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Employing the Laplace Residual Power Series Method to Solve (1+1)- and (2+1)-Dimensional Time-Fractional Nonlinear Differential Equations

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Abstract: In this paper, we present a highly efficient analytical method that combines the Laplace transform and the residual power series approach to approximate solutions of nonlinear time-fractional partial differential equations (PDEs). First, we derive the analytical method for a general form of fractional partial differential equations. Then, we apply the proposed method to find approximate solutions to the time-fractional coupled Berger equations, the time-fractional coupled Korteweg–de Vries equations and time-fractional Whitham–Broer–Kaup equations. Secondly, we extend the proposed method to solve the two-dimensional time-fractional coupled Navier–Stokes equations. The proposed method is validated through various test problems, measuring quality and efficiency using error norms E_2 and E_∞, and compared to existing methods.

Keywords: differential equations; Laplace transform; residual power series; time-fractional differential equations

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1. Introduction

Fractional calculus (FC) extends classical calculus to explore derivatives and integrals of non-integer order, allowing for a wide range of applications and real-life phenomena. Furthermore, FC has become a crucial tool in several fields, including engineering, solid-state physics, signal and image processing, chemistry, biology, ecology, stochastic-based finance, economics, control theory, fiber optics, and viscoelasticity [1–5]. Although many of these problems have been studied using fractional ordering in the literature, many models using fractional differential operators remain to be solved. Therefore, fractional differential equations (FDEs) have drawn the attention of several researchers in developing several analytical and numerical methods for linear and nonlinear problems and discussing dynamical systems. [6–8]. Sene and Fall [9] proposed the homotopy perturbation Laplace transform method of obtaining the approximate solution of the fractional diffusion equations. Tamsir and Srivastava [10] suggested the fractional reduced differential transform method to study analytically linear and nonlinear time-fractional order Klein–Gordon equations. Sahu and Jena [11] employed the Laplace Adomian decomposition technique to analyze a numerical study with the SDIQR mathematical model of COVID-19 for infected migrants in Odisha. Owolabi et al. [12] proposed the Laplace transform–homotopy perturbation method to simulate the time-dependent predator–prey model of Lotka–Volterra. Jawarneh et al. [13] introduced the new transform iteration method and the residual power series transform method to solve fractional nonlinear system Korteweg–de Vries (KdV) equations.

The Laplace residual power series (LRPS) approach is a highly efficient and accurate method for approximating solutions of nonlinear fractional-order partial differential equations (NFPDEs). This approach combines residual power series analysis with the Laplace transformation to provide a practical and fast convergence solution for linear and nonlinear problems. In this approach, the given equations are transferred into Laplace space, constructing fractional power series solutions to the new form of the equations and then using the inverse Laplace transform to obtain the solutions of the original equations. This method has been successfully applied to various equations, yielding accurate and convergent solutions, such as neutral fractional pantograph equations [14], temporal-fractional Drinfeld–Sokolov–Wilson systems [15], coupled fractional neutron diffusion equations [16], time-fractional reaction–diffusion models [17], nonlinear time-fractional Kolmogorov and Rosenau–Hyman models [18], three-dimensional fractional Helmholtz equations [19], fractional Riccati differential equations [20], and nonlinear time-fractional coupled Boussinesq–Burger equations [21].

In this work, we aim to accomplish three primary objectives. Firstly, we aim to develop the LRPS method to derive the analytical solution for a general form of (1+1)-dimensional NFPDEs and use it to solve various time-fractional coupled differential equations. Secondly, we aim to expand the application of the proposed approach to address (2+1)-dimensional time-fractional nonlinear coupled Navier–Stokes equations. Lastly, we aim to provide numerical and graphical solutions for different λ values to demonstrate the effectiveness of LRPS solutions compared to other methodologies, such as Laplace Adomian decomposition (LADM), the Laplace variational iteration method (LVIM), the residual differential transformation method (RDTM), and the Chebyshev method. Our findings highlight the simplicity, accuracy, and practical applicability of the proposed method.

The paper is organized as follows: in Section 2, we define key concepts and terminology. In Section 3, we present the proposed method and demonstrate its applicability to find analytical solutions of some nonlinear time-fractional coupled differential equations. Then, we explain the generalized LRPS method for the (2+1)-dimensional time-fractional coupled Navier–Stokes equations Section 4. Finally, we summarize our findings in Section 5.

2. Basic Concepts

In this section, we will present some basic concepts of the fractional derivative of order λ, where $\lambda > 0$. Although there are various definitions of fractional derivatives available, Riemann–Liouville and Caputo fractional derivatives are the most commonly used ones in the literature. So, the fractional derivative used in this study is in the Caputo meaning.

Definition 1 ([1])**.** *The Riemann–Liouville fractional integral operator of order $\lambda \geq 0$ is defined by*

$$J_t^{\lambda}\psi(\varkappa, t) = \begin{cases} \frac{1}{\Gamma(\lambda)}\int_0^t (t-\tau)^{\lambda-1}\psi(\varkappa,\tau)d\tau, & \lambda > 0, \\ \psi(\varkappa, t), & \lambda = 0. \end{cases}, \tag{1}$$

Definition 2 ([1])**.** *For n to be the smallest integer that exceeds λ, the Caputo time-fractional derivative operator of order $\lambda > 0, n-1 < \lambda \leq 1, n \in \mathbb{N}$ is defined as*

$$D_t^{\lambda}\psi(\varkappa, t) = J^{n-\lambda}D^n\psi(\varkappa, t) = \begin{cases} \dfrac{1}{\Gamma(n-\lambda)}\displaystyle\int_0^t (t-\tau)^{n-\lambda-1}\frac{\partial^n\psi(\varkappa,t)}{\partial t^n}d\tau, & n-1 < \lambda < n, \\ \dfrac{\partial^n\psi(\varkappa,t)}{\partial t^n}, & \lambda = n \in \mathbb{N}. \end{cases} \tag{2}$$

Definition 3 ([16]). *Let $\psi(\varkappa,t)$ be a continuous function on $I \times [0,\infty)$ and of exponential order δ. Then, the Laplace transform of the function $\psi(\varkappa,t)$ is denoted and defined as follows:*

$$\Psi(\varkappa,s) = \mathfrak{L}[\psi(\varkappa,t)] := \int_0^\infty e^{-st}\psi(\varkappa,t)dt, s > \delta, \quad (3)$$

whereas the inverse Laplace transform of the function $\Psi(\varkappa,s)$ is defined as follows:

$$\psi(\varkappa,t) = \mathfrak{L}^{-1}[\Psi(\varkappa,s)] := \int_{c-i\infty}^{c+i\infty} e^{st}\Psi(\varkappa,s)ds, c = \mathrm{Re}(s) > c_0, \quad (4)$$

where c_0 lies in the right half plane of the absolute convergence of the Laplace integral.

Assuming $\Psi(\varkappa,s) = \mathfrak{L}[\psi(\varkappa,t)]$, $\Phi(\varkappa,s) = \mathfrak{L}[\Phi(\varkappa,t)]$, ζ_1, $\zeta_2 \in \mathbb{R}$, we summarize the Laplace transform and its inverse below, highlighting their most prominent features.

1. $\mathfrak{L}[\zeta_1\psi(\varkappa,t) + \zeta_2\phi(\varkappa,t)] = \zeta_1\Psi(\varkappa,s) + \zeta_2\Phi(\varkappa,s)$.
2. $\mathfrak{L}^{-1}[\zeta_1\Psi(\varkappa,s) + \zeta_2\Phi(\varkappa,s)] = \zeta_1\psi(\varkappa,t) + \zeta_2\phi(\varkappa,t)$.
3. $\mathfrak{L}[e^{a\,t}\psi(\varkappa,t)] = \Psi(\varkappa,s-a)$,
4. $\mathfrak{L}\left[t^{m\lambda}\right] = \dfrac{\Gamma(m\lambda+1)}{s^{m\lambda+1}}, \lambda > -1$.

In the following lemma, we introduce several essential characteristics of the Laplace transform and the fractional derivative in the Caputo sense.

Lemma 1 ([16]). *Let $\psi(\varkappa,t)$ be a continuous function on $I \times [0,\infty)$ and of exponential orders δ, and $\Psi(\varkappa,s) = \mathfrak{L}[\psi(\varkappa,t)]$. Then,*

(i)- $\lim_{s\to\infty} s\Psi(\varkappa,s) = \psi(\varkappa,0), \varkappa \in I;$

(ii)- $\mathfrak{L}\left[J_t^\lambda\psi(\varkappa,t)\right] = s^{\lambda-1}\,\Psi(\varkappa,s), \lambda > 0;$

(iii)- $\mathfrak{L}\left[D_t^\lambda\psi(\varkappa,t)\right] = s^\lambda\,\Psi(\varkappa,s) - \sum_{k=0}^{n-1} s^{\lambda-k-1}\partial_t^k\psi(\varkappa,0), n-1 < \lambda < n;$

(iv)- $\mathfrak{L}\left[D_t^{m\lambda}\psi(\varkappa,t)\right] = s^{m\lambda}\,\Psi(\varkappa,s) - \sum_{k=0}^{m-1} s^{(m-k)\lambda-1}D_t^{k\lambda}\psi(\varkappa,0), 0 < \lambda < 1.$

where $D_t^{m\lambda} = D_t^\lambda \cdot D_t^\lambda \ldots D_t^\lambda$ (m-times).

Theorem 1. *Let $\psi(\varkappa,t)$ be continuous on $I \times [0,\infty)$ and of exponential order δ. Suppose that the function $\Psi(\varkappa,s) = \mathfrak{L}[\psi(\varkappa,t)]$ has the following fractional expansion:*

$$\Psi(\varkappa,s) = \sum_{n=0}^\infty \frac{f_n(\varkappa)}{s^{n\lambda+1}}, 0 < \lambda \leq 1, \varkappa \in I, s > \delta, \quad (5)$$

then $f_n(\varkappa) = D_t^{n\lambda}\psi(\varkappa,0)$.

3. Derivation LRPS Method

In this section, we discuss how to construct the solutions to some nonlinear coupled fractional partial differential equations using the LRPS method. The main algorithm of this method for solving nonlinear NFPDEs can be summarized by applying the Laplace transform to the mentioned equation and using the expansion as given in Theorem 1 to represent the solution of Laplace NFPDEs. Then, the coefficients of this expansion are determined similarly to the RPS method but with a new vision and a new analysis. Finally, we apply the inverse Laplace transform and obtain a solution to this problem in the original space.

3.1. The (1+1)-Dimensional Time-Fractional Coupled Differential Equation

Consider the following coupled fractional equation in the general form

$$D_t^{\lambda} u(x,t) = R_1\Big(u,v,D_x u,D_x v,D_x^2 u,D_x^2 v,\dots\Big) + N_1\Big(u,v,D_x u,D_x v,D_x^2 u,D_x^2 v,\dots\Big), \quad (6)$$
$$D_t^{\lambda} v(x,t) = R_2\Big(u,v,D_x u,D_x v,D_x^2 u,D_x^2 v,\dots\Big) + N_2\Big(u,v,D_x u,D_x v,D_x^2 u,D_x^2 v,\dots\Big). \quad (7)$$

Subject to the initial conditions

$$u(x,0) = f_0(x), \quad (8)$$
$$v(x,0) = g_0(x), \quad (9)$$

where $D_t^{\lambda} = \dfrac{\partial^{\lambda}}{\partial t^{\lambda}}$ is the Caputo derivative, $D_x^n = \frac{\partial^n}{\partial x^n}$, $n = 1,2,\dots$, and R_1, R_2 and N_1, N_2 are linear and nonlinear operators, respectively, and $0 < \lambda \leq 1$.

By utilizing the Laplace transform on Equations (6)–(9), we obtain

$$\mathfrak{L}\Big[D_t^{\lambda} u(x,t)\Big] = \mathfrak{L}\Big[R_1\Big(u,v,D_x u,D_x v,D_x^2 u,D_x^2 v,\dots\Big)\Big] + \mathfrak{L}\Big[N_1\Big(u,v,D_x u,D_x v,D_x^2 u,D_x^2 v,\dots\Big)\Big], \quad (10)$$
$$\mathfrak{L}\Big[D_t^{\lambda} v(x,t)\Big] = \mathfrak{L}\Big[R_2\Big(u,v,D_x u,D_x v,D_x^2 u,D_x^2 v,\dots\Big)\Big] + \mathfrak{L}\Big[N_2\Big(u,v,D_x u,D_x v,D_x^2 u,D_x^2 v,\dots\Big)\Big]. \quad (11)$$

Using the fact that $\mathfrak{L}[D_t^{\lambda} u(x,t)] = s^{\lambda}\mathfrak{L}[u(x,t)] - s^{\lambda-1}u(x,0) = s^{\lambda}\mathfrak{L}[u(x,t)] - s^{\lambda-1}f_0(x)$ and $\mathfrak{L}[D_t^{\lambda} v(x,t)] = s^{\lambda}\mathfrak{L}[v(x,t)] - s^{\lambda-1}v(x,0) = s^{\lambda}\mathfrak{L}[v(x,t)] - s^{\lambda-1}g_0(x)$, we can write Equations (10) and (11) as

$$\begin{aligned} U(x,s) =& \frac{f_0(x)}{s} + \frac{1}{s^{\lambda}} R_1\Big(U,V,D_x U,D_x V,D_x^2 U,D_x^2 V,\dots\Big) \\ &+ \frac{1}{s^{\lambda}}\mathfrak{L}\Big[N_1\Big(\mathfrak{L}^{-1}[U],\mathfrak{L}^{-1}[V],\mathfrak{L}^{-1}[D_x U],\mathfrak{L}^{-1}[D_x V],\mathfrak{L}^{-1}\Big[D_x^2 U\Big],\mathfrak{L}^{-1}\Big[D_x^2 V\Big],\dots\Big)\Big], \quad (12) \\ V(x,t) =& \frac{g_0(x)}{s} + \frac{1}{s^{\lambda}} R_2\Big(U,V,D_x U,D_x V,D_x^2 U,D_x^2 V,\dots\Big) \\ &+ \frac{1}{s^{\lambda}}\mathfrak{L}\Big[N_2\Big(\mathfrak{L}^{-1}[U],\mathfrak{L}^{-1}[V],\mathfrak{L}^{-1}[D_x U],\mathfrak{L}^{-1}[D_x V],\mathfrak{L}^{-1}\Big[D_x^2 U\Big],\mathfrak{L}^{-1}\Big[D_x^2 V\Big],\dots\Big)\Big]. \quad (13) \end{aligned}$$

where $U(x,s) = \mathfrak{L}[u(x,t)]$, $V(x,s) = \mathfrak{L}[v(x,t)]$. Now, we assume that both $U(x,s)$ and $V(x,s)$ have fractional power series representations as follows:

$$U(x,s) = \sum_{n=0}^{\infty} \frac{f_n(x)}{s^{n\lambda+1}}, \quad (14)$$
$$V(x,s) = \sum_{n=0}^{\infty} \frac{g_n(x)}{s^{n\lambda+1}}. \quad (15)$$

The k-th truncated series of Equations (14) and (15) take the forms

$$U_k(x,s) = \sum_{n=0}^{k} \frac{f_n(x)}{s^{n\lambda+1}}, \quad (16)$$
$$V_k(x,s) = \sum_{n=0}^{k} \frac{g_n(x)}{s^{n\lambda+1}}, \quad (17)$$

where $f_0(x)$ nd $g_0(x)$ are the initial conditions given in Equations (8) and (9). To find the unknown coefficients of the series in Equations (12) and (13), we define the Laplace residual functions for the coupled equations in Equations (16) and (17) as follows:

$$\begin{aligned}\mathfrak{L}ResU(\varkappa,s) =& U(\varkappa,s) - \frac{f_0(\varkappa)}{s} - \frac{1}{s^\lambda}R_1\left(U,V,D_\varkappa U,D_\varkappa V,D_\varkappa^2 U,D_\varkappa^2 V,\dots\right)\\ &- \frac{1}{s^\lambda}\mathfrak{L}\left[N_1\left(\mathfrak{L}^{-1}[U],\mathfrak{L}^{-1}[V],\mathfrak{L}^{-1}[D_\varkappa U],\mathfrak{L}^{-1}[D_\varkappa V],\mathfrak{L}^{-1}\left[D_\varkappa^2 U\right],\mathfrak{L}^{-1}\left[D_\varkappa^2 V\right],\dots\right)\right],\end{aligned} \tag{18}$$

$$\begin{aligned}\mathfrak{L}ResV(\varkappa,s) =& V(\varkappa,t) - \frac{g_0(\varkappa)}{s} - \frac{1}{s^\lambda}R_2\left(U,V,D_\varkappa U,D_\varkappa V,D_\varkappa^2 U,D_\varkappa^2 V,\dots\right)\\ &- \frac{1}{s^\lambda}\mathfrak{L}\left[N_2\left(\mathfrak{L}^{-1}[U],\mathfrak{L}^{-1}[V],\mathfrak{L}^{-1}[D_\varkappa U],\mathfrak{L}^{-1}[D_\varkappa V],\mathfrak{L}^{-1}\left[D_\varkappa^2 U\right],\mathfrak{L}^{-1}\left[D_\varkappa^2 V\right],\dots\right)\right].\end{aligned} \tag{19}$$

For the k-th Laplace residual function, we have

$$\begin{aligned}\mathfrak{L}ResU_k(\varkappa,s) =& U_k(\varkappa,s) - \frac{f_0(\varkappa)}{s} - \frac{1}{s^\lambda}R_1\left(U_k,V_k,D_\varkappa U_k,D_\varkappa V_k,D_\varkappa^2 U_k,D_\varkappa^2 V_k,\dots\right)\\ &- \frac{1}{s^\lambda}\mathfrak{L}\left[N_1\left(\mathfrak{L}^{-1}[U_k],\mathfrak{L}^{-1}[V_k],\mathfrak{L}^{-1}[D_\varkappa U_k],\mathfrak{L}^{-1}[D_\varkappa V_k],\mathfrak{L}^{-1}\left[D_\varkappa^2 U_k\right],\mathfrak{L}^{-1}\left[D_\varkappa^2 V_k\right],\dots\right)\right],\end{aligned} \tag{20}$$

$$\begin{aligned}\mathfrak{L}ResV_k(\varkappa,s) =& V_k(\varkappa,t) - \frac{g_0(\varkappa)}{s} - \frac{1}{s^\lambda}R_2\left(U_k,V_k,D_\varkappa U_k,D_\varkappa V_k,D_\varkappa^2 U_k,D_\varkappa^2 V_k,\dots\right)\\ &- \frac{1}{s^\lambda}\mathfrak{L}\left[N_2\left(\mathfrak{L}^{-1}[U_k],\mathfrak{L}^{-1}[V_k],\mathfrak{L}^{-1}[D_\varkappa U_k],\mathfrak{L}^{-1}[D_\varkappa V_k],\mathfrak{L}^{-1}\left[D_\varkappa^2 U_k\right],\mathfrak{L}^{-1}\left[D_\varkappa^2 V_k\right],\dots\right)\right].\end{aligned} \tag{21}$$

Substituting Equations (16) and (17) into Equations (20) and (21), we obtain

$$\begin{aligned}\mathfrak{L}ResU_k(\varkappa,s) =& \sum_{n=1}^{k}\frac{f_n(\varkappa)}{s^{n\lambda+1}} - \frac{1}{s^\lambda}R_1\left(\sum_{n=0}^{k}\frac{f_n(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{f_n'(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n'(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{f_n''(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n''(\varkappa)}{s^{n\lambda+1}},\dots\right)\\ &- \frac{1}{s^\lambda}\mathfrak{L}\left[N_1\left(\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{f_n(\varkappa)}{s^{n\lambda+1}}\right],\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{g_n(\varkappa)}{s^{n\lambda+1}}\right],\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{f_n'(\varkappa)}{s^{n\lambda+1}}\right],\right.\right.\\ &\left.\left.\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{g_n'(\varkappa)}{s^{n\lambda+1}}\right],\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{f_n''(\varkappa)}{s^{n\lambda+1}}\right],\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{g_n''(\varkappa)}{s^{n\lambda+1}}\right],\dots\right)\right],\end{aligned} \tag{22}$$

$$\begin{aligned}\mathfrak{L}ResV_k(\varkappa,s) =& \sum_{n=1}^{k}\frac{g_n(\varkappa)}{s^{n\lambda+1}} - \frac{1}{s^\lambda}R_2\left(\sum_{n=0}^{k}\frac{f_n(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{f_n'(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n'(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{f_n''(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n''(\varkappa)}{s^{n\lambda+1}},\dots\right)\\ &- \frac{1}{s^\lambda}\mathfrak{L}\left[N_2\left(\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{f_n(\varkappa)}{s^{n\lambda+1}}\right],\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{g_n(\varkappa)}{s^{n\lambda+1}}\right],\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{f_n'(\varkappa)}{s^{n\lambda+1}}\right],\right.\right.\\ &\left.\left.\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{g_n'(\varkappa)}{s^{n\lambda+1}}\right],\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{f_n''(\varkappa)}{s^{n\lambda+1}}\right],\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{g_n''(\varkappa)}{s^{n\lambda+1}}\right],\dots\right)\right].\end{aligned} \tag{23}$$

Using $\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{f_n(\varkappa)}{s^{n\lambda+1}}\right] = \sum_{n=0}^{k}\frac{t^{n\lambda}f_n(\varkappa)}{\Gamma(n\lambda+1)}$ in Equations (22) and (23), we obtain

$$\begin{aligned}\mathfrak{L}ResU_k(\varkappa,s) =& \sum_{n=1}^{k}\frac{f_n(\varkappa)}{s^{n\lambda+1}} - \frac{1}{s^\lambda}R_1\left(\sum_{n=0}^{k}\frac{f_n(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{f_n'(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n'(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{f_n''(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n''(\varkappa)}{s^{n\lambda+1}},\dots\right)\\ &- \frac{1}{s^\lambda}\mathfrak{L}\left[N_1\left(\sum_{n=0}^{k}\frac{t^{n\lambda}f_n(\varkappa)}{\Gamma(n\lambda+1)},\sum_{n=0}^{k}\frac{t^{n\lambda}g_n(\varkappa)}{\Gamma(n\lambda+1)},\sum_{n=0}^{k}\frac{t^{n\lambda}f_n'(\varkappa)}{\Gamma(n\lambda+1)},\right.\right.\\ &\left.\left.\sum_{n=0}^{k}\frac{t^{n\lambda}g_n'(\varkappa)}{\Gamma(n\lambda+1)},\sum_{n=0}^{k}\frac{t^{n\lambda}f_n''(\varkappa)}{\Gamma(n\lambda+1)},\sum_{n=0}^{k}\frac{t^{n\lambda}g_n''(\varkappa)}{\Gamma(n\lambda+1)},\dots\right)\right],\end{aligned} \tag{24}$$

$$\mathfrak{L}ResV_k(\varkappa,s) = \sum_{n=1}^{k}\frac{g_n(\varkappa)}{s^{n\lambda+1}} - \frac{1}{s^\lambda}R_2\left(\sum_{n=0}^{k}\frac{f_n(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{f_n'(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n'(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{f_n''(\varkappa)}{s^{n\lambda+1}},\sum_{n=0}^{k}\frac{g_n''(\varkappa)}{s^{n\lambda+1}},\dots\right)$$

$$-\frac{1}{s^{\lambda}}\mathfrak{L}\Bigg[N_2\Bigg(\sum_{n=0}^{k}\frac{t^{n\lambda}f_n(x)}{\Gamma(n\lambda+1)},\sum_{n=0}^{k}\frac{t^{n\lambda}g_n(x)}{\Gamma(n\lambda+1)},\sum_{n=0}^{k}\frac{t^{n\lambda}f'_n(x)}{\Gamma(n\lambda+1)},$$
$$\sum_{n=0}^{k}\frac{t^{n\lambda}g'_n(x)}{\Gamma(n\lambda+1)},\sum_{n=0}^{k}\frac{t^{n\lambda}f''_n(x)}{\Gamma(n\lambda+1)},\sum_{n=0}^{k}\frac{t^{n\lambda}g''_n(x)}{\Gamma(n\lambda+1)},\dots\Bigg)\Bigg]. \tag{25}$$

The next step is to solve the following system to calculate $f_k(x)$ and $g_k(x)$, $k=1,2,\dots$

$$\lim_{s\longrightarrow\infty} s^{k\lambda+1}\mathfrak{L}ResU_k(x,s)=0, \tag{26}$$
$$\lim_{s\longrightarrow\infty} s^{k\lambda+1}\mathfrak{L}ResV_k(x,s)=0. \tag{27}$$

Finally, by substituting the series solution $f_k(x)$ and $g_k(x)$ obtained from Equations (26) and (27) into Equations (16) and (17) and taking the inverse Laplace transform, we obtain the solutions of system (6)–(9) as follows:

$$u(x,t)=\sum_{n=0}^{\infty}\frac{t^{n\lambda}f_n(x)}{\Gamma(n\lambda+1)}, \tag{28}$$
$$v(x,t)=\sum_{n=0}^{\infty}\frac{t^{n\lambda}g_n(x)}{\Gamma(n\lambda+1)}. \tag{29}$$

3.2. Illustrative Examples

This section presents three important examples of the LRPS method to demonstrate its performance and efficiency. Throughout this paper, we used the Wolfram Mathematica 14 software package to compute numerical results.

Example 1. *Consider the following coupled time-fractional Burger equations [22–24]:*

$$D_t^{\lambda}u(x,t)=a\,D_x^2u(x,t)+b\,u(x,t)D_xu(x,t)-c\,u(x,t)D_xv(x,t)-c\,v(x,t)D_xu(x,t), \tag{30}$$
$$D_t^{\lambda}v(x,t)=\rho\,D_x^2v(x,t)+\gamma\,v(x,t)D_xv(x,t)-\epsilon\,u(x,t)D_xv(x,t)-\epsilon\,v(x,t)D_xu(x,t). \tag{31}$$

Subject to the initial conditions

$$u(x,0)=f_0(x)=v(x,0)=g_0(x). \tag{32}$$

In this system, we have

$$R_1\Big(u,v,D_xu,D_xv,D_x^2u,D_x^2v,\dots\Big)=a\,D_x^2u(x,t),$$
$$R_2\Big(u,v,D_xu,D_xv,D_x^2u,D_x^2v,\dots\Big)=\rho\,D_x^2v(x,t),$$
$$N_1\Big(u,v,D_xu,D_xv,D_x^2u,D_x^2v,\dots\Big)=b\,u(x,t)D_xu(x,t)-c\,u(x,t)D_xv(x,t)-c\,v(x,t)D_xu(x,t),$$
$$N_2\Big(u,v,D_xu,D_xv,D_x^2u,D_x^2v,\dots\Big)=\gamma\,v(x,t)D_xv(x,t)-\epsilon\,u(x,t)D_xv(x,t)-\epsilon\,v(x,t)D_xu(x,t).$$

Applying system (24) and (25), we obtain

$$\mathfrak{L}ResU_k(x,s)=\sum_{n=1}^{k}\frac{f_n(x)}{s^{n\lambda+1}}-\frac{a}{s^{\lambda}}\sum_{n=0}^{k}\frac{f''_n(x)}{s^{n\lambda+1}}-\frac{1}{s^{\lambda}}\mathfrak{L}\Bigg[b\sum_{n=0}^{k}\frac{t^{n\lambda}f_n(x)}{\Gamma(n\lambda+1)}\sum_{n=0}^{k}\frac{t^{n\lambda}f'_n(x)}{\Gamma(n\lambda+1)}$$
$$-c\sum_{n=0}^{k}\frac{t^{n\lambda}f_n(x)}{\Gamma(n\lambda+1)}\sum_{n=0}^{k}\frac{t^{n\lambda}g'_n(x)}{\Gamma(n\lambda+1)}-c\sum_{n=0}^{k}\frac{t^{n\lambda}g_n(x)}{\Gamma(n\lambda+1)}\sum_{n=0}^{k}\frac{t^{n\lambda}f'_n(x)}{\Gamma(n\lambda+1)}\Bigg], \tag{33}$$
$$\mathfrak{L}ResV_k(x,s)=\sum_{n=1}^{k}\frac{g_n(x)}{s^{n\lambda+1}}-\frac{\rho}{s^{\lambda}}\sum_{n=0}^{k}\frac{g''_n(x)}{s^{n\lambda+1}}-\frac{1}{s^{\lambda}}\mathfrak{L}\Bigg[\gamma\sum_{n=0}^{k}\frac{t^{n\lambda}g_n(x)}{\Gamma(n\lambda+1)}\sum_{n=0}^{k}\frac{t^{n\lambda}g'_n(x)}{\Gamma(n\lambda+1)}$$

$$-\epsilon \sum_{n=0}^{k} \frac{t^{n\lambda} f_n(x)}{\Gamma(n\lambda+1)} \sum_{n=0}^{k} \frac{t^{n\lambda} g_n'(x)}{\Gamma(n\lambda+1)} - \epsilon \sum_{n=0}^{k} \frac{t^{n\lambda} g_n(x)}{\Gamma(n\lambda+1)} \sum_{n=0}^{k} \frac{t^{n\lambda} f_n'(x)}{\Gamma(n\lambda+1)} \Bigg]. \tag{34}$$

If we take $k = 1$, *we obtain*

$$\begin{aligned} \mathfrak{L}ResU_1(x,s) = {} & \frac{f_1(x)}{s^{\lambda+1}} - \frac{a}{s^{\lambda}}\left(\frac{f_0''(x)}{s} + \frac{f_1''(x)}{s^{\lambda+1}}\right) - \frac{1}{s^{\lambda}}\mathfrak{L}\Bigg[b\left(f_0(x) + \frac{t^{\lambda} f_1(x)}{\Gamma(\lambda+1)}\right)\left(f_0'(x) + \frac{t^{\lambda} f_1'(x)}{\Gamma(\lambda+1)}\right) \\ & - c\left(f_0(x) + \frac{t^{\lambda} f_1(x)}{\Gamma(\lambda+1)}\right)\left(g_0'(x) + \frac{t^{\lambda} g_1'(x)}{\Gamma(\lambda+1)}\right) - c\left(g_0(x) + \frac{t^{\lambda} g_1(x)}{\Gamma(\lambda+1)}\right)\left(f_0'(x) + \frac{t^{\lambda} f_1'(x)}{\Gamma(\lambda+1)}\right)\Bigg], \end{aligned} \tag{35}$$

$$\begin{aligned} \mathfrak{L}ResV_k(x,s) = {} & \frac{g_1(x)}{s^{\lambda+1}} - \frac{1}{s^{\lambda}}\left(\frac{g_0''(x)}{s} + \frac{g_1''(x)}{s^{\lambda+1}}\right) - \frac{\rho}{s^{\lambda}}\mathfrak{L}\Bigg[\gamma\left(g_0(x) + \frac{t^{\lambda} g_1(x)}{\Gamma(\lambda+1)}\right)\left(g_0'(x) + \frac{t^{\lambda} g_1'(x)}{\Gamma(\lambda+1)}\right) \\ & - \epsilon\left(f_0(x) + \frac{t^{\lambda} f_1(x)}{\Gamma(\lambda+1)}\right)\left(g_0'(x) + \frac{t^{\lambda} g_1'(x)}{\Gamma(\lambda+1)}\right) - \epsilon\left(g_0(x) + \frac{t^{\lambda} g_1(x)}{\Gamma(\lambda+1)}\right)\left(f_0'(x) + \frac{t^{\lambda} f_1'(x)}{\Gamma(\lambda+1)}\right)\Bigg]. \end{aligned} \tag{36}$$

Next, by solving the system $\begin{array}{l} \lim\limits_{s\longrightarrow\infty} s^{\lambda+1}\mathfrak{L}ResU_1(x,s) = 0, \\ \lim\limits_{s\longrightarrow\infty} s^{\lambda+1}\mathfrak{L}ResV_1(x,s) = 0, \end{array}$ *for* $f_1(x),\ g_1(x)$, *one can obtain:*

$$f_1(x) = a f_0''(x) + b f_0(x) f_0'(x) - c\, f_0(x) g_0'(x) - c\, g_0(x) f_0'(x), \tag{37}$$

$$g_1(x) = \rho\, g_0''(x) + \gamma\, g_0(x) g_0'(x) - \epsilon\, g_0(x) f_0'(x) - \epsilon\, f_0(x) g_0'(x). \tag{38}$$

In the same way, continuing to solve (26) *and* (27) *for every* $f_k(x),\ g_k(x),\ \ k = 2, 3, \dots$ *and as a special case when* $a = 1, b = 2, c = 1, \rho = 1, \gamma = 2,\ \epsilon = 1$ *and* $f_0(x) = g_0(x) = sin(x)$, *we obtain*

$$\begin{aligned} f_1(x) &= -sin(x), & g_1(x) &= -sin(x), \\ f_2(x) &= sin(x), & g_2(x) &= sin(x), \\ f_3(x) &= -sin(x), & g_3(x) &= -sin(x), \\ f_4(x) &= sin(x), & g_4(x) &= sin(x), \dots. \end{aligned}$$

Substituting in Equations (28) *and* (29), *we obtain*

$$u(x,t) = \sin(x) - \frac{t^{\lambda}\sin(x)}{\Gamma(\lambda+1)} + \frac{t^{2\lambda}\sin(x)}{\Gamma(2\lambda+1)} - \frac{t^{3\lambda}\sin(x)}{\Gamma(3\lambda+1)} + \frac{t^{4\lambda}\sin(x)}{\Gamma(4\lambda+1)} - \frac{t^{5\lambda}\sin(x)}{\Gamma(5\lambda+1)} + \dots \tag{39}$$

$$v(x,t) = \sin(x) - \frac{t^{\lambda}\sin(x)}{\Gamma(\lambda+1)} + \frac{t^{2\lambda}\sin(x)}{\Gamma(2\lambda+1)} - \frac{t^{3\lambda}\sin(x)}{\Gamma(3\lambda+1)} + \frac{t^{4\lambda}\sin(x)}{\Gamma(4\lambda+1)} - \frac{t^{5\lambda}\sin(x)}{\Gamma(5\lambda+1)} + \dots \tag{40}$$

Table 1 compares the results of the proposed method with the results of other existing methods at $\lambda = 1, -5 \le x \le 5$. *In comparison with the other methods, this method is more accurate.*

Table 1. The L_2-norm errors for the suggested methods when $-5 \le x \le 5$ of $u(x,t) = v(x,t)$ for Example 1 in comparison with the results of [24].

t	LADM [24]	LVIM [24]	RDTM [24]	Present Method
0.01	$1.9098963 \times 10^{-12}$	$1.9098963 \times 10^{-12}$	$1.9098875 \times 10^{-12}$	1.22125×10^{-15}
0.05	5.9294056×10^{-9}	5.9294056×10^{-9}	5.9294056×10^{-9}	1.81315×10^{-11}
0.10	1.8818028×10^{-7}	1.8818029×10^{-7}	1.8818029×10^{-7}	1.15222×10^{-9}
0.50	5.5141181×10^{-4}	5.5139119×10^{-4}	5.5141181×10^{-4}	1.70339×10^{-5}
1.00	1.6348008×10^{-2}	1.6094187×10^{-2}	1.6348008×10^{-2}	1.02052×10^{-3}

Example 2. *Consider the time-fractional coupled KdV equation [25–28]*

$$\begin{aligned} D_t^{\lambda} u(\varkappa,t) &= a_1 D_{\varkappa}^3 u(\varkappa,t) + 6a_1 u(\varkappa,t) D_{\varkappa} u(\varkappa,t) + 2b_1 v(\varkappa,t) D_{\varkappa} v(\varkappa,t), \\ D_t^{\lambda} v(\varkappa,t) &= -D_{\varkappa}^3 v(\varkappa,t) - 3u(\varkappa,t) D_{\varkappa} v(\varkappa,t), \end{aligned} \tag{41}$$

with the initial conditions

$$\begin{aligned} u(\varkappa,0) &= f_0(\varkappa) = \frac{-\rho^2(1+a_1)}{3+6a_1} + \frac{4\rho^2 e^{\rho\varkappa}}{(1+e^{\rho\varkappa})^2}, \\ v(\varkappa,0) &= g_0(\varkappa) = \frac{d e^{\rho\varkappa}}{(1+e^{\rho\varkappa})^2}, \end{aligned} \qquad a \le \varkappa \le b, \tag{42}$$

where $c = \dfrac{-a_1\rho^2}{1+2a_1}$, $d = -\rho^2\sqrt{\dfrac{-24\,a_1}{b_1}}$, $a_1 b_1 < 0$, *and* ρ *is a constant. The exact solutions of this system at* $\lambda = 1$ *are given sa* $u(\varkappa,t) = \dfrac{-\rho^2(1+a_1)}{3+6a_1} + \dfrac{4\rho^2 e^{\rho(\varkappa+ct)}}{\left(1+e^{\rho(\varkappa+ct)}\right)^2}$, $v(\varkappa,t) = \dfrac{d e^{\rho(\varkappa+ct)}}{\left(1+e^{\rho(\varkappa+ct)}\right)^2}$.

In this system, we have

$$\begin{aligned} R_1\left(u, v, D_{\varkappa}u, D_{\varkappa}v, D_{\varkappa}^2 u, D_{\varkappa}^2 v, \ldots\right) &= a_1 D_{\varkappa}^3 u(\varkappa,t), \\ R_2\left(u, v, D_{\varkappa}u, D_{\varkappa}v, D_{\varkappa}^2 u, D_{\varkappa}^2 v, \ldots\right) &= -D_{\varkappa}^3 v(\varkappa,t), \\ N_1\left(u, v, D_{\varkappa}u, D_{\varkappa}v, D_{\varkappa}^2 u, D_{\varkappa}^2 v, \ldots\right) &= 6a_1 u(\varkappa,t) D_{\varkappa} u(\varkappa,t) + 2b_1 v(\varkappa,t) D_{\varkappa} v(\varkappa,t), \\ N_2\left(u, v, D_{\varkappa}u, D_{\varkappa}v, D_{\varkappa}^2 u, D_{\varkappa}^2 v, \ldots\right) &= -3u(\varkappa,t) D_{\varkappa} v(\varkappa,t). \end{aligned}$$

So, the system (24) and (25) for Equations (41) and (42) can be written as follows:

$$\begin{aligned} \mathfrak{L}ResU_k(\varkappa,s) = {} & \sum_{n=1}^{k} \frac{f_n(\varkappa)}{s^{n\lambda+1}} - \frac{a_1}{s^{\lambda}} \sum_{n=0}^{k} \frac{f_n^{(3)}(\varkappa)}{s^{n\lambda+1}} \\ & - \frac{1}{s^{\lambda}} \mathfrak{L}\left[6a_1 \sum_{n=0}^{k} \frac{t^{n\lambda} f_n(\varkappa)}{\Gamma(n\lambda+1)} \sum_{n=0}^{k} \frac{t^{n\lambda} f_n'(\varkappa)}{\Gamma(n\lambda+1)} + 2b_1 \sum_{n=0}^{k} \frac{t^{n\lambda} g_n(\varkappa)}{\Gamma(n\lambda+1)} \sum_{n=0}^{k} \frac{t^{n\lambda} g_n'(\varkappa)}{\Gamma(n\lambda+1)} \right], \end{aligned} \tag{43}$$

$$\mathfrak{L}ResV_k(\varkappa,s) = \sum_{n=1}^{k} \frac{g_n(\varkappa)}{s^{n\lambda+1}} + \frac{1}{s^{\lambda}} \sum_{n=0}^{k} \frac{g_n^{(3)}(\varkappa)}{s^{n\lambda+1}} + \frac{3}{s^{\lambda}} \mathfrak{L}\left[\sum_{n=0}^{k} \frac{t^{n\lambda} f_n(\varkappa)}{\Gamma(n\lambda+1)} \sum_{n=0}^{k} \frac{t^{n\lambda} g_n'(\varkappa)}{\Gamma(n\lambda+1)} \right]. \tag{44}$$

For $k = 1$, *we get*

$$\begin{aligned} \mathfrak{L}ResU_1(\varkappa,s) = {} & \frac{f_1(\varkappa)}{s^{\lambda+1}} - \frac{a_1}{s^{\lambda}} \left(\frac{f_0^{(3)}(\varkappa)}{s} + \frac{f_1^{(3)}(\varkappa)}{s^{\lambda+1}} \right) - \frac{1}{s^{\lambda}} \mathfrak{L}\left[6a_1 \left(f_0(\varkappa) + \frac{t^{\lambda} f_1(\varkappa)}{\Gamma(\lambda+1)} \right) \times \right. \\ & \left. \left(f_0'(\varkappa) + \frac{t^{\lambda} f_1'(\varkappa)}{\Gamma(\lambda+1)} \right) + 2b_1 \left(g_0(\varkappa) + \frac{t^{\lambda} g_1(\varkappa)}{\Gamma(\lambda+1)} \right) \left(g_0'(\varkappa) + \frac{t^{\lambda} g_1'(\varkappa)}{\Gamma(\lambda+1)} \right) \right], \end{aligned} \tag{45}$$

$$\mathfrak{L}ResV_1(\varkappa,s) = \frac{g_1(\varkappa)}{s^{\lambda+1}} + \frac{1}{s^{\lambda}} \left(\frac{g_0^{(3)}(\varkappa)}{s} + \frac{g_1^{(3)}(\varkappa)}{s^{\lambda+1}} \right) + \frac{3}{s^{\lambda}} \mathfrak{L}\left[\left(f_0(\varkappa) + \frac{t^{\lambda} f_1(\varkappa)}{\Gamma(\lambda+1)} \right) \left(g_0'(\varkappa) + \frac{t^{\lambda} g_1'(\varkappa)}{\Gamma(\lambda+1)} \right) \right]. \tag{46}$$

Solving the system $\begin{aligned} \lim_{s\to\infty} s^{\lambda+1} \mathfrak{L}ResU_1(\varkappa,s) = 0 \\ \lim_{s\to\infty} s^{\lambda+1} \mathfrak{L}ResV_1(\varkappa,s) = 0 \end{aligned}$ *for* $f_1(\varkappa)$, $g_1(\varkappa)$, *we obtain*

$$\begin{aligned} f_1(\varkappa) &= a_1 f_0^{(3)}(\varkappa) + 6a_1 f_0(\varkappa)\, f_0'(\varkappa) + 2b_1 g_0(\varkappa)\, g_0'(\varkappa), \\ g_1(\varkappa) &= -g_0^{(3)}(\varkappa) - 3f_0(\varkappa)\, g_0'(\varkappa). \end{aligned}$$

Similarly, we can obtain both $f_k(\varkappa)$ and $g_k(\varkappa)$ for each $k = 2, 3 \ldots$. As a particular case, if we substitute by the initial conditions $f_0(\varkappa), g_(\varkappa)$ and $a_1 = -1.5, b_1 = 0.1, \rho = 0.1$, in obtained solutions, we obtain

$$f_1(\varkappa) = \frac{-0.00003\left(e^{0.1\varkappa} + e^{0.2\varkappa} - e^{0.3\varkappa} - e^{0.4\varkappa}\right)}{\left(1 + e^{0.1\varkappa}\right)^5}, \tag{47}$$

$$g_1(\varkappa) = \frac{-0.000142302\left(e^{0.1\varkappa} + e^{0.2\varkappa} - e^{0.3\varkappa} - e^{0.4\varkappa}\right)}{\left(1 + e^{0.1\varkappa}\right)^5}, \tag{48}$$

$$f_2(\varkappa) = \frac{\frac{9}{4}\left(\left(e^{0.1\varkappa} + e^{\varkappa}\right) + 3\left(e^{0.2\varkappa} + e^{0.9\varkappa}\right) - 6\left(e^{0.3\varkappa} + e^{0.8\varkappa}\right) - 42\left(e^{0.4\varkappa} + e^{0.7\varkappa}\right) - 84\left(e^{0.5\varkappa} + e^{0.6\varkappa}\right)\right) \times 10^{-8}}{\left(1 + e^{0.1\varkappa}\right)^{11}}, \tag{49}$$

$$g_2(\varkappa) = \frac{\left(1.07\left(e^{0.1\varkappa} + e^{\varkappa}\right) + 3.2\left(e^{0.2\varkappa} + e^{0.9\varkappa}\right) - 6.4\left(e^{0.3\varkappa} + e^{0.8\varkappa}\right) - 44.8\left(e^{0.4\varkappa} + e^{0.7\varkappa}\right) - 89.7\left(e^{0.5\varkappa} + e^{0.6\varkappa}\right)\right) \times 10^{-7}}{\left(1 + e^{0.1\varkappa}\right)^{11}}. \tag{50}$$

$$\vdots$$

To obtain the solutions, we substitute the values of $f_k(\varkappa)$ and $g_k(\varkappa), k = 1, 2, \ldots$ into Equations (28) and (29):

$$\begin{aligned} u(\varkappa, \mathfrak{t}) = {} & -0.000833333 + \frac{0.04e^{0.1(\varkappa+0.)}}{\left(e^{0.1(\varkappa+0.)} + 1\right)^2} - \frac{0.00003\mathfrak{t}^{\lambda}\left(e^{0.1\varkappa} + e^{0.2\varkappa} - e^{0.3\varkappa} - e^{0.4\varkappa}\right)}{\Gamma(\lambda+1)\left(1 + e^{0.1\varkappa}\right)^5} \\ & + \frac{\frac{9}{4} \times 10^{-8}\mathfrak{t}^{2\lambda}\left(\left(e^{0.1\varkappa} + e^{\varkappa}\right) + 3\left(e^{0.2\varkappa} + e^{0.9\varkappa}\right) - 6\left(e^{0.3\varkappa} + e^{0.8\varkappa}\right) - 42\left(e^{0.4\varkappa} + e^{0.7\varkappa}\right) - 84\left(e^{0.5\varkappa} + e^{0.6\varkappa}\right)\right)}{\Gamma(\lambda+1)\left(1 + e^{0.1\varkappa}\right)^{11}} + \ldots \\ v(\varkappa, \mathfrak{t}) = {} & \frac{0.189737e^{0.1(\varkappa+0.)}}{\left(e^{0.1(\varkappa+0.)} + 1\right)^2} - \frac{0.000142302\mathfrak{t}^{\lambda}\left(e^{0.1\varkappa} + e^{0.2\varkappa} - e^{0.3\varkappa} - e^{0.4\varkappa}\right)}{\Gamma(\lambda+1)\left(1 + e^{0.1\varkappa}\right)^5} \\ & + \frac{10^{-7}\,\mathfrak{t}^{2\lambda}\left(1.07\left(e^{0.1\varkappa} + e^{\varkappa}\right) + 3.2\left(e^{0.2\varkappa} + e^{0.9\varkappa}\right) - 6.4\left(e^{0.3\varkappa} + e^{0.8\varkappa}\right) - 44.8\left(e^{0.4\varkappa} + e^{0.7\varkappa}\right) - 89.7\left(e^{0.5\varkappa} + e^{0.6\varkappa}\right)\right)}{\Gamma(\lambda+1)\left(1 + e^{0.1\varkappa}\right)^{11}} + \ldots \end{aligned}$$

Table 2 displays the error norms computed at different space and time levels, indicating acceptable accuracy with the current method at $\lambda = 1$.

Table 2. Maximum error norms for different values of $\varkappa$ and $\mathfrak{t}$ of the suggested methods for $u(\varkappa, \mathfrak{t})$ and $v(\varkappa, \mathfrak{t})$ corresponds to Example 2 at $\lambda = 1$.

$\varkappa$	$\mathfrak{t}$	E_u	E_v
-5	0.1	2.966377×10^{-16}	1.408595×10^{-15}
	0.4	1.885471×10^{-14}	8.94354×10^{-14}
	0.7	1.010737×10^{-13}	4.79429×10^{-13}
	1	2.946948×10^{-13}	1.397861×10^{-12}
-2.5	0.1	1.700029×10^{-16}	8.049117×10^{-16}
	0.4	1.076743×10^{-14}	5.106332×10^{-14}
	0.7	5.771252×10^{-14}	2.737463×10^{-13}
	1	1.68289×10^{-13}	7.982642×10^{-13}
0	0.1	1.734723×10^{-18}	6.938894×10^{-18}
	0.4	5.20417×10^{-18}	2.081668×10^{-17}
	0.7	3.122502×10^{-17}	1.457168×10^{-16}
	1	1.301043×10^{-16}	6.175616×10^{-16}
2.5	0.1	1.682682×10^{-16}	7.979728×10^{-16}
	0.4	1.075875×10^{-14}	5.102863×10^{-14}
	0.7	5.765353×10^{-14}	2.734757×10^{-13}
	1	1.6806×10^{-13}	7.971679×10^{-13}

Table 2. *Cont.*

$\varkappa$	$\mathfrak{t}$	E_u	E_v
5	0.1	2.94903×10^{-16}	1.401657×10^{-15}
	0.4	1.885644×10^{-14}	8.944234×10^{-14}
	0.7	1.01039×10^{-13}	4.792694×10^{-13}
	1	2.945543×10^{-13}	1.397195×10^{-12}

Since the exact solutions do not exist for varied values of λ, we need to confirm the validity of our method by measuring absolute two-step errors $|U_n - U_{n-1}|$ and $|V_n - V_{n-1}|$. For the sake of comparison, the constants have been assumed to be $a_1 = -1, b_1 = \frac{3}{2}$, $\mathfrak{t} = 0.1$ and $\lambda = 0.5, 0.3$, and the results are listed in Table 3 in comparison to the results of the Chebyshev method [25]. Figure 1 shows the surface graphs of the approximate LRPS and the exact solutions for Equations (41) and (42) when $\varkappa \in [-5, 5], \mathfrak{t} \in [0, 1]$ and $\lambda = 1$. These subfigures clearly show that the approximate solutions $U(\varkappa, \mathfrak{t})$ and $V(\varkappa, \mathfrak{t})$ are close to the exact solutions.

Table 3. Comparison of error norms $|U_n - U_{n-1}| = |V_n - V_{n-1}|$ with the result obtained by Chebyshev method [25] for Example 2 with $\mathfrak{t} = 0.1$.

$\varkappa$	**Present Method**		**Chebyshev Method [25]**	
	$\lambda = 0.5$ $|U_2 - U_1|$	$\lambda = 0.3$ $|U_5 - U_4|$	$\lambda = 0.5$ $|U_2 - U_1|$	$\lambda = 0.3$ $|V_5 - V_4|$
0.1	4.9995×10^{-10}	1.406×10^{-9}	1.73826×10^{-5}	1.90199×10^{-5}
0.2	4.998×10^{-10}	1.405×10^{-9}	6.64154×10^{-5}	7.24797×10^{-5}
0.3	4.9955×10^{-10}	1.404×10^{-9}	1.140569×10^{-4}	1.234857×10^{-4}
0.4	4.992×10^{-10}	1.403×10^{-9}	1.090816×10^{-4}	1.150670×10^{-4}
0.5	4.9875×10^{-10}	1.402×10^{-9}	6.6253×10^{-6}	2.4281×10^{-6}
0.6	4.982×10^{-10}	1.4×10^{-9}	2.072694×10^{-4}	2.439404×10^{-4}
0.7	4.9755×10^{-10}	1.399×10^{-9}	4.911051×10^{-4}	5.623051×10^{-4}
0.8	4.968×10^{-10}	1.397×10^{-9}	7.233840×10^{-4}	8.208567×10^{-4}
0.9	4.9596×10^{-10}	1.394×10^{-9}	6.78061×10^{-4}	7.66036×10^{-4}

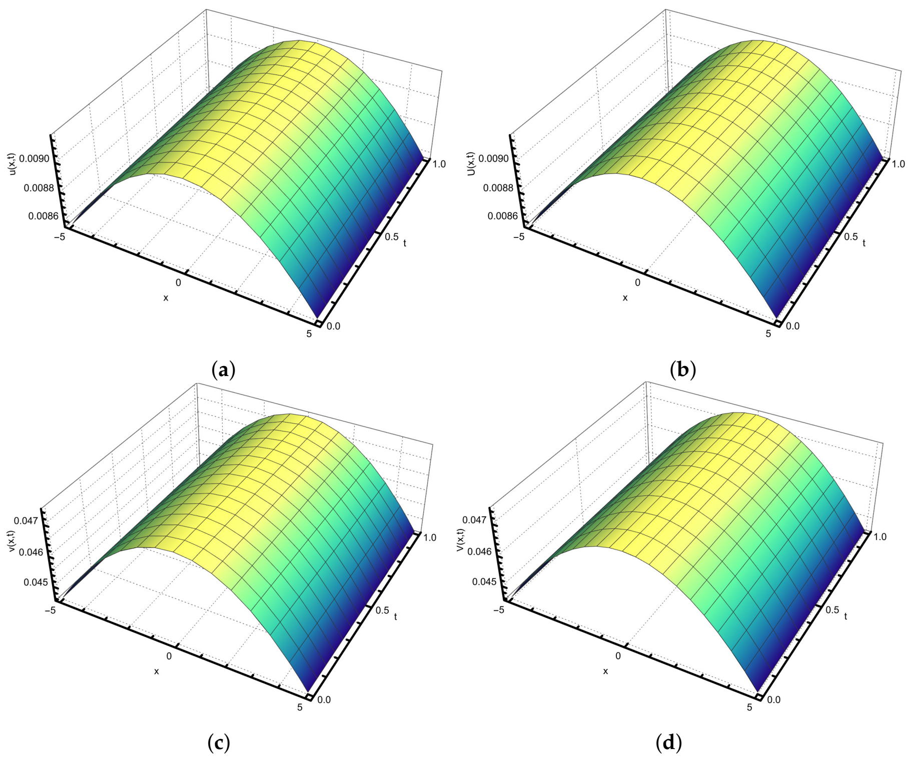


Figure 1. Comparison between the exact solutions (**a**,**c**) and the approximate solutions (**b**,**d**) of $u(\varkappa, \mathfrak{t})$ and $v(\varkappa, \mathfrak{t})$ for Example 2 at $\lambda = 1$, $\varkappa \in [-5, 5]$ and $\mathfrak{t} \in [0, 1]$.

Example 3. *Consider the nonlinear time-fractional coupled Whitham–Broer–Kaup equations [29–31]:*

$$\begin{aligned} D_t^{\lambda}u(\varkappa,t) &= -u(\varkappa,t)D_{\varkappa}u(\varkappa,t) - D_{\varkappa}v(\varkappa,t) - \xi D_{\varkappa}^2 u(\varkappa,t), \\ D_t^{\lambda}v(\varkappa,t) &= -u(\varkappa,t)D_{\varkappa}v(\varkappa,t) - v(\varkappa,t)D_{\varkappa}u(\varkappa,t) + \xi D_{\varkappa}^2 v(\varkappa,t) - \eta D_{\varkappa}^3 u(\varkappa,t), \end{aligned} \quad 0 < \lambda \leq 1. \tag{51}$$

Subject to the initial conditions

$$\begin{aligned} u(\varkappa,0) &= f_0(\varkappa) = \tfrac{1}{2}(1 - 16\tanh(-2\varkappa)), \\ v(\varkappa,0) &= g_0(\varkappa) = 16\Big(1 - \tanh^2(-2\varkappa)\Big), \end{aligned} \quad \varkappa \in [a,b]. \tag{52}$$

The linear and nonlinear parts of this system are

$$\begin{aligned} R_1\Big(u,v,D_{\varkappa}u,D_{\varkappa}v,D_{\varkappa}^2u,D_{\varkappa}^2v,\dots\Big) &= -D_{\varkappa}v(\varkappa,t) - \xi D_{\varkappa}^2 u(\varkappa,t), \\ R_2\Big(u,v,D_{\varkappa}u,D_{\varkappa}v,D_{\varkappa}^2u,D_{\varkappa}^2v,\dots\Big) &= \xi D_{\varkappa}^2 v(\varkappa,t) - \eta D_{\varkappa}^3 u(\varkappa,t), \\ N_1\Big(u,v,D_{\varkappa}u,D_{\varkappa}v,D_{\varkappa}^2u,D_{\varkappa}^2v,\dots\Big) &= -u(\varkappa,t)D_{\varkappa}u(\varkappa,t), \\ N_2\Big(u,v,D_{\varkappa}u,D_{\varkappa}v,D_{\varkappa}^2u,D_{\varkappa}^2v,\dots\Big) &= -u(\varkappa,t)D_{\varkappa}v(\varkappa,t) - v(\varkappa,t)D_{\varkappa}u(\varkappa,t). \end{aligned}$$

So, applying the system (24) and (25) for Equations (51) and (52), we obtain

$$\mathfrak{L}ResU_k(\varkappa,s) = \sum_{n=1}^{k}\frac{f_n(\varkappa)}{s^{n\lambda+1}} + \frac{1}{s^{\lambda}}\left(\sum_{n=0}^{k}\frac{g_n'(\varkappa)}{s^{n\lambda+1}} + \xi\sum_{n=0}^{k}\frac{f_n''(\varkappa)}{s^{n\lambda+1}}\right) + \frac{1}{s^{\lambda}}\mathfrak{L}\left[\sum_{n=0}^{k}\frac{t^{n\lambda}f_n(\varkappa)}{\Gamma(n\lambda+1)}\sum_{n=0}^{k}\frac{t^{n\lambda}f_n'(\varkappa)}{\Gamma(n\lambda+1)}\right], \tag{53}$$

$$\begin{aligned} \mathfrak{L}ResV_k(\varkappa,s) &= \sum_{n=1}^{k}\frac{g_n(\varkappa)}{s^{n\lambda+1}} + \frac{1}{s^{\lambda}}\left(\xi\sum_{n=0}^{k}\frac{g_n''(\varkappa)}{s^{n\lambda+1}} - \eta\sum_{n=0}^{k}\frac{f_n^{(3)}(\varkappa)}{s^{n\lambda+1}}\right) \\ &\quad + \frac{1}{s^{\lambda}}\mathfrak{L}\left[\sum_{n=0}^{k}\frac{t^{n\lambda}f_n(\varkappa)}{\Gamma(n\lambda+1)}\sum_{n=0}^{k}\frac{t^{n\lambda}g_n'(\varkappa)}{\Gamma(n\lambda+1)} + \sum_{n=0}^{k}\frac{t^{n\lambda}g_n(\varkappa)}{\Gamma(n\lambda+1)}\sum_{n=0}^{k}\frac{t^{n\lambda}f_n'(\varkappa)}{\Gamma(n\lambda+1)}\right]. \end{aligned} \tag{54}$$

To determine $f_1(\varkappa)$ and $g_1(\varkappa)$, we consider $k = 1$, which yields to

$$\begin{aligned} \mathfrak{L}ResU_1(\varkappa,s) &= \frac{f_1(\varkappa)}{s^{\lambda+1}} + \frac{1}{s^{\lambda}}\left(\frac{g_0'(\varkappa)}{s} + \frac{g_1'(\varkappa)}{s^{\lambda+1}} + \xi\frac{f_0''(\varkappa)}{s} + \xi\frac{f_1''(\varkappa)}{s^{\lambda+1}}\right) \\ &\quad + \frac{1}{s^{\lambda}}\mathfrak{L}\left[\left(f_0(\varkappa) + \frac{t^{\lambda}f_1(\varkappa)}{\Gamma(\lambda+1)}\right)\left(f_0'(\varkappa) + \frac{t^{\lambda}f_1'(\varkappa)}{\Gamma(\lambda+1)}\right)\right], \end{aligned} \tag{55}$$

$$\begin{aligned} \mathfrak{L}ResV_1(\varkappa,s) &= \frac{g_1(\varkappa)}{s^{\lambda+1}} + \frac{1}{s^{\lambda}}\left(\xi\frac{g_0''(\varkappa)}{s} + \xi\frac{g_1''(\varkappa)}{s^{\lambda+1}} - \eta\frac{f_0^{(3)}(\varkappa)}{s} - \eta\frac{f_1^{(3)}(\varkappa)}{s^{\lambda+1}}\right) \\ &\quad + \frac{1}{s^{\lambda}}\mathfrak{L}\left[\left(f_0(\varkappa) + \frac{t^{\lambda}f_1(\varkappa)}{\Gamma(\lambda+1)}\right)\left(g_0'(\varkappa) + \frac{t^{\lambda}g_1'(\varkappa)}{\Gamma(\lambda+1)}\right) + \left(g_0(\varkappa) + \frac{t^{\lambda}g_1(\varkappa)}{\Gamma(\lambda+1)}\right)\left(f_0'(\varkappa) + \frac{t^{\lambda}f_1'(\varkappa)}{\Gamma(\lambda+1)}\right)\right]. \end{aligned} \tag{56}$$

Now, to determine $f_1(\varkappa)$ and $g_1(\varkappa)$, we multiply the Equations (55) and (56) by $s^{\lambda+1}$ and $s^{\lambda+1}$, respectively, and then solve recursively the the system $\begin{aligned} \lim_{s\longrightarrow\infty} s^{\lambda+1}\mathfrak{L}ResU_1(\varkappa,s) = 0, \\ \lim_{s\longrightarrow\infty} s^{\lambda+1}\mathfrak{L}ResV_1(\varkappa,s) = 0, \end{aligned}$ for $f_1(\varkappa)$ and $g(\varkappa)$, we obtain

$$\begin{aligned} f_1(\varkappa) &= -f_0(\varkappa)f_0'(\varkappa) - g_0'(\varkappa) - \xi f_0''(\varkappa), \\ g_1(\varkappa) &= -g_0(\varkappa)f_0'(\varkappa) - f_0(\varkappa)g_0'(\varkappa) + \xi g_0''(\varkappa) - \eta f_0^{(3)}(\varkappa). \end{aligned}$$

Similarly, we determine $f_k(\varkappa)$ and $g_k(\varkappa), k = 2,3,\dots$. The following are the first few elements of the sequence $f_k(\varkappa), g_k(\varkappa)$ when $\xi = 1, \eta = 3$.

$$f_1(\varkappa) = -8sech^2(2\varkappa), \qquad g_1(\varkappa) = 32sech^2(2\varkappa)\tanh(2\varkappa),$$
$$f_2(\varkappa) = -16sech^2(2\varkappa)\tanh(2\varkappa), \qquad g_2(\varkappa) = 32(\cosh(4\varkappa) - 2)sech^4(2\varkappa),$$
$$f_3(\varkappa) = 16sech^4(2\varkappa)\left(2 - \cosh(4\varkappa) - 32\tanh(2\varkappa) + \frac{4^{\lambda+2}\Gamma(\lambda+\frac{1}{2})\tanh(2\varkappa)}{\sqrt{\pi}\Gamma(\lambda+1)}\right),$$
$$g_3(\varkappa) = 16sech^6(2\varkappa)\left(128\cosh(4\varkappa) - 10\sinh(4\varkappa) + \sinh(8\varkappa) - 192 + \frac{32(3-2\cosh(4\varkappa))\Gamma(2\lambda+1)}{\Gamma(\lambda+1)^2}\right),$$
$$\vdots$$

Consequently, by substituting in Equations (28) and (29), one can write the approximate solutions for the system (51) and (52) as the following expansion:

$$u(\varkappa,\mathfrak{t}) = \frac{1}{2}(1 - 16\tanh(-2\varkappa)) + \frac{-8\mathfrak{t}^{\lambda}sech^2(2\varkappa)}{\Gamma(\lambda+1)} + \frac{-16\mathfrak{t}^{2\lambda}sech^2(2\varkappa)\tanh(2\varkappa)}{\Gamma(2\lambda+1)}$$
$$+ 16sech^4(2\varkappa)\left(\frac{2 - \cosh(4\varkappa) - 32\tanh(2\varkappa)}{\Gamma(3\lambda+1)} + \frac{4^{\lambda+2}\Gamma(\lambda+\frac{1}{2})\tanh(2\varkappa)}{\sqrt{\pi}\Gamma(\lambda+1)\Gamma(3\lambda+1)}\right) + \dots,$$
$$v(\varkappa,\mathfrak{t}) = 16\left(1 - \tanh^2(-2\varkappa)\right) + \frac{32\mathfrak{t}^{\lambda}sech^2(2\varkappa)\tanh(2\varkappa)}{\Gamma(\lambda+1)} + \frac{32\mathfrak{t}^{2\lambda}(\cosh(4\varkappa)-2)sech^4(2\varkappa)}{\Gamma(2\lambda+1)}$$
$$+ 16sech^6(2\varkappa)\left(\frac{128\cosh(4\varkappa) - 10\sinh(4\varkappa) + \sinh(8\varkappa) - 192}{\Gamma(3\lambda+1)} + \frac{32(3-2\cosh(4\varkappa))\Gamma(2\lambda+1)}{\Gamma(\lambda+1)^2\Gamma(3\lambda+1)}\right) + \dots.$$

Table 4 summarizes the maximum absolute errors for the obtained solutions of system (51) and (52) computed at different values of $\varkappa$ and $\mathfrak{t}$. Additionally, Figure 2 shows the behavior of the approximate solutions and compares them with the exact solution. The numerical and graphical results demonstrate the harmony and convergence between the approximate and exact solutions.

Table 4. Maximum error norms for different values of $\varkappa$ and $\mathfrak{t}$ for $u(\varkappa,\mathfrak{t})$ and $v(\varkappa,\mathfrak{t})$ corresponds to Example 3 at $\xi = 1, \eta = 3$ and $\lambda = 1$.

$\varkappa$	$\mathfrak{t}$	E_u	E_v
−5	0.1	1.776357×10^{-15}	1.052059×10^{-14}
	0.2	1.758593×10^{-13}	7.095397×10^{-13}
	0.3	1.965539×10^{-12}	7.866877×10^{-12}
	0.4	1.075939×10^{-11}	4.304191×10^{-11}
−2	0.1	4.541549×10^{-10}	1.778161×10^{-9}
	0.2	2.828785×10^{-8}	1.108183×10^{-7}
	0.3	3.137571×10^{-7}	1.229783×10^{-6}
	0.4	1.717497×10^{-6}	6.734969×10^{-6}
1	0.1	7.134204×10^{-10}	7.901643×10^{-8}
	0.2	4.941075×10^{-9}	5.45743×10^{-6}
	0.3	4.701765×10^{-7}	6.711423×10^{-5}
	0.4	5.989914×10^{-6}	4.071898×10^{-5}
4	0.1	1.634248×10^{-13}	6.577691×10^{-13}
	0.2	1.085887×10^{-11}	4.34379×10^{-11}
	0.3	1.274785×10^{-10}	5.099124×10^{-10}
	0.4	7.386944×10^{-10}	2.954758×10^{-9}

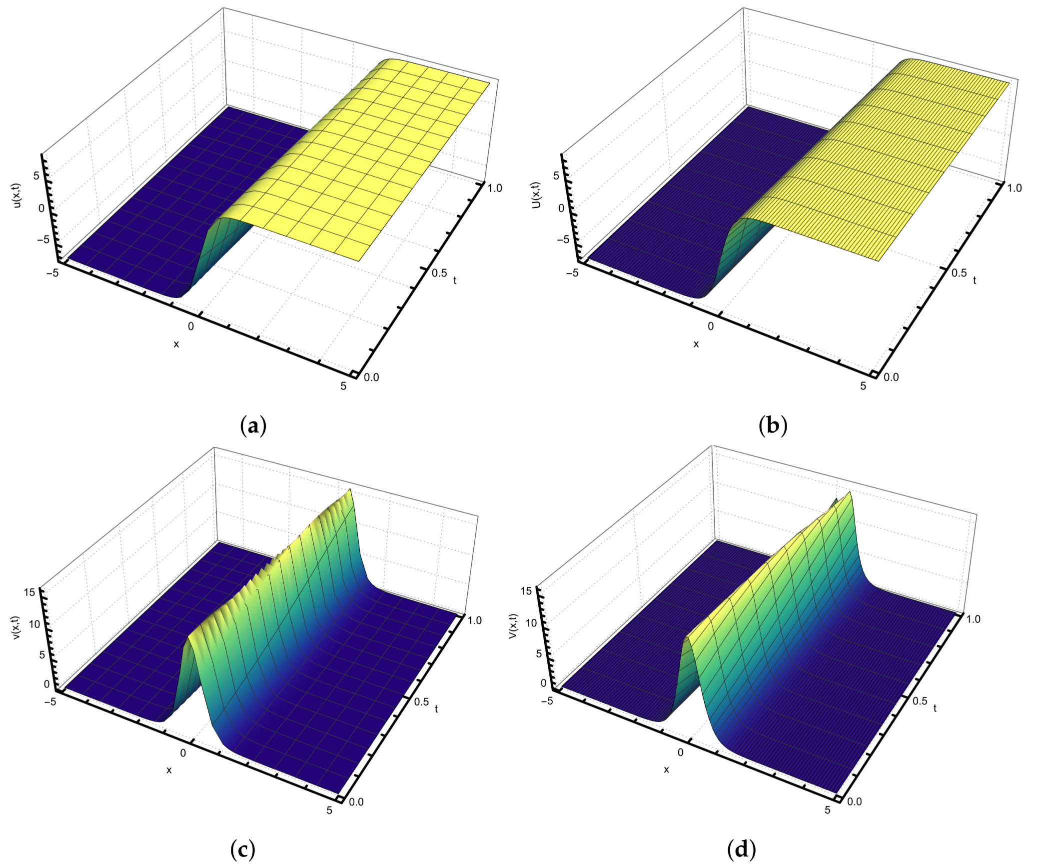


Figure 2. Comparison between the exact solutions (**a**,**c**) and the approximate solutions (**b**,**d**) of $u(\varkappa, t)$ and $v(\varkappa, t)$ for Example 3 at $\lambda = 1$, $\varkappa \in [-5, 5]$ and $t \in [0, 1]$.

4. The (2+1)-Dimensional Time-Fractional Coupled Differential Equation

In this section, we applied the LRPS method to solve the two dimensional coupled fractional Navier–Stokes equations of the form

$$\begin{aligned} &D_t^{\lambda} u(\varkappa,\omega,t) + u(\varkappa,\omega,t)u_{\varkappa}(\varkappa,\omega,t) + v(\varkappa,\omega,t)u_{\omega}(\varkappa,\omega,t) = \rho_0(u_{\varkappa\varkappa}(\varkappa,\omega,t) + u_{\omega\omega}(\varkappa,\omega,t)), \\ &D_t^{\lambda} v(\varkappa,\omega,t) + u(\varkappa,\omega,t)v_{\varkappa}(\varkappa,\omega,t) + v(\varkappa,\omega,t)v_{\omega}(\varkappa,\omega,t) = \rho_0(v_{\varkappa\varkappa}(\varkappa,\omega,t) + v_{\omega\omega}(\varkappa,\omega,t)), \end{aligned} \quad 0 < \lambda \leq 1. \tag{57}$$

Subject to the initial conditions

$$\begin{aligned} v(\varkappa,\omega,0) &= f_0(\varkappa,\omega), \\ u(\varkappa,\omega,0) &= g_0(\varkappa,\omega). \end{aligned} \tag{58}$$

Applying the Laplace transform to Equations (57) and (58), we obtain

$$\mathcal{L}\Big[D_t^{\lambda} u(\varkappa,\omega,t)\Big] = \rho_0 \mathcal{L}[u_{\varkappa\varkappa}(\varkappa,\omega,t) + u_{\omega\omega}(\varkappa,\omega,t)] - \mathcal{L}[u(\varkappa,\omega,t)u_{\varkappa}(\varkappa,\omega,t) + v(\varkappa,\omega,t)u_{\omega}(\varkappa,\omega,t)], \tag{59}$$

$$\mathcal{L}\Big[D_t^{\lambda} v(\varkappa,\omega,t)\Big] = \rho_0 \mathcal{L}[v(\varkappa,\omega,t) + v_{\omega\omega}(\varkappa,\omega,t)] - \mathcal{L}[u(\varkappa,\omega,t)v_{\varkappa}(\varkappa,\omega,t) + v(\varkappa,\omega,t)v_{\omega}(\varkappa,\omega,t)]. \tag{60}$$

Using $\mathcal{L}[D_t^{\lambda} u(\varkappa,\omega,t)] = s^{\lambda}\mathcal{L}[u(\varkappa,\omega,t)] - s^{\lambda-1}u(\varkappa,\omega,0) = s^{\lambda}\mathcal{L}[u(\varkappa,\omega,t)] - s^{\lambda-1}f_0(\varkappa,\omega)$ and $\mathcal{L}[D_t^{\lambda} v(\varkappa,\omega,t)] = s^{\lambda}\mathcal{L}[v(\varkappa,\omega,t)] - s^{\lambda-1}v(\varkappa,\omega,0) = s^{\lambda}\mathcal{L}[v(\varkappa,\omega,t)] - s^{\lambda-1}g_0(\varkappa,\omega)$, we can write Equations (59) and (60) as

$$\begin{aligned} U(\varkappa,\omega,s) =& \frac{f_0(\varkappa,\omega)}{s} + \frac{\rho_0}{s^{\lambda}}\Big(D_{\varkappa}^2 U(\varkappa,\omega,s) + D_{\omega}^2 U(\varkappa,\omega,s)\Big) \\ & - \frac{1}{s^{\lambda}}\mathcal{L}\Big[\mathcal{L}^{-1}[U(\varkappa,\omega,s)]\mathcal{L}^{-1}[D_{\varkappa}U(\varkappa,\omega,s)] + \mathcal{L}^{-1}[V(\varkappa,\omega,s)]\mathcal{L}^{-1}[D_{\omega}U(\varkappa,\omega,s)]\Big], \end{aligned} \tag{61}$$

$$V(x,w,t) = \frac{g_0(x,w)}{s} + \frac{\rho_0}{s^\lambda}\Big(D_x^2 V(x,w,s) + D_w^2 V(x,w,s)\Big) - \frac{1}{s^\lambda}\mathfrak{L}\Big[\mathfrak{L}^{-1}[U(x,w,s)]\mathfrak{L}^{-1}[D_x V(x,w,s)] + \mathfrak{L}^{-1}[V(x,w,s)]\mathfrak{L}^{-1}[D_w V(x,w,s)]\Big], \tag{62}$$

where $U(x,w,s) = \mathfrak{L}[u(x,w,t)]$, $V(x,w,s) = \mathfrak{L}[v(x,w,t)]$. By writing transformed functions $U(x,w,s)$ and $V(x,w,s)$ as fractional power series representations, we obtain

$$U(x,w,s) = \sum_{n=0}^{\infty} \frac{f_n(x,w)}{s^{n\lambda+1}}, \tag{63}$$

$$V(x,w,s) = \sum_{n=0}^{\infty} \frac{g_n(x,w)}{s^{n\lambda+1}}. \tag{64}$$

The k-th truncated series of Equations (63) and (64) take the forms

$$U_k(x,w,s) = \sum_{n=0}^{k} \frac{f_n(x,w)}{s^{n\lambda+1}}, \tag{65}$$

$$V_k(x,w,s) = \sum_{n=0}^{k} \frac{g_n(x,w)}{s^{n\lambda+1}}, \tag{66}$$

where $f_0(x,w)$ nd $g_0(x,w)$ are the initial conditions. To find the unknown coefficients of the series in Equations (61) and (62), we define the Laplace residual functions for the coupled equations in Equations (65) and (66) as follows:

$$\mathfrak{L}ResU(x,w,s) = U(x,w,s) - \frac{f_0(x,w)}{s} - \frac{\rho_0}{s^\lambda}\Big(D_x^2 U(x,w,s) + D_w^2 U(x,w,s)\Big) + \frac{1}{s^\lambda}\mathfrak{L}\Big[\mathfrak{L}^{-1}[U(x,w,s)]\mathfrak{L}^{-1}[D_x U(x,w,s)] + \mathfrak{L}^{-1}[V(x,w,s)]\mathfrak{L}^{-1}[D_w U(x,w,s)]\Big], \tag{67}$$

$$\mathfrak{L}ResV(x,w,s) = V(x,w,t) - \frac{g_0(x,w)}{s} - \frac{\rho_0}{s^\lambda}\Big(D_x^2 V(x,w,s) + D_w^2 V(x,w,s)\Big) + \frac{1}{s^\lambda}\mathfrak{L}\Big[\mathfrak{L}^{-1}[U(x,w,s)]\mathfrak{L}^{-1}[D_x V(x,w,s)] + \mathfrak{L}^{-1}[V(x,w,s)]\mathfrak{L}^{-1}[D_w V(x,w,s)]\Big]. \tag{68}$$

For the k-th Laplace residual function, we have

$$\mathfrak{L}ResU_k(x,w,s) = U_k(x,w,s) - \frac{f_0(x,w)}{s} - \frac{\rho_0}{s^\lambda}\Big(D_x^2 U_k(x,w,s) + D_w^2 U_k(x,w,s)\Big) + \frac{1}{s^\lambda}\mathfrak{L}\Big[\mathfrak{L}^{-1}[U_k(x,w,s)]\mathfrak{L}^{-1}[D_x U_k(x,w,s)] + \mathfrak{L}^{-1}[V_k(x,w,s)]\mathfrak{L}^{-1}[D_w U_k(x,w,s)]\Big], \tag{69}$$

$$\mathfrak{L}ResV_k(x,w,s) = V_k(x,w,t) - \frac{g_0(x,w)}{s} - \frac{\rho_0}{s^\lambda}\Big(D_x^2 V_k(x,w,s) + D_w^2 V_k(x,w,s)\Big) + \frac{1}{s^\lambda}\mathfrak{L}\Big[\mathfrak{L}^{-1}[U_k(x,w,s)]\mathfrak{L}^{-1}[D_x V_k(x,w,s)] + \mathfrak{L}^{-1}[V_k(x,w,s)]\mathfrak{L}^{-1}[D_w V_k(x,w,s)]\Big]. \tag{70}$$

By substituting Equations (65) and (66) into Equations (69) and (70), we obtain

$$\mathfrak{L}ResU_k(x,w,s) = \sum_{n=1}^{k} \frac{f_n(x,w)}{s^{n\lambda+1}} - \frac{\rho_0}{s^\lambda}\left(\sum_{n=0}^{k} \frac{(f_n)(x,w)}{s^{n\lambda+1}} + \sum_{n=0}^{k} \frac{(f_n)_{ww}(x,w)}{s^{n\lambda+1}}\right) + \frac{1}{s^\lambda}\mathfrak{L}\left[\mathfrak{L}^{-1}\left[\sum_{n=0}^{k} \frac{f_n(x,w)}{s^{n\lambda+1}}\right]\mathfrak{L}^{-1}\left[\sum_{n=0}^{k} \frac{(f_n)_x(x,w)}{s^{n\lambda+1}}\right] + \mathfrak{L}^{-1}\left[\sum_{n=0}^{k} \frac{g_n(x,w)}{s^{n\lambda+1}}\right]\mathfrak{L}^{-1}\left[\sum_{n=0}^{k} \frac{(f_n)_w(x,w)}{s^{n\lambda+1}}\right]\right], \tag{71}$$

$$\mathfrak{L}ResV_k(x,w,s) = \sum_{n=1}^{k} \frac{g_n(x,w)}{s^{n\lambda+1}} - \frac{\rho_0}{s^\lambda}\left(\sum_{n=0}^{k} \frac{(g_n)_{xx}(x,w)}{s^{n\lambda+1}} + \sum_{n=0}^{k} \frac{(g_n)_{ww}(x,w)}{s^{n\lambda+1}}\right)$$

$$-\frac{1}{s^{\lambda}}\mathfrak{L}\left[\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{f_n(x,w)}{s^{n\lambda+1}}\right]\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{(g_n)_x(x,w)}{s^{n\lambda+1}}\right]+\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{g_n(x,w)}{s^{n\lambda+1}}\right]\mathfrak{L}^{-1}\left[\sum_{n=0}^{k}\frac{(g_n)_w(x,w)}{s^{n\lambda+1}}\right]\right]. \quad (72)$$

The last system can be written as

$$\mathfrak{L}ResU_k(x,w,s)=\sum_{n=1}^{k}\frac{f_n(x,w)}{s^{n\lambda+1}}-\frac{\rho_0}{s^{\lambda}}\left(\sum_{n=0}^{k}\frac{(f_n)_{xx}(x,w)}{s^{n\lambda+1}}+\sum_{n=0}^{k}\frac{(f_n)_{ww}(x,w)}{s^{n\lambda+1}}\right)$$
$$+\frac{1}{s^{\lambda}}\mathfrak{L}\left[\sum_{n=0}^{k}\frac{t^{n\lambda}f_n(x,w)}{\Gamma(n\lambda+1)}\sum_{n=0}^{k}\frac{t^{n\lambda}(f_n)_x(x,w)}{\Gamma(n\lambda+1)}+\sum_{n=0}^{k}\frac{t^{n\lambda}g_n(x,w)}{\Gamma(n\lambda+1)}\sum_{n=0}^{k}\frac{t^{n\lambda}(f_n)_w(x,w)}{\Gamma(n\lambda+1)}\right], \quad (73)$$
$$\mathfrak{L}ResV_k(x,w,s)=\sum_{n=1}^{k}\frac{g_n(x,w)}{s^{n\lambda+1}}-\frac{\rho_0}{s^{\lambda}}\left(\sum_{n=0}^{k}\frac{(g_n)_{xx}(x,w)}{s^{n\lambda+1}}+\sum_{n=0}^{k}\frac{(g_n)_{ww}(x,w)}{s^{n\lambda+1}}\right)$$
$$-\frac{1}{s^{\lambda}}\mathfrak{L}\left[\sum_{n=0}^{k}\frac{t^{n\lambda}f_n(x,w)}{\Gamma(n\lambda+1)}\sum_{n=0}^{k}\frac{t^{n\lambda}(g_n)_x(x,w)}{\Gamma(n\lambda+1)}+\sum_{n=0}^{k}\frac{t^{n\lambda}g_n(x,w)}{\Gamma(n\lambda+1)}\sum_{n=0}^{k}\frac{t^{n\lambda}(g_n)_w(x,w)}{\Gamma(n\lambda+1)}\right]. \quad (74)$$

To determine $f_1(x,w)$ and $g_1(x,w)$, we consider $k=1$ in Equations (73) and (74) and obtain

$$\mathfrak{L}ResU_1(x,w,s)=\frac{f_1(x,w)}{s^{\lambda+1}}-\frac{\rho_0}{s^{\lambda}}\left(\frac{f_{0_{xx}}(x,w)}{s}+\frac{f_{1_{xx}}(x,w)}{s^{\lambda+1}}+\frac{f_{0_{ww}}(x,w)}{s}+\frac{f_{1_{ww}}(x,w)}{s^{\lambda+1}}\right)$$
$$+\frac{1}{s^{\lambda}}\mathfrak{L}\Bigg[f_0(x,w)f_{0_x}(x,w)+\frac{t^{\lambda}(f_1(x,w)f_{0_x}(x,w)+f_0(x,w)f_{1_x}(x,w))}{\Gamma(\lambda+1)}+\frac{t^{2\lambda}f_1(x,w)f_{1_x}(x,w)}{(\Gamma(\lambda+1))^2}$$
$$+g_0(x,w)f_{0_w}(x,w)+\frac{t^{\lambda}g_1(x,w)f_{0_w}(x,w)}{\Gamma(\lambda+1)}+\frac{t^{\lambda}g_0(x,w)f_{1_w}(x,w)}{\Gamma(\lambda+1)}+\frac{t^{\lambda+\lambda}g_1(x,w)f_{1_w}(x,w)}{(\Gamma(\lambda+1))^2}\Bigg], \quad (75)$$
$$\mathfrak{L}ResV_1(x,w,s)=\frac{g_1(x,w)}{s^{\lambda+1}}-\frac{\rho_0}{s^{\lambda}}\left(\frac{g_{0_{xx}}(x,w)}{s}+\frac{g_{1_{xx}}(x,w)}{s^{\lambda+1}}+\frac{g_{0_{ww}}(x,w)}{s}+\frac{g_{1_{ww}}(x,w)}{s^{\lambda+1}}\right)$$
$$-\frac{1}{s^{\lambda}}\mathfrak{L}\Bigg[f_0(x,w)g_{0_x}(x,w)+\frac{t^{\lambda}f_1(x,w)g_{0_x}(x,w)}{\Gamma(\lambda+1)}+\frac{t^{\lambda}f_0(x,w)g_{1_x}(x,w)}{\Gamma(\lambda+1)}+\frac{t^{\lambda+\lambda}f_1(x,w)g_{1_x}(x,w)}{(\Gamma(\lambda+1))^2}$$
$$+g_0(x,w)f_{0_w}(x,w)+\frac{t^{\lambda}(g_1(x,w)f_{0_w}(x,w)+g_0(x,w)g_{1_w}(x,w))}{\Gamma(\lambda+1)}+\frac{t^{2\lambda}g_1(x,w)g_{1_w}(x,w)}{(\Gamma(\lambda+1))^2}\Bigg]. \quad (76)$$

Solving the system $\begin{array}{l}\lim_{s\longrightarrow\infty} s^{\lambda+1}\mathfrak{L}ResU_1(x,w,s)=0,\\ \lim_{s\longrightarrow\infty} s^{\lambda+1}\mathfrak{L}ResV_1(x,w,s)=0,\end{array}$ for $f_1(x,w)$ and $g_1(x,w)$, we obtain

$$f_1(x,w)=-g_0(x,w)\frac{\partial f_0(x,w)}{\partial w}+\rho\frac{\partial^2 f_0(x,w)}{\partial w^2}-f_0(x,w)\frac{\partial f_0(x,w)}{\partial x}+\rho\frac{\partial^2 f_0(x,w)}{\partial x^2}, \quad (77)$$
$$g_1(x,w)=-g_0(x,w)\frac{\partial g_0(x,w)}{\partial w}+\rho\frac{\partial^2 g_0(x,w)}{\partial w^2}-f_0(x,w)\frac{\partial g_0(x,w)}{\partial x}+\rho\frac{\partial^2 g_0(x,w)}{\partial x^2}. \quad (78)$$

Continuing in that manner to calculate $f_k(x,w)$ and $g_k(x,w)$, $k=2,3,\ldots,$ we solve the following system for each $k=2,3,\ldots$

$$\lim_{s\longrightarrow\infty} s^{k\lambda+1}\mathfrak{L}ResU_k(x,w,s)=0, \quad (79)$$
$$\lim_{s\longrightarrow\infty} s^{k\lambda+1}\mathfrak{L}ResV_k(x,w,s)=0. \quad (80)$$

Finally, by substituting the series solution $f_k(x,w)$ and $g_k(x,w)$, $k=1,2,\ldots$ obtained from Equations (77)–(80) into Equations (65) and (66) and taking the inverse Laplace transform, we obtain the solutions of system (57) and (58) as follows:

$$u(\varkappa,\mathfrak{w},\mathfrak{t}) = \sum_{n=0}^{\infty} \frac{\mathfrak{t}^{n\lambda} f_n(\varkappa,\mathfrak{w})}{\Gamma(n\lambda+1)}, \tag{81}$$

$$v(\varkappa,\mathfrak{w},\mathfrak{t}) = \sum_{n=0}^{\infty} \frac{\mathfrak{t}^{n\lambda} g_n(\varkappa,\mathfrak{w})}{\Gamma(n\lambda+1)}. \tag{82}$$

Example 4. *Let us assume two-dimensional incompressible time-fractional Navier–Stokes equations as [32,33]*

$$\begin{array}{l} D_{\mathfrak{t}}^{\lambda} u + u\frac{\partial u}{\partial \varkappa} + v\frac{\partial u}{\partial \mathfrak{w}} = \frac{1}{2}\left(\frac{\partial^2 u}{\partial \varkappa^2} + \frac{\partial^2 u}{\partial \mathfrak{w}^2}\right), \\ D_{\mathfrak{t}}^{\lambda} v + u\frac{\partial v}{\partial \varkappa} + v\frac{\partial v}{\partial \mathfrak{w}} = \frac{1}{2}\left(\frac{\partial^2 v}{\partial \varkappa^2} + \frac{\partial^2 v}{\partial \mathfrak{w}^2}\right), \end{array}, \quad 0 < \lambda \leq 1. \tag{83}$$

Subject to the initial conditions

$$\begin{array}{l} v(\varkappa,\mathfrak{w},0) = \sin(\varkappa+\mathfrak{w}), \\ u(\varkappa,\mathfrak{w},0) = -\sin(\varkappa+\mathfrak{w}). \end{array} \tag{84}$$

According to the discussion and obtained results in Section 4, Equations (77)–(80), the series coefficients are as follows:

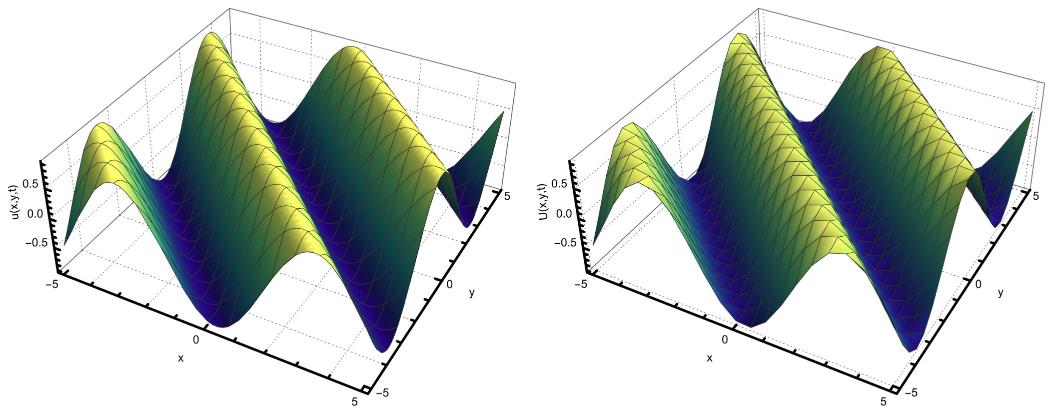
$$\begin{array}{l} f_1(\varkappa,\mathfrak{w}) = \sin(\varkappa+\mathfrak{w}),\ g_1(\varkappa,\mathfrak{w}) = -\sin(\varkappa+\mathfrak{w}), \\ f_2(\varkappa,\mathfrak{w}) = -\sin(\varkappa+\mathfrak{w}),\ g_2(\varkappa,\mathfrak{w}) = \sin(\varkappa+\mathfrak{w}), \\ f_3(\varkappa,\mathfrak{w}) = \sin(\varkappa+\mathfrak{w}),\ g_3(\varkappa,\mathfrak{w}) = -\sin(\varkappa+\mathfrak{w}), \\ f_4(\varkappa,\mathfrak{w}) = -\sin(\varkappa+\mathfrak{w}),\ g_4(\varkappa,\mathfrak{w}) = \sin(\varkappa+\mathfrak{w}), \\ f_5(\varkappa,\mathfrak{w}) = \sin(\varkappa+\mathfrak{w}),\ g_5(\varkappa,\mathfrak{w}) = -\sin(\varkappa+\mathfrak{w}) \ldots. \end{array}$$

Using Equations (81) and (82), we obtain

$$U(\varkappa,\mathfrak{w},\mathfrak{t}) = -\sin(\varkappa+\mathfrak{w}) + \frac{\mathfrak{t}^{\lambda}\sin(\varkappa+\mathfrak{w})}{\Gamma(\lambda+1)} - \frac{\mathfrak{t}^{2\lambda}\sin(\varkappa+\mathfrak{w})}{\Gamma(2\lambda+1)} + \frac{\mathfrak{t}^{3\lambda}\sin(\varkappa+\mathfrak{w})}{\Gamma(3\lambda+1)} - \frac{\mathfrak{t}^{4\lambda}\sin(\varkappa+\mathfrak{w})}{\Gamma(4\lambda+1)} + \frac{\mathfrak{t}^{5\lambda}\sin(\varkappa+\mathfrak{w})}{\Gamma(5\lambda+1)} - \ldots$$

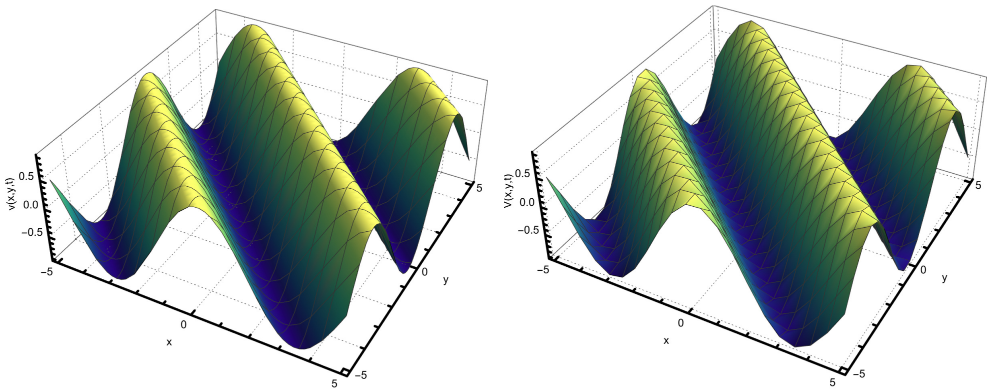
$$V(\varkappa,\mathfrak{w},\mathfrak{t}) = \sin(\varkappa+\mathfrak{w}) - \frac{\mathfrak{t}^{\lambda}\sin(\varkappa+\mathfrak{w})}{\Gamma(\lambda+1)} + \frac{\mathfrak{t}^{2\lambda}\sin(\varkappa+\mathfrak{w})}{\Gamma(2\lambda+1)} - \frac{\mathfrak{t}^{3\lambda}\sin(\varkappa+\mathfrak{w})}{\Gamma(3\lambda+1)} + \frac{\mathfrak{t}^{4\lambda}\sin(\varkappa+\mathfrak{w})}{\Gamma(4\lambda+1)} - \frac{\mathfrak{t}^{5\lambda}\sin(\varkappa+\mathfrak{w})}{\Gamma(5\lambda+1)} + \ldots$$

The efficiency of the proposed algorithm for Example 4 is shown in Figure 3. These subfigures depict surfaces of approximate and exact solutions for systems (83) and (84) at $\mathfrak{t} = 0.1,\ \lambda = 1$ *and* $\varkappa,\mathfrak{w} \in [-5,5]$.



(**a**) (**b**)

Figure 3. *Cont.*



(**c**) (**d**)

Figure 3. Comparison between the exact solutions (**a**,**c**) and the approximate solutions (**b**,**d**) of $u(\varkappa, \mathfrak{t})$ and $v(\varkappa, \mathfrak{t})$ for Example 4 at $\mathfrak{t} = 0.1,\ \lambda = 1$, and $\varkappa, w \in [-5, 5]$.

5. Conclusions

In the present study, the LRPS method is successfully applied to find the analytical solution of the (1+1)- and (2+1)-dimensional time-fractional coupled differential equations. The obtained results demonstrate the reliability and simplicity of the method. The proposed technique has the advantage of reducing the size of computation needed to figure out the coefficients in a power series form. The proposed expansion in our study allowed us to obtain a series solution for the equations in Laplace transform space. In comparison with other techniques, LRPS method is a competent tool to obtain the analytical solution of coupled nonlinear time-fractional partial differential equations.

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