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Performance Analysis of Fully Intuitionistic Fuzzy Multi-Objective Multi-Item Solid Fractional Transportation Model

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Abstract: An investigation is conducted in this paper into a performance analysis of fully intuitionistic fuzzy multi-objective multi-item solid fractional transport model (FIF-MMSFTM). It is to be anticipated that the parameters of the conveyance model will be imprecise by virtue of numerous uncontrollable factors. The model under consideration incorporates intuitionistic fuzzy (IF) quantities of shipments, costs and profit coefficients, supplies, demands, and transport. The FIF-MMSFTM that has been devised is transformed into a linear form through a series of operations. The accuracy function and ordering relations of IF sets are then used to reduce the linearized model to a concise multi-objective multi-item solid transportation model (MMSTM). Furthermore, an examination is conducted on several theorems that illustrate the correlation between the FIF-MMSFTM and its corresponding crisp model, which is founded upon linear, hyperbolic, and parabolic membership functions. A numerical example was furnished to showcase the efficacy and feasibility of the suggested methodology. The numerical data acquired indicates that the linear, hyperbolic, and parabolic models require fewer computational resources to achieve the optimal solution. The parabolic model has the greatest number of iterations, in contrast to the hyperbolic model which has the fewest. Additionally, the elapsed run time for the three models is a negligible amount of time: 0.2, 0.15, and 1.37 s, respectively. In conclusion, suggestions for future research are provided.

Keywords: intuitionistic fuzzy set; multi-objective optimization; accuracy function fractional transportation problem

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1. Introduction

Transportation issues (TP) have been the subject of considerable scholarly attention [1–7] due to their critical significance in supply chain management and logistics (e.g., cost reduction and service quality improvement) [4]. Hitchcock first introduced the classic Transportation Problems (TPs) [8], which Shell subsequently expanded to Solid Transportation Problems (STPs) [9], which integrated a novel set of conveyances. This development resulted in three-dimensional TPs that more closely resemble real-world applications. Subsequently, scholars investigated STPs through various lenses. Haley [10] proposed a solution procedure for STPs using a modified distribution method. Patel and Tripathy [11] examined a computationally preferable approach for STPs with mixed constraints. Baidya et al. [12] introduced the concept of a safety factor in TPs and scrutinized an STP that utilized fuzzy numbers to represent imprecise parameters. Chen et al. [13] addressed STPs in fuzzy and interval environments. In addition, Chen et al. [14] provided STPs with an entropy function implemented in an ambiguous circumstance.

A multi-objective transportation problem (MOTP), which evaluates distinct objectives at various dimensions and is inherently contradictory, frequently reflects the practical situation. The task of simultaneously optimizing all objectives while adhering to specified constraints presents a formidable obstacle. Numerous disciplines, including engineering, economics, and logistics, use MOTP to balance competing objectives through trade-offs in order to achieve optimal outcomes. Bit et al. [3] were the first to apply fuzzy linear programming to multi-objective STP solutions. Also, Kundu et al. [15] looked into multi-objective STP in uncertain settings and the vagueness of parameters related to problems like this that can happen when there is not enough information or when the financial markets are unstable. This entailed representing unit transportation costs as random, fuzzy, and hybrid variables, respectively.

Practically speaking, it is common for businesses to manufacture multiple products in an effort to maximize profits. Afterwards, businesses distribute these products to various destinations using diverse modes of conveyance. Multiple elements are therefore typically required to solve STPs. The STP with multiple items has been the subject of extensive research, including that of Liu et al. [16], who examined a single-objective, multi-item fixed-charge STP having ambiguous inputs.

At this time, decision-makers (DMs) are primarily concerned with the accurate determination of parameter values [4]. Given the intrinsic ambiguity, it is imperative to establish clear parameters for the model, such as the objective function coefficients and constraints [17,18]. As a result, it is logical to solicit a variety of descriptive perspectives from authorities and thought leaders regarding these parameters, which may be considered imprecise data [18,19]. Uncertainty can be caused by a variety of uncontrollable factors. For example, DMs' initial omission of transportation costs could result in unpredictability regarding cost estimation [19]. As a result of fierce competition, market conditions in the contemporary business landscape are inherently volatile, generating erratic demand for recently introduced products. Furthermore, there may be uncertainties concerning the accessibility of resources at the origin as a result of a multitude of factors. It is possible that the necessary quantity of resources will not be available for transportation [19,20]. Moreover, when a requester requires additional resources, the supplier's resource allocation becomes uncertain. Requesters' rapid modification of demands via email or mobile communication contributes to the inherent uncertainty [19].

Substantial advancements in the domain of network design have led to a multitude of approaches in managing uncertainty. Researchers have classified various approaches into three categories: intuitionistic, imprecise, and uncertain. The works cited in the text are as follows: Ghosh et al. [21], Rizk-Allah et al. [22], Biswas et al. [23], and Si-faoui and Aïder [24]. A TP denotes itself as "fully fuzzy" when every parameter, including decision variables, is fuzzy. Several researchers, including Jalil et al. [25] and Ebrahimnejad [26], have investigated entirely fuzzy TPs with respect to individual items. Ziqan et al. [27] investigated completely fuzzy linear systems using fuzzy numbers. When faced with imprecise constraints such as availability, demand, and conveyance capacity, fuzzy numbers are frequently employed to represent them. On the contrary, traditional Fuzzy Set (FS) theory only considers gratification in relation to fuzziness, neglecting to consider dissatisfaction. To get around this problem, Atanassov came up with a better framework called the Intuitionistic Fuzzy Set (IFS). This framework combined levels of membership and non-membership, which made it easier to handle uncertainty [28]. Prior scholars such as Ebrahimnejad and Verdegay [29], Midya et al. [30], and Roy and Midya [31] have integrated IFS into the formulations of TP. Among the various forms of IFNs, we frequently use triangular or trapezoidal IFNs to control ambiguity.

A number of scholars have emphasized seminal articles concerning TPs and their possible uses. FST has become a fundamental principle in improving transportation frameworks, specifically with regard to their operational implementations [32,33]. Ammar and Youness [2] conducted an investigation into MOTPs using imprecise numbers. Fuzzy programming techniques have tackled MOTPs with a variety of non-linear membership functions [18]. Lee and Abd Elwahed [1] presented an implementation of Fuzzy Goal Programming (FGP) for MOTPs. We employed FGP in conjunction with non-linear membership to address MOTPs [34,35]. Gupta and Kumar [36] proposed a method for handling linear MOTPs with ambiguous characteristics. Roy and Mahapatra [37] introduced MOTPs utilizing interval and formula within the stochastic environment. Roy et al. [19] investigated multi-choice TPs with exponential distribution. Mahapatra et al. [38] exhibited an extremist value distribution incorporating multi-choice stochastic TPs. Maity and Roy [39] utilized the utility function approach to analyze MOTPs under multi-decision conditions. Maity and Roy [40] also proposed an alternative method for dealing with MOTPs in the presence of nonlinear demand and cost variables. All of the parameters in the method for solving MOTPs proposed by Gupta and Kumar [36] are interval fuzzy numbers. Kocken et al. [41] introduced a compensatory fuzzy method for fuzzy parameter MOTP resolution. Roy et al. [42] examined an innovative method for resolving IF MOTPs. In their work, Mahajan and Gupta [17] put forth an entirely IF MOTP that features a multitude of membership functions.

In various practical scenarios, such as analyzing the economic aspects of transportation projects and management situations, TPs with ratio objective functions often serve as performance metrics. Some ratio goals in TPs are to obtain the best overall statistical transportation charges to overall average transportation charges, the best gross returns to gross operations, the best project resources to principal, and the best overall tariffs to overall general costs on leech [43]. Cetin and Tiryaki proposed a fuzzy approach for fractional MOTPs, utilizing a generalized version of Dinkelbach's algorithm. In a setting with fuzzy random hybridized ambiguity, Nasseri and Bavandi demonstrated multi-choice linear programming, as well as its application to a multicommodity TP [44]. El Sayed and Abo-Sinna presented an innovative method for a fully IF fractional MOTP, representing all parameters and variables as IFNs [45]. El Sayed and Baky presented a multi-choice fractional stochastic MOTP [46]. Additionally, Devnath et al. introduced a fully fuzzy multi-item, fixed charge in dimensions with breakability during transport [47]. Mondal et al. considered an IF sustainable multi-objective multi-item multi-choice step fixed-charge STP [48]. Chhibber et al. [49] presented a literature review on fuzzy TPs, which are non-linear IF multi-objective problems. Malik et al. exhibited a method for dealing with fully interval-valued via goal programming IF MOTPs [50]. In their paper [51], Bind et al. described a means of solving an enduring multi-objective multi-item 4D STP that uses triangular intuitionistic fuzzy numbers (TIFN).

A thorough review of the literature reveals a potentially effective approach in the quest for solutions to the complex issues presented by nonlinear TPs: the use of global optimization techniques. Scholars have investigated a variety of approaches to address the inherent complexity of nonlinear TPs in this domain. After conducting a thorough examination and synthesis of prior research, it becomes indisputable that the implementation of global optimization strategies has considerable capacity to provide efficacious resolutions. Researchers have devised various global optimization methods to resolve non-linear TPs, including the branch and reduce method, the branch and cut method, and a hybrid approach that combines local and global search strategies [52]. We have implemented mixed integer nonlinear programming methods, such as the extended cutting plane, branch and reduce, branch and cut, and simple branch and bound, to address nonlinear discrete TPs [53]. This methodology not only acknowledges the nonlinear characteristics of TPs, but it also surpasses the constraints of traditional optimization techniques by incorporating sophisticated algorithms and expanding the search space. Through the integration of knowledge from multiple fields, the proposed methodology emerges as a strong contender, providing notable benefits in comparison to conventional optimization methods, including improved precision, scalability, and resilience.

This study aims to introduce the FIF-MMSFTM, a model that has not previously been described in the literature. We have developed a model that optimizes the profit-to-cost ratio of a product shipment unit quantity from its source to its destination through a specific mode. An IFN represents each coefficient and parameter to mitigate the challenges associated with data acquisition during the modeling phase of real-world TPs. Additionally, we construct three distinct solution models using linear, parabolic, and hyperbolic functions, respectively, to equip the DM with the ability to distinguish among various solutions. Furthermore, the FIF-MMSFTM provides the solution in the form of IFNs with considerably fewer computational efforts, a considerably shorter runtime, and less precise numbers. Ultimately, we can expand the present framework to include a large-scale FIF-MMSFTM.

Although a multitude of studies have investigated STPs in a variety of contexts, it is apparent that linear and single-objective STPs may not be adequate to tackle the intricacies of the real world. In light of this constraint, we proposed a FIF-MMSFTM to bridge this void. This model integrates IF factors, including supplies, demands, transportation, and shipped quantity, profit, and cost coefficients. Firstly, we conceptualize a transformation procedure that converts the model into a linear format. Subsequently, we employ the ordering relations and accuracy function of IFS to further simplify the linearized model and generate a crisp MSTM. In our solution model, we build upon Zimmerman's approach [54] in order to optimize membership functions while minimizing non-membership functions. We incorporate linear, hyperbolic, and parabolic membership into the solution model. We provide a numeric illustration to showcase the practicality and effectiveness of the proposed methodology.

This article is structured as follows: After the introduction, Section 2 provides key ideas and preliminary work. Section 3 establishes the creation of the FIF-MMSFTM. Section 4 outlines the technique of linearizing the FIF-MMSFTM. Sections 5 and 6 elaborate on the development of the suggested strategy and an algorithm for solution procedures, respectively. Section 7 presents an example to demonstrate the applicability of the suggested methodology. Lastly, the paper concludes with some key insights.

2. Preliminaries and Notions

This section outlines the main concepts of IFSs and IFNs [17,28,55,56]. Table 1 displays the abbreviations and nomenclature used in the current study.

Table 1. Abbreviations and nomenclature.

FIF-MMSFTM	Fully intuitionistic fuzzy multi-objective multi-item solid fractional transportation model
IF	Intuitionistic fuzzy
MMSTM	Multi-objective multi-item solid transportation model
TP	Transportation problems
STP	Solid transportation problems
MOTP	Multi-objective transportation problem
DM	Decision-makers
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
FGP	Fuzzy goal programming
IFN	Intuitionistic fuzzy numbers
TIFN	Triangular intuitionistic fuzzy number
AF	Accuracy function
LMF	Linear membership functions
HMF	Hyperbolic membership functions
PMF	Parabolic membership functions

Definition 1. *An IFS* $\overset{\sim I}{A}$ *in X is a set of ordered triples* $\overset{\sim I}{A} = \left\{ \left(x, \mu_{\underset{A}{\sim I}}(x), v_{\underset{A}{\sim I}}(x) \right) \middle| x \in X \right\}$, *where* $\mu_{\underset{A}{\sim I}}(x),\ v_{\underset{A}{\sim I}}(x) : X \to [0,\ 1]$ *are functions such that* $0 \leq \mu_{\underset{A}{\sim I}}(x) + v_{\underset{A}{\sim I}}(x) \leq 1,\ \ \forall x \in X.$ *The value of* $\mu_{\underset{A}{\sim I}}(x)$ *acts as the grade of membership and* $v_{\underset{A}{\sim I}}(x)$ *acts as the grade of non-membership*

of the element $x \in X$ *being in* $\overset{\sim I}{A}$. $h(x) = 1 - \mu_{\overset{\sim I}{A}}(x) - v_{\overset{\sim I}{A}}(x)$ *represents the grade of hesitation for the element* x *in* $\overset{\sim I}{A}$ *[28].*

Definition 2. *A TIFN* $\overset{\sim I}{A} = (a_1, a_2, a_3; a_1', a_2, a_3')$ *(Figure 1) is an IFS with membership and non-membership as follows [17]:*

$$\mu_{\overset{\sim I}{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 < x \le a_2, \\ 1, & x = a_2, \\ \frac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3, \\ 0, & otherwise, \end{cases} \tag{1}$$

$$\mu_{\overset{\sim I}{v}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'}, & a_1' < x \le a_2, \\ 0, & x = a_2, \\ \frac{x-a_2}{a_3'-a_2}, & a_2 \le x \le a_3', \\ 1, & otherwise, \end{cases} \tag{2}$$

where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$.

Definition 3. *A TIFNs* $\overset{\sim I}{A} = (a_1, a_2, a_3; a_1', a_2, a_3')$ *is thought to be a positive TIFN iff,* $a_1' \ge 0$ *[17,42,56].*

Definition 4. *Arithmetic operations on TIFNs [42,56]*

Let $\overset{\sim I}{A} = (a_1, a_2, a_3; a_1', a_2, a_3')$ *and* $\overset{\sim I}{B} = (b_1, b_2, b_3; b_1', b_2, b_3')$ *be a positive TIFN then;*

Addition: $\overset{\sim I}{A} + \overset{\sim I}{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3; a_1' + b_1', a_2 + b_2, a_3' + b_3')$;

Subtraction: $\overset{\sim I}{A} - \overset{\sim I}{B} = (a_1 - b_3, a_2 + b_2, a_3 + b_1; a_1' - b_3', a_2 + b_2, a_3' - b_1')$;

Multiplication: $\overset{\sim I}{A} \times \overset{\sim I}{B} = (a_1 b_1, a_2 b_2, a_3 b_3; a_1' b_1', a_2' b_2', a_3' b_3')$.

Definition 5. *(The Accuracy function) (AF) Let* $\overset{\sim I}{A} = (a_1, a_2, a_3; a_1', a_2, a_3')$ *be TIFN. The score function for the membership* $\mu_{\overset{\sim I}{A}}$ *and non-membership* $v_{\overset{\sim I}{A}}$ *function is indicated by* $S\left(\mu_{\overset{\sim I}{A}}\right)$ $S\left(v_{\overset{\sim I}{A}}\right)$ *and obtained as* $S\left(\mu_{\overset{\sim I}{A}}\right) = \frac{a_1+2a_2+a_3}{4}$, $S\left(v_{\overset{\sim I}{A}}\right) = \frac{a_1'+2a_2+a_3'}{4}$. *The AF of* $\overset{\sim I}{A}$ *is symbolized by* $\Re\left(\overset{\sim I}{A}\right)$ *and established as follows [42,56,57]:*

$$\Re\left(\overset{\sim I}{A}\right) = \frac{S\left(\mu_{\overset{\sim I}{A}}\right) + S\left(v_{\overset{\sim I}{A}}\right)}{2} = \frac{a_1 + 2a_2 + a_3 + a_1' + 2a_2 + a_3'}{8}$$

Theorem 1. *Let* $\overset{\sim I}{A}$ *and* $\overset{\sim I}{B}$ *be two TIFNs. Then* $\Re\left(\overset{\sim I}{A} + t\overset{\sim I}{B}\right) = \Re\left(\overset{\sim I}{A}\right) + t\Re\left(\overset{\sim I}{B}\right)$ *for all* $t \in R$ *[55].*

The Proof of Theorem 1 can be easily found in [45,55].

Definition 6. *Ordering of TIFNs, consider* $\overset{\sim I}{A} = (a_1, a_2, a_3; a_1', a_2, a_3')$ *and* $\overset{\sim I}{B} = (b_1, b_2, b_3; b_1', b_2, b_3')$ *then [29,31,40]*

(a) $\Re\left(\overset{\sim I}{A}\right) \ge \Re\left(\overset{\sim I}{B}\right)$ iff $\overset{\sim I}{A} \ge \overset{\sim I}{B}$;

(b) $\Re\left(\overset{\sim I}{A}\right) \leq \Re\left(\overset{\sim I}{B}\right)$ iff $\overset{\sim I}{A} \leq \overset{\sim I}{B}$;

(c) $\Re\left(\overset{\sim I}{A}\right) = \Re\left(\overset{\sim I}{B}\right)$ iff $\overset{\sim I}{A} = \overset{\sim I}{B}$;

(d) $min\left(\overset{\sim I}{A}, \overset{\sim I}{B}\right) = \overset{\sim I}{A}$, if $\overset{\sim I}{A} \leq \overset{\sim I}{B}$ or $\overset{\sim I}{B} \geq \overset{\sim I}{A}$;

(e) $max\left(\overset{\sim I}{A}, \overset{\sim I}{B}\right) = \overset{\sim I}{A}$, if $\overset{\sim I}{A} \geq \overset{\sim I}{B}$ or $\overset{\sim I}{B} \leq \overset{\sim I}{A}$.

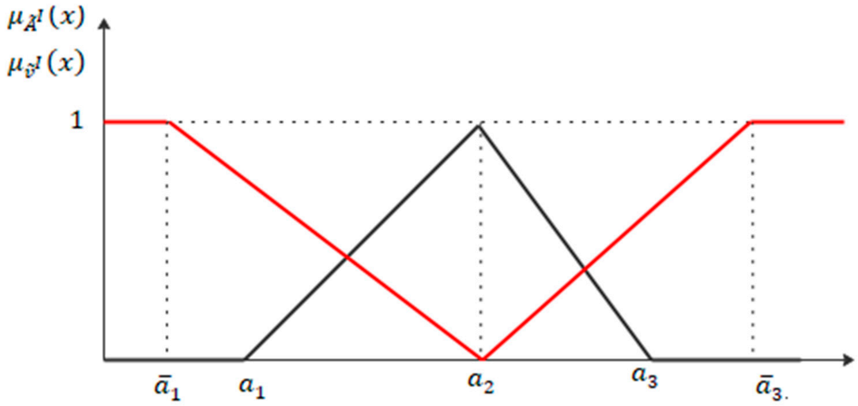

Figure 1. Triangular intuitionistic fuzzy number.

3. Problem Formulation

In real-world TPs, the constructing phase often encounters inaccuracies in transportation parameters due to incomplete data and market fluctuations. We employ a FIF-MMSFTM, where TINFs represent IFNs, to address this quantitatively. Suppose there are sources, destinations, and conveyances. This paper uses the following notations in the mathematical model development for the proposed FIF-MMSFTM, as shown in Table 2. We can express the FIF-MMSFTM's mathematical formulation as follows [4,47,51]:

$$max \quad \overset{\sim I}{Z}_q\left(\overset{\sim I}{x}\right) = \frac{\overset{\sim I}{N}_q\left(\overset{\sim I}{x}\right)}{\overset{\sim I}{D}_q\left(\overset{\sim I}{x}\right)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\sum_{p=1}^{l} \overset{\sim(q)I}{c}_{ijkp}\overset{\sim I}{x}_{ijkp} + \overset{\sim(q)I}{\alpha}_0}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\sum_{p=1}^{l} \overset{\sim(q)I}{d}_{ijkp}\overset{\sim I}{x}_{ijkp} + \overset{\sim(q)I}{\beta}_0}, \qquad q = 12, \ldots, Q \tag{3}$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{k} \overset{\sim I}{x}_{ijkp} \leq \overset{\sim I}{a}_{ip}, \qquad i = 1,2,\ldots,m, \quad p = 1,2,\ldots,l, \tag{4}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} \overset{\sim I}{x}_{ijkp} \geq \overset{\sim I}{b}_{jp}, \qquad j = 1,2,\ldots,n, \quad p = 1,2,\ldots,l, \tag{5}$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} \overset{\sim I}{x}_{ijkp} \leq \overset{\sim I}{e}_{k}, \qquad k = 1,2,\ldots,k, \tag{6}$$

$$\overset{\sim I}{x}_{ijkp} \geq \overset{\sim I}{0}, \qquad \forall \quad i,\ j,\ k,\ p. \tag{7}$$

Table 2. Notations.

Index	i	Index for the i^{th}-source;
	j	Index for the j^{th}-destinations;
	k	Index for the k^{th}-conveyances;
	p	Index for the p^{th}-item;
	q	Index for the q^{th}- objective function;

Table 2. *Cont.*

Sets	m	Number of sources;
	n	Number of destinations;
	k	Number of conveyances;
	l	Number of items.
Parameters	$\overset{\sim(q)I}{c}_{ijkp}$	IF profit gained from shipment unit quantity of p^{th} product from i^{th} source to j^{th} destination via k^{th} conveyance;
	$\overset{\sim(q)I}{d}_{ijkp}$	IF cost per unit of shipment of p^{th} product from i^{th} source to j^{th} destination via k^{th} conveyance;
	$\overset{\sim(q)I}{\alpha}_0$	IF fixed profit associated with p^{th} product;
	$\overset{\sim(q)I}{\beta}_0$	IF fixed cost associated with p^{th} product;
	$\overset{\sim I}{a}_{ip}$	IF supply of p^{th} product at source i;
	$\overset{\sim I}{b}_{jp}$	IF demand of p^{th} product at destination j;
	$\overset{\sim I}{e}_k$	IF capacity of k^{th} conveyance.
Decision variables	$\overset{\sim I}{x}_{ijkp}$	IF quantity shipped of p^{th} product from i^{th} source to j^{th} destination by k^{th} conveyance;
Optimization objectives	$\overset{\sim I}{Z}_q\left(\overset{\sim I}{x}\right)$	Objective function with IF parameters.

where $\overset{\sim(q)I}{c}_{ijkp} = \left(c^{1(q)}_{ijkp}, c^{2(q)}_{ijkp}, c^{3(q)}_{ijkp}; c'^{1(q)}_{ijkp}, c^{2(q)}_{ijkp}, c'^{3(q)}_{ijkp}\right)$ is a TIFN that represents the revenue from shipping unit amount of p^{th} goods via k^{th} conveyance from i^{th} source to j^{th} destination. Also, $\overset{\sim(q)I}{d}_{ijkp} = \left(d^{1(q)}_{ijkp}, d^{2(q)}_{ijkp}, d^{3(q)}_{ijkp}; d'^{1(q)}_{ijkp}, d^{2(q)}_{ijkp}, d'^{3(q)}_{ij}\right)$ is a TIFN that indicates the cost of shipping a unit of p^{th} product from i^{th} source to j^{th} destination via k^{th} conveyance. $\overset{\sim(q)I}{\alpha}_0$, $\overset{\sim(q)I}{\beta}_0$ are assumed to be TIFNs that denote the fixed profit and cost associated with p^{th} product, i^{th} source, j^{th} destination, and k^{th} conveyance, respectively. $\overset{\sim I}{x}_{ijkp} = \left(x^1_{ijkp}, x^2_{ijkp}, x^3_{ijkp}; x'^1_{ijkp}, x^2_{ijkp}, x'^3_{ijkp}\right)$ denotes a TIFN quantity shipped of p^{th} product from i^{th} source to j^{th} destination via k^{th} conveyance. $\overset{\sim I}{a}_{ip} = \left(a^1_{ip}, a^2_{ip}, a^3_{ip}; a'^1_{ip}, a^2_{ip}, a'^3_{ip}\right)$ represents the accessible TIF supply at i^{th} source, and $\overset{\sim I}{b}_{jp} = \left(b^1_{jp}, b^2_{jp}, b^3_{jp}; b'^1_{jp}, b^2_{jp}, b'^3_{jp}\right)$ alludes to the accessible TIF demand at j^{th} destination for p^{th} product. $\overset{\sim I}{e}_k = \left(e^1_k, e^2_k, e^3_k; e'^1_k, e^2_k, e'^3_k\right)$ is the total TIF capacity of k^{th} conveyance. Additionally, we hypothesize that $\overset{\sim I}{D}_q\left(\overset{\sim I}{x}\right) > \overset{\sim I}{0}$, $q = 1, 2, .., Q$, $\overset{\sim I}{a}_{ip} > \overset{\sim I}{0}$, $\forall i, p$; $\overset{\sim I}{b}_{jp} > \overset{\sim I}{0}$, $\forall j, p$; $\overset{\sim(q)I}{c}_{ijkp} > \overset{\sim I}{0}$, $\overset{\sim(q)I}{\alpha}_0 > \overset{\sim I}{0}$, $\overset{\sim(q)I}{\beta}_0 > \overset{\sim I}{0}$, $\forall i, j, k, p$, and the gross supply is equivalent to or above the value of the gross demand as well as overall conveyance capacities that are higher than or comparable to overall needs.

$$\sum_{i=1}^{m} \overset{\sim I}{a}_{ip} \geq \sum_{j=1}^{n} \overset{\sim I}{b}_{jp}, \quad \forall\, i,\, p \quad and \qquad \sum_{k=1}^{p} \overset{\sim I}{e}_k \geq \sum_{p=1}^{l} \sum_{j=1}^{n} \overset{\sim I}{b}_{jp}, \tag{8}$$

As a necessary and sufficient condition for the existence of a feasible solution to the FIF-MMSFTM, the discrepancy (8) is thought to exist. Let S be a convex, compact, and feasible set that is described with Equations (4)–(7), and $S \neq \varnothing$.

Definition 7. *A point $\overset{\sim *I}{x}_{ijkp} \in S$ is claimed to be an IF weakly efficient solution iff there is no $\overset{\sim I}{x}_{ijkp} \in S$ such that* $\dfrac{\overset{\sim I}{N}_q\left(\overset{\sim I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_q\left(\overset{\sim I}{x}_{ijkp}\right)} > \dfrac{\overset{\sim I}{N}_q\left(\overset{\sim *I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_q\left(\overset{\sim *I}{x}_{ijkp}\right)}, \qquad q = 1, 2, \ldots, Q.$

Definition 8. *A point $\overset{\sim *I}{x}_{ijkp} \in S$ is claimed to be an IF strongly efficient solution iff there is no $\overset{\sim I}{x}_{ijkp} \in S$ such that* $\dfrac{\overset{\sim I}{N}_q\left(\overset{\sim I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_q\left(\overset{\sim I}{x}_{ijkp}\right)} \geq \dfrac{\overset{\sim I}{N}_q\left(\overset{\sim *I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_q\left(\overset{\sim *I}{x}_{ijkp}\right)}, \qquad q = 1, 2, \ldots, Q.$

4. Linearization Procedure

The FIF-MMSFTM can be converted into FIF-MMSTM based on Charnes and Cooper [6] strategy by introducing a new variable $\overset{\sim I}{w} = \overset{\sim I}{t}\overset{\sim I}{x};\ \overset{\sim I}{t} > 0^I$, [6,58,59] since $\overset{\sim I}{D}_q\left(\overset{\sim I}{x}\right) > 0^I$; $\overset{\sim (q)I}{c}_{ijkp} > 0^I$; $q = 1, 2, \ldots, Q$, and $\overset{\sim I}{x}_{ijkp} \geq 0^I$. Thus, we take into account the lowest value of $\frac{\overset{\sim I}{1}}{\overset{\sim I}{D}_q\left(\overset{\sim I}{x}\right)} = \overset{\sim I}{t}$, i.e.,

$$\bigcap \frac{\overset{\sim I}{1}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\sum_{p=1}^{l} \overset{\sim (q)I}{d}_{ijkp}\overset{\sim I}{x}_{ijkp} + \overset{\sim (q)I}{\beta}_0} = \overset{\sim I}{t}, \qquad q = 1, 2, \ldots, Q, \tag{9}$$

In general, it can be formulated as follows:

$$\frac{\overset{\sim I}{1}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\sum_{p=1}^{l} \overset{\sim (q)I}{d}_{ijkp}\overset{\sim I}{x}_{ijkp} + \overset{\sim (q)I}{\beta}_0} \geq \overset{\sim I}{t}, \qquad q = 1, 2, \ldots, Q, \tag{10}$$

Following the stratification of the change $\overset{\sim I}{w} = \overset{\sim I}{t}\overset{\sim I}{x};\ \overset{\sim I}{t} > 0^I$, and using Equation (10), FIF-MMSFTM transforms into a comparable FIF-MMSTM as follows [47,58,59]:

$$max\ \ \overset{\sim I}{Z}_q\left(\overset{\sim I}{x}\right) = \overset{\sim I}{t}\,\overset{\sim I}{N}_q\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right), \qquad q = 12, \ldots, Q \tag{11}$$

subject to

$$\overset{\sim I}{t}\,\overset{\sim I}{D}_q\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right) \leq \overset{\sim I}{1}, \qquad \forall i,\ j, k, p; \qquad q = 12, \ldots, Q \tag{12}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{k}\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right) \leq \overset{\sim I}{a}_{ip}, \qquad i = 1, 2, \ldots, m, \quad p = 1, 2, \ldots, l, \tag{13}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k}\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right) \geq \overset{\sim I}{b}_{jp}, \qquad j = 1, 2, \ldots, n, \quad p = 1, 2, \ldots, l, \tag{14}$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right) \leq \overset{\sim I}{e}_{k}, \qquad k = 1, 2, \ldots, k, \tag{15}$$

$$\overset{\sim I}{w}_{ijkp} \geq 0^I, \quad \overset{\sim I}{t} > 0^I, \qquad i = 1, 2, \ldots, m, \qquad j = 1, 2, \ldots, n. \tag{16}$$

Remark 1. *The IF efficient solution for the model* $(11) - (16)$ $\left(\frac{\overset{\sim *I}{w}_{ijkp}}{\overset{\sim *I}{t}}\right) = \left(\left(\frac{w^*_{ijkp}}{t^*}\right)^1, \left(\frac{w^*_{ijkp}}{t^*}\right)^2, \left(\frac{w^*_{ijkp}}{t^*}\right)^3; \left(\frac{w^*_{ijkp}}{t^*}\right)^{\prime 1}, \left(\frac{w^*_{ijkp}}{t^*}\right)^2, \left(\frac{w^*_{ijkp}}{t^*}\right)^{\prime 3}\right) \in G$*, where G is the collection of constraints* $(12) - (16)$*, meets the following conditions:*

1. $\sum_{j=1}^{n}\sum_{k=1}^{k}\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right) \leq \overset{\sim I}{a}_{ip}, i = 1, 2, \ldots, m, p = 1, 2, \ldots, l;$
2. $\sum_{i=1}^{m}\sum_{k=1}^{k}\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right) \geq \overset{\sim I}{b}_{jp}, j = 1, 2, \ldots, n, p = 1, 2, \ldots, l;$
3. $\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right) \leq \overset{\sim I}{e}_{k}, k = 1, 2, \ldots, k;$
4. $\overset{\sim *I}{w}_{ijkp} \geq 0^I; \overset{\sim *I}{t} > 0^I\ i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, k; p = 1, 2, \ldots, l;$
5. $\forall\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right) \in S,$ *we have that* $\frac{\overset{\sim I}{N}_q\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right)}{\overset{\sim I}{D}_q\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right)} \leq \frac{\overset{\sim I}{N}_q\left(\frac{\overset{\sim *I}{w}_{ijkp}}{\overset{\sim *I}{t}}\right)}{\overset{\sim I}{D}_q\left(\frac{\overset{\sim *I}{w}_{ijkp}}{\overset{\sim *I}{t}}\right)}.$

Theorem 2. *The solution* $\overset{\sim *I}{x}$ *for the Equations* $(3)-(7)$ *is an IF efficient solution iff the solution* $\left(\overset{\sim *I}{w}, \overset{\sim *I}{t}\right)$ *for the Equations* $(11)-(16)$ *is also an IF efficient solution.*

Proof. Contrariwise, suppose that $\overset{\sim *I}{x}_{ijkp} = \frac{\overset{\sim *I}{w}_{ijkp}}{\overset{\sim *I}{t}}$ is a solution which is a fuzzy efficient solution for Equations (3)–(7), but $\left(\overset{\sim *I}{w}_{ijkp}, \overset{\sim *I}{t}\right)$ is a solution that is not a fuzzy efficient solution for Equations (11)–(16). Therefore, there should be $\overset{\sim I}{w}_{ijkp} \in S$ such that the following is true:

$$\overset{\sim I}{t}\,\overset{\sim I}{N}_q\left(\frac{\overset{\sim *I}{w}_{ijkp}}{\overset{\sim *I}{t}}\right) \leq \overset{\sim I}{t}\,\overset{\sim I}{N}_q\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right), \qquad q = 1, 2, \ldots, Q; \quad \text{and} \tag{17}$$

$$\overset{\sim I}{t}\,\overset{\sim I}{N}_v\left(\frac{\overset{\sim *I}{w}_{ijkp}}{\overset{\sim *I}{t}}\right) < \overset{\sim I}{t}\,\overset{\sim I}{N}_v\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right), \quad \text{for at least one } v, \quad v \in \{1, 2, \ldots, Q\} \tag{18}$$

Making use of Chakraborty and Gupta's innovation [47,59], based on information provided by Arya et al., [58], we infer the following:

$$\overset{\sim I}{t}\,\overset{\sim I}{N}_q\left(\overset{\sim *I}{x}_{ijkp}\right) \leq \overset{\sim I}{t}\,\overset{\sim I}{N}_q\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right), \qquad q = 1, 2, \ldots, Q; \quad \text{and} \tag{19}$$

$$\overset{\sim I}{t}\,\overset{\sim I}{N}_v\left(\overset{\sim *I}{x}_{ijkp}\right) < \overset{\sim I}{t}\,\overset{\sim I}{N}_v\left(\frac{\overset{\sim I}{w}_{ijkp}}{\overset{\sim I}{t}}\right), \quad \text{for at least one } v, \; v \in \{1, 2, \ldots, Q\} \tag{20}$$

This deduces that

$$\frac{\overset{\sim I}{N}_q\left(\overset{\sim *I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_q\left(\overset{\sim *I}{x}_{ijkp}\right)} \leq \frac{\overset{\sim I}{N}_q\left(\overset{\sim I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_q\left(\overset{\sim I}{x}_{ijkp}\right)}, \qquad q = 1, 2, \ldots, Q; \tag{21}$$

$$\text{and} \quad \frac{\overset{\sim I}{N}_v\left(\overset{\sim *I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_v\left(\overset{\sim *I}{x}_{ijkp}\right)} \leq \frac{\overset{\sim I}{N}_v\left(\overset{\sim I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_v\left(\overset{\sim I}{x}_{ijkp}\right)}, \quad \textit{for at least one } v, \quad v \in \{1, 2, \ldots, Q\} \tag{22}$$

This demonstrates that $\overset{\sim *I}{x}_{ijkp}$ is not an IF efficient solution for the Equations (3)–(7). This disproves our hypothesis and therefore assumes that $\left(\overset{\sim *I}{w}_{ijkp}, \overset{\sim *I}{t}\right)$ is an IF efficient solution for the Equations (11)–(16).

Conversely, suppose that $\left(\overset{\sim *I}{w}_{ijkp}, \overset{\sim *I}{t}\right)$ is an IF efficient solution to Equations (11)–(16) and $\overset{\sim *I}{x}_{ijkp}$ is not an IF efficient solution to Equations (3)–(7). Then, there need to $\overset{\sim I}{x}_{ijkp} \in S$

$$\frac{\overset{\sim I}{N}_q\left(\overset{\sim *I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_q\left(\overset{\sim *I}{x}_{ijkp}\right)} \leq \frac{\overset{\sim I}{N}_q\left(\overset{\sim I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_q\left(\overset{\sim I}{x}_{ijkp}\right)}, \qquad q = 1, 2, \ldots, Q; \tag{23}$$

$$\text{and} \quad \frac{\overset{\sim I}{N}_v\left(\overset{\sim *I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_v\left(\overset{\sim *I}{x}_{ijkp}\right)} \leq \frac{\overset{\sim I}{N}_v\left(\overset{\sim I}{x}_{ijkp}\right)}{\overset{\sim I}{D}_v\left(\overset{\sim I}{x}_{ijkp}\right)}, \quad \text{for at least one } v, \quad v \in \{1, 2, \ldots, Q\} \tag{24}$$

Applying the conversion $\tilde{w}^{*I}_{ijkp} = \tilde{t}^{*I}\tilde{x}^{*I}_{ijkp}$, we conclude the following:

$$\tilde{t}^{I}\tilde{N}^{I}_{q}\left(\frac{\tilde{w}^{*I}_{ijkp}}{\tilde{t}^{*I}}\right) \leq \frac{\tilde{N}^{I}_{q}\left(\tilde{x}^{I}_{ijkp}\right)}{\tilde{D}^{I}_{q}\left(\tilde{x}^{I}_{ijkp}\right)}, \qquad q = 1,2,\ldots,Q; \quad \text{and} \tag{25}$$

$$\tilde{t}^{I}\tilde{N}^{I}_{v}\left(\frac{\tilde{w}^{*I}_{ijkp}}{\tilde{t}^{*I}}\right) < \frac{\tilde{N}^{I}_{v}\left(\tilde{x}^{I}_{ijkp}\right)}{\tilde{D}^{I}_{v}\left(\tilde{x}^{I}_{ijkp}\right)}, \qquad \text{for at least one } v,\ \ v \in \{1,2,\ldots,Q\} \tag{26}$$

It contradicts the fact that the solution $\left(\tilde{w}^{*I}_{ijkp}, \tilde{t}^{*I}\right)$ of Equations (11)–(16) is an IF efficient solution. Hence, the solution $\tilde{x}^{*I}_{ijkp}$ of Equations (3)–(7) is an IF efficient solution. This proves the theorem. □

5. Conception of a Suggested Strategy

Upon developing the real-world FIF-MMSFTM (3)–(7), we proceed to linearize the issue using the transformation outlined in the preceding section. This necessitates the introduction of a novel IF variable. We can reform the crisp model as follows [47] by applying the AF to each objective function, employing arithmetic operations, and using the order of TIFN.

$$max\ \Re\left(\tilde{Z}^{I}_{q}\right) = \left\{\begin{array}{l} \Re\left(\left(t^1, t^2,\ t^3; t'^1, t^2, t'^3\right)\left[\tilde{N}^{I}_{1}\left(\frac{w^1_{ijkp}}{t^1}, \frac{w^2_{ijkp}}{t^2}, \frac{w^3_{ijkp}}{t^3}; \frac{w'^1_{ijkp}}{t'^1}, \frac{w^2_{ijkp}}{t^2}, \frac{w'^3_{ijkp}}{t'^3}\right)\right]\right) \\ \Re\left(\left(t^1, t^2,\ t^3; t'^1, t^2, t'^3\right)\left[\tilde{N}^{I}_{2}\left(\frac{w^1_{ijkp}}{t^1}, \frac{w^2_{ijkp}}{t^2}, \frac{w^3_{ijkp}}{t^3}; \frac{w'^1_{ijkp}}{t'^1}, \frac{w^2_{ijkp}}{t^2}, \frac{w'^3_{ijkp}}{t'^3}\right)\right]\right) \\ \vdots \\ \Re\left(\left(t^1, t^2,\ t^3; t'^1, t^2, t'^3\right)\left[\tilde{N}^{I}_{q}\left(\frac{w^1_{ijkp}}{t^1}, \frac{w^2_{ijkp}}{t^2}, \frac{w^3_{ijkp}}{t^3}; \frac{w'^1_{ijkp}}{t'^1}, \frac{w^2_{ijkp}}{t^2}, \frac{w'^3_{ijkp}}{t'^3}\right)\right]\right) \end{array}\right\}$$

subject to

$$\begin{gathered}
\Re\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\sum_{p=1}^{l} d^{1(q)}_{ijkp} w^1_{ijkp} + \beta_0^{1(q)} t^1\right) \leq 1; \qquad \Re\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\sum_{p=1}^{l} d^{2(q)}_{ijkp} w^2_{ijkp} + \beta_0^{2(q)} t^2\right) \leq 1 \\
\Re\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\sum_{p=1}^{l} d^{3(q)}_{ijkp} w^3_{ijkp} + \beta_0^{3(q)} t^3\right) \leq 1, \qquad q = 1,2,\ldots,Q \\
\Re\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\sum_{p=1}^{l} d'^{1(q)}_{ijkp} w'^1_{ijkp} + \beta_0'^{1(q)} t'^1\right) \leq 1; \quad \Re\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\sum_{p=1}^{l} d^{2(q)}_{ijkp} w^2_{ijkp} + \beta_0^{2(q)} t^2\right) \leq 1 \\
\Re\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\sum_{p=1}^{l} d'^{3(q)}_{ijkp} w^3_{ijkp} + \beta_0'^{3(q)} t'^3\right) \leq 1, \qquad q = 1,2,\ldots,Q \\
\Re\left(\sum_{j=1}^{n}\sum_{k=1}^{k} w^1_{ijkp} - a^1_{ip} t^1\right) \leq 0; \qquad \Re\left(\sum_{j=1}^{n}\sum_{k=1}^{k} w^2_{ijkp} - a^2_{ip} t^2\right) \leq 0; \quad \forall\ i,\ \ p \\
\Re\left(\sum_{j=1}^{n}\sum_{k=1}^{k} w^3_{ijkp} - a^3_{ip} t^3\right) \leq 0; \qquad i = 1,2,\ldots,m, \quad p = 1,2,\ldots,l, \\
\Re\left(\sum_{j=1}^{n}\sum_{k=1}^{k} w'^1_{ijkp} - a'^1_{ip} t'^1\right) \leq 0; \qquad \Re\left(\sum_{j=1}^{n}\sum_{k=1}^{k} w^2_{ijkp} - a^2_{ip} t^2\right) \leq 0; \quad \forall\ i,\ \ p \\
\Re\left(\sum_{j=1}^{n}\sum_{k=1}^{k} w'^3_{ijkp} - a'^3_{ip} t'^3\right) \leq 0; \qquad i = 1,2,\ldots,m, \quad p = 1,2,\ldots,l,
\end{gathered} \tag{27}$$

$$\Re\left(\sum_{i=1}^{m}\sum_{k=1}^{k} w_{ijkp}^{1} - b_{jp}^{1}t^{1}\right) \geq 1; \quad \Re\left(\sum_{i=1}^{m}\sum_{k=1}^{k} w_{ijkp}^{2} - b_{jp}^{2}t^{2}\right) \geq 1, \quad j = 1,2,\ldots,n, \quad p = 1,2,\ldots,l,$$

$$\Re\left(\sum_{i=1}^{m}\sum_{k=1}^{k} w_{ijkp}^{3} - b_{jp}^{3}t^{3}\right) \geq 1; \qquad j = 1,2,\ldots,n, \quad p = 1,2,\ldots,l,$$

$$\Re\left(\sum_{i=1}^{m}\sum_{k=1}^{k} w_{ijkp}^{\prime 1} - b_{jp}^{\prime 1}t^{\prime 1}\right) \geq 1; \quad \Re\left(\sum_{i=1}^{m}\sum_{k=1}^{k} w_{ijkp}^{2} - b_{jp}^{2}t^{2}\right) \geq 1, \quad j = 1,2,\ldots,n, \quad p = 1,2,\ldots,l,$$

$$\Re\left(\sum_{i=1}^{m}\sum_{k=1}^{k} w_{ijkp}^{\prime 3} - b_{jp}^{\prime 3}t^{\prime 3}\right) \geq 1; \qquad j = 1,2,\ldots,n, \quad p = 1,2,\ldots,l,$$

$$\Re\left(\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ijkp}^{1} - e_{k}^{1}t^{1}\right) \leq 1; \qquad \Re\left(\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ijkp}^{2} - e_{k}^{2}t^{2}\right) \leq 1 \quad k = 1,2,\ldots,k,$$

$$\Re\left(\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ijkp}^{3} - e_{k}^{3}t^{3}\right) \leq 1; \qquad k = 1,2,\ldots,k,$$

$$\Re\left(\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ijkp}^{\prime 1} - e_{k}^{\prime 1}t^{\prime 1}\right) \leq 1; \qquad \Re\left(\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ijkp}^{2} - e_{k}^{2}t^{2}\right) \leq 1 \quad k = 1,2,\ldots,k,$$

$$\Re\left(\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ijkp}^{\prime 3} - e_{k}^{\prime 3}t^{\prime 3}\right) \leq 1; \qquad k = 1,2,\ldots,k,$$

$$w_{ijkp}^{\prime 1} \geq 0; \quad w_{ijkp}^{1} - w_{ijkp}^{\prime 1} \geq 0; \quad w_{ijkp}^{\prime 2} - w_{ijkp}^{1} \geq 0; \quad w_{ijkp}^{2} - w_{ijkp}^{\prime 2} = 0; \quad w_{ijkp}^{3} - w_{ijkp}^{2} \geq 0;$$
$$w_{ijkp}^{\prime 3} - w_{ijkp}^{3} \geq 0, \ t^{\prime 1} > 0, t^{1} - t^{\prime 1} > 0; \ t^{\prime 2} - t^{1} > 0; \ t^{2} - t^{\prime 2} = 0; \ t^{3} - t^{2} > 0; \ t^{\prime 3} - t^{3} > 0,$$

The model (27) represents the linearized form of the FIF-MMSFTM. Consequently, various techniques can address the resulting FIF-MMSTM to derive an efficient solution. Examining the collection of limitations in model (27) indicated by $\boldsymbol{G}_1$, we first define the appropriate objective for the single objective function. The optimal value for each objective is determined and referred to as the desired or most satisfactory level denoted by U_q. Let $U_q = max\,\Re\left(\overset{\sim I}{Z_q}\right);\ \ L_q = min\,\Re\left(\overset{\sim I}{Z_q}\right) \quad q = 1,2,\ldots,Q, \quad L_q$ represent the least desirable level of satisfaction for the q^{th} objective. Subsequently, model (27) is transformed to a FGP model, with the objective of attaining the single optimum U_q. However, some flexibility is allowed, and this degree of indulgence is determined by the minimal value L_q. The FGP model is formulated as follows [17]:

$$Find \quad \left\{w_{ijkp},\ t;\ \ i = 12,\ldots,m;\ \ j = 1,2,\ldots,n; \quad p = 1,2,\ldots,l; \quad k = 1,2,\ldots,k\ \right\} \tag{28}$$

subject to

$$w_{ijkp}, \quad t \in \boldsymbol{G}_1 \tag{29}$$

$$\Re\left(\overset{\sim I}{Z_q}\left(w_{ijkp},\ t\right)\right) \approx U_q, \qquad q = 1,2,\ldots,Q \tag{30}$$

where the constraint $\Re\left(\overset{\sim I}{Z_q}\left(w_{ijkp},\ t\right)\right) \approx U_q,\ q = 1,2,\ldots,Q$ is an IF constraint that includes IF parity that may be addressed by using a membership role. This membership role can be linear, parabolic, or hyperbolic according to the DM's preferences. Assuming that $\Re\left(\overset{\sim I}{Z_q}\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\right)$ denoted by $Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)$ [17,20].

5.1. Linear Membership Function Model

In the literature on solving fuzzy mathematical programming problems, a key assumption is often made about applying a linear membership function (LMF) in the decision-making method. People primarily favor this choice because of its simplicity and ease of implementation. It involves defining the upper and lower bounds of respect for decision parameters. Typically, refs. [17,20,42] represent linear membership and non-membership functions, as illustrated in Figure 2.

$$\mu^L\left(Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right) = \begin{cases} 0, & Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \leq L_q, \\ \frac{Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) - L_q}{U_q - L_q}, & L_q \leq Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \leq U_q, \\ 1, & Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \geq U_q, \end{cases} \tag{31}$$

$$v^L\left(Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right) = \begin{cases} 1, & Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \leq L_q, \\ \frac{U_q - Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)}{U_q - L_q}, & L_q \leq Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \leq U_q, \\ 0, & Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \geq U_q, \end{cases} \tag{32}$$

Let $\gamma = min\left\{\mu^L\left(Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right), q = 1, 2, \ldots, Q\right\}$ and $\vartheta = max\left\{v^L\left(Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right), q = 1, 2, \ldots, Q\right\}$. Stated differently, $\mu^L\left(Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right) \geq \gamma$ and $v^L\left(Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right) \leq \vartheta, \forall q$. Based on Zimmerman's method [54], which offers to maximize the lowest acceptance grade while simultaneously minimizing the greatest rejection degree, we use LMF to develop the model as follows:

$$\begin{aligned} & max \quad \gamma - \vartheta \\ & subject\ to \\ & Z_1\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) - \gamma(U_1 - L_1) \geq L_1, \\ & Z_2\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) - \gamma(U_2 - L_2) \geq L_2, \\ & \vdots \\ & Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) - \gamma(U_q - L_q) \geq L_q, \\ & Z_1\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) + \vartheta(U_1 - L_1) \geq U_1, \\ & Z_2\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) + \vartheta(U_2 - L_2) \geq U_2, \\ & \vdots \\ & Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) + \vartheta(U_q - L_q) \geq U_q, \\ & \gamma \geq \vartheta, \quad \gamma + \vartheta \leq 1, \qquad \gamma, \vartheta \in [0,\ 1] \\ & w_{ijkp},\ t\ \in \boldsymbol{G}_1 \end{aligned} \tag{33}$$

Theorem 3. *A unique optimal solution of the LMF model is an efficient solution of the model (33).*

Proof. Allow $\left(\hat{w}_{ijkp},\ \hat{t}, \hat{\gamma},\ \hat{\vartheta}\right)$ to represent the singular best solution for LMF. Then, for any feasible $\left(w_{ijkp},\ t, \gamma,\ \vartheta\right)$ of the LMF problem, $(\hat{\gamma} - \hat{\vartheta}) > (\gamma - \vartheta)$. On the other hand, assume that the model (33), with respect to $\left(\hat{w}_{ijkp},\ \hat{t}, \hat{\gamma},\ \hat{\vartheta}\right)$, is not efficiently solved. Then, there exists w^*_{ijkp}, $t^*\left(w^*_{ijkp} \neq \hat{w}_{ijkp}\ and\ t^* \neq \hat{t}\right)$ feasible to model (33), such that $Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right) \leq Z_q\left(w^*_{ijkp}, t^*\right);\ \ \forall\, q = 12, \ldots, Q$, and $Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right) < Z_q\left(w^*_{ijkp}, t^*\right)$ for a minimum of one q. Therefore,

$$\frac{U_q - Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right)}{U_q - L_q} \geq \frac{U_q - Z_q\left(w^*_{ijkp}, t^*\right)}{U_q - L_q}, \qquad \forall\, q = 12, \ldots, Q \quad \text{and}$$

$$\frac{U_q - Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right)}{U_q - L_q} > \frac{U_q - Z_q\left(w^*_{ijkp}, t^*\right)}{U_q - L_q}, \qquad \text{for at least one } q.$$

$$\text{Thus,} \quad \underset{q}{max}\left(\frac{U_q - Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right)}{U_q - L_q}\right) > (\geq)\ \underset{q}{max}\left(\frac{U_q - Z_q\left(w^*_{ijkp}, t^*\right)}{U_q - L_q}\right).$$

$$\text{Let } \vartheta^* = \underset{q}{max}\left(\frac{U_q - Z_q\left(w^*_{ijkp}, t^*\right)}{U_q - L_q}\right), \ \text{then} \quad \hat{\vartheta} > (\geq)\vartheta^*.$$

$$\text{Similarly,} \quad \frac{Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right) - L_q}{U_q - L_q} \leq \frac{Z_q\left(w^*_{ijkp}, t^*\right) - L_q}{U_q - L_q}, \qquad \forall\, q = 12, \ldots, Q \quad \text{and}$$

$$\frac{Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right)-L_q}{U_q-L_q}<\frac{Z_q\left(w^*_{ijkp},t^*\right)-L_q}{U_q-L_q},\qquad \text{for at least one } q.$$

$$\text{Thus,}\quad \underset{q}{min}\left(\frac{Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right)-L_q}{U_q-L_q}\right)\leq(<)\ \underset{q}{min}\left(\frac{Z_q\left(w^*_{ijkp},t^*\right)-L_q}{U_q-L_q}\right).$$

$$\text{Suppose } \gamma^*=\underset{q}{min}\left(\frac{Z_q\left(w^*_{ijkp},t^*\right)-L_q}{U_q-L_q}\right),\ \text{which means}\quad \hat{\gamma}-\hat{\vartheta}<\gamma^*-\vartheta^*.$$

There is no unique ideal solution. It is an efficient solution of model (33), as it contradicts the truth that $\left(\hat{w}_{ijkp},\ \hat{t},\hat{\gamma},\ \hat{\vartheta}\right)$ is the single optimal solution of LMF. □

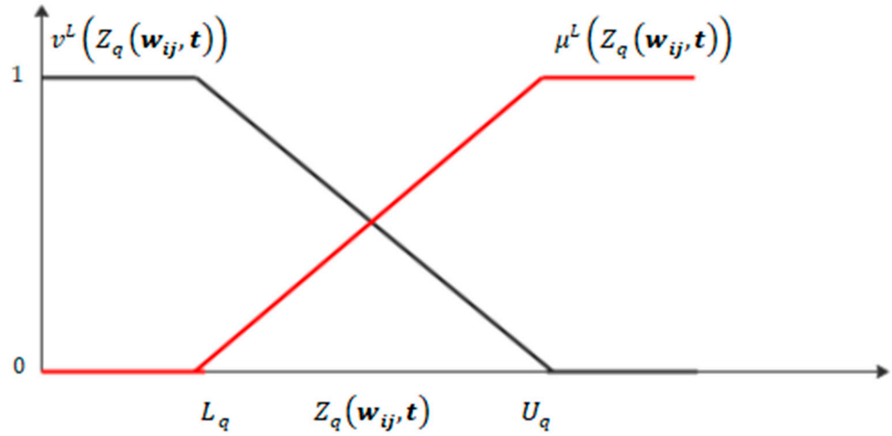

Figure 2. Linear membership and non-membership functions.

5.2. Hyperbolic Membership Function Model

The LMF may not adequately capture decision-making in many real-world situations, as the degree of approval or opposition for a given target may vary non-constantly. Nonlinear membership functions offer a more precise representation of DMs' behavior under certain conditions. Drawing from references [17,20,42], we can express the hyperbolic membership and non-membership function (HMF) (Figure 3) as follows:

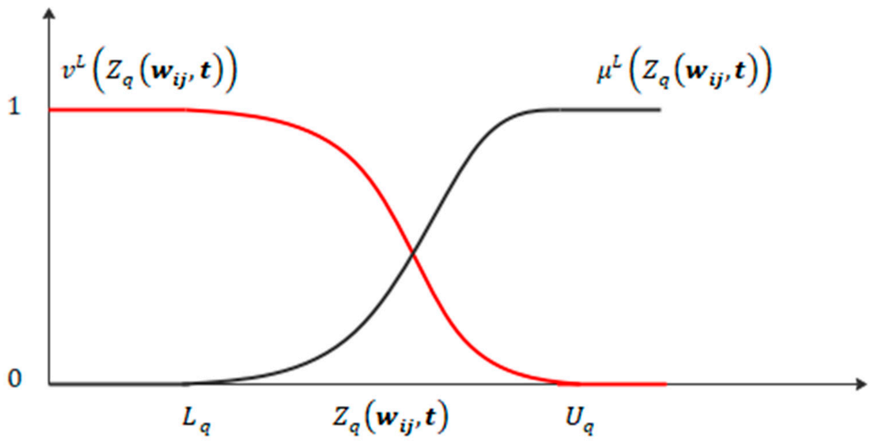

Figure 3. Hyperbolic membership and non-membership functions.

$$\mu^H\left(Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\right)=\begin{cases}0, & Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\leq L_q,\\ \frac{1}{2}\tanh\left(\alpha_q\left(Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)-\frac{U_q+L_q}{2}\right)\right)+\frac{1}{2}, & L_q\leq Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\leq U_q,\\ 1, & Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\geq U_q,\end{cases}\tag{34}$$

$$v^H\left(Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\right)=\begin{cases}1, & Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\le L_q,\\ \frac{1}{2}\tanh\left(\alpha_q\left(\frac{U_q+L_q}{2}-Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\right)\right)+\frac{1}{2}, & L_q\le Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\le U_q,\\ 0, & Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\ge U_q,\end{cases}\quad(35)$$

where $\alpha_q = \frac{6}{U_q-L_q}, q = 1,2,..,Q$. Let $\gamma = min\left\{\mu^H\left(Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\right), q=1,2,\ldots,Q\right\}$ and $\vartheta = max\left\{v^H\left(Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\right), q=1,2,\ldots,Q\right\}$. Consequently, we create the crisp model utilizing the HMF as follows:

$$\begin{aligned}&max\quad \gamma-\vartheta\\&subject\ to\\&\alpha_1\,Z_1\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)-\tanh^{-1}(2\gamma-1)\ge\tfrac{\alpha_1}{2}(U_1+L_1),\\&\alpha_2\,Z_2\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)-\tanh^{-1}(2\gamma-1)\ge\tfrac{\alpha_2}{2}(U_2+L_2),\\&\vdots\\&\alpha_q\,Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)-\tanh^{-1}(2\gamma-1)\ge\tfrac{\alpha_q}{2}\left(U_q+L_q\right),\\&\alpha_1\,Z_1\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)+\tanh^{-1}(2\vartheta-1)\ge\tfrac{\alpha_1}{2}(U_1+L_1),\\&\alpha_2\,Z_2\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)+\tanh^{-1}(2\vartheta-1)\ge\tfrac{\alpha_2}{2}(U_2+L_2),\\&\vdots\\&\alpha_q\,Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)+\tanh^{-1}(2\vartheta-1)\ge\tfrac{\alpha_q}{2}\left(U_q+L_q\right),\\&\gamma\ge\vartheta,\quad\gamma+\vartheta\le 1,\qquad\gamma,\vartheta\in[0,\ 1]\\&w_{ijkp},\ t\ \in\boldsymbol{G}_1\end{aligned}\quad(36)$$

The HMF exhibits a concave segment at one end and a convex segment at the other. The convex shape typically represents scenarios where the DM requires a higher level of satisfaction margin. Conversely, the concave portion signifies situations where a smaller margin of fulfillment suffices.

The preceding model is comparable to:

$$\begin{aligned}&max\quad \delta-\beta\\&subject\ to\\&\alpha_1\,Z_1\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)-\delta\ge\tfrac{\alpha_1}{2}(U_1+L_1),\\&\alpha_2\,Z_2\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)-\delta\ge\tfrac{\alpha_2}{2}(U_2+L_2),\\&\vdots\\&\alpha_q\,Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)-\delta\ge\tfrac{\alpha_q}{2}\left(U_q+L_q\right),\\&\alpha_1\,Z_1\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)+\beta\ge\tfrac{\alpha_1}{2}(U_1+L_1),\\&\alpha_2\,Z_2\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)+\beta\ge\tfrac{\alpha_2}{2}(U_2+L_2),\\&\vdots\\&\alpha_q\,Z_q\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)+\beta\ge\tfrac{\alpha_q}{2}\left(U_q+L_q\right),\\&\delta\ge\beta,\quad\beta\ge 0,\quad\delta\ge 0,\quad\text{and}\quad\alpha_q=\tfrac{6}{U_q-L_q},\quad q=1,2,\ldots,Q,\\&w_{ijkp},\ t\ \in\boldsymbol{G}_1\end{aligned}\quad(37)$$

where $\delta=\tanh^{-1}(2\gamma-1);\beta=\tanh^{-1}(2\vartheta-1)$, as γ and ϑ are proportional to $tanh^{-1}(2\gamma-1)$ and $\tanh^{-1}(2\vartheta-1)$. The aforementioned approach is more applicable to real-world scenarios because it addresses uncertain fractional MOTPs. Additionally, the model keeps costs and non-negative variables, making it applicable in real-world scenarios.

Theorem 4. *A unique optimal solution of the HMF model is an efficient solution of the model (37).*

Proof. Consider $\left(\hat{w}_{ijkp},\ \hat{t},\hat{\gamma},\ \hat{\vartheta}\right)$ the unique optimal solution of HMF. By paradox, postulate that $\left(\hat{w}_{ijkp},\ \hat{t},\hat{\gamma},\ \hat{\vartheta}\right)$ is not an efficient solution to the model (37). Then, there exist $w^*_{ijkp},\quad t^*\left(w^*_{ijkp}\ne\hat{w}_{ijkp}\ and\ t^*\ne\hat{t}\right)$ feasible to model (37), such that

$Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right) \leq Z_q\left(w^*_{ijkp}, t^*\right);\ \ \forall\, q = 12, \ldots, Q$, and $Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right) < Z_q\left(w^*_{ijkp}, t^*\right)$ for a minimum of one q. So,

$$\tanh\left(\alpha_q\left(Z_q\left(w^*_{ijkp}, t^*\right) - \frac{U_q + L_q}{2}\right)\right) \geq \tanh\left(\alpha_q\left(Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right) - \frac{U_q + L_q}{2}\right)\right),\quad q = 1, 2, \ldots, Q,$$

$$\tanh\left(\alpha_q\left(Z_q\left(w^*_{ijkp}, t^*\right) - \frac{U_q + L_q}{2}\right)\right) > \tanh\left(\alpha_q\left(Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right) - \frac{U_q + L_q}{2}\right)\right), q = 1, 2, \ldots, Q,$$

So,

$$\underset{q}{min}\left(\tanh\left(\alpha_q\left(Z_q\left(w^*_{ijkp}, t^*\right) - \frac{U_q + L_q}{2}\right)\right)\right) > (\geq)\ \underset{q}{min}\left(\tanh\left(\alpha_q\left(Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right) - \frac{U_q + L_q}{2}\right)\right)\right).$$

$$\text{If } \gamma^* = \underset{q}{min}\left(\frac{1}{2}\tanh\left(\alpha_q\left(Z_q\left(w^*_{ijkp}, t^*\right) - \frac{U_q + L_q}{2}\right)\right) + \frac{1}{2}\right), \quad \text{then} \quad \gamma^* > (\geq)\hat{\gamma}.$$

In a similar manner, we obtain

$$\vartheta^* = \underset{q}{max}\left(\tanh\left(\alpha_q\left(\frac{U_q + L_q}{2} - Z_q\left(w^*_{ijkp}, t^*\right)\right)\right)\right), \quad \text{then} \quad \hat{\vartheta} > (\geq)\vartheta^*.$$

Hence, $\hat{\gamma} - \hat{\vartheta} < \gamma^* - \vartheta^*$. This runs counter to the fact that $\left(\hat{w}_{ijkp},\ \hat{t}, \hat{\gamma},\ \hat{\vartheta}\right)$ is the unique optimal solution of HMF. □

5.3. Parabolic Membership Function Model

The parabolic membership and non-membership functions (PMF) (Figure 4) are expressed as follows [17,20,42]:

$$\mu^P\left(Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right) = \begin{cases} 0, & Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \leq L_q, \\ 1 - \left(\frac{U_q - Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)}{U_q - L_q}\right)^2, & L_q \leq Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \leq U_q, \\ 1, & Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \geq U_q, \end{cases} \tag{38}$$

$$v^P\left(Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right) = \begin{cases} 1, & Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \leq L_q, \\ 1 - \left(\frac{Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) - L_q}{U_q - L_q}\right)^2, & L_q \leq Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \leq U_q, \\ 0, & Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) \geq U_q, \end{cases} \tag{39}$$

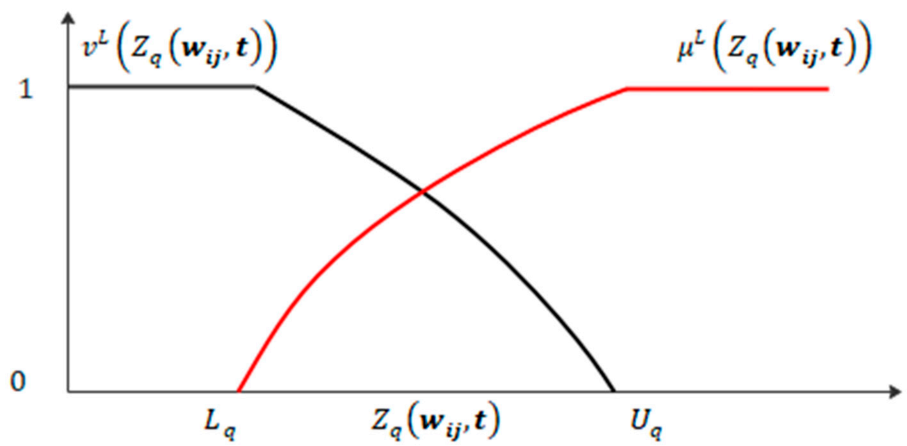

Figure 4. Parabolic membership and non-membership functions.

Let $\gamma = min\left\{\mu^P\left(Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right), q = 1, 2, \ldots, Q\right\}$ and $\vartheta = max\left\{v^P\left(Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right), q = 1, 2, \ldots, Q\right\}$. Consequently, we formulate the crisp model utilizing the PMF in accordance with Zimmerman technique [58] as follows:

$$\begin{array}{l} max \quad \gamma - \vartheta \\ subject\ to \\ \quad \left(U_1 - Z_1\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right)^2 \leq (1-\gamma)\,(U_1 - L_1)^2 \\ \quad \left(U_2 - Z_2\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right)^2 \leq (1-\gamma)\,(U_2 - L_2)^2 \\ \quad \vdots \\ \quad \left(U_q - Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right)^2 \leq (1-\gamma)\,(U_q - L_q)^2 \\ \quad \left(Z_1\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) - L_1\right)^2 \geq (1-\vartheta)(U_1 - L_1)^2 \\ \quad \left(Z_2\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) - L_2\right)^2 \geq (1-\vartheta)(U_2 - L_2)^2 \\ \quad \vdots \\ \quad \left(Z_q\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right) - L_q\right)^2 \geq (1-\vartheta)(U_q - L_q)^2 \\ \quad \gamma \geq \vartheta, \quad \gamma + \vartheta \leq 1, \qquad \gamma, \vartheta \in [0,\ 1] \\ \quad w_{ijkp},\ t\ \in \boldsymbol{G}_1 \end{array} \tag{40}$$

The model described above relies on the PMF, as linear membership and non-membership functions are often inadequate for representing real-world scenarios accurately. Moreover, the model maintains constraints on costs and ensures non-negative variables, thereby enhancing its applicability to real-world problems.

Theorem 5. *A unique optimal solution of the PMF model is an efficient solution of the model (40).*

Proof. Consider $\left(\hat{w}_{ijkp},\ \hat{t}, \hat{\gamma},\ \hat{\vartheta}\right)$ the unique optimal solution of PMF. By paradox, assume $\left(\hat{w}_{ijkp},\ \hat{t}, \hat{\gamma},\ \hat{\vartheta}\right)$ is not an efficient solution to the model (40). Next, there are w^*_{ijkp}, $t^*\left(w^*_{ijkp} \neq \hat{w}_{ijkp}\ and\ t^* \neq \hat{t}\right)$ feasible to model (40), such that $Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right) \leq Z_q\left(w^*_{ijkp}, t^*\right);\ \ \forall\, q = 12, \ldots, Q$, and $Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right) < Z_q\left(w^*_{ijkp}, t^*\right)$ for a minimum of one q. Thus,

$$1 - \left(\frac{U_q - Z_q\left(w^*_{ijkp}, t^*\right)}{U_q - L_q}\right)^2 \geq 1 - \left(\frac{U_q - Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right)}{U_q - L_q}\right)^2, \qquad q = 1, 2, \ldots, Q,$$

$$1 - \left(\frac{U_q - Z_q\left(w^*_{ijkp}, t^*\right)}{U_q - L_q}\right)^2 > 1 - \left(\frac{U_q - Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right)}{U_q - L_q}\right)^2, \qquad q = 1, 2, \ldots, Q,$$

$$\min_q\left(1 - \left(\frac{U_q - Z_q\left(w^*_{ijkp}, t^*\right)}{U_q - L_q}\right)^2\right) > (\geq)\ \min_q\left(1 - \left(\frac{U_q - Z_q\left(\hat{w}_{ijkp},\ \hat{t}\right)}{U_q - L_q}\right)^2\right).$$

$$\text{If } \gamma^* = \min_q\left(1 - \left(\frac{U_q - Z_q\left(w^*_{ijkp}, t^*\right)}{U_q - L_q}\right)^2\right), \quad \text{then} \quad \gamma^* > (\geq)\hat{\gamma}.$$

In a similar manner, we obtain

$$\vartheta^* = \max_q\left(1 - \left(\frac{Z_q\left(w^*_{ijkp}, t^*\right) - L_q}{U_q - L_q}\right)^2\right), \text{ then } \quad \hat{\vartheta} > (\geq)\vartheta^*.$$

Hence, $\hat{\gamma} - \hat{\vartheta} < \gamma^* - \vartheta^*$. This runs counter to the fact that $\left(\hat{w}_{ijkp},\ \hat{t}, \hat{\gamma},\ \hat{\vartheta}\right)$ is the unique optimal solution of PMF. □

6. An Algorithm for Solving the FIF-MMSFTM via Various Membership Functions

The following Algorithm 1 describes the FIF-MMSFTM solution procedures as outlined in Section 5:

Algorithm 1: Solving the FIF-MMSFTM via Various Membership Functions

Input: IF parameters
Output: IF solution.

Step 1.	Set up the IF parameter-based issue, referred to as FIF-MMSFTM;
Step 2.	Execute the linearization strategy to obtain FIF-MMSFTM. Model (30)–(34);
Step 3.	Formulate the crisp model (45) using the AF $\Re\left(\tilde{Z}_q^I\left(\boldsymbol{w_{ijkp}}, \boldsymbol{t}\right)\right) q = 1, 2, \ldots, Q$, and arithmetic operations;
Step 4.	Develop the FGP version of the FIF-MMSFTM as in models (46)–(48);
Step 5.	Use Zimmerman's technique to define and solve the LHF (51). If the DM is fulfilled, proceed to Step 8; Otherwise, proceed to the Following Step;
Step 6.	Evaluate the HMF model (55). If the DM meets their requirements, proceed to Step 8, otherwise advance to the next step;
Step 7.	Go to the following step after resolving the PMF model (58);
Step 8.	End with the acquired IF solution.

7. Numerical Illustration

Consider a TP with 2 origins, 2 destinations, and 2 conveyances between supply and demand. Every parameter was considered as TIFN. Here, the emblems $\tilde{a}_{ip}^I$, $\tilde{b}_{jp}^I$, and $\tilde{e}_k^I$ are employed to indicate the IF power of supply, the IF demand prerequisite, and the IF capacity of transportation, respectively. The details for the FIF-MMSFTP are provided in Tables 3–10.

Table 3. $C_{ijk1}^{(1)}$ profit gained from shipment unit quantity.

i	*j*		*j*	
	1	**2**	**1**	**2**
1	$(1,2,4;0,2,6)$	$(2,3,5;1,3,7)$	$(7,9,11;6,9,12)$	$(2,5,7;1,5,8)$
2	$(3,5,8;1,5,10)$	$(4,5,7;2,5,9)$	$(3,5,8;1,5,10)$	$(5,7,9;4,7,12)$
k	**1**		**2**	

Table 4. $C_{ijk2}^{(1)}$ profit gained from shipment unit quantity.

i	*j*		*j*	
	1	**2**	**1**	**2**
1	$(6,8,10;5,8,12)$	$(4,5,7;2,5,9)$	$(3,5,8;1,5,10)$	$(3,5,8;1,5,10)$
2	$(7,9,11;6,9,12)$	$(2,3,5;1,3,7)$	$(6,8,10;5,8,12)$	$(4,5,7;2,5,9)$
k	**1**		**2**	

Table 5. $d_{ijk1}^{(1)}$ the expense per unit of shipment.

i	*j*		*j*	
	1	**2**	**1**	**2**
1	$(1,2,3;0,2,4)$	$(2,3,5;1,3,6)$	$(2,3,5;1,3,6)$	$(2,4,6;1,4,7)$
2	$(3,5,6;2,5,7)$	$(1,2,3;0,2,4)$	$(3,5,8;1,5,10)$	$(1,3,5;0,3,6)$
k	**1**		**2**	

Table 6. $d_{ijk2}^{(1)}$ the expense per unit of shipment.

i	*j*		*j*	
	1	**2**	**1**	**2**
1	$(2, 3, 5; 1, 3, 6)$	$(1, 2, 3; 0, 2, 4)$	$(3, 5, 8; 1, 5, 10)$	$(1, 3, 5; 0, 3, 6)$
2	$(4, 5, 6; 3, 5, 7)$	$(3, 5, 8; 1, 5, 10)$	$(1, 2, 3; 0, 2, 4)$	$(1, 2, 3; 0, 2, 4)$
k	**1**		**2**	

Table 7. $C_{ijk1}^{(2)}$ profit gained from shipment unit quantity.

i	*j*		*j*	
	1	**2**	**1**	**2**
1	$(3, 5, 8; 1, 5, 10)$	$(4, 5, 7; 2, 5, 9)$	$(6, 8, 10; 5, 8, 12)$	$(3, 5, 8; 1, 5, 10)$
2	$(7, 9, 11; 6, 9, 12)$	$(7, 9, 11; 6, 9, 14)$	$(5, 7, 9; 4, 7, 12)$	$(8, 10, 12; 7, 10, 15)$
k	**1**		**2**	

Table 8. $C_{ijk2}^{(2)}$ profit gained from shipment unit quantity.

i	*j*		*j*	
	1	**2**	**1**	**2**
1	$(7, 8, 9; 5, 8, 10)$	$(1, 3, 5; 0, 3, 7)$	$(3, 5, 8; 1, 5, 10)$	$(8, 10, 12; 7, 10, 13)$
2	$(6, 9, 11; 5, 9, 12)$	$(7, 9, 11; 6, 9, 14)$	$(10, 11, 12; 9, 11, 13)$	$(1, 3, 5; 0, 3, 7)$
k	**1**		**2**	

Table 9. $d_{ijk1}^{(2)}$ the expense per unit of shipment.

i	*j*		*j*	
	1	**2**	**1**	**2**
1	$(1, 2, 3; 0, 2, 4)$	$(2, 3, 4; 1, 3, 5)$	$(2, 3, 4; 1, 3, 5)$	$(4, 5, 6; 3, 5, 7)$
2	$(3, 5, 7; 1, 5, 8)$	$(1, 2, 3; 0, 2, 4)$	$(3, 5, 6; 2, 5, 7)$	$(1, 2, 3; 0, 2, 4)$
k	**1**		**2**	

Table 10. $d_{ijk2}^{(2)}$ the expense per unit of shipment.

i	*j*		*j*	
	1	**2**	**1**	**2**
1	$(2, 3, 4; 1, 3, 5)$	$(2, 3, 4; 1, 3, 5)$	$(5, 6, 7; 3, 5, 8)$	$(1, 2, 3; 0, 2, 4)$
2	$(4, 5, 6; 3, 5, 7)$	$(1, 2, 3; 0, 2, 4)$	$(4, 5, 6; 3, 5, 7)$	$(2, 3, 4; 1, 3, 5)$
k	**1**		**2**	

$$\overset{\sim I}{a}_{11} = (15, 20, 30; 10, 20, 40); \quad \overset{\sim I}{a}_{12} = (5, 18, 25; 1, 18, 28); \quad \overset{\sim I}{a}_{21} = (4, 15, 20; 2, 15, 40)$$
$$\overset{\sim I}{a}_{22} = (10, 30, 40; 5, 30, 50); \quad \overset{\sim I}{b}_{11} = (1, 8, 12; 0, 8, 14); \quad \overset{\sim I}{b}_{12} = (2, 5, 10; 1, 5, 15)$$
$$\overset{\sim I}{b}_{21} = (3, 7, 15; 1, 7, 16); \quad \overset{\sim I}{b}_{22} = (4, 8, 10; 2, 8, 12); \quad \overset{\sim I}{e}_{1} = (15, 20, 25; 10, 20, 30)$$
$$\overset{\sim I}{e}_{1} = (18, 25, 30; 15, 25, 40)$$

Solution: First, predicated on the suggested modification $\overset{\sim I}{w} = \overset{\sim I}{t}\overset{\sim I}{x}$, the FIF-MMSFTM is transformed into the FIF-MMSTM as follows:

$$\begin{aligned} \boldsymbol{max}\ \ \overset{\sim I}{Z}_1\left(\overset{\sim I}{w},\overset{\sim I}{t}\right) = & \overset{\sim I(1)}{c}_{1111}\overset{\sim I}{w}_{1111} + \overset{\sim I(1)}{c}_{1211}\overset{\sim I}{w}_{1211} + \overset{\sim I(1)}{c}_{2111}\overset{\sim I}{w}_{2111} + \overset{\sim I(1)}{c}_{2211}\overset{\sim I}{w}_{2211} \\ & + \overset{\sim I(1)}{c}_{1121}\overset{\sim I}{w}_{1121} + \overset{\sim I(1)}{c}_{1221}\overset{\sim I}{w}_{1221} + \overset{\sim I(1)}{c}_{2121}\overset{\sim I}{w}_{2121} + \overset{\sim I(1)}{c}_{2221}\overset{\sim I}{w}_{2221} + \overset{\sim I(1)}{c}_{1112}\overset{\sim I}{w}_{1112} \\ & + \overset{\sim I(1)}{c}_{1212}\overset{\sim I}{w}_{1212} + \overset{\sim I(1)}{c}_{2112}\overset{\sim I}{w}_{2112} + \overset{\sim I(1)}{c}_{2212}\overset{\sim I}{w}_{2212} + \overset{\sim I(1)}{c}_{1122}\overset{\sim I}{w}_{1122} + \overset{\sim I(1)}{c}_{1222}\overset{\sim I}{w}_{1222} \\ & + \overset{\sim I(1)}{c}_{2122}\overset{\sim I}{w}_{2122} + \overset{\sim I(1)}{c}_{2222}\overset{\sim I}{w}_{2222} + \overset{\sim I(1)}{\alpha_0}\overset{\sim I}{t}, \end{aligned}$$

$$\begin{aligned} \boldsymbol{max}\ \ \overset{\sim I}{Z}_2\left(\overset{\sim I}{w},\overset{\sim I}{t}\right) = & \overset{\sim I(2)}{c}_{1111}\overset{\sim I}{w}_{1111} + \overset{\sim I(2)}{c}_{1211}\overset{\sim I}{w}_{1211} + \overset{\sim I(2)}{c}_{2111}\overset{\sim I}{w}_{2111} + \overset{\sim I(2)}{c}_{2211}\overset{\sim I}{w}_{2211} \\ & + \overset{\sim I(2)}{c}_{1121}\overset{\sim I}{w}_{1121} + \overset{\sim I(2)}{c}_{1221}\overset{\sim I}{w}_{1221} + \overset{\sim I(2)}{c}_{2121}\overset{\sim I}{w}_{2121} + \overset{\sim I(2)}{c}_{2221}\overset{\sim I}{w}_{2221} + \overset{\sim I(2)}{c}_{1112}\overset{\sim I}{w}_{1112} \\ & + \overset{\sim I(2)}{c}_{1212}\overset{\sim I}{w}_{1212} + \overset{\sim I(2)}{c}_{2112}\overset{\sim I}{w}_{2112} + \overset{\sim I(2)}{c}_{2212}\overset{\sim I}{w}_{2212} + \overset{\sim I(2)}{c}_{1122}\overset{\sim I}{w}_{1122} + \overset{\sim I(2)}{c}_{1222}\overset{\sim I}{w}_{1222} \\ & + \overset{\sim I(2)}{c}_{2122}\overset{\sim I}{w}_{2122} + \overset{\sim I(2)}{c}_{2222}\overset{\sim I}{w}_{2222} + \overset{\sim I(2)}{\alpha_0}\overset{\sim I}{t}, \end{aligned}$$

Subject to

$$\begin{aligned} & \overset{\sim I(1)}{d}_{1111}\overset{\sim I}{w}_{1111} + \overset{\sim I(1)}{d}_{1211}\overset{\sim I}{w}_{1211} + \overset{\sim I(1)}{d}_{2111}\overset{\sim I}{w}_{2111} + \overset{\sim I(1)}{d}_{2211}\overset{\sim I}{w}_{2211} + \overset{\sim I(1)}{d}_{1121}\overset{\sim I}{w}_{1121} + \overset{\sim I(1)}{d}_{1221}\overset{\sim I}{w}_{1221} \\ & + \overset{\sim I(1)}{d}_{2121}\overset{\sim I}{w}_{2121} + \overset{\sim I(1)}{d}_{2221}\overset{\sim I}{w}_{2221} + \overset{\sim I(1)}{d}_{1112}\overset{\sim I}{w}_{1112} + \overset{\sim I(1)}{d}_{1212}\overset{\sim I}{w}_{1212} + \overset{\sim I(1)}{d}_{2112}\overset{\sim I}{w}_{2112} + \overset{\sim I(1)}{d}_{2212}\overset{\sim I}{w}_{2212} \\ & + \overset{\sim I(1)}{d}_{1122}\overset{\sim I}{w}_{1122} + \overset{\sim I(1)}{d}_{1222}\overset{\sim I}{w}_{1222} + \overset{\sim I(1)}{d}_{2122}\overset{\sim I}{w}_{2122} + d^{I(1)}_{2222}\overset{\sim I}{w}_{2222} + \overset{\sim I(1)}{\beta_0}\overset{\sim I}{t} \le \overset{\sim I}{1}, \\[1ex] & \overset{\sim I(2)}{d}_{1111}\overset{\sim I}{w}_{1111} + \overset{\sim I(2)}{d}_{1211}\overset{\sim I}{w}_{1211} + \overset{\sim I(2)}{d}_{2111}\overset{\sim I}{w}_{2111} + \overset{\sim I(2)}{d}_{2211}\overset{\sim I}{w}_{2211} + \overset{\sim I(2)}{d}_{1121}\overset{\sim I}{w}_{1121} + \overset{\sim I(2)}{d}_{1221}\overset{\sim I}{w}_{1221} \\ & + \overset{\sim I(2)}{d}_{2121}\overset{\sim I}{w}_{2121} + \overset{\sim I(2)}{d}_{2221}\overset{\sim I}{w}_{2221} + \overset{\sim I(2)}{d}_{1112}\overset{\sim I}{w}_{1112} + \overset{\sim I(2)}{d}_{1212}\overset{\sim I}{w}_{1212} + \overset{\sim I(2)}{d}_{2112}\overset{\sim I}{w}_{2112} + \overset{\sim I(2)}{d}_{2212}\overset{\sim I}{w}_{2212} \\ & + \overset{\sim I(2)}{d}_{1122}\overset{\sim I}{w}_{1122} + \overset{\sim I(2)}{d}_{1222}\overset{\sim I}{w}_{1222} + \overset{\sim I(2)}{d}_{2122}\overset{\sim I}{w}_{2122} + d^{I(2)}_{2222}\overset{\sim I}{w}_{2222} + \overset{\sim I(2)}{\beta_0}\overset{\sim I}{t} \le \overset{\sim I}{1}, \\ & \overset{\sim I}{w}_{1111} + \overset{\sim I}{w}_{1121} + \overset{\sim I}{w}_{1211} + \overset{\sim I}{w}_{1221} - \overset{\sim I}{a}_{11}\overset{\sim I}{t} \le \overset{\sim I}{0} \\ & \overset{\sim I}{w}_{1112} + \overset{\sim I}{w}_{1122} + \overset{\sim I}{w}_{1212} + \overset{\sim I}{w}_{1222} - \overset{\sim I}{a}_{12}\overset{\sim I}{t} \le \overset{\sim I}{0} \\ & \overset{\sim I}{w}_{2111} + \overset{\sim I}{w}_{2121} + \overset{\sim I}{w}_{2211} + \overset{\sim I}{w}_{2221} - \overset{\sim I}{a}_{21}\overset{\sim I}{t} \le \overset{\sim I}{0} \\ & \overset{\sim I}{w}_{2112} + \overset{\sim I}{w}_{2122} + \overset{\sim I}{w}_{2212} + \overset{\sim I}{w}_{2222} - \overset{\sim I}{a}_{22}\overset{\sim I}{t} \le \overset{\sim I}{0} \\ & \overset{\sim I}{w}_{1111} + \overset{\sim I}{w}_{1112} + \overset{\sim I}{w}_{2111} + \overset{\sim I}{w}_{2112} - \overset{\sim I}{b}_{11}\overset{\sim I}{t} \ge \overset{\sim I}{0} \\ & \overset{\sim I}{w}_{1121} + \overset{\sim I}{w}_{1122} + \overset{\sim I}{w}_{2121} + \overset{\sim I}{w}_{2122} - \overset{\sim I}{b}_{12}\overset{\sim I}{t} \ge \overset{\sim I}{0} \\ & \overset{\sim I}{w}_{1211} + \overset{\sim I}{w}_{1212} + \overset{\sim I}{w}_{2211} + \overset{\sim I}{w}_{2212} - \overset{\sim I}{b}_{21}\overset{\sim I}{t} \ge \overset{\sim I}{0} \\ & \overset{\sim I}{w}_{1221} + \overset{\sim I}{w}_{1222} + \overset{\sim I}{w}_{2221} + \overset{\sim I}{w}_{2222} - \overset{\sim I}{b}_{22}\overset{\sim I}{t} \ge \overset{\sim I}{0} \\ & \overset{\sim I}{w}_{1111} + \overset{\sim I}{w}_{1211} + \overset{\sim I}{w}_{2111} + \overset{\sim I}{w}_{2211} + \overset{\sim I}{w}_{1112} + \overset{\sim I}{w}_{1212} + \overset{\sim I}{w}_{2112} + \overset{\sim I}{w}_{2212} - \overset{\sim I}{e}_{1}\overset{\sim I}{t} \le \overset{\sim I}{0} \\ & \overset{\sim I}{w}_{1121} + \overset{\sim I}{w}_{1221} + \overset{\sim I}{w}_{2121} + \overset{\sim I}{w}_{2221} + \overset{\sim I}{w}_{1122} + \overset{\sim I}{w}_{1222} + \overset{\sim I}{w}_{2122} + \overset{\sim I}{w}_{2222} - \overset{\sim I}{e}_{2}\overset{\sim I}{t} \le \overset{\sim I}{0} \\ & \overset{\sim I}{w}_{ijkp} \ge \overset{\sim I}{0}, \quad \forall\, i,j,k,p; \qquad \overset{\sim I}{t} > \overset{\sim I}{0}, \end{aligned}$$

Then, the crisp model was obtained by applying the AF, ordering relation and arithmetic operations as follows:

$$max\,\Re\left(\widetilde{Z}_1^I\left(w_{ijkp},\ t\right)\right)=\frac{1}{8}\left[\begin{array}{l} w^1_{1111}+4w^2_{1111}+4w^3_{1111}+0w'^1_{1111}+4w'^2_{1111}+6w'^3_{1111}+2w^1_{1211}+6w^2_{1211}+5w^3_{1211}\\ +w'^1_{1211}+6w'^2_{1211}+7w'^3_{1211}+3w^1_{2111}+10w^2_{2111}+8w^3_{2111}+w'^1_{2111}+10w'^2_{2111}+10w'^3_{2111}\\ +4w^1_{2211}+10w^2_{2211}+7w^3_{2211}+2w'^1_{2211}+10w'^2_{2211}+9w'^3_{2211}+7w^1_{1121}+18w^2_{1121}\\ +11w^3_{1121}+6w'^1_{1121}+18w'^2_{1121}+12w'^3_{1121}+2w^1_{1221}+10w^2_{1221}+7w^3_{1221}+w'^1_{1221}\\ +10w'^2_{1221}+8w'^3_{1221}+3w^1_{2121}+10w^2_{2121}+8w^3_{2121}+w'^1_{2121}+10w'^2_{2121}+10w'^3_{2121}\\ +5w^1_{2221}+14w^2_{2221}+9w^3_{2221}+4w'^1_{2221}+14w'^2_{2221}+12w'^3_{2221}6w^1_{1112}+16w^2_{1112}+10w^3_{1112}\\ +5w'^1_{1112}+16w'^2_{1112}+12w'^3_{1112}+4w^1_{1212}+10w^2_{1212}+7w^3_{1212}+2w'^1_{1212}+10w'^2_{1212}+9w'^3_{1212}\\ +7w^1_{2112}+18w^2_{2112}+11w^3_{2112}+6w'^1_{2112}+18w'^2_{2112}+12w'^3_{2112}+2w^1_{2212}+6w^2_{2212}+5w^3_{2212}\\ +1w'^1_{2212}+6w'^2_{2212}+7w'^3_{2212}+3w^1_{1122}+10w^2_{1122}+8w^3_{1122}+w'^1_{1122}+10w'^2_{1122}+10w'^3_{1122}\\ +3w^1_{1222}+10w^2_{1222}+8w^3_{1222}+w'^1_{1222}+10w'^2_{1222}+10w'^3_{1222}+6w^1_{2122}+16w^2_{2122}+10w^3_{2122}\\ +5w'^1_{2122}+16w'^2_{2122}+12w'^3_{2122}+4w^1_{2222}+10w^2_{2222}+7w^3_{2222}+2w'^1_{2222}+10w'^2_{2222}+9w'^3_{2222}\\ +150t^1+200t^2+250t^3+100t'^1+200t'^2+300t'^3 \end{array}\right]$$

$$max\,\Re\left(\widetilde{Z}_2^I\left(w_{ijkp},\ t\right)\right)=\frac{1}{8}\left[\begin{array}{l} 3w^1_{1111}+10w^2_{1111}+8w^3_{1111}+w'^1_{1111}+10w'^2_{1111}+10w'^3_{1111}+4w^1_{1211}+10w^2_{1211}+7w^3_{1211}\\ +2w'^1_{1211}+10w'^2_{1211}+9w'^3_{1211}+7w^1_{2111}+18w^2_{2111}+11w^3_{2111}+6w'^1_{2111}+18w'^2_{2111}+12w'^3_{2111}\\ +7w^1_{2211}+18w^2_{2211}+11w^3_{2211}+6w'^1_{2211}+18w'^2_{2211}+14w'^3_{2211}+6w^1_{1121}+16w^2_{1121}\\ +10w^3_{1121}+5w'^1_{1121}+16w'^2_{1121}+12w'^3_{1121}+3w^1_{1221}+10w^2_{1221}+8w^3_{1221}+w'^1_{1221}\\ +10w'^2_{1221}+10w'^3_{1221}+5w^1_{2121}+14w^2_{2121}+9w^3_{2121}+4w'^1_{2121}+14w'^2_{2121}+12w'^3_{2121}\\ +8w^1_{2221}+20w^2_{2221}+12w^3_{2221}+7w'^1_{2221}+20w'^2_{2221}+15w'^3_{2221}+7w^1_{1112}+16w^2_{1112}+9w^3_{1112}\\ +5w'^1_{1112}+16w'^2_{1112}+10w'^3_{1112}+w^1_{1212}+6w^2_{1212}+5w^3_{1212}+0w'^1_{1212}+6w'^2_{1212}+7w'^3_{1212}\\ +6w^1_{2112}+18w^2_{2112}+11w^3_{2112}+5w'^1_{2112}+18w'^2_{2112}+12w'^3_{2112}+7w^1_{2212}+18w^2_{2212}+11w^3_{2212}\\ +6w'^1_{2212}+18w'^2_{2212}+14w'^3_{2212}+3w^1_{1122}+10w^2_{1122}+8w^3_{1122}+w'^1_{1122}+10w'^2_{1122}+10w'^3_{1122}\\ +8w^1_{1222}+20w^2_{1222}+12w^3_{1222}+7w'^1_{1222}+20w'^2_{1222}+13w'^3_{1222}+10w^1_{2122}+22w^2_{2122}+12w^3_{2122}\\ +9w'^1_{2122}+22w'^2_{2122}+13w'^3_{2122}+w^1_{2222}+6w^2_{2222}+5w^3_{2222}+0w'^1_{2222}+6w'^2_{2222}+7w'^3_{2222}\\ +250t^1+300t^2+350t^3+200t'^1+300t'^2+400t'^3 \end{array}\right]$$

Subject to

$$\frac{1}{8}\left[\begin{array}{l} w^1_{1111}+4w^2_{1111}+3w^3_{1111}+0w'^1_{1111}+4w'^2_{1111}+4w'^3_{1111}+2w^1_{1211}+6w^2_{1211}+5w^3_{1211}\\ +w'^1_{1211}+6w'^2_{1211}+6w'^3_{1211}+3w^1_{2111}+10w^2_{2111}+6w^3_{2111}+2w'^1_{2111}+10w'^2_{2111}+7w'^3_{2111}\\ +w^1_{2211}+4w^2_{2211}+3w^3_{2211}+0w'^1_{2211}+4w'^2_{2211}+4w'^3_{2211}+2w^1_{1121}+6w^2_{1121}\\ +5w^3_{1121}+w'^1_{1121}+6w'^2_{1121}+6w'^3_{1121}+2w^1_{1221}+8w^2_{1221}+6w^3_{1221}+w'^1_{1221}\\ +8w'^2_{1221}+7w'^3_{1221}+3w^1_{2121}+10w^2_{2121}+8w^3_{2121}+w'^1_{2121}+10w'^2_{2121}+10w'^3_{2121}\\ +w^1_{2221}+6w^2_{2221}+5w^3_{2221}+0w'^1_{2221}+6w'^2_{2221}+6w'^3_{2221}+2w^1_{1112}+6w^2_{1112}+5w^3_{1112}\\ +w'^1_{1112}+6w'^2_{1112}+6w'^3_{1112}+w^1_{1212}+4w^2_{1212}+3w^3_{1212}+0w'^1_{1212}+4w'^2_{1212}+4w'^3_{1212}\\ +4w^1_{2112}+10w^2_{2112}+6w^3_{2112}+3w'^1_{2112}+10w'^2_{2112}+7w'^3_{2112}+3w^1_{2212}+10w^2_{2212}+8w^3_{2212}\\ +1w'^1_{2212}+10w'^2_{2212}+10w'^3_{2212}+3w^1_{1122}+10w^2_{1122}+8w^3_{1122}+w'^1_{1122}+10w'^2_{1122}+10w'^3_{1122}\\ +w^1_{1222}+6w^2_{1222}+5w^3_{1222}+0w'^1_{1222}+6w'^2_{1222}+6w'^3_{1222}+w^1_{2122}+4w^2_{2122}+3w^3_{2122}\\ +0w'^1_{2122}+4w'^2_{2122}+4w'^3_{2122}+w^1_{2222}+4w^2_{2222}+3w^3_{2222}+0w'^1_{2222}+4w'^2_{2222}+4w'^3_{2222}\\ +50t^1+100t^2+150t^3+20t'^1+100t'^2+200t'^3 \end{array}\right]\leq 1$$

$$\frac{1}{8}\left[\begin{array}{l} w^1_{1111}+4w^2_{1111}+3w^3_{1111}+0w'^1_{1111}+4w'^2_{1111}+4w'^3_{1111}+2w^1_{1211}+6w^2_{1211}+4w^3_{1211}\\ +w'^1_{1211}+6w'^2_{1211}+5w'^3_{1211}+3w^1_{2111}+10w^2_{2111}+7w^3_{2111}+w'^1_{2111}+10w'^2_{2111}+8w'^3_{2111}\\ +w^1_{2211}+4w^2_{2211}+3w^3_{2211}+0w'^1_{2211}+4w'^2_{2211}+4w'^3_{2211}+2w^1_{1121}+6w^2_{1121}\\ +4w^3_{1121}+w'^1_{1121}+6w'^2_{1121}+5w'^3_{1121}+4w^1_{1221}+10w^2_{1221}+6w^3_{1221}+3w'^1_{1221}\\ +10w'^2_{1221}+7w'^3_{1221}+3w^1_{2121}+10w^2_{2121}+6w^3_{2121}+2w'^1_{2121}+10w'^2_{2121}+7w'^3_{2121}\\ +w^1_{2221}+4w^2_{2221}+3w^3_{2221}+0w'^1_{2221}+4w'^2_{2221}+4w'^3_{2221}+2w^1_{1112}+6w^2_{1112}+4w^3_{1112}\\ +w'^1_{1112}+6w'^2_{1112}+5w'^3_{1112}+2w^1_{1212}+6w^2_{1212}+4w^3_{1212}+w'^1_{1212}+6w'^2_{1212}+5w'^3_{1212}\\ +4w^1_{2112}+10w^2_{2112}+6w^3_{2112}+3w'^1_{2112}+10w'^2_{2112}+7w'^3_{2112}+w^1_{2212}+4w^2_{2212}+3w^3_{2212}\\ +0w'^1_{2212}+4w'^2_{2212}+4w'^3_{2212}+5w^1_{1122}+12w^2_{1122}+7w^3_{1122}+3w'^1_{1122}+12w'^2_{1122}+8w'^3_{1122}\\ +w^1_{1222}+4w^2_{1222}+3w^3_{1222}+0w'^1_{1222}+4w'^2_{1222}+4w'^3_{1222}+4w^1_{2122}+10w^2_{2122}+6w^3_{2122}\\ +3w'^1_{2122}+10w'^2_{2122}+7w'^3_{2122}+2w^1_{2222}+6w^2_{2222}+4w^3_{2222}+w'^1_{2222}+6w'^2_{2222}+5w'^3_{2222}\\ +150t^1+200t^2+250t^3+100t'^1+200t'^2+300t'^3 \end{array}\right]\leq 1$$

$$\left[\begin{array}{l} w^1_{1111}+2w^2_{1111}+w^3_{1111}+w'^1_{1111}+2w'^2_{1111}+w'^3_{1111}+w^1_{1121}+2w^2_{1121}+w^3_{1121}+w'^1_{1121}\\ +2w'^2_{1121}+w'^3_{1121}+w^1_{1211}+2w^2_{1211}+w^3_{1211}+w'^1_{1211}+2w'^2_{1211}+w'^3_{1211}+w^1_{1221}+2w^2_{1221}\\ +w^3_{1221}+w'^1_{1221}+2w'^2_{1221}+w'^3_{1221}-15t^1-40t^2-30t^3-10t'^1-40t'^2-40t'^3 \end{array}\right]\leq 0$$

$$\left[\begin{array}{l} w^1_{1112}+2w^2_{1112}+w^3_{1112}+w'^1_{1112}+2w'^2_{1112}+w'^3_{1112}+w^1_{1122}+2w^2_{1122}+w^3_{1122}+w'^1_{1122}\\ +2w'^2_{1122}+w'^3_{1122}+w^1_{1212}+2w^2_{1212}+w^3_{1212}+w'^1_{1212}+2w'^2_{1212}+w'^3_{1212}+w^1_{1222}+2w^2_{1222}\\ +w^3_{1222}+w'^1_{1222}+2w'^2_{1222}+w'^3_{1222}-5t^1-36t^2-25t^3-1t'^1-36t'^2-28t'^3 \end{array}\right]\leq 0$$

$$\left[\begin{array}{l} w^1_{2111}+2w^2_{2111}+w^3_{2111}+w'^1_{2111}+2w'^2_{2111}+w'^3_{2111}+w^1_{2121}+2w^2_{2121}+w^3_{2121}+w'^1_{2121} \\ +2w'^2_{2121}+w'^3_{2121}+w^1_{2211}+2w^2_{2211}+w^3_{2211}+w'^1_{2211}+2w'^2_{2211}+w'^3_{2211}+w^1_{2221}+2w^2_{2221} \\ +w^3_{2221}+w'^1_{2221}+2w'^2_{2221}+w'^3_{2221}-4t^1-30t^2-20t^3-2t'^1-30t'^2-40t'^3 \end{array}\right] \le 0$$

$$\left[\begin{array}{l} w^1_{2112}+2w^2_{2112}+w^3_{2112}+w'^1_{2112}+2w'^2_{2112}+w'^3_{2112}+w^1_{2122}+2w^2_{2122}+w^3_{2122}+w'^1_{2122} \\ +2w'^2_{2122}+w'^3_{2122}+w^1_{2212}+2w^2_{2212}+w^3_{2212}+w'^1_{2212}+2w'^2_{2212}+w'^3_{2212}+w^1_{2222}+2w^2_{2222} \\ +w^3_{2222}+w'^1_{2222}+2w'^2_{2222}+w'^3_{2222}-10t^1-60t^2-40t^3-5t'^1-60t'^2-50t'^3 \end{array}\right] \le 0$$

$$\left[\begin{array}{l} w^1_{1111}+2w^2_{1111}+w^3_{1111}+w'^1_{1111}+2w'^2_{1111}+w'^3_{1111}+w^1_{1121}+2w^2_{1121}+w^3_{1121}+w'^1_{1121} \\ +2w'^2_{1121}+w'^3_{1121}+w^1_{2111}+2w^2_{2111}+w^3_{2111}+w'^1_{2111}+2w'^2_{2111}+w'^3_{2111}+w^1_{2121}+2w^2_{2121} \\ +w^3_{2121}+w'^1_{2121}+2w'^2_{2121}+w'^3_{2121}-1t^1-16t^2-12t^3-0t'^1-16t'^2-14t'^3 \end{array}\right] \ge 0$$

$$\left[\begin{array}{l} w^1_{1112}+2w^2_{1112}+w^3_{1112}+w'^1_{1112}+2w'^2_{1112}+w'^3_{1112}+w^1_{1122}+2w^2_{1122}+w^3_{1122}+w'^1_{1122} \\ +2w'^2_{1122}+w'^3_{1122}+w^1_{2112}+2w^2_{2112}+w^3_{2112}+w'^1_{2112}+2w'^2_{2112}+w'^3_{2112}+w^1_{2122}+2w^2_{2122} \\ +w^3_{2122}+w'^1_{2122}+2w'^2_{2122}+w'^3_{2122}-2t^1-10t^2-10t^3-1t'^1-10t'^2-15t'^3 \end{array}\right] \ge 0$$

$$\left[\begin{array}{l} w^1_{1211}+2w^2_{1211}+w^3_{1211}+w'^1_{1211}+2w'^2_{1211}+w'^3_{1211}+w^1_{1221}+2w^2_{1221}+w^3_{1221}+w'^1_{1221} \\ +2w'^2_{1221}+w'^3_{1221}+w^1_{2211}+2w^2_{2211}+w^3_{2211}+w'^1_{2211}+2w'^2_{2211}+w'^3_{2211}+w^1_{2221}+2w^2_{2221} \\ +w^3_{2221}+w'^1_{2221}+2w'^2_{2221}+w'^3_{2221}-3t^1-14t^2-15t^3-1t'^1-14t'^2-16t'^3 \end{array}\right] \ge 0$$

$$\left[\begin{array}{l} w^1_{1212}+2w^2_{1212}+w^3_{1212}+w'^1_{1212}+2w'^2_{1212}+w'^3_{1212}+w^1_{1222}+2w^2_{1222}+w^3_{1222}+w'^1_{1222} \\ +2w'^2_{1222}+w'^3_{1222}+w^1_{2212}+2w^2_{2212}+w^3_{2212}+w'^1_{2212}+2w'^2_{2212}+w'^3_{2212}+w^1_{2222}+2w^2_{2222} \\ +w^3_{2222}+w'^1_{2222}+2w'^2_{2222}+w'^3_{2222}-4t^1-16t^2-10t^3-2t'^1-16t'^2-12t'^3 \end{array}\right] \ge 0$$

$$\left[\begin{array}{l} w^1_{1111}+2w^2_{1111}+w^3_{1111}+w'^1_{1111}+2w'^2_{1111}+w'^3_{1111}+w^1_{1211}+2w^2_{1211}+w^3_{1211}+w'^1_{1211}+2w'^2_{1211} \\ +w'^3_{1211}+w^1_{2111}+2w^2_{2111}+w^3_{2111}+w'^1_{2111}+2w'^2_{2111}+w'^3_{2111}+w^1_{2211}+2w^2_{2211}+w^3_{2211}+w'^1_{2211} \\ +2w'^2_{2211}+w'^3_{2211}+w^1_{1112}+2w^2_{1112}+w^3_{1112}+w'^1_{1112}+2w'^2_{1112}+w'^3_{1112}+w^1_{1212}+2w^2_{1212}+w^3_{1212} \\ +w'^1_{1212}+2w'^2_{1212}+w'^3_{1212}+w^1_{2112}+2w^2_{2112}+w^3_{2112}+w'^1_{2112}+2w'^2_{2112}+w'^3_{2112}+w^1_{2212}+2w^2_{2212} \\ +w^3_{2212}+w'^1_{2212}+2w'^2_{2212}+w'^3_{2212}-15t^1-40t^2-25t^3-10t'^1-40t'^2-30t'^3 \end{array}\right] \le 0$$

$$\left[\begin{array}{l} w^1_{1121}+2w^2_{1121}+w^3_{1121}+w'^1_{1121}+2w'^2_{1121}+w'^3_{1121}+w^1_{1221}+2w^2_{1221}+w^3_{1221}+w'^1_{1221}+2w'^2_{1221} \\ +w'^3_{1221}+w^1_{2121}+2w^2_{2121}+w^3_{2121}+w'^1_{2121}+2w'^2_{2121}+w'^3_{2121}+w^1_{2221}+2w^2_{2221}+w^3_{2221}+w'^1_{2221} \\ +2w'^2_{2221}+w'^3_{2221}+w^1_{1122}+2w^2_{1122}+w^3_{1122}+w'^1_{1122}+2w'^2_{1122}+w'^3_{1122}+w^1_{1222}+2w^2_{1222}+w^3_{1222} \\ +w'^1_{1222}+2w'^2_{1222}+w'^3_{1222}+w^1_{2122}+2w^2_{2122}+w^3_{2122}+w'^1_{2122}+2w'^2_{2122}+w'^3_{2122}+w^1_{2222}+2w^2_{2222} \\ +w^3_{2222}+w'^1_{2222}+2w'^2_{2222}+w'^3_{2222}-18t^1-50t^2-30t^3-15t'^1-50t'^2-40t'^3 \end{array}\right] \le 0$$

$$w'^1_{ijkp} \ge 0;\ w^1_{ijkp}-w'^1_{ijkp} \ge 0;\ \ w^2_{ijkp}-w^1_{ijkp} \ge 0;\ \ w^2_{ijkp}-w'^2_{ijkp}=0;\ \ w^3_{ijkp}-w^2_{ijkp} \ge 0;$$

$$w'^3_{ij}-w^3_{ij} \ge 0; \quad i=1,2; \quad j=1,2; \quad k=1,2; \quad p=1,2.$$

$$t'^1>0; \quad t^1-t'^1>0; \quad t^2-t^1>0; \quad t^2-t'^2=0; \quad t^3-t^2>0; \quad t'^3-t^3>0;$$

Then, the individual maximum $U_q = max\Re\left(\widetilde{Z}^I_q\left(w_{ij},t\right)\right), q=1,2$ and personal minimal $L_q = min\Re\left(\widetilde{Z}^I_q\left(w_{ij},t\right)\right), q=1,2$ are obtained as in Table 11.

Table 11. Individual minimum and maximum values.

Objective Function	$\Re\left(\widetilde{\boldsymbol{Z}}^I_1\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\right)$	$\Re\left(\widetilde{\boldsymbol{Z}}^I_2\left(\boldsymbol{w_{ijkp}},\boldsymbol{t}\right)\right)$
$U_q = max$	1.806118	2.37213
$L_q = min$	0.002937	0.00389

The following formulation of precise identical structures with various membership functions can be made based on Zimmerman's methodology:

LMF:

max$\gamma - \vartheta$
subject to

$$\frac{1}{8}\left[\begin{array}{l} w^1_{1111} + 4w^2_{1111} + 4w^3_{1111} + 0w'^1_{1111} + 4w'^2_{1111} + 6w'^3_{1111} + 2w^1_{1211} + 6w^2_{1211} + 5w^3_{1211} \\ +w'^1_{1211} + 6w'^2_{1211} + 7w'^3_{1211} + 3w^1_{2111} + 10w^2_{2111} + 8w^3_{2111} + w'^1_{2111} + 10w'^2_{2111} + 10w'^3_{2111} \\ +4w^1_{2211} + 10w^2_{2211} + 7w^3_{2211} + 2w'^1_{2211} + 10w'^2_{2211} + 9w'^3_{2211} + 7w^1_{1121} + 18w^2_{1121} \\ +11w^3_{1121} + 6w'^1_{1121} + 18w'^2_{1121} + 12w'^3_{1121} + 2w^1_{1221} + 10w^2_{1221} + 7w^3_{1221} + w'^1_{1221} \\ +10w'^2_{1221} + 8w'^3_{1221} + 3w^1_{2121} + 10w^2_{2121} + 8w^3_{2121} + w'^1_{2121} + 10w'^2_{2121} + 10w'^3_{2121} \\ +5w^1_{2221} + 14w^2_{2221} + 9w^3_{2221} + 4w'^1_{2221} + 14w'^2_{2221} + 12w'^3_{2221} 6w^1_{1112} + 16w^2_{1112} + 10w^3_{1112} \\ +5w'^1_{1112} + 16w'^2_{1112} + 12w'^3_{1112} + 4w^1_{1212} + 10w^2_{1212} + 7w^3_{1212} + 2w'^1_{1212} + 10w'^2_{1212} + 9w'^3_{1212} \\ +7w^1_{2112} + 18w^2_{2112} + 11w^3_{2112} + 6w'^1_{2112} + 18w'^2_{2112} + 12w'^3_{2112} + 2w^1_{2212} + 6w^2_{2212} + 5w^3_{2212} \\ +1w'^1_{2212} + 6w'^2_{2212} + 7w'^3_{2212} + 3w^1_{1122} + 10w^2_{1122} + 8w^3_{1122} + w'^1_{1122} + 10w'^2_{1122} + 10w'^3_{1122} \\ +3w^1_{1222} + 10w^2_{1222} + 8w^3_{1222} + w'^1_{1222} + 10w'^2_{1222} + 10w'^3_{1222} + 6w^1_{2122} + 16w^2_{2122} + 10w^3_{2122} \\ +5w'^1_{2122} + 16w'^2_{2122} + 12w'^3_{2122} + 4w^1_{2222} + 10w^2_{2222} + 7w^3_{2222} + 2w'^1_{2222} + 10w'^2_{2222} + 9w'^3_{2222} \\ +150t^1 + 200t^2 + 250t^3 + 100t'^1 + 200t'^2 + 300t'^3 \end{array}\right] - 1.803\gamma \geq 0.002937,$$

$$\frac{1}{8}\left[\begin{array}{l} 3w^1_{1111} + 10w^2_{1111} + 8w^3_{1111} + w'^1_{1111} + 10w'^2_{1111} + 10w'^3_{1111} + 4w^1_{1211} + 10w^2_{1211} + 7w^3_{1211} \\ +2w'^1_{1211} + 10w'^2_{1211} + 9w'^3_{1211} + 7w^1_{2111} + 18w^2_{2111} + 11w^3_{2111} + 6w'^1_{2111} + 18w'^2_{2111} + 12w'^3_{2111} \\ +7w^1_{2211} + 18w^2_{2211} + 11w^3_{2211} + 6w'^1_{2211} + 18w'^2_{2211} + 14w'^3_{2211} + 6w^1_{1121} + 16w^2_{1121} \\ +10w^3_{1121} + 5w'^1_{1121} + 16w'^2_{1121} + 12w'^3_{1121} + 3w^1_{1221} + 10w^2_{1221} + 8w^3_{1221} + w'^1_{1221} \\ +10w'^2_{1221} + 10w'^3_{1221} + 5w^1_{2121} + 14w^2_{2121} + 9w^3_{2121} + 4w'^1_{2121} + 14w'^2_{2121} + 12w'^3_{2121} \\ +8w^1_{2221} + 20w^2_{2221} + 12w^3_{2221} + 7w'^1_{2221} + 20w'^2_{2221} + 15w'^3_{2221} + 7w^1_{1112} + 16w^2_{1112} + 9w^3_{1112} \\ +5w'^1_{1112} + 16w'^2_{1112} + 10w'^3_{1112} + w^1_{1212} + 6w^2_{1212} + 5w^3_{1212} + 0w'^1_{1212} + 6w'^2_{1212} + 7w'^3_{1212} \\ +6w^1_{2112} + 18w^2_{2112} + 11w^3_{2112} + 5w'^1_{2112} + 18w'^2_{2112} + 12w'^3_{2112} + 7w^1_{2212} + 18w^2_{2212} + 11w^3_{2212} \\ +6w'^1_{2212} + 18w'^2_{2212} + 14w'^3_{2212} + 3w^1_{1122} + 10w^2_{1122} + 8w^3_{1122} + w'^1_{1122} + 10w'^2_{1122} + 10w'^3_{1122} \\ +8w^1_{1222} + 20w^2_{1222} + 12w^3_{1222} + 7w'^1_{1222} + 20w'^2_{1222} + 13w'^3_{1222} + 10w^1_{2122} + 22w^2_{2122} + 12w^3_{2122} \\ +9w'^1_{2122} + 22w'^2_{2122} + 13w'^3_{2122} + w^1_{2222} + 6w^2_{2222} + 5w^3_{2222} + 0w'^1_{2222} + 6w'^2_{2222} + 7w'^3_{2222} \\ +250t^1 + 300t^2 + 350t^3 + 200t'^1 + 300t'^2 + 400t'^3 \end{array}\right] - 2.36824\gamma \geq 0.00389,$$

$$\frac{1}{8}\left[\begin{array}{l} w^1_{1111} + 4w^2_{1111} + 4w^3_{1111} + 0w'^1_{1111} + 4w'^2_{1111} + 6w'^3_{1111} + 2w^1_{1211} + 6w^2_{1211} + 5w^3_{1211} \\ +w'^1_{1211} + 6w'^2_{1211} + 7w'^3_{1211} + 3w^1_{2111} + 10w^2_{2111} + 8w^3_{2111} + w'^1_{2111} + 10w'^2_{2111} + 10w'^3_{2111} \\ +4w^1_{2211} + 10w^2_{2211} + 7w^3_{2211} + 2w'^1_{2211} + 10w'^2_{2211} + 9w'^3_{2211} + 7w^1_{1121} + 18w^2_{1121} \\ +11w^3_{1121} + 6w'^1_{1121} + 18w'^2_{1121} + 12w'^3_{1121} + 2w^1_{1221} + 10w^2_{1221} + 7w^3_{1221} + w'^1_{1221} \\ +10w'^2_{1221} + 8w'^3_{1221} + 3w^1_{2121} + 10w^2_{2121} + 8w^3_{2121} + w'^1_{2121} + 10w'^2_{2121} + 10w'^3_{2121} \\ +5w^1_{2221} + 14w^2_{2221} + 9w^3_{2221} + 4w'^1_{2221} + 14w'^2_{2221} + 12w'^3_{2221} 6w^1_{1112} + 16w^2_{1112} + 10w^3_{1112} \\ +5w'^1_{1112} + 16w'^2_{1112} + 12w'^3_{1112} + 4w^1_{1212} + 10w^2_{1212} + 7w^3_{1212} + 2w'^1_{1212} + 10w'^2_{1212} + 9w'^3_{1212} \\ +7w^1_{2112} + 18w^2_{2112} + 11w^3_{2112} + 6w'^1_{2112} + 18w'^2_{2112} + 12w'^3_{2112} + 2w^1_{2212} + 6w^2_{2212} + 5w^3_{2212} \\ +1w'^1_{2212} + 6w'^2_{2212} + 7w'^3_{2212} + 3w^1_{1122} + 10w^2_{1122} + 8w^3_{1122} + w'^1_{1122} + 10w'^2_{1122} + 10w'^3_{1122} \\ +3w^1_{1222} + 10w^2_{1222} + 8w^3_{1222} + w'^1_{1222} + 10w'^2_{1222} + 10w'^3_{1222} + 6w^1_{2122} + 16w^2_{2122} + 10w^3_{2122} \\ +5w'^1_{2122} + 16w'^2_{2122} + 12w'^3_{2122} + 4w^1_{2222} + 10w^2_{2222} + 7w^3_{2222} + 2w'^1_{2222} + 10w'^2_{2222} + 9w'^3_{2222} \\ +150t^1 + 200t^2 + 250t^3 + 100t'^1 + 200t'^2 + 300t'^3 \end{array}\right] + 1.803\vartheta \geq 1.806118,$$

$$\frac{1}{8}\left[\begin{array}{l} 3w^1_{1111} + 10w^2_{1111} + 8w^3_{1111} + w'^1_{1111} + 10w'^2_{1111} + 10w'^3_{1111} + 4w^1_{1211} + 10w^2_{1211} + 7w^3_{1211} \\ +2w'^1_{1211} + 10w'^2_{1211} + 9w'^3_{1211} + 7w^1_{2111} + 18w^2_{2111} + 11w^3_{2111} + 6w'^1_{2111} + 18w'^2_{2111} + 12w'^3_{2111} \\ +7w^1_{2211} + 18w^2_{2211} + 11w^3_{2211} + 6w'^1_{2211} + 18w'^2_{2211} + 14w'^3_{2211} + 6w^1_{1121} + 16w^2_{1121} \\ +10w^3_{1121} + 5w'^1_{1121} + 16w'^2_{1121} + 12w'^3_{1121} + 3w^1_{1221} + 10w^2_{1221} + 8w^3_{1221} + w'^1_{1221} \\ +10w'^2_{1221} + 10w'^3_{1221} + 5w^1_{2121} + 14w^2_{2121} + 9w^3_{2121} + 4w'^1_{2121} + 14w'^2_{2121} + 12w'^3_{2121} \\ +8w^1_{2221} + 20w^2_{2221} + 12w^3_{2221} + 7w'^1_{2221} + 20w'^2_{2221} + 15w'^3_{2221} + 7w^1_{1112} + 16w^2_{1112} + 9w^3_{1112} \\ +5w'^1_{1112} + 16w'^2_{1112} + 10w'^3_{1112} + w^1_{1212} + 6w^2_{1212} + 5w^3_{1212} + 0w'^1_{1212} + 6w'^2_{1212} + 7w'^3_{1212} \\ +6w^1_{2112} + 18w^2_{2112} + 11w^3_{2112} + 5w'^1_{2112} + 18w'^2_{2112} + 12w'^3_{2112} + 7w^1_{2212} + 18w^2_{2212} + 11w^3_{2212} \\ +6w'^1_{2212} + 18w'^2_{2212} + 14w'^3_{2212} + 3w^1_{1122} + 10w^2_{1122} + 8w^3_{1122} + w'^1_{1122} + 10w'^2_{1122} + 10w'^3_{1122} \\ +8w^1_{1222} + 20w^2_{1222} + 12w^3_{1222} + 7w'^1_{1222} + 20w'^2_{1222} + 13w'^3_{1222} + 10w^1_{2122} + 22w^2_{2122} + 12w^3_{2122} \\ +9w'^1_{2122} + 22w'^2_{2122} + 13w'^3_{2122} + w^1_{2222} + 6w^2_{2222} + 5w^3_{2222} + 0w'^1_{2222} + 6w'^2_{2222} + 7w'^3_{2222} \\ +250t^1 + 300t^2 + 350t^3 + 200t'^1 + 300t'^2 + 400t'^3 \end{array}\right] + 2.36824\vartheta \geq 2.37213,$$

$$\gamma \geq \vartheta, \quad \gamma + \vartheta \leq 1, \qquad \gamma, \vartheta \in [0,\ 1]$$

and all the constraints in the

HMF model:

$$\begin{array}{l} max\delta - \beta \\ subject\ to \end{array}$$

$$\frac{3.32745}{8}\left[\begin{array}{l}
w^{1}_{1111}+4w^{2}_{1111}+4w^{3}_{1111}+0w'^{1}_{1111}+4w'^{2}_{1111}+6w'^{3}_{1111}+2w^{1}_{1211}+6w^{2}_{1211}+5w^{3}_{1211}\\
+w'^{1}_{1211}+6w'^{2}_{1211}+7w'^{3}_{1211}+3w^{1}_{2111}+10w^{2}_{2111}+8w^{3}_{2111}+w'^{1}_{2111}+10w'^{2}_{2111}+10w'^{3}_{2111}\\
+4w^{1}_{2211}+10w^{2}_{2211}+7w^{3}_{2211}+2w'^{1}_{2211}+10w'^{2}_{2211}+9w'^{3}_{2211}+7w^{1}_{1121}+18w^{2}_{1121}\\
+11w^{3}_{1121}+6w'^{1}_{1121}+18w'^{2}_{1121}+12w'^{3}_{1121}+2w^{1}_{1221}+10w^{2}_{1221}+7w^{3}_{1221}+w'^{1}_{1221}\\
+10w'^{2}_{1221}+8w'^{3}_{1221}+3w^{1}_{2121}+10w^{2}_{2121}+8w^{3}_{2121}+w'^{1}_{2121}+10w'^{2}_{2121}+10w'^{3}_{2121}\\
+5w^{1}_{2221}+14w^{2}_{2221}+9w^{3}_{2221}+4w'^{1}_{2221}+14w'^{2}_{2221}+12w'^{3}_{2221}6w^{1}_{1112}+16w^{2}_{1112}+10w^{3}_{1112}\\
+5w'^{1}_{1112}+16w'^{2}_{1112}+12w'^{3}_{1112}+4w^{1}_{1212}+10w^{2}_{1212}+7w^{3}_{1212}+2w'^{1}_{1212}+10w'^{2}_{1212}+9w'^{3}_{1212}\\
+7w^{1}_{2112}+18w^{2}_{2112}+11w^{3}_{2112}+6w'^{1}_{2112}+18w'^{2}_{2112}+12w'^{3}_{2112}+2w^{1}_{2212}+6w^{2}_{2212}+5w^{3}_{2212}\\
+1w'^{1}_{2212}+6w'^{2}_{2212}+7w'^{3}_{2212}+3w^{1}_{1122}+10w^{2}_{1122}+8w^{3}_{1122}+w'^{1}_{1122}+10w'^{2}_{1122}+10w'^{3}_{1122}\\
+3w^{1}_{1222}+10w^{2}_{1222}+8w^{3}_{1222}+w'^{1}_{1222}+10w'^{2}_{1222}+10w'^{3}_{1222}+6w^{1}_{2122}+16w^{2}_{2122}+10w^{3}_{2122}\\
+5w'^{1}_{2122}+16w'^{2}_{2122}+12w'^{3}_{2122}+4w^{1}_{2222}+10w^{2}_{2222}+7w^{3}_{2222}+2w'^{1}_{2222}+10w'^{2}_{2222}+9w'^{3}_{2222}\\
+150t^{1}+200t^{2}+250t^{3}+100t'^{1}+200t'^{2}+300t'^{3}
\end{array}\right]-\delta\geq 3,$$

$$\frac{2.534}{8}\left[\begin{array}{l}
3w^{1}_{1111}+10w^{2}_{1111}+8w^{3}_{1111}+w'^{1}_{1111}+10w'^{2}_{1111}+10w'^{3}_{1111}+4w^{1}_{1211}+10w^{2}_{1211}+7w^{3}_{1211}\\
+2w'^{1}_{1211}+10w'^{2}_{1211}+9w'^{3}_{1211}+7w^{1}_{2111}+18w^{2}_{2111}+11w^{3}_{2111}+6w'^{1}_{2111}+18w'^{2}_{2111}+12w'^{3}_{2111}\\
+7w^{1}_{2211}+18w^{2}_{2211}+11w^{3}_{2211}+6w'^{1}_{2211}+18w'^{2}_{2211}+14w'^{3}_{2211}+6w^{1}_{1121}+16w^{2}_{1121}\\
+10w^{3}_{1121}+5w'^{1}_{1121}+16w'^{2}_{1121}+12w'^{3}_{1121}+3w^{1}_{1221}+10w^{2}_{1221}+8w^{3}_{1221}+w'^{1}_{1221}\\
+10w'^{2}_{1221}+10w'^{3}_{1221}+5w^{1}_{2121}+14w^{2}_{2121}+9w^{3}_{2121}+4w'^{1}_{2121}+14w'^{2}_{2121}+12w'^{3}_{2121}\\
+8w^{1}_{2221}+20w^{2}_{2221}+12w^{3}_{2221}+7w'^{1}_{2221}+20w'^{2}_{2221}+15w'^{3}_{2221}+7w^{1}_{1112}+16w^{2}_{1112}+9w^{3}_{1112}\\
+5w'^{1}_{1112}+16w'^{2}_{1112}+10w'^{3}_{1112}+w^{1}_{1212}+6w^{2}_{1212}+5w^{3}_{1212}+0w'^{1}_{1212}+6w'^{2}_{1212}+7w'^{3}_{1212}\\
+6w^{1}_{2112}+18w^{2}_{2112}+11w^{3}_{2112}+5w'^{1}_{2112}+18w'^{2}_{2112}+12w'^{3}_{2112}+7w^{1}_{2212}+18w^{2}_{2212}+11w^{3}_{2212}\\
+6w'^{1}_{2212}+18w'^{2}_{2212}+14w'^{3}_{2212}+3w^{1}_{1122}+10w^{2}_{1122}+8w^{3}_{1122}+w'^{1}_{1122}+10w'^{2}_{1122}+10w'^{3}_{1122}\\
+8w^{1}_{1222}+20w^{2}_{1222}+12w^{3}_{1222}+7w'^{1}_{1222}+20w'^{2}_{1222}+13w'^{3}_{1222}+10w^{1}_{2122}+22w^{2}_{2122}+12w^{3}_{2122}\\
+9w'^{1}_{2122}+22w'^{2}_{2122}+13w'^{3}_{2122}+w^{1}_{2222}+6w^{2}_{2222}+5w^{3}_{2222}+0w'^{1}_{2222}+6w'^{2}_{2222}+7w'^{3}_{2222}\\
+250t^{1}+300t^{2}+350t^{3}+200t'^{1}+300t'^{2}+400t'^{3}
\end{array}\right]-\delta\geq 3,$$

$$\frac{3.32745}{8}\left[\begin{array}{l}
w^{1}_{1111}+4w^{2}_{1111}+4w^{3}_{1111}+0w'^{1}_{1111}+4w'^{2}_{1111}+6w'^{3}_{1111}+2w^{1}_{1211}+6w^{2}_{1211}+5w^{3}_{1211}\\
+w'^{1}_{1211}+6w'^{2}_{1211}+7w'^{3}_{1211}+3w^{1}_{2111}+10w^{2}_{2111}+8w^{3}_{2111}+w'^{1}_{2111}+10w'^{2}_{2111}+10w'^{3}_{2111}\\
+4w^{1}_{2211}+10w^{2}_{2211}+7w^{3}_{2211}+2w'^{1}_{2211}+10w'^{2}_{2211}+9w'^{3}_{2211}+7w^{1}_{1121}+18w^{2}_{1121}\\
+11w^{3}_{1121}+6w'^{1}_{1121}+18w'^{2}_{1121}+12w'^{3}_{1121}+2w^{1}_{1221}+10w^{2}_{1221}+7w^{3}_{1221}+w'^{1}_{1221}\\
+10w'^{2}_{1221}+8w'^{3}_{1221}+3w^{1}_{2121}+10w^{2}_{2121}+8w^{3}_{2121}+w'^{1}_{2121}+10w'^{2}_{2121}+10w'^{3}_{2121}\\
+5w^{1}_{2221}+14w^{2}_{2221}+9w^{3}_{2221}+4w'^{1}_{2221}+14w'^{2}_{2221}+12w'^{3}_{2221}6w^{1}_{1112}+16w^{2}_{1112}+10w^{3}_{1112}\\
+5w'^{1}_{1112}+16w'^{2}_{1112}+12w'^{3}_{1112}+4w^{1}_{1212}+10w^{2}_{1212}+7w^{3}_{1212}+2w'^{1}_{1212}+10w'^{2}_{1212}+9w'^{3}_{1212}\\
+7w^{1}_{2112}+18w^{2}_{2112}+11w^{3}_{2112}+6w'^{1}_{2112}+18w'^{2}_{2112}+12w'^{3}_{2112}+2w^{1}_{2212}+6w^{2}_{2212}+5w^{3}_{2212}\\
+1w'^{1}_{2212}+6w'^{2}_{2212}+7w'^{3}_{2212}+3w^{1}_{1122}+10w^{2}_{1122}+8w^{3}_{1122}+w'^{1}_{1122}+10w'^{2}_{1122}+10w'^{3}_{1122}\\
+3w^{1}_{1222}+10w^{2}_{1222}+8w^{3}_{1222}+w'^{1}_{1222}+10w'^{2}_{1222}+10w'^{3}_{1222}+6w^{1}_{2122}+16w^{2}_{2122}+10w^{3}_{2122}\\
+5w'^{1}_{2122}+16w'^{2}_{2122}+12w'^{3}_{2122}+4w^{1}_{2222}+10w^{2}_{2222}+7w^{3}_{2222}+2w'^{1}_{2222}+10w'^{2}_{2222}+9w'^{3}_{2222}\\
+150t^{1}+200t^{2}+250t^{3}+100t'^{1}+200t'^{2}+300t'^{3}
\end{array}\right]+\beta\geq 3,$$

$$\frac{2.534}{8}\left[\begin{array}{l}
3w^{1}_{1111}+10w^{2}_{1111}+8w^{3}_{1111}+w'^{1}_{1111}+10w'^{2}_{1111}+10w'^{3}_{1111}+4w^{1}_{1211}+10w^{2}_{1211}+7w^{3}_{1211}\\
+2w'^{1}_{1211}+10w'^{2}_{1211}+9w'^{3}_{1211}+7w^{1}_{2111}+18w^{2}_{2111}+11w^{3}_{2111}+6w'^{1}_{2111}+18w'^{2}_{2111}+12w'^{3}_{2111}\\
+7w^{1}_{2211}+18w^{2}_{2211}+11w^{3}_{2211}+6w'^{1}_{2211}+18w'^{2}_{2211}+14w'^{3}_{2211}+6w^{1}_{1121}+16w^{2}_{1121}\\
+10w^{3}_{1121}+5w'^{1}_{1121}+16w'^{2}_{1121}+12w'^{3}_{1121}+3w^{1}_{1221}+10w^{2}_{1221}+8w^{3}_{1221}+w'^{1}_{1221}\\
+10w'^{2}_{1221}+10w'^{3}_{1221}+5w^{1}_{2121}+14w^{2}_{2121}+9w^{3}_{2121}+4w'^{1}_{2121}+14w'^{2}_{2121}+12w'^{3}_{2121}\\
+8w^{1}_{2221}+20w^{2}_{2221}+12w^{3}_{2221}+7w'^{1}_{2221}+20w'^{2}_{2221}+15w'^{3}_{2221}+7w^{1}_{1112}+16w^{2}_{1112}+9w^{3}_{1112}\\
+5w'^{1}_{1112}+16w'^{2}_{1112}+10w'^{3}_{1112}+w^{1}_{1212}+6w^{2}_{1212}+5w^{3}_{1212}+0w'^{1}_{1212}+6w'^{2}_{1212}+7w'^{3}_{1212}\\
+6w^{1}_{2112}+18w^{2}_{2112}+11w^{3}_{2112}+5w'^{1}_{2112}+18w'^{2}_{2112}+12w'^{3}_{2112}+7w^{1}_{2212}+18w^{2}_{2212}+11w^{3}_{2212}\\
+6w'^{1}_{2212}+18w'^{2}_{2212}+14w'^{3}_{2212}+3w^{1}_{1122}+10w^{2}_{1122}+8w^{3}_{1122}+w'^{1}_{1122}+10w'^{2}_{1122}+10w'^{3}_{1122}\\
+8w^{1}_{1222}+20w^{2}_{1222}+12w^{3}_{1222}+7w'^{1}_{1222}+20w'^{2}_{1222}+13w'^{3}_{1222}+10w^{1}_{2122}+22w^{2}_{2122}+12w^{3}_{2122}\\
+9w'^{1}_{2122}+22w'^{2}_{2122}+13w'^{3}_{2122}+w^{1}_{2222}+6w^{2}_{2222}+5w^{3}_{2222}+0w'^{1}_{2222}+6w'^{2}_{2222}+7w'^{3}_{2222}\\
+250t^{1}+300t^{2}+350t^{3}+200t'^{1}+300t'^{2}+400t'^{3}
\end{array}\right]+\beta\geq 3,$$

$$\delta\geq\beta,\quad \beta\geq 0,\quad \delta\geq 0,$$

and all the constraints

where $\delta=\tanh^{-1}(2\gamma-1)$; and $\beta=\tanh^{-1}(2\vartheta-1)$.

PMF model:

$max\,\gamma-\vartheta$
subject to

$$\left[1.806118-\frac{1}{8}\left[\begin{array}{l}
w^{1}_{1111}+4w^{2}_{1111}+4w^{3}_{1111}+0w'^{1}_{1111}+4w'^{2}_{1111}+6w'^{3}_{1111}+2w^{1}_{1211}+6w^{2}_{1211}+5w^{3}_{1211}\\
+w'^{1}_{1211}+6w'^{2}_{1211}+7w'^{3}_{1211}+3w^{1}_{2111}+10w^{2}_{2111}+8w^{3}_{2111}+w'^{1}_{2111}+10w'^{2}_{2111}+10w'^{3}_{2111}\\
+4w^{1}_{2211}+10w^{2}_{2211}+7w^{3}_{2211}+2w'^{1}_{2211}+10w'^{2}_{2211}+9w'^{3}_{2211}+7w^{1}_{1121}+18w^{2}_{1121}\\
+11w^{3}_{1121}+6w'^{1}_{1121}+18w'^{2}_{1121}+12w'^{3}_{1121}+2w^{1}_{1221}+10w^{2}_{1221}+7w^{3}_{1221}+w'^{1}_{1221}\\
+10w'^{2}_{1221}+8w'^{3}_{1221}+3w^{1}_{2121}+10w^{2}_{2121}+8w^{3}_{2121}+w'^{1}_{2121}+10w'^{2}_{2121}+10w'^{3}_{2121}\\
+5w^{1}_{2221}+14w^{2}_{2221}+9w^{3}_{2221}+4w'^{1}_{2221}+14w'^{2}_{2221}+12w'^{3}_{2221}6w^{1}_{1112}+16w^{2}_{1112}+10w^{3}_{1112}\\
+5w'^{1}_{1112}+16w'^{2}_{1112}+12w'^{3}_{1112}+4w^{1}_{1212}+10w^{2}_{1212}+7w^{3}_{1212}+2w'^{1}_{1212}+10w'^{2}_{1212}+9w'^{3}_{1212}\\
+7w^{1}_{2112}+18w^{2}_{2112}+11w^{3}_{2112}+6w'^{1}_{2112}+18w'^{2}_{2112}+12w'^{3}_{2112}+2w^{1}_{2212}+6w^{2}_{2212}+5w^{3}_{2212}\\
+1w'^{1}_{2212}+6w'^{2}_{2212}+7w'^{3}_{2212}+3w^{1}_{1122}+10w^{2}_{1122}+8w^{3}_{1122}+w'^{1}_{1122}+10w'^{2}_{1122}+10w'^{3}_{1122}\\
+3w^{1}_{1222}+10w^{2}_{1222}+8w^{3}_{1222}+w'^{1}_{1222}+10w'^{2}_{1222}+10w'^{3}_{1222}+6w^{1}_{2122}+16w^{2}_{2122}+10w^{3}_{2122}\\
+5w'^{1}_{2122}+16w'^{2}_{2122}+12w'^{3}_{2122}+4w^{1}_{2222}+10w^{2}_{2222}+7w^{3}_{2222}+2w'^{1}_{2222}+10w'^{2}_{2222}+9w'^{3}_{2222}\\
+150t^{1}+200t^{2}+250t^{3}+100t'^{1}+200t'^{2}+300t'^{3}
\end{array}\right]\right]^{2}\leq 3.2515(1-\gamma)$$

$$\left[2.3721-\frac{1}{8}\left[\begin{array}{l}
3w^{1}_{1111}+10w^{2}_{1111}+8w^{3}_{1111}+w'^{1}_{1111}+10w'^{2}_{1111}+10w'^{3}_{1111}+4w^{1}_{1211}+10w^{2}_{1211}+7w^{3}_{1211}\\
+2w'^{1}_{1211}+10w'^{2}_{1211}+9w'^{3}_{1211}+7w^{1}_{2111}+18w^{2}_{2111}+11w^{3}_{2111}+6w'^{1}_{2111}+18w'^{2}_{2111}+12w'^{3}_{2111}\\
+7w^{1}_{2211}+18w^{2}_{2211}+11w^{3}_{2211}+6w'^{1}_{2211}+18w'^{2}_{2211}+14w'^{3}_{2211}+6w^{1}_{1121}+16w^{2}_{1121}\\
+10w^{3}_{1121}+5w'^{1}_{1121}+16w'^{2}_{1121}+12w'^{3}_{1121}+3w^{1}_{1221}+10w^{2}_{1221}+8w^{3}_{1221}+w'^{1}_{1221}\\
+10w'^{2}_{1221}+10w'^{3}_{1221}+5w^{1}_{2121}+14w^{2}_{2121}+9w^{3}_{2121}+4w'^{1}_{2121}+14w'^{2}_{2121}+12w'^{3}_{2121}\\
+8w^{1}_{2221}+20w^{2}_{2221}+12w^{3}_{2221}+7w'^{1}_{2221}+20w'^{2}_{2221}+15w'^{3}_{2221}+7w^{1}_{1112}+16w^{2}_{1112}+9w^{3}_{1112}\\
+5w'^{1}_{1112}+16w'^{2}_{1112}+10w'^{3}_{1112}+w^{1}_{1212}+6w^{2}_{1212}+5w^{3}_{1212}+0w'^{1}_{1212}+6w'^{2}_{1212}+7w'^{3}_{1212}\\
+6w^{1}_{2112}+18w^{2}_{2112}+11w^{3}_{2112}+5w'^{1}_{2112}+18w'^{2}_{2112}+12w'^{3}_{2112}+7w^{1}_{2212}+18w^{2}_{2212}+11w^{3}_{2212}\\
+6w'^{1}_{2212}+18w'^{2}_{2212}+14w'^{3}_{2212}+3w^{1}_{1122}+10w^{2}_{1122}+8w^{3}_{1122}+w'^{1}_{1122}+10w'^{2}_{1122}+10w'^{3}_{1122}\\
+8w^{1}_{1222}+20w^{2}_{1222}+12w^{3}_{1222}+7w'^{1}_{1222}+20w'^{2}_{1222}+13w'^{3}_{1222}+10w^{1}_{2122}+22w^{2}_{2122}+12w^{3}_{2122}\\
+9w'^{1}_{2122}+22w'^{2}_{2122}+13w'^{3}_{2122}+w^{1}_{2222}+6w^{2}_{2222}+5w^{3}_{2222}+0w'^{1}_{2222}+6w'^{2}_{2222}+7w'^{3}_{2222}\\
+250t^{1}+300t^{2}+350t^{3}+200t'^{1}+300t'^{2}+400t'^{3}
\end{array}\right]\right]^{2}\leq 5.61(1-\gamma)$$

$$\left[\frac{1}{8}\left[\begin{array}{l}
w^{1}_{1111}+4w^{2}_{1111}+4w^{3}_{1111}+0w'^{1}_{1111}+4w'^{2}_{1111}+6w'^{3}_{1111}+2w^{1}_{1211}+6w^{2}_{1211}+5w^{3}_{1211}\\
+w'^{1}_{1211}+6w'^{2}_{1211}+7w'^{3}_{1211}+3w^{1}_{2111}+10w^{2}_{2111}+8w^{3}_{2111}+w'^{1}_{2111}+10w'^{2}_{2111}+10w'^{3}_{2111}\\
+4w^{1}_{2211}+10w^{2}_{2211}+7w^{3}_{2211}+2w'^{1}_{2211}+10w'^{2}_{2211}+9w'^{3}_{2211}+7w^{1}_{1121}+18w^{2}_{1121}\\
+11w^{3}_{1121}+6w'^{1}_{1121}+18w'^{2}_{1121}+12w'^{3}_{1121}+2w^{1}_{1221}+10w^{2}_{1221}+7w^{3}_{1221}+w'^{1}_{1221}\\
+10w'^{2}_{1221}+8w'^{3}_{1221}+3w^{1}_{2121}+10w^{2}_{2121}+8w^{3}_{2121}+w'^{1}_{2121}+10w'^{2}_{2121}+10w'^{3}_{2121}\\
+5w^{1}_{2221}+14w^{2}_{2221}+9w^{3}_{2221}+4w'^{1}_{2221}+14w'^{2}_{2221}+12w'^{3}_{2221}6w^{1}_{1112}+16w^{2}_{1112}+10w^{3}_{1112}\\
+5w'^{1}_{1112}+16w'^{2}_{1112}+12w'^{3}_{1112}+4w^{1}_{1212}+10w^{2}_{1212}+7w^{3}_{1212}+2w'^{1}_{1212}+10w'^{2}_{1212}+9w'^{3}_{1212}\\
+7w^{1}_{2112}+18w^{2}_{2112}+11w^{3}_{2112}+6w'^{1}_{2112}+18w'^{2}_{2112}+12w'^{3}_{2112}+2w^{1}_{2212}+6w^{2}_{2212}+5w^{3}_{2212}\\
+1w'^{1}_{2212}+6w'^{2}_{2212}+7w'^{3}_{2212}+3w^{1}_{1122}+10w^{2}_{1122}+8w^{3}_{1122}+w'^{1}_{1122}+10w'^{2}_{1122}+10w'^{3}_{1122}\\
+3w^{1}_{1222}+10w^{2}_{1222}+8w^{3}_{1222}+w'^{1}_{1222}+10w'^{2}_{1222}+10w'^{3}_{1222}+6w^{1}_{2122}+16w^{2}_{2122}+10w^{3}_{2122}\\
+5w'^{1}_{2122}+16w'^{2}_{2122}+12w'^{3}_{2122}+4w^{1}_{2222}+10w^{2}_{2222}+7w^{3}_{2222}+2w'^{1}_{2222}+10w'^{2}_{2222}+9w'^{3}_{2222}\\
+150t^{1}+200t^{2}+250t^{3}+100t'^{1}+200t'^{2}+300t'^{3}
\end{array}\right]\right]^{2}\geq 3.2515(1-\vartheta)$$

$$\left[\frac{1}{8}\left[\begin{array}{l}
3w^{1}_{1111}+10w^{2}_{1111}+8w^{3}_{1111}+w'^{1}_{1111}+10w'^{2}_{1111}+10w'^{3}_{1111}+4w^{1}_{1211}+10w^{2}_{1211}+7w^{3}_{1211}\\
+2w'^{1}_{1211}+10w'^{2}_{1211}+9w'^{3}_{1211}+7w^{1}_{2111}+18w^{2}_{2111}+11w^{3}_{2111}+6w'^{1}_{2111}+18w'^{2}_{2111}+12w'^{3}_{2111}\\
+7w^{1}_{2211}+18w^{2}_{2211}+11w^{3}_{2211}+6w'^{1}_{2211}+18w'^{2}_{2211}+14w'^{3}_{2211}+6w^{1}_{1121}+16w^{2}_{1121}\\
+10w^{3}_{1121}+5w'^{1}_{1121}+16w'^{2}_{1121}+12w'^{3}_{1121}+3w^{1}_{1221}+10w^{2}_{1221}+8w^{3}_{1221}+w'^{1}_{1221}\\
+10w'^{2}_{1221}+10w'^{3}_{1221}+5w^{1}_{2121}+14w^{2}_{2121}+9w^{3}_{2121}+4w'^{1}_{2121}+14w'^{2}_{2121}+12w'^{3}_{2121}\\
+8w^{1}_{2221}+20w^{2}_{2221}+12w^{3}_{2221}+7w'^{1}_{2221}+20w'^{2}_{2221}+15w'^{3}_{2221}+7w^{1}_{1112}+16w^{2}_{1112}+9w^{3}_{1112}\\
+5w'^{1}_{1112}+16w'^{2}_{1112}+10w'^{3}_{1112}+w^{1}_{1212}+6w^{2}_{1212}+5w^{3}_{1212}+0w'^{1}_{1212}+6w'^{2}_{1212}+7w'^{3}_{1212}\\
+6w^{1}_{2112}+18w^{2}_{2112}+11w^{3}_{2112}+5w'^{1}_{2112}+18w'^{2}_{2112}+12w'^{3}_{2112}+7w^{1}_{2212}+18w^{2}_{2212}+11w^{3}_{2212}\\
+6w'^{1}_{2212}+18w'^{2}_{2212}+14w'^{3}_{2212}+3w^{1}_{1122}+10w^{2}_{1122}+8w^{3}_{1122}+w'^{1}_{1122}+10w'^{2}_{1122}+10w'^{3}_{1122}\\
+8w^{1}_{1222}+20w^{2}_{1222}+12w^{3}_{1222}+7w'^{1}_{1222}+20w'^{2}_{1222}+13w'^{3}_{1222}+10w^{1}_{2122}+22w^{2}_{2122}+12w^{3}_{2122}\\
+9w'^{1}_{2122}+22w'^{2}_{2122}+13w'^{3}_{2122}+w^{1}_{2222}+6w^{2}_{2222}+5w^{3}_{2222}+0w'^{1}_{2222}+6w'^{2}_{2222}+7w'^{3}_{2222}\\
+250t^{1}+300t^{2}+350t^{3}+200t'^{1}+300t'^{2}+400t'^{3}
\end{array}\right]\right]^{2}\geq 5.61(1-\vartheta)$$

$$\gamma\geq\vartheta,\quad \gamma+\vartheta\leq 1,\qquad \gamma,\vartheta\in[0,\ 1]$$

and all the constraints.

The membership and non-membership values for any model are obtained by resolving the three crisp issues. Additionally, as shown in Table 12, IF solutions and intuitionistic objective values are also achieved. The numerical models are carried with a PC having a Core i5 CPU@2.8 GH, 8 GB of RAM and a 64-bit operating system. The numerical models are solved using Lingo programming software: LINGO 19.

Table 12. The IF solutions and intuitionistic objective values.

Membership Function Type	Intuitionistic Fuzzy Solution	Intuitionistic Objective Value
Linear	$\tilde{x}^I_{1111} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$; $\tilde{x}^I_{1121} = \begin{pmatrix} 9.655, 9.655, 9.655; \\ 9.655, 9.655, 3.283 \end{pmatrix}$ $\tilde{x}^I_{2111} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$; $\tilde{x}^I_{2211} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{1211} = \begin{pmatrix} 0,0,0; \\ 0,0,0, \end{pmatrix}$; $\tilde{x}^I_{1221} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2121} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2221} = \begin{pmatrix} 0,0,0; \\ 0,0,69.37 \end{pmatrix}$ $\tilde{x}^I_{1112} = \begin{pmatrix} 15.158, 15.158, 15.158; \\ 15.158, 15.158, 5.154 \end{pmatrix}$ $\tilde{x}^I_{1212} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2112} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2212} = \begin{pmatrix} 2.99, 2.99, 2.99; \\ 2.99, 2.99, 26.02 \end{pmatrix}$ $\tilde{x}^I_{1122} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{1222} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2122} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2222} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$	$\tilde{Z}^I_1 = \begin{pmatrix} 1.055, 2.202, 4.814; \\ 0.255, 2.202, 29.618 \end{pmatrix}$ $\tilde{Z}^I_2 = \begin{pmatrix} 1.34,\ 2.095,\ 3.476; \\ 0.473,\ 2.095,\ 15.566 \end{pmatrix}$
Hyperbolic	$\tilde{x}^I_{1111} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$; $\tilde{x}^I_{1121} = \begin{pmatrix} 6.857, 6.857, 6.857; \\ 0, 6.857, 6.857 \end{pmatrix}$ $\tilde{x}^I_{2111} = \begin{pmatrix} 0.429, 0.429, 0.429; \\ 0, 0.429, 0.429 \end{pmatrix}$; $\tilde{x}^I_{1221} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{1211} = \begin{pmatrix} 0,0,0; \\ 0,0,0, \end{pmatrix}$; $\tilde{x}^I_{2211} = \begin{pmatrix} 0,0,31.5; \\ 0,0,31.5 \end{pmatrix}$ $\tilde{x}^I_{2121} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2221} = \begin{pmatrix} 0,0,30; \\ 0,0,30 \end{pmatrix}$ $\tilde{x}^I_{1112} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{1212} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2112} = \begin{pmatrix} 0,0,31.755; \\ 0,0,31.755 \end{pmatrix}$ $\tilde{x}^I_{2212} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{1122} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{1222} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2122} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2222} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$	$\tilde{Z}^I_1 = \begin{pmatrix} 0.32,\ 2.15, 17.98; \\ 0.129,\ 2.15,\ 70.55 \end{pmatrix}$ $\tilde{Z}^I_2 = \begin{pmatrix} 0.449,\ 1.611,\ 8.964; \\ 0.248,\ 1.611,\ 17.594 \end{pmatrix}$
Parabolic	$\tilde{x}^I_{1111} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$; $\tilde{x}^I_{1121} = \begin{pmatrix} 9.655, 9.655, 9.655; \\ 9.655, 9.655, 3.283 \end{pmatrix}$ $\tilde{x}^I_{2111} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$; $\tilde{x}^I_{2211} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{1211} = \begin{pmatrix} 0,0,0; \\ 0,0,0, \end{pmatrix}$; $\tilde{x}^I_{1221} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2121} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2221} = \begin{pmatrix} 0,0,0; \\ 0,0,69.37 \end{pmatrix}$ $\tilde{x}^I_{1112} = \begin{pmatrix} 15.158, 15.158, 15.158; \\ 15.158, 15.158, 5.154 \end{pmatrix}$ $\tilde{x}^I_{1212} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2112} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2212} = \begin{pmatrix} 2.99, 2.99, 2.99; \\ 2.99, 2.99, 26.02 \end{pmatrix}$ $\tilde{x}^I_{1122} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{1222} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2122} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$ $\tilde{x}^I_{2222} = \begin{pmatrix} 0,0,0; \\ 0,0,0 \end{pmatrix}$	$\tilde{Z}^I_1 = \begin{pmatrix} 1.055, 2.202, 4.814; \\ 0.255, 2.202, 29.618; \end{pmatrix}$ $\tilde{Z}^I_2 = \begin{pmatrix} 1.34,\ 2.095,\ 3.476; \\ 0.473,\ 2.095,\ 15.566 \end{pmatrix}$

Thus, for the LMF, HMF, and PMF structures, $\mu_{\tilde{Z}_1^{I}}(x)$ and $v_{\tilde{Z}_1^{I}}(x)$ may be constructed.

$$\mu_{\tilde{Z}_1^{I}}(x) = \begin{cases} \frac{x-1.055}{2.202-1.055} & 1.055 < x \leq 2.202, \\ 1 & x = 2.202, \\ \frac{4.814-x}{4.814-2.202}, & 2.202 \leq x \leq 4.814, \\ 0, & otherwise, \end{cases}$$

$$v_{\tilde{Z}_1^{I}}(x) = \begin{cases} \frac{2.202-x}{2.202-0.255} & 0.255 < x \leq 2.202, \\ 0 & x = 2.202 \\ \frac{x-2.202}{29.618-2.202}, & 2.202 \leq x \leq 29.618, \\ 1, & otherwise, \end{cases}$$

$$\mu_{\tilde{Z}_2^{I}}(x) = \begin{cases} \frac{x-1.34}{2.095-1.34} & 1.34 < x \leq 2.095, \\ 1 & x = 2.095, \\ \frac{3.476-x}{3.476-2.095}, & 2.095 \leq x \leq 3.476, \\ 0, & otherwise, \end{cases}$$

$$v_{\tilde{Z}_2^{I}}(x) = \begin{cases} \frac{2.095-x}{2.095-0.473} & 0.473 < x \leq 2.095, \\ 0 & x = 2.095, \\ \frac{x-2.095}{15.566-2.095}, & 2.095 \leq x \leq 15.566, \\ 1, & otherwise, \end{cases}$$

Also, for the HMF models

$$\mu_{\tilde{Z}_1^{I}}(x) = \begin{cases} \frac{x-0.32}{2.15-0.32} & 0.32 < x \leq 2.15, \\ 1 & x = 2.15, \\ \frac{17.98-x}{17.98-2.15}, & 2.15 \leq x \leq 17.98, \\ 0, & otherwise, \end{cases}$$

$$v_{\tilde{Z}_1^{I}}(x) = \begin{cases} \frac{2.15-x}{2.15-0.129} & 0.129 < x \leq 2.15, \\ 0 & x = 2.15, \\ \frac{x-2.15}{70.55-2.15}, & 2.15 \leq x \leq 70.55, \\ 1, & otherwise, \end{cases}$$

$$\mu_{\tilde{Z}_2^{I}}(x) = \begin{cases} \frac{x-0.449}{1.611-0.449} & 0.449 < x \leq 1.611, \\ 1 & x = 1.611, \\ \frac{8.964-x}{8.964-1.611}, & 1.611 \leq x \leq 8.964, \\ 0, & otherwise, \end{cases}$$

$$v_{\tilde{Z}_2^{I}}(x) = \begin{cases} \frac{1.611-x}{1.611-0.248} & 0.248 < x \leq 1.611, \\ 0 & x = 1.611, \\ \frac{x-1.611}{17.594-1.611}, & 1.611 \leq x \leq 17.594, \\ 1, & otherwise, \end{cases}$$

Table 13 provides an evaluation of the various membership models. The HMF is greater than the parabolic and linear ones. In the three presented solution models, both degrees of refusal and acceptance were considered. Numerical data for the LMF, HMF, and PMF models were exhibited in Table 14 to ensure the applicability and computational efficiency of the proposed models for solving FIF-MMSFTM. The total solver iteration and elapsed run time per second were exhibited. The HMF model has the smallest number of iterations, while the PMF model has the largest number of iterations. It is clear that the elapsed runtime of the HMF model is smaller than that of the LMF and PMF models. Also, the model class, nonlinear variables, integer variables, and a total number of constraints were provided. This study that is being presented has broad applications in a variety of logistical and supply chain management organizations. When single-type ambiguity is insufficient to describe specific parameters during any logistic operation, the suggested approach can be beneficial in handling two-fold ambiguity (multi-choice and IFN).

Table 13. Comparison of the different membership approaches.

Membership Function	γ	ϑ	$\gamma-\vartheta$
Linear	0.971	0.029	0.942
Hyperbolic	1	0	1
Parabolic	0.9428	0.0572	0.8856

Table 14. Numerical data of LMF, HMF, and PMF models.

	LMF Model	HMF Model	PMF Model
Total solver iteration	26	10	103
Elapsed runtime seconds	0.2	0.15	1.37
Model class	LP	LP	QP
Total variables	104	104	104
Nonlinear variables	0	0	102
Integer variables	0	0	0
Total constraints	129	133	135

8. Conclusions

Through this research, the FIF-MMSFTM was formed. Given the evolving market policies, we anticipate the quantities shipped, profits, cost coefficients, supplies, demands, and transportation parameters to exhibit TIFNs. Analyzing TPs within the IF domain is becoming increasingly relevant and practically applicable compared to conventional fuzzy domains. To address this, the developed FIF-MMSFTM is transformed into a linear model by introducing a new variable to tackle the nonlinearity of fractional objective functions. Subsequently, the model is converted into a crisp version using the AF and ordering relations of the IFS. Extending Zimmerman's approach is realized to maximize membership and minimize non-membership functions within the solution model. In many real-world situations, the degree of approval or opposition to a given target may vary non-constantly. Nonlinear membership (HMF and PMF) functions offer a more precise representation of DMs' behavior under certain conditions. Furthermore, in the proposed solution model, three different membership functions, namely LMF, PMF, and HMF, were utilized. For the presented LMF, HMF, and PMF models, the proof is that the unique optimal solution of each model is an efficient solution of the original transportation model. The main advantage of the proposed model is that it can easily be applied in various uncertain domains, requiring less computational effort to obtain the optimal solution. The elapsed run time is very short, at 0.2, 0.15, and 1.37 s for the presented LMF, HMF, and PMF models, respectively. Also, each model has 104 variables and 129 constraints.

Numerous unexplored areas to investigate in the field of solid fraction transportation, in our opinion, will be investigated in the future. The following lists a few of these points:

- Multi-choice multi-objective multi-item solid fractional transportation models.
- Fully IF multi-objective fractional programming problems;
- Bi-level supply chain model under an IF environment.

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References

1. Abd El-Wahed, W.F.; Lee, S.M. Interactive fuzzy goal programming for multi-objective transportation problems. *Int. J. Manag. Sci.* **2006**, *34*, 158–166. [CrossRef]
2. Ammar, E.E.; Youness, E.A. Study on multi-objective transportation problem with fuzzy numbers. *Appl. Math. Comput.* **2005**, *166*, 241–253.
3. Bit, A.K.; Biswal, M.P.; Alam, S.S. Fuzzy programming approach to multi-criteria decision-making transportation problem. *Fuzzy Sets Syst.* **1992**, *50*, 135–141. [CrossRef]
4. Cetin, N.; Tiryaki, F. A Fuzzy Approach Using Generalized Dinkelbach's Algorithm for Multi-objective Linear Fractional Transportation Problem. *Math. Probl. Eng.* **2014**, *2014*, 702319. [CrossRef]
5. Chanas, S.; Kuchta, D. A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. *Fuzzy Sets Syst.* **1996**, *82*, 299–305. [CrossRef]
6. Charnes, A.; Cooper, W.W. Programming with linear fractional functional. *Nav. Res. Logist. Q.* **1962**, *9*, 181–186. [CrossRef]
7. Diaz, J.A. Finding a complete description of all efficient solutions to a multi-objective transportation problem. *Ekon. Mat. Obz.* **1979**, *15*, 62–73.
8. Hitchcock, F.L. The distribution of a product from several sources to numerous localities. *J. Math. Phys.* **1941**, *20*, 224–230. [CrossRef]
9. Shell, E. Distribution of a product by several properties, directorate of management analysis. In Proceedings of the Second Symposium in Linear Programming, Washington, DC, USA, 27–29 January 1955; Volume 2, pp. 615–642.
10. Haley, K. New methods in mathematical programming—The solid transportation problem. *Oper. Res.* **1962**, *10*, 448–463. [CrossRef]
11. Patel, G.; Tripathy, J. The solid transportation problem and its variants. *Int. J. Manag. Syst.* **1989**, *5*, 17–36.
12. Baidya, A.; Bera, U.K.; Maiti, M. A solid transportation problem with safety factor under different uncertainty environments. *J. Uncertain. Anal. Appl.* **2013**, *1*, 18. [CrossRef]
13. Chen, S.M.; Phuong, B.D.H. Fuzzy time series forecasting based on optimal partitions of intervals and optimal weighting vectors. *Knowl. Based Syst.* **2017**, *118*, 204–216. [CrossRef]
14. Chen, B.; Liu, Y.; Zhou, T. An entropy based solid transportation problem in uncertain environment. *J. Ambient. Intell. Hum. Comput.* **2019**, *10*, 357–363. [CrossRef]
15. Kundu, P.; Kar, S.; Maiti, M. Multi-objective solid transportation problems with budget constraint in uncertain environment. *Int. J. Syst. Sci.* **2014**, *45*, 1668–1682. [CrossRef]
16. Liu, L.; Zhang, B.; Ma, W. Uncertain programming models for fixed charge multi-item solid transportation problem. *Soft Comput.* **2018**, *22*, 5825–5833. [CrossRef]
17. Mahajan, S.; Gupta, K.S. On fully intuitionistic fuzzy multi-objective transportation problems using different membership functions. *Ann. Oper. Res.* **2019**, *296*, 211–241. [CrossRef]
18. Verma, R.; Biswal, M.P.; Biswas, A. Fuzzy programming technique to solve multi-objective transportation problems with some non-linear membership functions. *Fuzzy Sets Syst.* **1997**, *91*, 37–43. [CrossRef]
19. Roy, S.K.; Mahapatra, D.R.; Biswal, M.P. Multi-choice stochastic transportation problem with exponential distribution. *J. Uncertain. Syst.* **2012**, *6*, 200–213.
20. Singh, S.K.; Yadav, S.P. Intuitionistic fuzzy multi-objective linear programming problem with various membership functions. *Ann. Oper. Res.* **2018**, *269*, 693–707. [CrossRef]
21. Ghosh, S.; Roy, S.K.; Ebrahimnejad, A.; Verdegay, J.L. Multi-objective fully intuitionistic fuzzy fixed-charge solid transportation problem. *Complex Intell. Syst.* **2021**, *7*, 1009–1023. [CrossRef]
22. Rizk-Allah, R.M.; Hassanien, A.E.; Elhoseny, M. A multi-objective transportation model under neutrosophic environment. *Comput. Electr. Eng.* **2018**, *69*, 705–719. [CrossRef]
23. Biswas, A.; Shaikh, A.A.; Niaki, S.T.A. Multi-objective non-linear fixed charge transportation problem with multiple modes of transportation in crisp and interval environments. *Appl. Soft Comput.* **2019**, *80*, 628–649. [CrossRef]
24. Sifaoui, T.; Aïder, M. Uncertain interval programming model for multi-objective multi-item fixed charge solid transportation problem with budget constraint and safety measure. *Soft Comput.* **2019**, *24*, 10123–10147. [CrossRef]
25. Jalil, S.A.; Sadia, S.; Javaid, S.; Ali, Q. A solution approach for solving fully fuzzy multiobjective solid transportation problem. *Int. J. Agric. Stat. Sci.* **2017**, *13*, 75–84.
26. Ebrahimnejad, A. An effective computational attempt for solving fully fuzzy linear programming using Molp problem. *J. Ind. Prod. Eng.* **2019**, *36*, 59–69. [CrossRef]
27. Ziqan, A.; Ibrahim, S.; Marabeh, M.; Qarariyah, A. Fully fuzzy linear systems with trapezoidal and hexagonal fuzzy numbers. *Granul. Comput.* **2021**, *7*, 229–238. [CrossRef]
28. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [CrossRef]
29. Ebrahimnejad, A.; Verdegay, J.L. A new approach for solving fully intuitionistic fuzzy transportation problems. *Fuzzy Optim. Decis. Mak.* **2018**, *17*, 447–474. [CrossRef]
30. Midya, S.; Roy, S.K.; Vincent, F.Y. Intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in a green supply chain. *Int. J. Mach. Learn. Cybern.* **2021**, *12*, 699–717. [CrossRef]

31. Roy, S.K.; Midya, S. Multi-objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment. *Appl. Intell.* **2019**, *49*, 3524–3538. [CrossRef]
32. Zadeh, L.A. Toward a generalized theory of uncertainty (GTU)—An outline. *Inf. Sci.* **2005**, *172*, 1–40. [CrossRef]
33. Zadeh, L.A. Is there a need for fuzzy logic? *Inf. Sci.* **2008**, *178*, 2751–2779. [CrossRef]
34. Zangiabadi, M.; Maleki, H.R. Fuzzy goal programming for multi-objective transportation problems. *J. Appl. Math. Comput.* **2007**, *24*, 449–460. [CrossRef]
35. Zangiabadi, M.; Maleki, H.R. Fuzzy goal programming technique to solve multi-objective transportation problems with some non-linear membership functions. *Iran. J. Fuzzy Syst.* **2013**, *10*, 61–74.
36. Gupta, A.; Kumar, A. A new method for solving linear multi-objective transportation problems with fuzzy parameters. *Appl. Math. Model.* **2012**, *36*, 1421–1430. [CrossRef]
37. Roy, S.K.; Mahapatra, D.R. Multi-objective interval valued transportation probabilistic problem involving lognormal. *Int. J. Math. Sci. Comput.* **2011**, *1*, 14–21.
38. Mahapatra, D.R.; Roy, S.K.; Biswal, M.P. Multi-choice stochastic transportation problem involving extreme value distribution. *Appl. Math. Model.* **2013**, *37*, 2230–2240. [CrossRef]
39. Maity, G.; Roy, S.K. Solving multi-choice multi-objective transportation problem: A utility function approach. *J. Uncertain. Anal. Appl.* **2014**, *2*, 11. [CrossRef]
40. Maity, G.; Roy, S.K. Solving a multi-objective transportation problem with nonlinear cost and multi-choice demand. *Int. J. Manag. Sci. Eng. Manag.* **2016**, *11*, 62–70. [CrossRef]
41. Kocken, H.G.; Ozlok, B.A.; Tiryaki, F. A compensatory fuzzy approach to multi-objective transportation problems with fuzzy parameters. *Eur. J. Pure Appl. Math.* **2014**, *7*, 369–386.
42. Roy, S.K.; Ebrahimnejad, A.; Verdegay, J.L.; Das, S. New approach for solving intuitionistic fuzzy multi-objective transportation problem. *Sadhana* **2018**, *43*, 3. [CrossRef]
43. Mahmoodirad, A.; Allahviranloo, T.; Niroomand, S. A new effective solution method for fully intuitionistic fuzzy transportation problem. *Soft Comput.* **2018**, *23*, 4521–4530. [CrossRef]
44. Nasseri, S.H.; Bavandi, S. Multi-choice Linear Programming in Fuzzy Random Hybrid Uncertainty Environment and Their Application in Multi-commodity Transportation Problem. *Fuzzy Inf. Eng.* **2020**, *12*, 109–122. [CrossRef]
45. El Sayed, M.A.; Abo-Sinna, M.A. A novel Approach for Fully Intuitionistic Fuzzy Multi-Objective Fractional Transportation Problem. *Alex. Eng. J.* **2021**, *60*, 1447–1463. [CrossRef]
46. El Sayed, M.A.; Baky, I.A. Multi-choice fractional stochastic multi-objective transportation problem. *Soft Comput.* **2023**, *27*, 11551–11567. [CrossRef]
47. Devnath, S.; Giri, P.K.; Mondal, S.S.; Maiti, M. Fully fuzzy multi-item two-stage fixed charge four-dimensional transportation problems with flexible constraints. *Granul. Comput.* **2022**, *7*, 779–797. [CrossRef]
48. Mondal, A.; Roy, S.K.; Midya, S. Intuitionistic fuzzy sustainable multi-objective multi-item multi-choice step fixed-charge solid transportation problem. *J. Ambient. Intell. Humaniz. Comput.* **2023**, *14*, 6975–6999. [CrossRef]
49. Chhibber, D.; Srivastava, P.K.; Bisht, D.C. From fuzzy transportation problem to non-linear intuitionistic fuzzy multi-objective transportation problem: A literature review. *Int. J. Model. Simul.* **2021**, *41*, 335–350. [CrossRef]
50. Malik, M.; Gupta, S.K. Goal programming technique for solving fully interval-valued intuitionistic fuzzy multiple objective transportation problems. *Soft Comput.* **2020**, *24*, 13955–13977. [CrossRef]
51. Bind, A.B.; Rani, D.; Goyal, K.K.; Ebrahimnejad, A. A solution approach for sustainable multi-objective multi-item 4D solid transportation problem involving triangular intuitionistic fuzzy parameters. *J. Clean. Prod.* **2023**, *414*, 137661. [CrossRef]
52. Klanšek, U.; Pšunder, M. solving the nonlinear transportation problem by global optimization. *Transport* **2010**, *25*, 314–324. [CrossRef]
53. Klanšek, U. solving the nonlinear discrete transportation problem MINLP optimization. *Transport* **2013**, *29*, 1–11. [CrossRef]
54. Zimmermann, H.J. *Fuzzy Set Theory and Its Applications*; Springer: Berlin/Heidelberg, Germany, 2001.
55. Gong, Z.; Zhangc, N.; Chiclanad, F. The optimization ordering model for intuitionistic fuzzy preference relations with utility functions. *Knowl.-Based Syst.* **2018**, *162*, 174–184. [CrossRef]
56. Mahapatra, G.S.; Roy, T.K. Intuitionistic Fuzzy Number and Its Arithmetic Operation with Application on System Failure. *J. Uncertain. Syst.* **2013**, *7*, 92–107.
57. Singh, V.P.; Sharma, K.; Chakraborty, D.; Ebrahimnejad, A. A novel multi objective bi-level programming problem under intuitionistic fuzzy environment and its application in production planning problem. *Complex Intell. Syst.* **2022**, *8*, 3263–3278. [CrossRef]
58. Arya, R.; Singh, P.; Kumari, S.; Obaidat, M.S. An approach for solving fully fuzzy multi-objective linear fractional optimization problems. *Soft Comput.* **2020**, *24*, 9105–9119. [CrossRef]
59. Chakraborty, M.; Gupta, S. Fuzzy mathematical programming for multi-objective linear fractional programming problem. *Fuzzy Sets Syst.* **2002**, *125*, 335–342. [CrossRef]