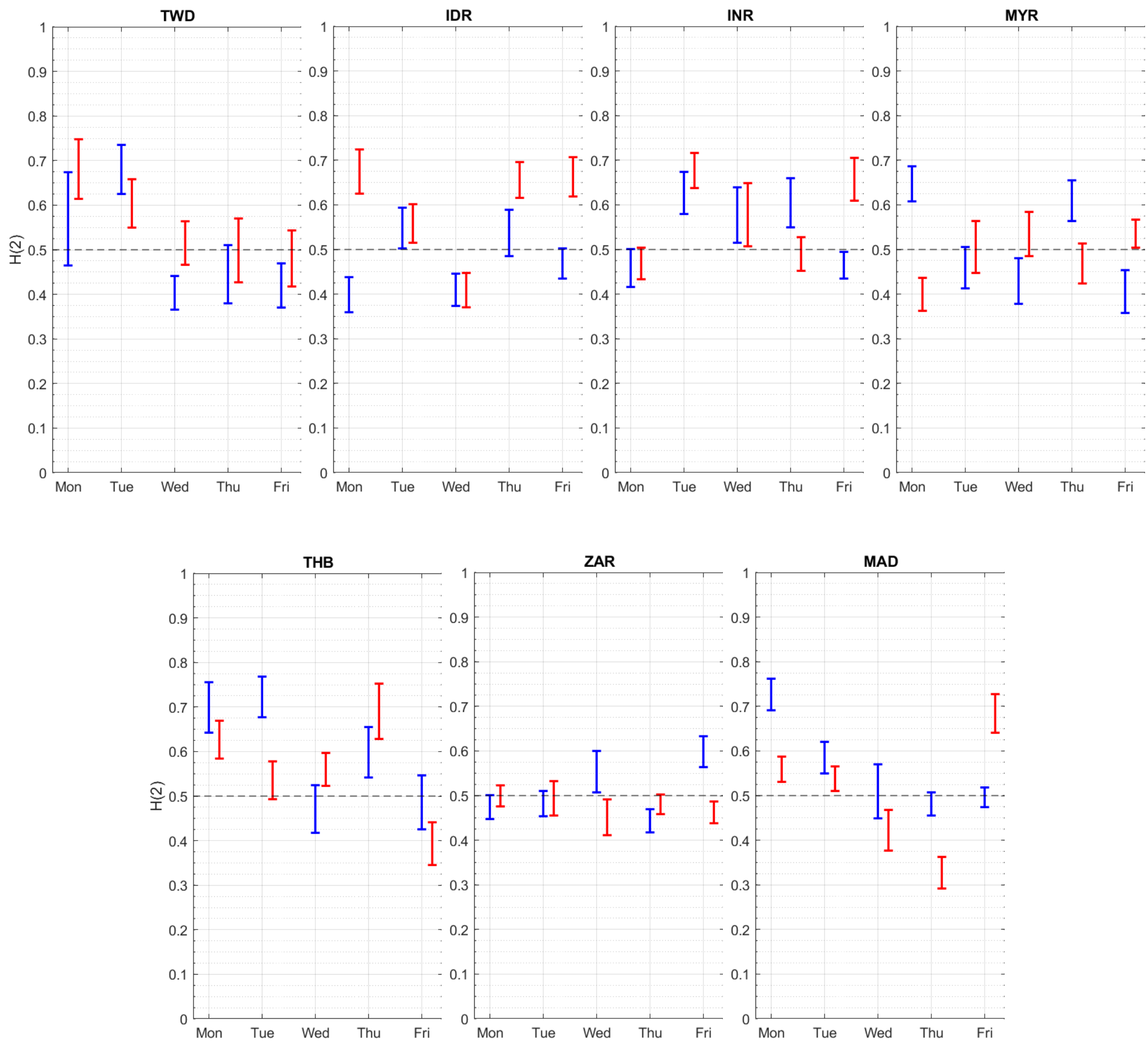

Figure 7. Hurst exponent ($H_x(2)$) and 95% confidence interval on each day for upward and downward trends for Asian and African currencies. Note: in blue, the $H_x(2)$ under upward trend; in red, under downward trend.

The two African currencies did not present any antipersistence under an upward trend (Figure 7). In the case of MAD, there was no random walk behavior under a downward trend, while ZAR presented random walk behavior on two days under an upward trend and three days under a downward trend. MAD presented asymmetry on three days of the week, while ZAR presented asymmetry on two days. Both African currencies had day-of-the-week effects for both trends.

Finally, the American currencies presented weekday effects under both trends for all currencies (Figure 8). In particular, there was asymmetry on three days of the week in the case of BRL (Mondays, Wednesdays, Thursdays) and PEN (Tuesdays, Thursdays, Fridays). There were no days with antipersistence in either of the two trends for CLP and COP. In the case of COP, there were four days with random walk behavior under an upward trend and three days under a downward trend. For BRL and CLP, there was only one day of random walk behavior under an upward trend and one day under a downward trend.

Table 5 presents the results for the multifractality degree under the upward and downward trends. In the case of the major currencies, all currencies for all days had a greater positive degree of multifractality under the downward trend, while under the upward trend the majority again had positive multifractality, except EUR (Tuesdays and Fridays), JPY (Wednesdays), CHF (Tuesdays), AUD (Wednesdays), and CAD (Mondays and Wednesdays). The results were also positive in the case of the European currencies. In almost all cases, multifractality was present under an upward trend (25 out of 35 days) and a downward trend (28 out of 35 days). Under a downward trend, SEK, DKK, and NOK all had a positive multifractality degree, while under an upward trend only RUB did. In the case of the Asian currencies, all had a multifractality degree under a downward trend except for KRW (Tuesdays and Wednesdays) and THB (Thursdays). In the case of an upward trend, the vast majority also had a positive multifractality degree, with the exceptions of KRW (Thursdays), ILS (Mondays and Wednesdays), INR (Thursdays), and THB (Tuesdays and Wednesdays). Under an upward trend, MAD did not present multifractality on Mondays or Tuesdays, while under a downward trend ZAR did not present multifractality on Wednesdays and MAD did not on Fridays. Both African currencies showed a positive multifractality degree in all other cases. Finally, in the case of the American currencies, a large majority had positive multifractality degrees for almost every day under a downward trend. Only in the case of MXN (Wednesdays and Fridays) and PEN (Mondays) was there no multifractality degree under the downward trend, while in the case of BRL (Mondays) it was negative. This negative multifractality degree of BRL was also present on Tuesdays under an upward trend, while ARS had a multifractality degree on all days and under both trends.

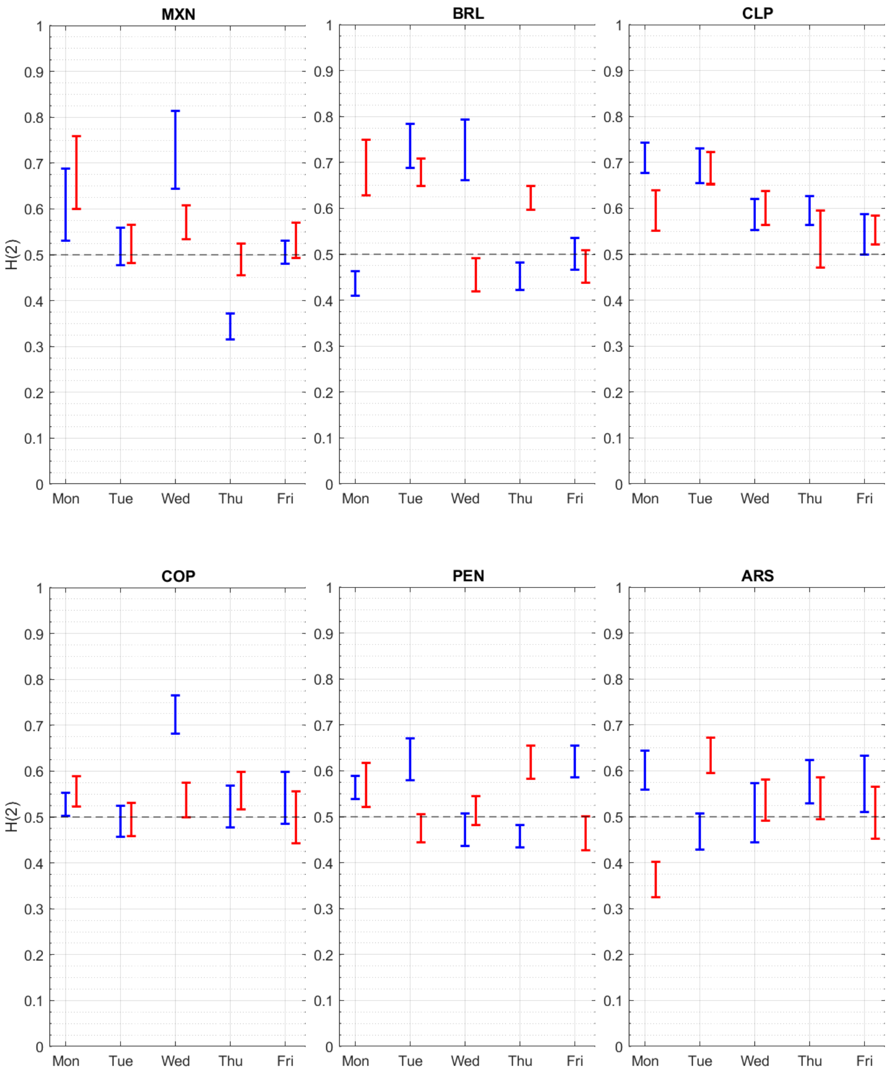

Figure 8. Hurst exponent ($H_x(2)$) and 95% confidence interval on each day for upward and downward trends for American currencies. Note: in blue, the $H_x(2)$ under upward trend; in red, under downward trend.

Table 5. Multifractal degree of currency price returns by day.

	Monday		Tuesday		Wednesday		Thrusday		Friday		Monday		Tuesday		Wednesday		Thrusday		Friday	
EUR	0.215	**	−0.128	0.542	**	0.365	**	0.025		0.457	**	**	0.355	**	0.164	**	0.433	**		
GBP	0.238	**	0.124	**	0.247	**	0.168	**	0.285	**	0.286	**	0.331	**	0.369	**	0.145	**	0.341	**
JPY	0.247	**	0.183	**	0.004	0.161	**	0.358	**	0.110	**	0.223	**	0.236	**	0.116	**	0.147	**	
CHF	0.296	**	0.044	0.425	**	0.081	*	0.286	**	0.381	**	0.346	**	0.423	**	0.133	**	0.264	**	
AUD	0.255	**	0.186	**	0.108	0.250	**	0.574	**	0.328	**	0.404	**	0.162	**	0.183	**	0.291	**	
CAD	0.031	0.481	**	0.117	0.168	**	0.313	**	0.255	**	0.179	**	0.171	**	0.078	0.310	**			
SEK	0.181	**	0.018	0.211	**	0.260	**	−0.059		0.267	**	0.181	**	0.324	**	0.222	**	0.297	**	
DKK	0.195	**	0.028	0.439	**	0.076	0.113	*	0.349	**	0.430	**	0.140	**	0.215	**	0.276	**		
NOK	0.033	0.103	*	0.235	**	0.097	**	0.051		0.647	**	0.136	**	0.187	**	0.214	**	0.381	**	
CZK	0.450	**	0.193	**	0.062	0.272	**	0.188	**	0.228	**	0.323	**	0.044	0.212	**	0.090			
PLN	0.097	*	0.081	0.460	**	0.309	**	0.184	*	0.207	**	0.137	0.039	0.330	**	0.313	**			
HUF	0.330	**	0.185	**	−0.142	0.414	**	0.041		0.184	**	−0.061	0.139	**	0.239	**	0.005			
RUB	0.592	**	0.588	**	0.251	**	0.522	**	0.597	**	0.586	**	0.332	**	0.073	0.293	**	0.457	**	
KRW	0.241	**	0.337	**	0.620	**	0.048	0.497	**	0.304	**	−0.057	0.052	0.585	**	0.347	**			
ILS	−0.018	0.289	**	0.086	0.231	**	0.286	**	0.105	**	0.328	**	0.312	**	0.134	**	0.455	**		
HKD	0.789	**	0.338	**	1.023	**	0.421	**	1.064	**	1.632	**	0.711	**	0.445	**	0.144	*	0.596	**
SGD	0.185	**	0.319	**	0.237	**	0.348	**	0.453	**	0.425	**	0.231	**	0.125	*	0.155	**	0.218	**
TWD	0.402	**	0.610	**	0.652	**	0.351	**	0.535	**	0.686	**	0.218	**	0.414	**	0.180	**	0.264	**
IDR	0.574	**	0.099	*	0.859	**	0.550	**	1.102	**	0.249	**	0.171	**	0.335	**	0.331	**	0.075	
INR	0.567	**	0.340	**	0.154	*	−0.010	0.249	**	0.320	**	0.081	*	0.258	**	0.292	**	0.110	*	
MYR	0.456	**	0.430	**	0.552	**	0.593	**	0.459	**	0.352	**	0.812	**	0.570	**	0.157	**	0.494	**
THB	0.226	**	−0.028	0.099	0.123	**	0.397	**	0.086	*	0.350	**	0.185	**	0.078	0.594	**			
ZAR	0.158	**	0.168	**	0.130	**	0.155	**	0.211	**	0.205	**	0.078	*	0.010	0.224	**	0.168	**	
MAD	0.062	−0.036	0.465	**	0.526	**	0.345	**	0.303	**	0.270	**	0.515	**	0.559	**	−0.009			
MXN	−0.093	0.249	**	0.043	0.384	**	0.274	**	0.162	*	0.246	**	0.023	0.195	**	0.065				
BRL	0.384	**	−0.120	*	0.015	0.246	**	0.286	**	−0.121	*	0.165	**	0.370	**	0.315	**	0.147	**	
CLP	0.212	**	0.171	**	−0.049	−0.034	0.189	**	0.345	**	0.226	**	0.241	**	0.193	**	0.404	**		
COP	0.930	**	0.261	**	0.505	**	0.600	**	0.117		0.466	**	0.575	**	0.574	**	0.202	**	0.799	**
PEN	0.527	**	−0.145	**	0.276	**	0.518	**	0.409	**	0.016	0.461	**	0.766	**	0.230	**	0.483	**	
ARS	0.441	**	0.397	**	0.613	**	0.743	**	0.257	*	0.474	**	0.176	**	0.801	**	0.928	**	0.402	**

Note: ** and * represent statistical significance at 5% and 1%, respectively.

4. Conclusions

The foreign exchange rates market is one of the most liquid and efficient markets. In this study, we address the efficiency analysis of this market by verifying the day-of-the-week effect through fractal analysis. The presence of fractality is evident in the return series of each day as well as when performing the analysis under both upward and downward trends. Thirty main world currencies from all continents were analyzed in this study, showing weekday effects according to their fractal behavior. These results show a form of market inefficiency, as the returns or price variations of each day of the analyzed currencies should have behaved similarly and tended towards random walks. The discovery of this fractal day-of-the-week effect in world currencies opens up a number of possibilities. It could allow innovative investment strategies to be generated and enhance investors' ability to make informed decisions about when to buy and sell currencies. This research has the potential to inspire new approaches to currency trading. The econometric models show that the day-of-the-week effects in the studied currencies do not fully align with previous studies. However, analyzing the Hurst exponent for each day reveals that there are still weekday effects in the fractal dimension.

The existence of multifractality among the days of the week for the world's currency exchange rates suggests that trading strategies could utilize this information to develop strategies that depend on the day of the week on which the trades take place.

The generation of trading strategies that use the fractal dimension with respect trading days is beyond the scope of this paper; nonetheless, it is deserving of further discussion and analysis in the literature. The potential number of strategies that could be built using these ideas is relatively large, and it is important to discuss which are the best and to compare them to both passive investment strategies and those that use other indicators.

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Data Availability Statement: Data are available from the authors on request.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

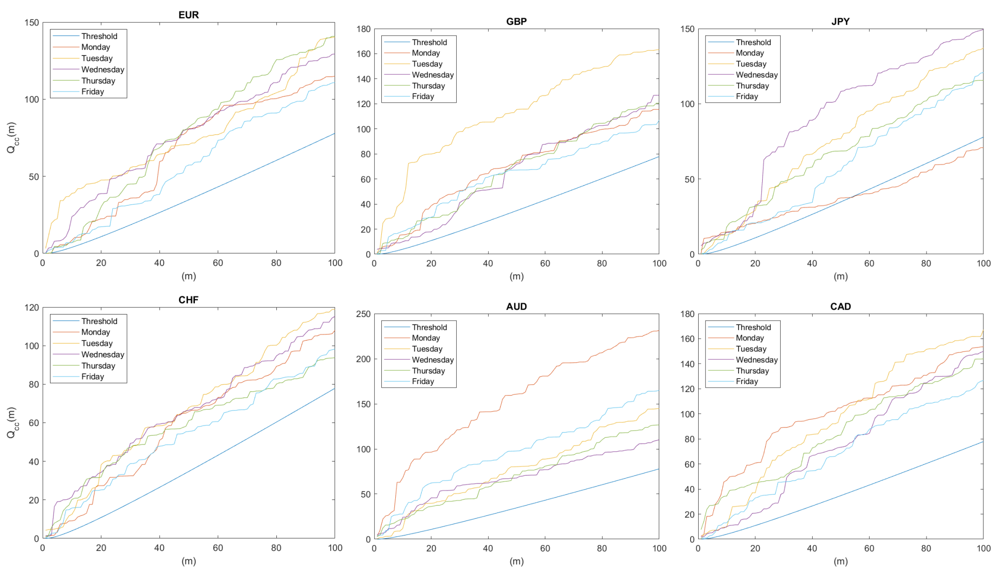

Figure A1. *Cont.*

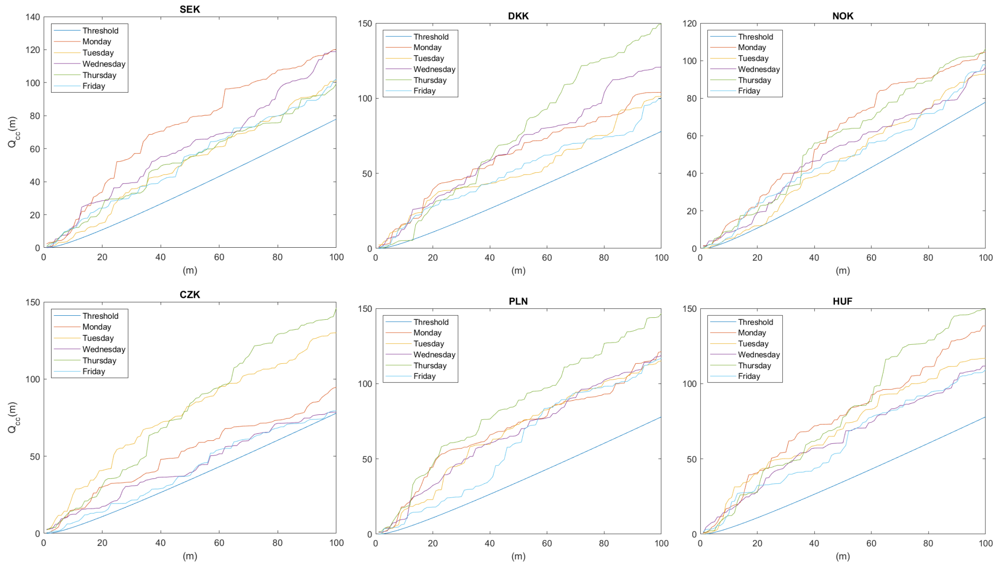

Figure A1. Cross-correlation statistics $Qcc(m)$ vs. the degrees of freedom m for the pairs studied.

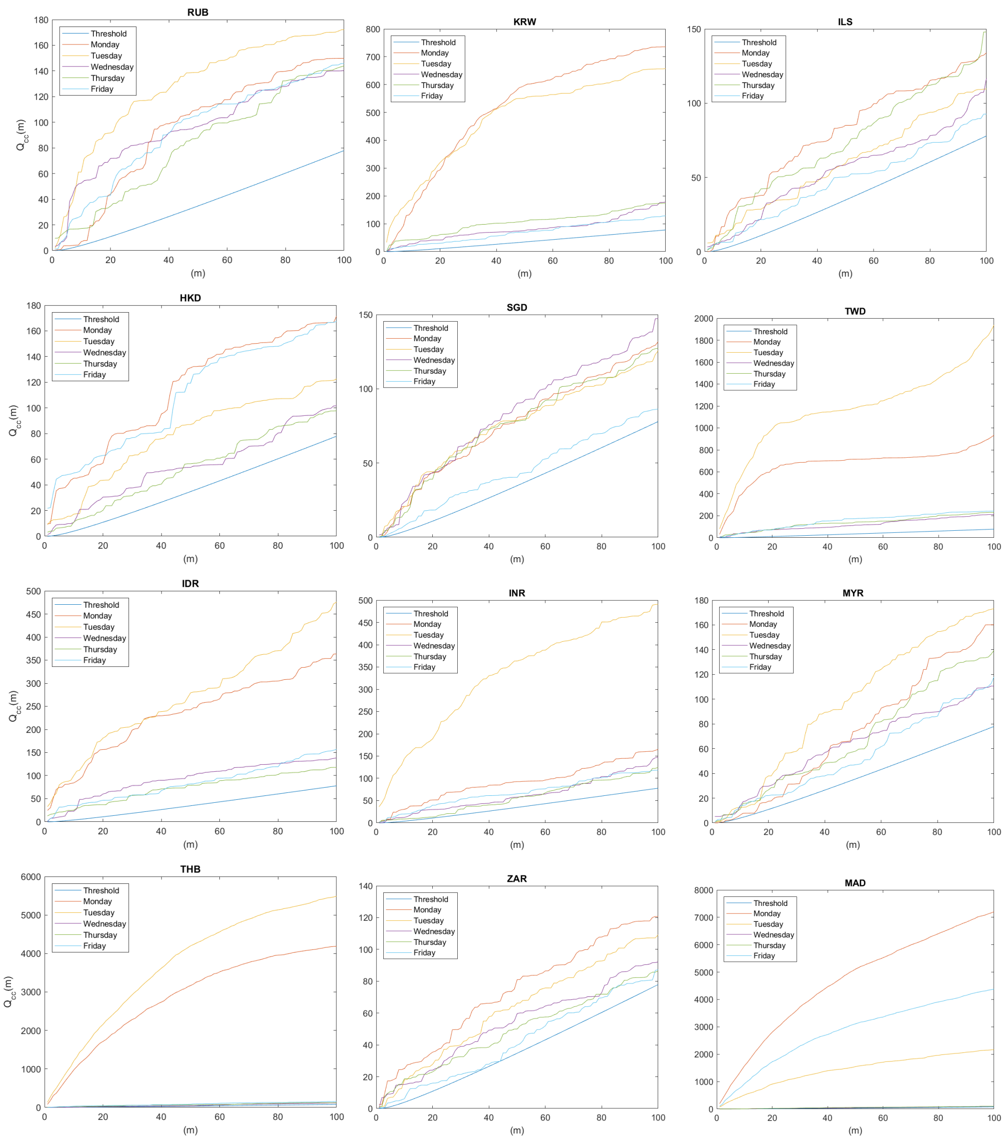

Figure A2. Cross-correlation statistics $Qcc(m)$ vs. the degrees of freedom m for the pairs studied.

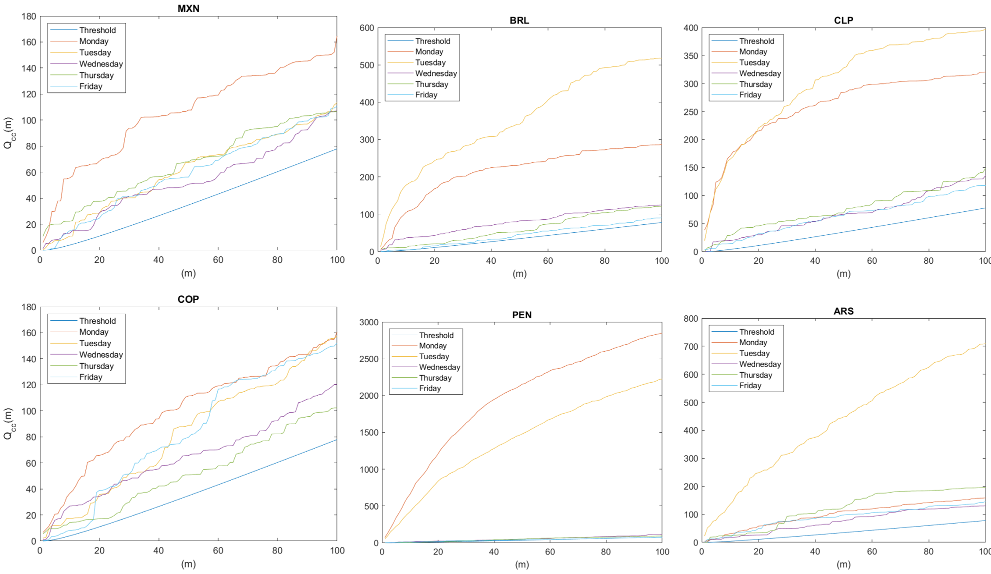

Figure A3. Cross-correlation statistics $Qcc(m)$ vs. the degrees of freedom m for the pairs studied.

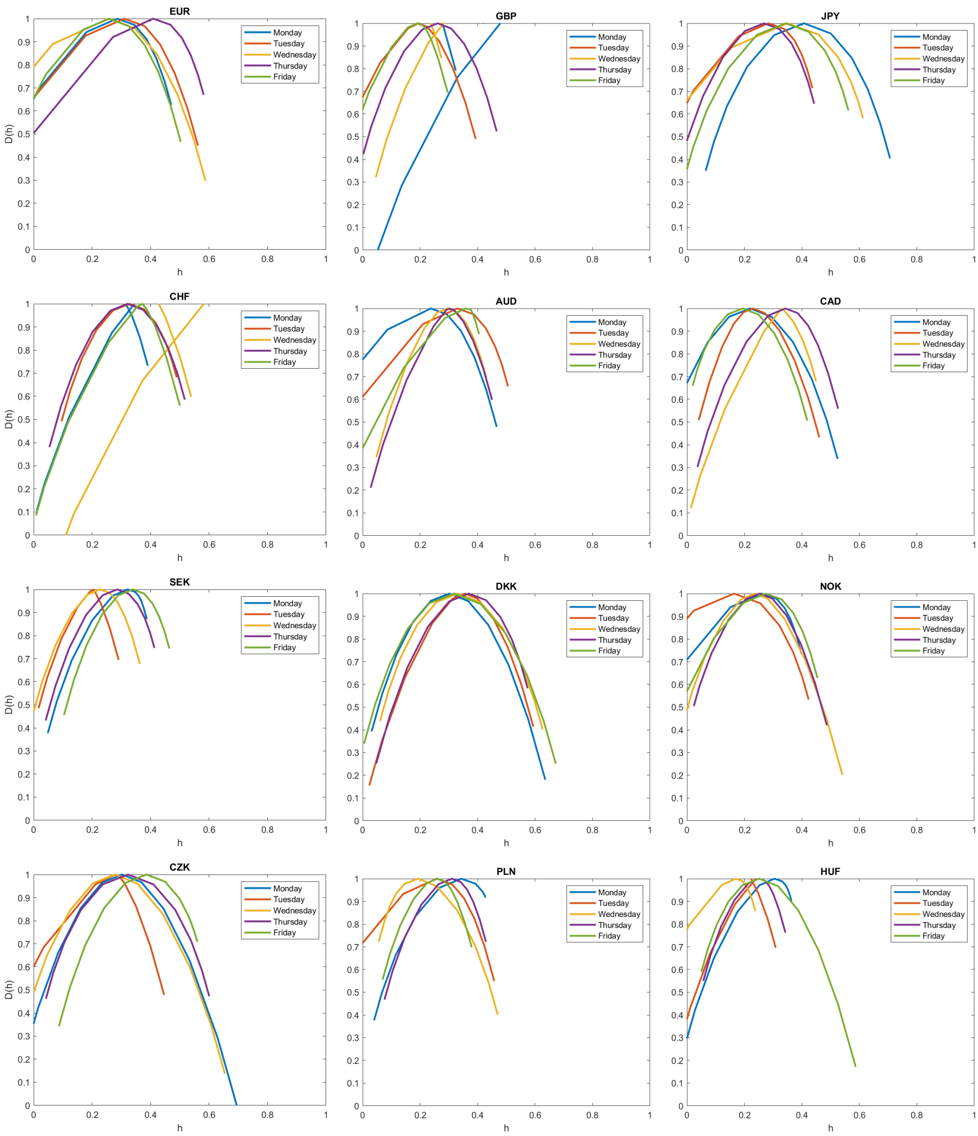

Figure A4. Multifractal spectra: distribution of scaling exponent for each return time series.

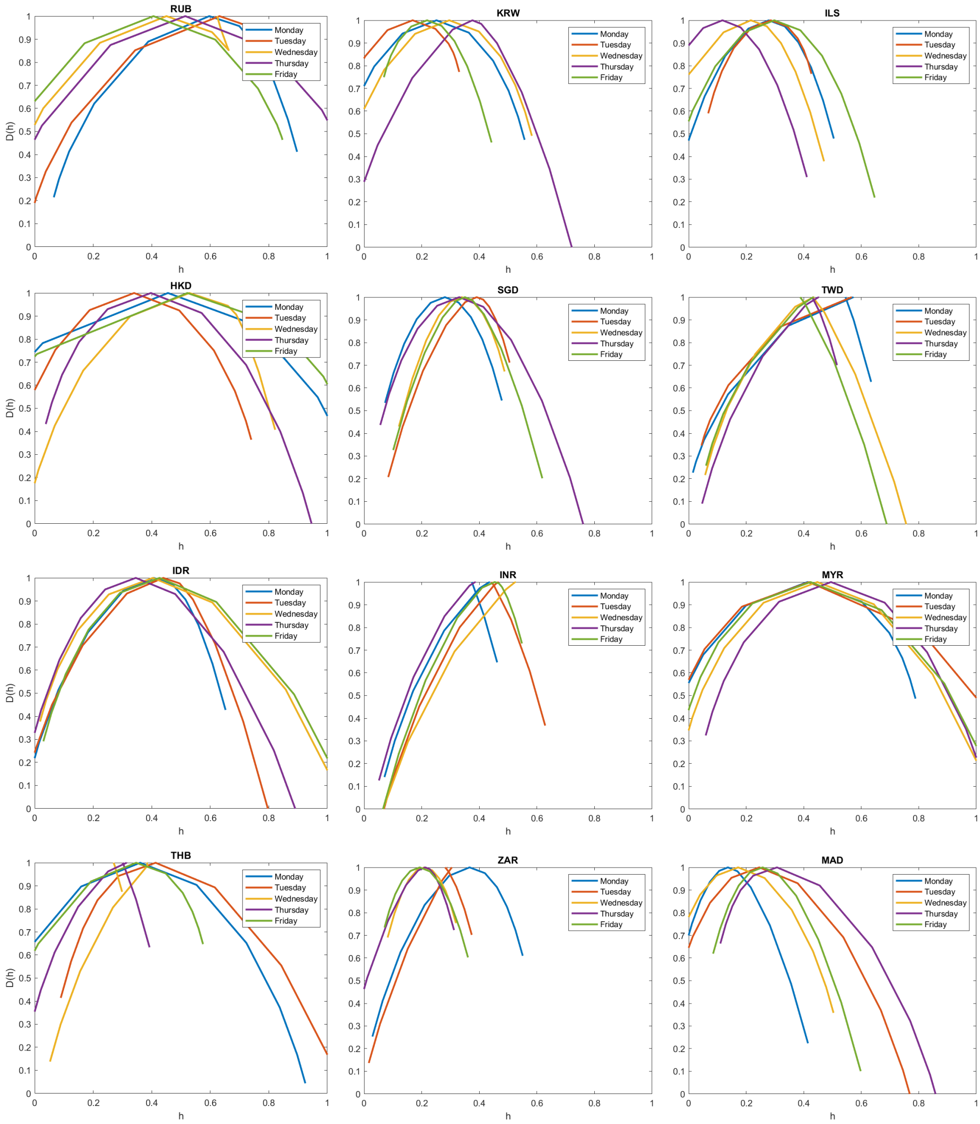

Figure A5. Multifractal Spectra: distribution of scaling exponent for each return time series.

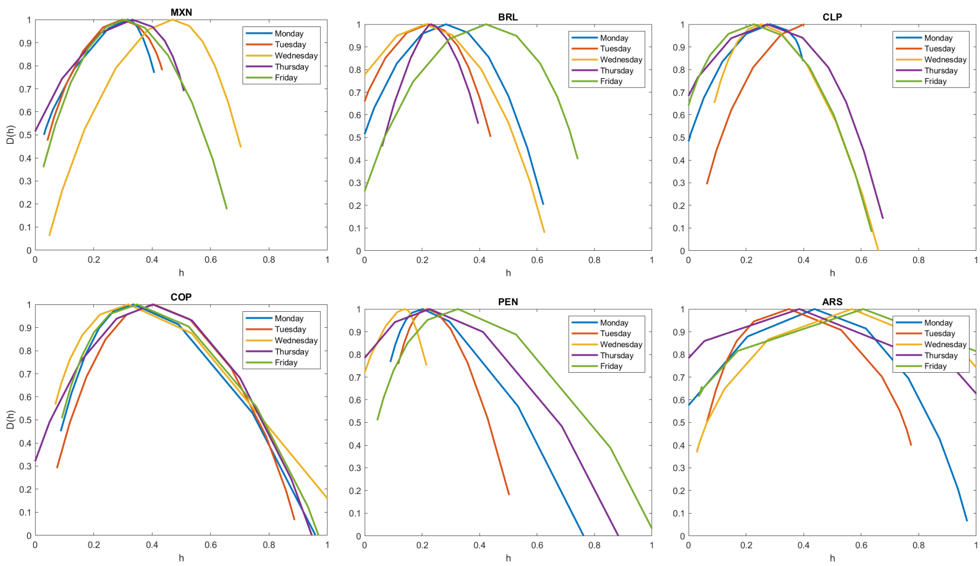

Figure A6. Multifractal spectra: distribution of scaling exponent for each return time series.

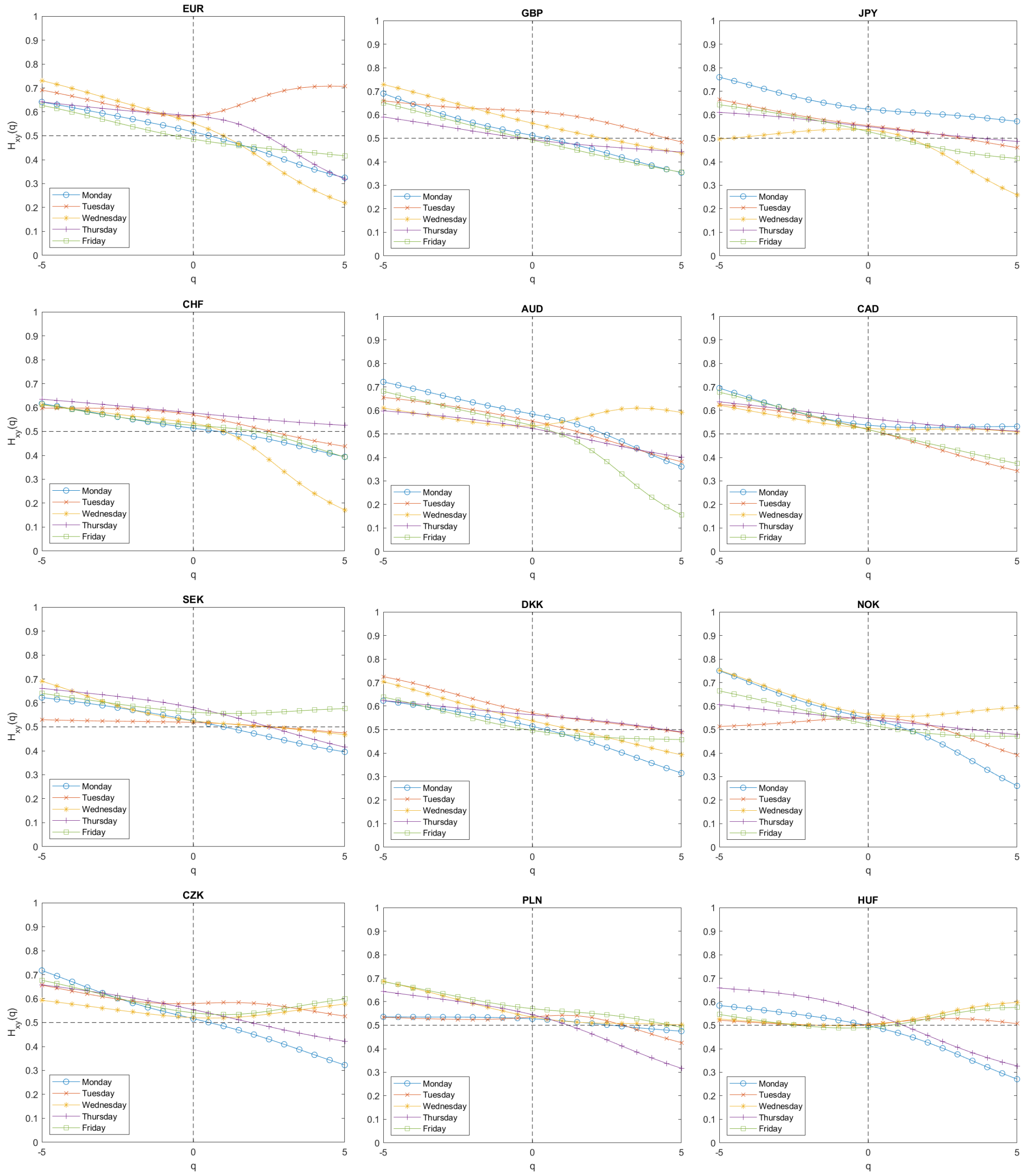

Figure A7. Cross-correlation exponent $H_x(q)$ vs. q.

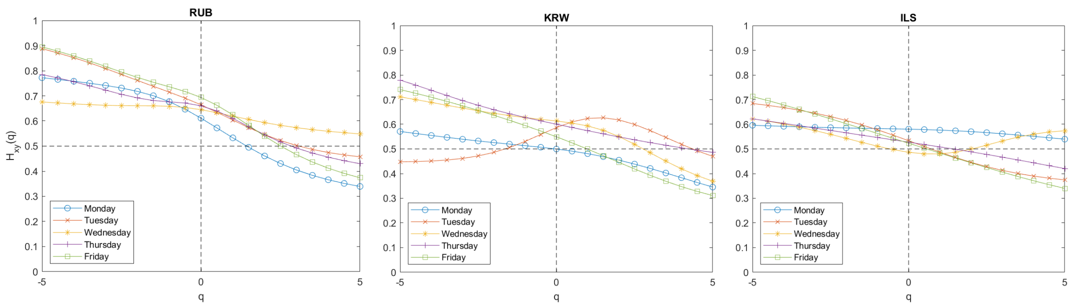

Figure A8. *Cont.*

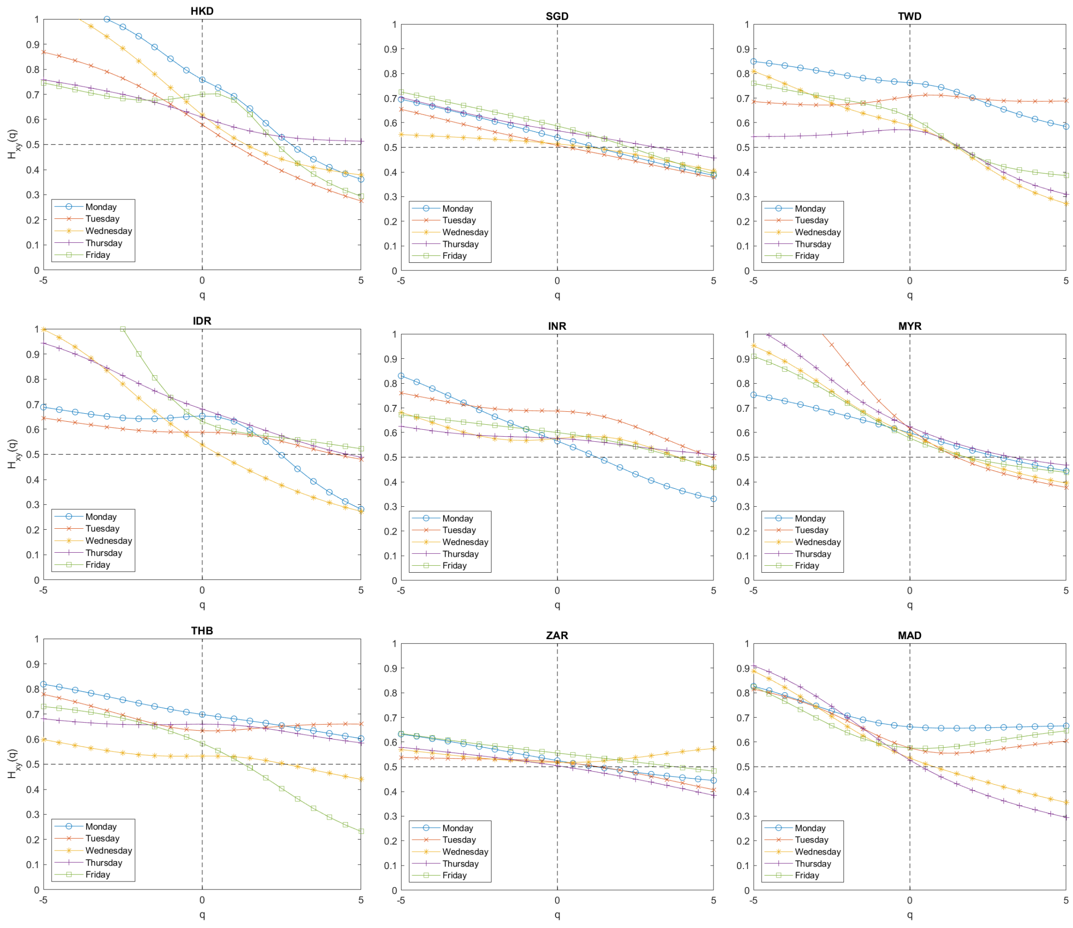

Figure A8. Cross-correlation exponent $H_x(q)$ vs. q.

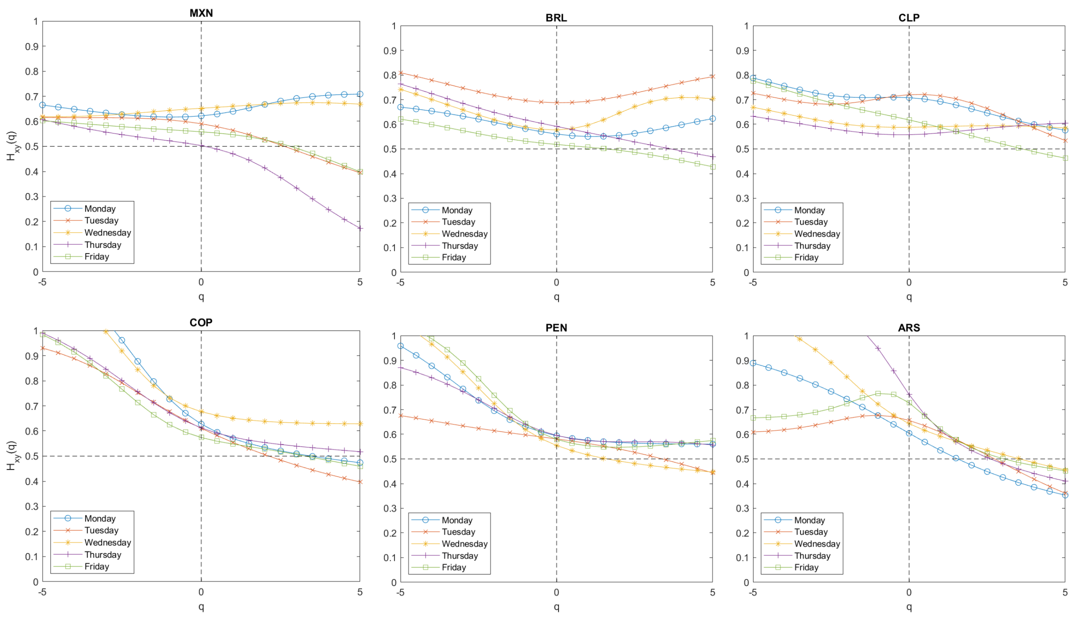

Figure A9. Cross-correlation exponent $H_x(q)$ vs. q.

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