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On the Application of Mann-Iterative Scheme with *h*-Convexity in the Generation of Fractals

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Abstract: Self-similarity is a common feature among mathematical fractals and various objects of our natural environment. Therefore, escape criteria are used to determine the dynamics of fractal patterns through various iterative techniques. Taking motivation from this fact, we generate and analyze fractals as an application of the proposed Mann iterative technique with h-convexity. By doing so, we develop an escape criterion for it. Using this established criterion, we set the algorithm for fractal generation. We use the complex function $f(x) = x^n + c^t$, with $n \geq 2, c \in \mathbb{C}$ and $t \in R$ to generate and compare fractals using both the Mann iteration and Mann iteration with h-convexity. We generalize the Mann iterative scheme using the convexity parameter $h(\alpha) = \alpha^2$ and provide the detailed representations of quadratic and cubic fractals. Our comparative analysis consistently proved that the Mann iteration with h-convexity significantly outperforms the standard Mann iteration scheme regarding speed and efficiency. It is noticeable that the average number of iterations required to perform the task using Mann iteration with h-convexity is significantly less than the classical Mann iteration scheme. Moreover, the relationship between the fractal patterns and the input parameters of the proposed iteration is extremely intricate.

Keywords: Mann iteration with h-convexity (MIH); escape criterion; complex polynomial function; Mandelbrot set (M-set)

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1. Introduction

The word 'fractal' originates from the Latin word 'fractus', meaning broken or fractured. This concept was introduced by Benoît Mandelbrot, who is widely recognized as the pioneer of fractal geometry [1]. Our environment is full of systems having a fractal nature. Reiterating their shapes at increasingly finer magnifications results in an ironic complexity. Trees, clouds, and mountains are examples of such fractals with medium complexity. The natural world exhibits fractal patterns in global as well as microscopic systems [2]. A Science Design Lab (SDL) [3] is established to create patterns relevant to environmental psychology, as our eyes can adapt to them utilizing the "fractal fluency" paradigm. The "relaxation floors" were built using fractals, which reflect the positive effects of gazing at nature. This initiative delved into the positive effects of the geometry of our natural environment. Furthermore, fractals have a wide range of applications across multiple scientific fields. In particular, they are used in science to study and understand various biological phenomena, such as the growth cycles of microbes (e.g., bacteria, syphilis microorganisms, and unicellular organisms) and the patterns of nerve fibers. In cryptography, fractal theory is a useful tool for reducing the size of images [4] and securing images with cryptography [5]. In physical science, fractals are utilized to explore and comprehend the fluid dynamics of turbulent flows. They are also employed in telecommunications for antenna design [6], as well as in modeling architectures [7], computer networks, and radar systems [8], all of which are an application of fractals.

In the early twentieth century, P. Fatou and G. Julia were the first researchers to investigate the iterative calculation of $G : m \longrightarrow m^2 + r$, where $m, r \in \mathbb{C}$. However, they were unable to plot the function's graph. This work was continued in 1985 by B. Mandelbrot, who successfully plotted the graph of $F : x \longrightarrow m^2 + r$. He created the Mandelbrot set by changing the values of m as well as r [9]. The M-set for $G : m \rightarrow m^s + r$ where $s \geq 2$ and $m, r \in \mathbb{C}$ is explained in ref. [10]. The J-set and M-set images for complex functions that are both rational and transcendental in nature were analyzed in ref. [11]. The anti-J-sets and anti-M-sets were later established by Crow et al. [12]. As a result, they created visuals that illustrate $\overline{m}^2 + r$ where $m, r \in \mathbb{C}$ have a tricorn shape.

Fractals are generated through iterative fixed-point schemes, and in this process, the theory of fixed points plays a crucial role. The fractals of higher dimensions were analyzed in refs. [13–15]. Different iterative schemes, such as the Ishikawa iterative scheme [16], the Mann iterative scheme [17], the Noor iterative scheme [18], the S-iterative scheme [19], the Picard–Thakur hybrid [20], and the four-step iteration [21], have been used to generate some beautiful fractals. Both the Mandelbrot set and the Julia set for complex logarithmic, exponential, and trigonometric functions produced by multiple iterations are discussed in refs. [22,23] and the references therein. Furthermore, CR-iteration [24] and SP-iteration [25] are utilized to create generalized J-sets and M-sets. The discussion pertains to fractals generated through complex polynomials and new orbits are generated in [26]. A comparative analysis of fractals produced by the hybrid Picard S-iteration is presented in [27]. According to generalized Jungck–Mann orbit, the explanation of filled J-sets has been given in [28]. The discussion pertains to fractals generated through the extended Jungck–Noor orbit in ref. [29]. Certain biomorphs are discussed in refs. [30–33].

The above review of the literature demonstrates that researchers were inspired by different iteration schemes because each new technique was better than the previous one in efficiency to generate fractals. Therefore, we create M-sets for the complex polynomial function $f(x) = x^k + c^t$, where $k \geq 2, c \in \mathbb{C}$ using Mann-iteration with h-convexity. To explain the results, we need some fundamental definitions of M-sets and J-sets, as well as several iterative schemes that are stated in Section 2. The main results are presented in Section 3. Section 4 contains the newly generated fractals with a comparison investigation that continuously demonstrated that the Mann iteration with h-convexity (MIH) performs better than the normal Mann iteration (MI) scheme in terms of speed and efficiency. We conclude that there is a very complex interaction between the input parameters of the suggested iteration and the fractal designs, which is discussed in Section 5.

2. Preliminaries

This section of the paper involves basic definitions and terminology required to present fractals and iterative methods.

Definition 1 (J-Set [34])**.** *Suppose that G_k refers to the collection of points within $\mathbb{C}$ so that $k : \mathbb{C} \rightarrow \mathbb{C}$ be a polynomial function of complex numbers with degree ≥ 2. When the orbit of $G_k \nrightarrow \infty$ as $i \rightarrow \infty$, the set G_k is known as a filled Julia set i.e.,*

$$G_k = \{y \in \mathbb{C} : \{|k^i(y)|\}_{i=0}^{\infty} \textit{ is confined}\}. \tag{1}$$

The collection of points along G_k is a called simple J-set.

Definition 2 (Mandelbrot Set [34])**.** *Mandelbrot's (M-set) set is the collection of all points c for which the corresponding Julia set (i.e., J-set) G_k is connected, i.e.,*

$$M = \{c \in \mathbb{C} : G_k \textit{ is connected}\}, \tag{2}$$

As a result M-set is stated as [35]:

$$M = \left\{c \in \mathbb{C} : \left\{k^i(0)\right\} \nrightarrow \infty as i \rightarrow \infty\right\}, \tag{3}$$

where the only particular point is 0, such that $\acute{k}(0) = 0$. *Thus, picking 0 as the starting point is the ideal research option.*

Consider the case where $\mathbb{C}$ is a set of complex numbers that contains at least one element and $\psi : \mathbb{C} \rightarrow \mathbb{C}$ be a complex transformation. Then, with respect to every $x_0 \in \mathbb{C}$, the following iterative schemes can be defined as:

Definition 3 (Mann iterative method [36]). *In 1953, W. Robert Mann proposed a fixed-point iterative scheme known as the Mann-iterative scheme. The Mann-iterative scheme was subsequently defined for complex spaces as:*

$$x_{i+1} = (1-\alpha)x_i + \alpha\psi(x_i), \tag{4}$$

where $\alpha \in (0,1]$ *as well as* $i = 0, 1, 2, \cdots$.

Definition 4 (Ishikawa Iterative Method [37]). *A two-step fixed-point iterative scheme known as the Ishikawa iteration was introduced by Shero Ishikawa in 1974. It can also be defined for complex spaces as:*

$$\begin{cases} r_{i+1} = (1-\alpha)r_i + \alpha\psi(s_i), \\ s_i = (1-\beta)r_i + \beta\psi(r_i), \end{cases} \tag{5}$$

where $\alpha, \beta \in (0,1]$ *and* $i = 0, 1, 2, \cdots$.

Definition 5 (h-convexity [38]). *A group of h-convex functions was introduced by Varosanec (2007), extending the ideas of s-convex functions, convex, P-function, and Godunova–Levin functions. These h-convex functions are characterized as non-negative functions,* $\psi : K \rightarrow R$ *that convinces the given condition.*

$$\psi(\beta s + (1-\beta)t) \leq h(\beta)\psi(s) + h(1-\beta)\psi(t) \tag{6}$$

where h is a function that has non-negative values, $\beta \in (0,1)$ *and* $s, t \in K$.

For further such details and fixed points, see [39].

3. Escape Criterion

The escape criterion plays a crucial role in the generation of fractals. This section presents the escape criterion for the Mann iterative scheme with h-convexity for the proposed complex function.

Theorem 1. *Suppose that* $F(x_k) = x^k + c^t$ *is a complex function with* $|x| \geq |c| > \left(\frac{2+|\theta|}{1-\alpha^2}\right)^{\frac{1}{n-1}}$, *where* $\alpha \in (0,1]$ *and* $c \in \mathbb{C}$. *The sequence of successive approximations* $\{x_k\}_{k\in\mathbb{N}}$ *is described as*

$$x_{k+1} = h(\alpha)x_k + h(1-\alpha)f(x_k)$$

where $h(x) = x^2$, *for* $k = 0, 1, 2, 3 \ldots$. *Then* $|x_k| \rightarrow \infty$ *as* $k \rightarrow \infty$.

Proof. Since $F(x_k) = x^k + c^t$, where $c \in \mathbb{C}$, $t \in \mathbb{R}$ then by the definition of the Mann-iterative scheme with h-convexity;

$$x_{k+1} = \alpha^2 x_k + \left(1-\alpha^2\right)F(x_k)$$

Initially, choosing $k = 0$, we have the following.

$$
\begin{aligned}
x_1 &= \alpha^2 x_0 + \left(1-\alpha^2\right) F(x_0) \\
x_1 &= \alpha^2 x_0 + \left(1-\alpha^2\right)\left(x_0^k + c^t\right) \\
|x_1| &= |\alpha^2 x_0 + \left(1-\alpha^2\right)\left(x_0^k + c^t\right)| \\
|x_1| &= |\left(1-\alpha^2\right)\left(x_0^k + c^t\right) + \alpha^2 x_0| \\
&\geq \left(1-\alpha^2\right)|x_0^k + c^t| - \alpha^2 |x_0| \\
&\geq \left(1-\alpha^2\right)|x_0^k| - \alpha^2|x_0| - \left(1-\alpha^2\right)|c^t| \\
&\geq \left(1-\alpha^2\right)|x_0^k| - \alpha^2|x_0| - \left(1-\alpha^2\right)|c\theta| \quad as \quad \theta = \frac{c^t}{c}
\end{aligned}
$$

Continuing in this way, we obtain

$$
\begin{aligned}
x_1 &\geq \left(1-\alpha^2\right)|x_0^k| - \alpha^2|x_0| - |c\theta| + \alpha^2|c\theta| \\
&\geq \left(1-\alpha^2\right)|x_0^k| - \alpha^2|x_0| - |x_0\theta| + \alpha^2|x_0\theta| \quad as \quad |x_0| \geq |c| \\
&\geq \left(1-\alpha^2\right)|x_0^k| - \alpha^2|x_0| - |x_0\theta| \quad as \quad \alpha^2|x_0| > 0 \\
&\geq |x_0|(\left(1-\alpha^2\right)|x_0^{k-1}| - \alpha^2 - |\theta|) \\
&\geq |x_0|(\left(1-\alpha^2\right)|x_0^{k-1}| - 1 - |\theta| \quad as \quad \alpha \in (0,1] \\
&\geq |x_0|(\left(1-\alpha^2\right)|x_0^{k-1}| - (1+|\theta|))
\end{aligned}
$$

Since $|x_0| > \left(\frac{2+|\theta|}{1-\alpha^2}\right)^{\frac{1}{k-1}}$, *this implies that* $(1-\alpha^2)|x_0^{k-1}| - (1+|\theta|) > 1$. *Thereafter, there is* $\lambda > 0$ *s that,* $(1-\alpha^2)|x_0^{k-1}| - (1+|\theta|) > 1+\lambda$. Thus, $|x_1| > (1+\lambda)|x_0^{k-1}|$. Specifically, $|x_1| > |x_0|$ we repeat this process up to kth iterates to achieve $|x_k| > (1+\lambda)^k \left|x_0^{n-1}\right|$

Hence, $|x_k| \to \infty$ as $k \to \infty$. □

Corollary 1. *Let us suppose that* $|c| > \left(\frac{2+|\theta|}{1-\alpha^2}\right)^{\frac{1}{k-1}}$*, then Mann's iterative orbital strategy via h-convexity jumps to infinity.*

Corollary 2. *Let us suppose that* $|x| > max[|c|, \left(\frac{2+|\theta|}{1-\alpha^2}\right)^{\frac{1}{k-1}}]$*; therefore, there exists* $\lambda > 0$*, so that* $|x_k| > (1+\lambda)^k |x|$*, and* $x_k \to \infty$ *when* $k \to \infty$*.*

Corollary 3. *Let us suppose that* $|x_k| > |x| > max[|c|, \left(\frac{2+|\theta|}{1-\alpha^2}\right)^{\frac{1}{k-1}}]$*, then for any* $k \geq 0$*, there are* $\lambda > 0$*, so that* $|x_{k+l}| > (1+\lambda)^{k+l}|x_{k+l}|$ *and* $|x_{k+l}| \to \infty$ *when* $k+l \to \infty$*.*

4. Applications in Fractals

Perhaps the most well-known object in fractal theory is the Mandelbrot set. It is regarded to be both the most beautiful and the most complicated thing ever made visible. This set explains how the dynamics of a quadratic polynomial alter depending on the complex parameter. The placement of the parameter relative to the Mandelbrot set reveals a lot about the polynomial's dynamical features. The Mandelbrot set is represented by a cubic polynomial with two critical orbits. As a result, their analysis of cubic polynomials is substantially more sophisticated than quadratic polynomials. In this section, Mandelbrot sets are presented using the proposed iteration method. To generate these Mandelbrot sets, it is essential to establish an execution criterion for the algorithm. There are several common algorithms used for generating such fractals, including the following:

- Distance measure [40];
- An algorithm by using potential functions [41];
- Escape criteria [42].

To visualize the Mandelbrot sets graphically, we use the escape criterion described in Algorithm 1. The visualizations were produced via computer "Intel(R) Core(TM) i7-7500U CPU @ 2.70GHz 2.90 GHz" in Matlab R2013a.

Algorithm 1: Geometry of Mandelbrot-Set

Input: $F(x) = x^n + c^t$ proposed function with $n \geq 2$ complex function, $A \subset \mathbb{C}$-occupied area, L-fixed number of iterates, $\alpha \in (0,1]$, $t \in \mathbb{R}$, $c \in \mathbb{C}$ and Coloursmap [0...C-1].
Output: M-Set
for $c \in A$ **do**
  $R = \left(\frac{2+|\theta|}{1-\alpha^2}\right)^{\frac{1}{k-1}}$, where $\theta = \frac{c^t}{c}$
  $k = 0$
  x_0–critical point
  while $k \leq L$ **do**
    $x_{k+1} = h(\alpha)x_k + h(1-\alpha)f(x_k)$, where $h(\alpha) = \alpha^2$
    if $|x_{k+1}| > R$ **then**
      break
    k=k+1
  $j = \lfloor (C-1)k/L \rfloor$
  colour x_0 with colurmap[j]

Mandelbrot Set

Here, we presented a discussion of several M-sets related to the function $f(x) = x^n + c^t$ at various n, within the trajectory of the suggested iteration. Here, M-sets have been generated for $n = 2, 3$ via Mann-iteration with h-convexity. In every graph, we set $K = 35$ (i.e., number of iterations is fixed) in the Algorithm 1.

Example 1. *In Figure 1, we visualize how M-sets follow the function $f(x) = x^n + c^t$ at $n = t = 2$ by varying the other parameters to attain attractive M-sets.*

Furthermore, in Figure 2 by fixing the parameter $\alpha = 0.70$ and varying the value of the other parameters, we attain attractive M-sets using Mann iteration with h-convexity.

Example 2. *In Figure 3, we use the function $f(z) = z^n + c^t$ at $n = 3; t = 2.00$, and vary the other parameters to produce cubic M-sets using Mann iteration.*

Moreover, in Figure 4, by fixing the parameter $\alpha = 0.70$ and varying the other parameters to produce cubic M-sets using Mann iteration with h-convexity. It can be observed that the position of the parameter relative to the Mandelbrot set informs a lot about the polynomial's dynamical properties. The Mandelbrot set is represented by a cubic polynomial $(n = 3)$ with two critical orbits. As a result, the apparel beauty of the Mandelbrot set using cubic polynomials $(n = 3)$ is far more intricate than quadratic polynomials $(n = 2)$.

We continue this way, and discuss the following examples by considering various values of α and t for simple and convex functions. We observe the number of iterations and the amount of time to generate fractals.

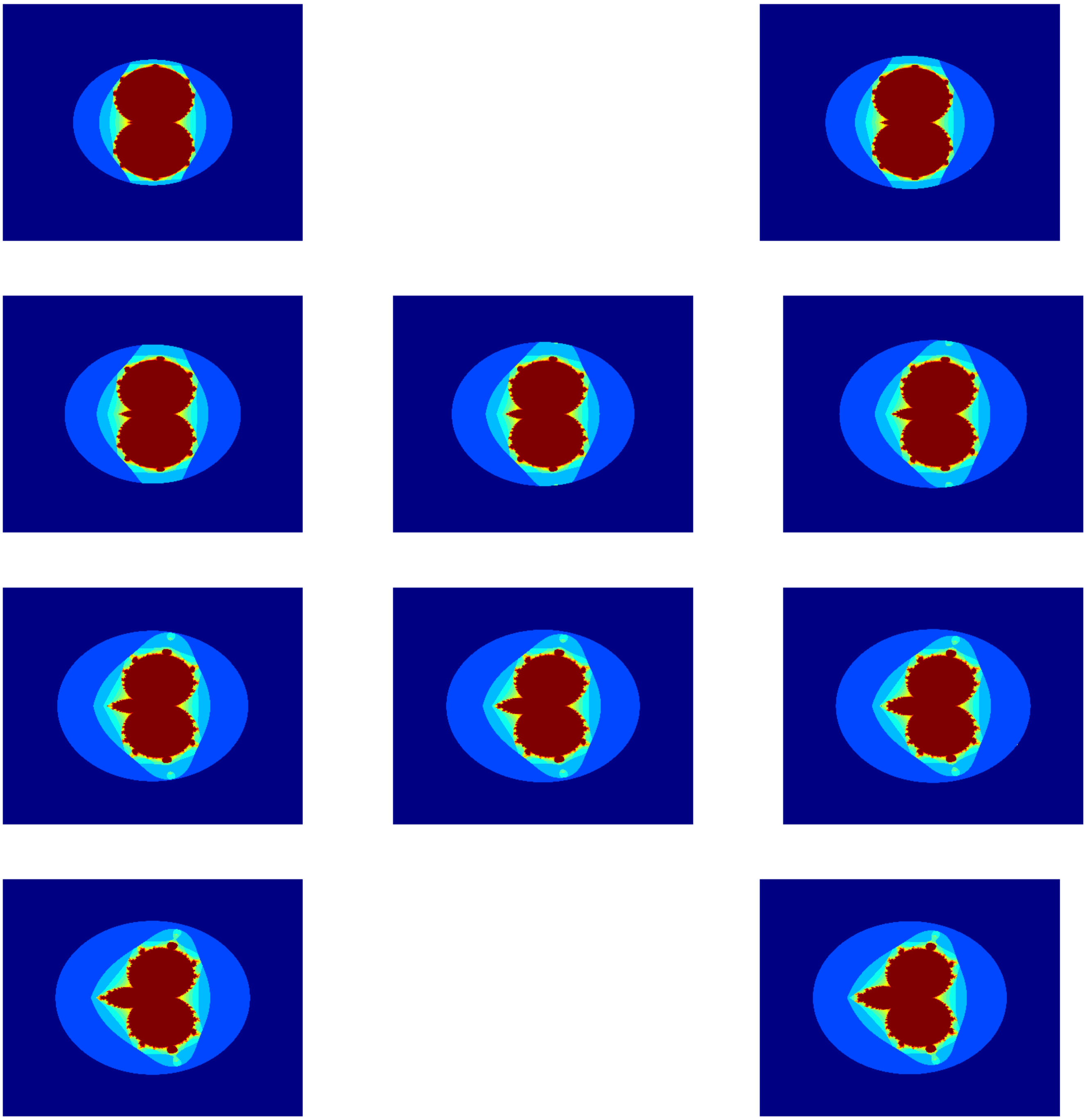

Figure 1. M-set for $f(x) = x^n + c^t$ via the Mann-iterative scheme; $t = 2; n = 2$.

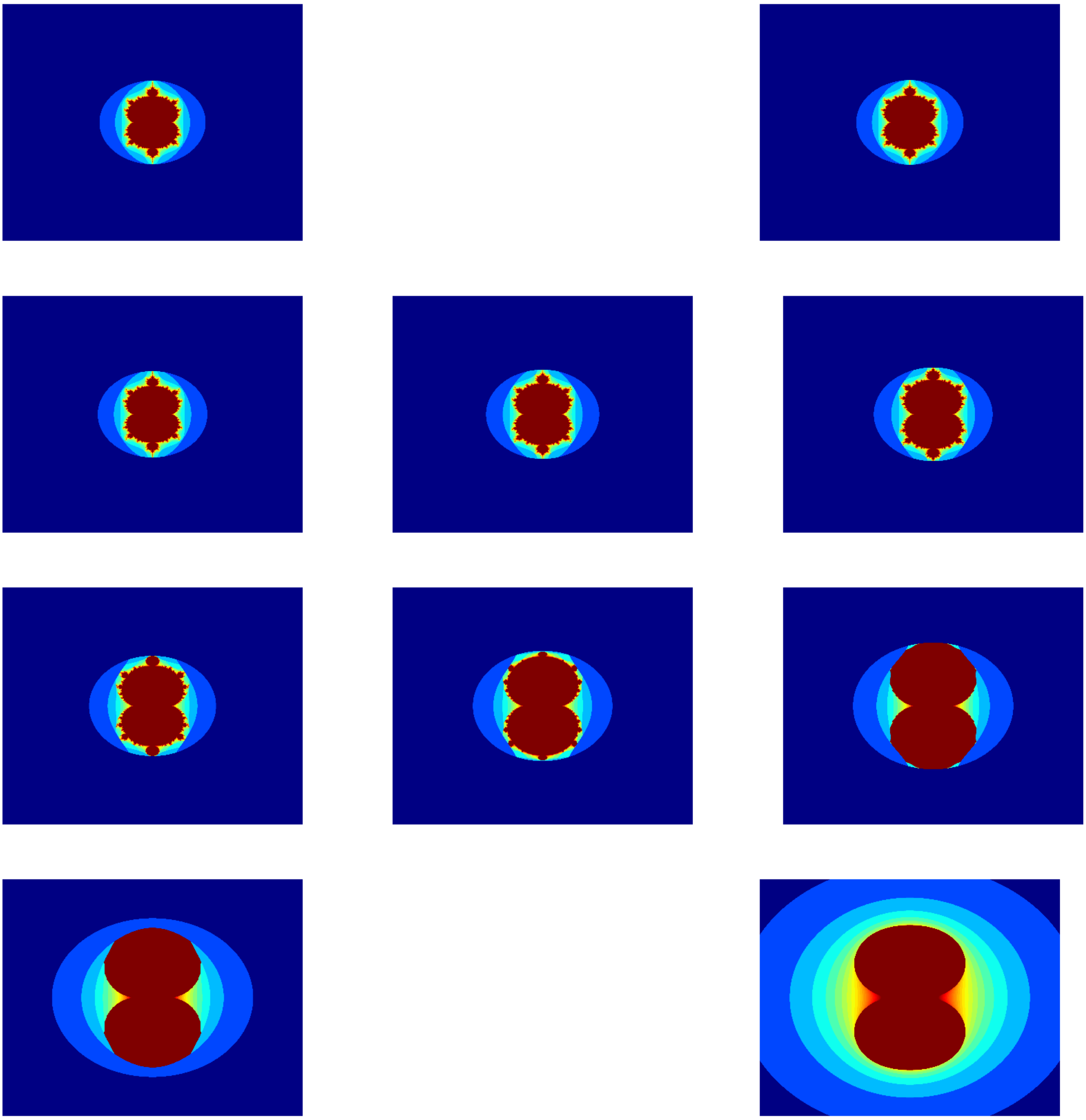

Figure 2. M-set for $f(x) = x^n + c^t$ via the Mann-iterative scheme with h-convexity; $\alpha = 0.7; n = 2$.

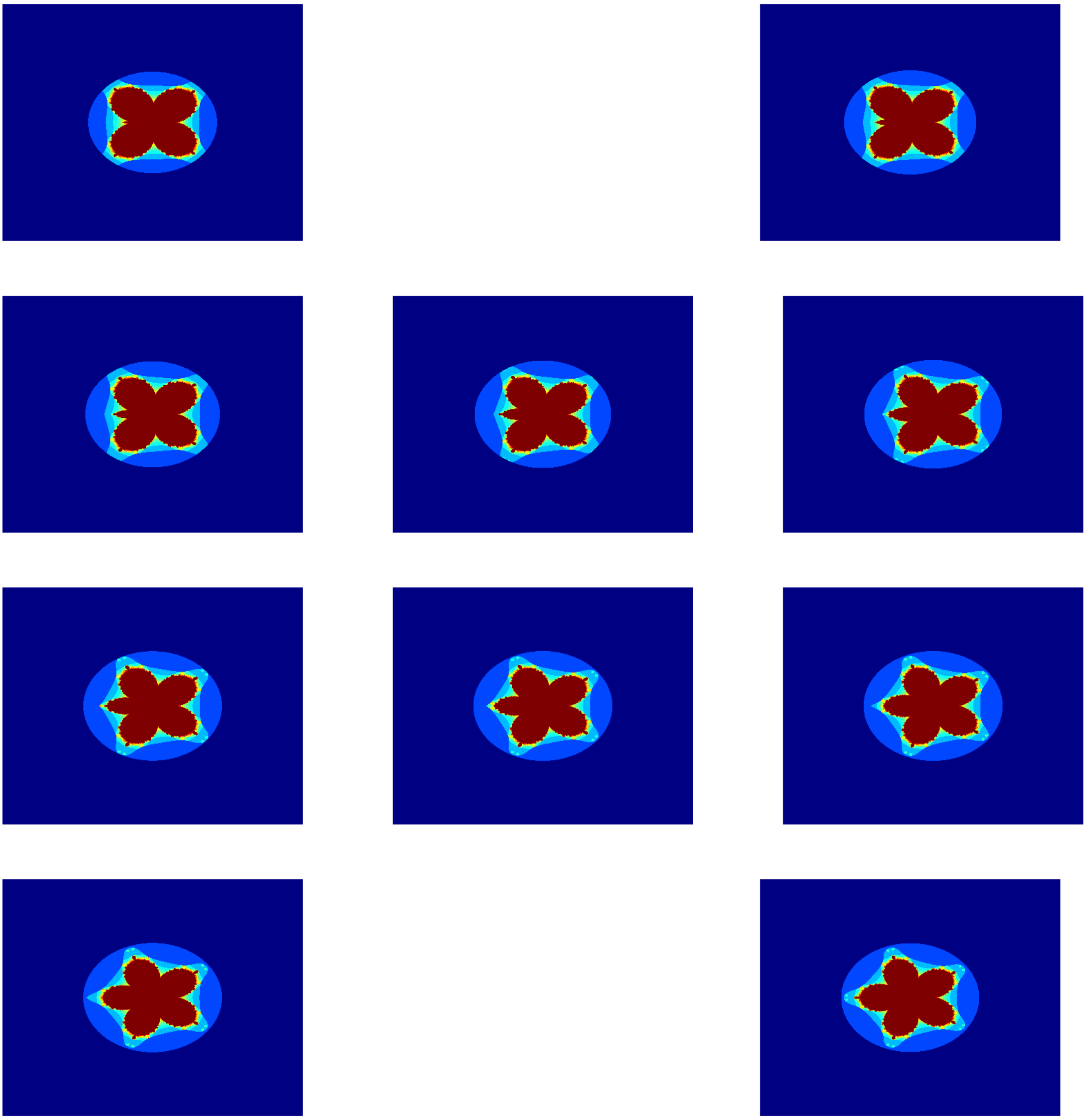

Figure 3. M-set for $f(x) = x^n + c^t$ via the Mann-iterative scheme; $t = 2; n = 3$.

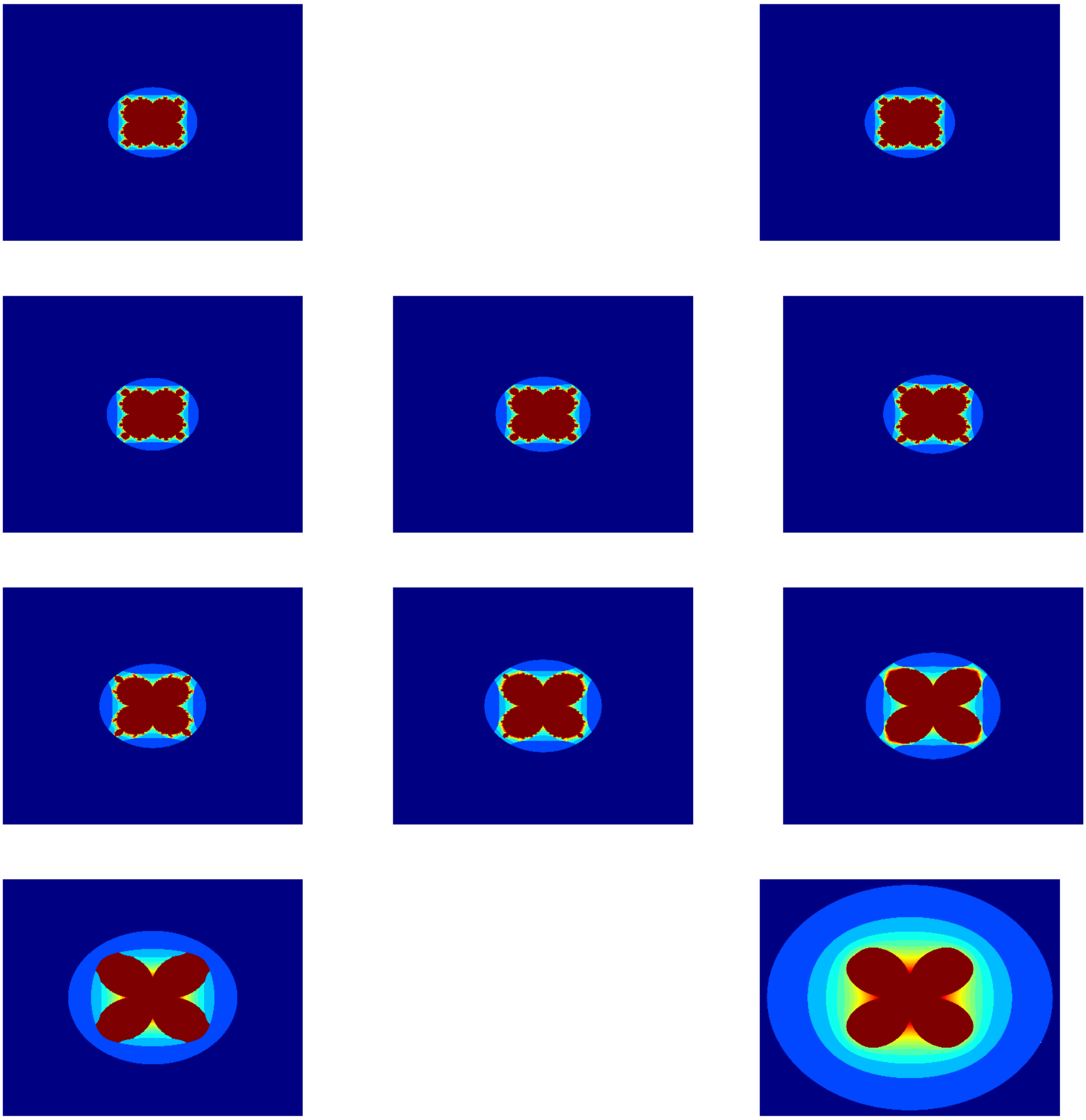

Figure 4. M-set for $f(x) = x^n + c^t$ via the Mann-iterative scheme with h-convexity; $\alpha = 0.7; n = 3$.

Example 3. *In the first ten figures of this example, we illustrate how the Mandelbrot sets follow the function $f(x) = x^n + c^t$ at $n = 2$. In Figures 5–14, we fix the parameter $t = 1$ and vary α to achieve visually appealing Mandelbrot sets. In these figures, the color bar represents the average number of iterations (ANI) required to generate each graph. It is evident from Table 1 that the Mann iterative scheme with h-convexity (MIH) outperforms the standard Mann iterative (MI) scheme.*

This table shows that the maximum number of iterations required for a simple Mann iteration is 810, which is significantly larger than 23 in the case of the proposed Mann iteration with h-convexity. Similarly, the time required to perform many iterations is also comparatively higher. Moreover, the images generated by our newly proposed Mann iteration with h-convexity are far better than the usual ones.

Table 1. Comparison table when $t = 1$ and $A = [-3, 3]^2$.

α	ANI for MI	ANI for MIH	Time (s) for MI	Time (s) for MIH
0.1	810	23	3.81	2.70
0.2	190	20	4.72	3.25
0.3	74	19	6.52	3.26
0.4	36	18	9.00	5.15
0.5	25	17	10.23	6.15

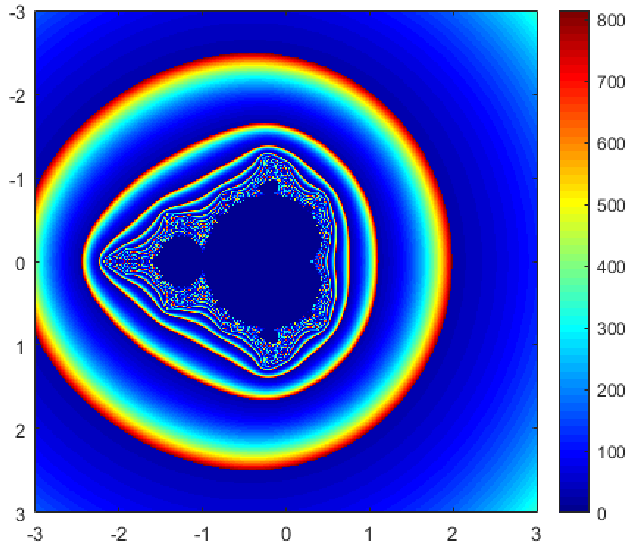

Figure 5. M-set using MI for $\alpha = 0.1$.

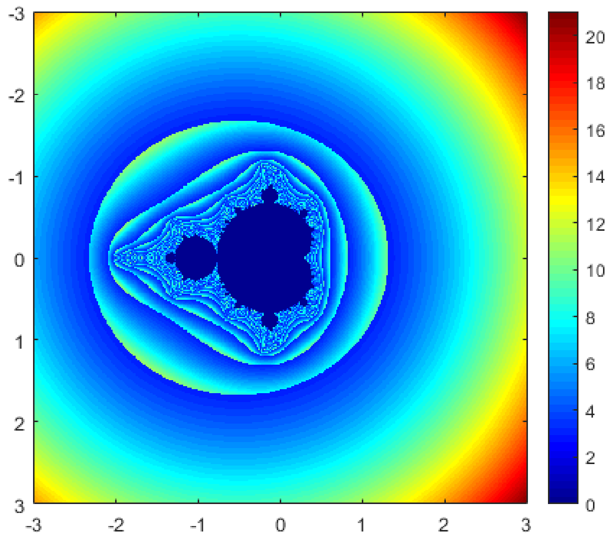

Figure 6. M-set using MIH for $\alpha = 0.1$.

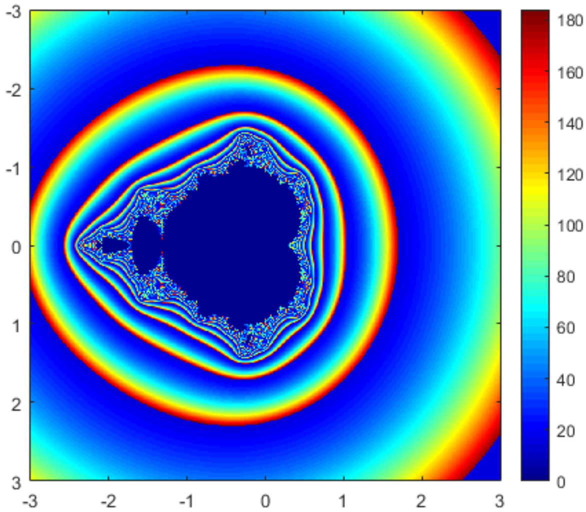

Figure 7. M-set using MI for $\alpha = 0.2$.

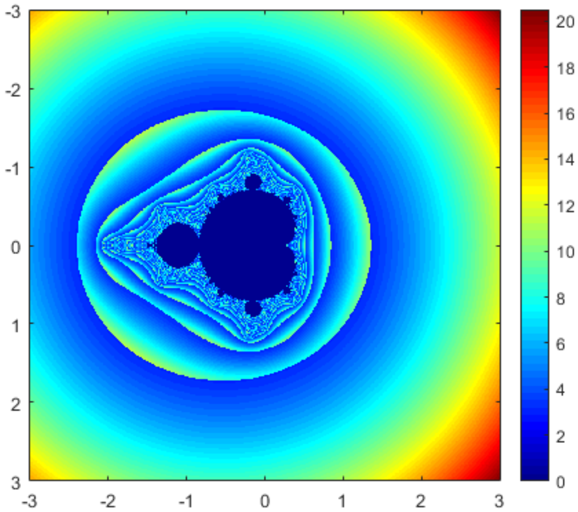

Figure 8. M-set using MIH for $\alpha = 0.2$.

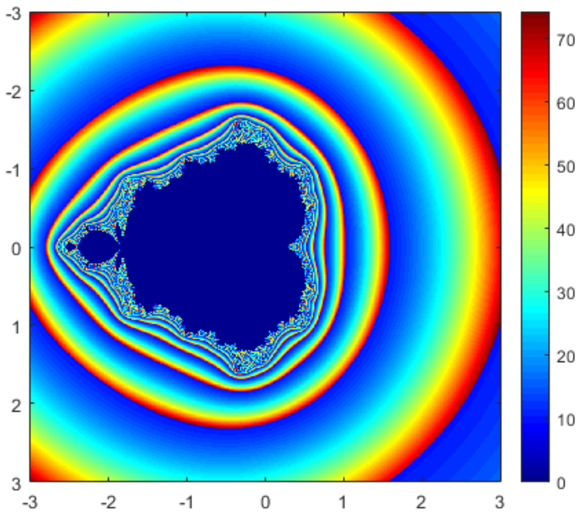

Figure 9. M-set using MI for $\alpha = 0.3$.

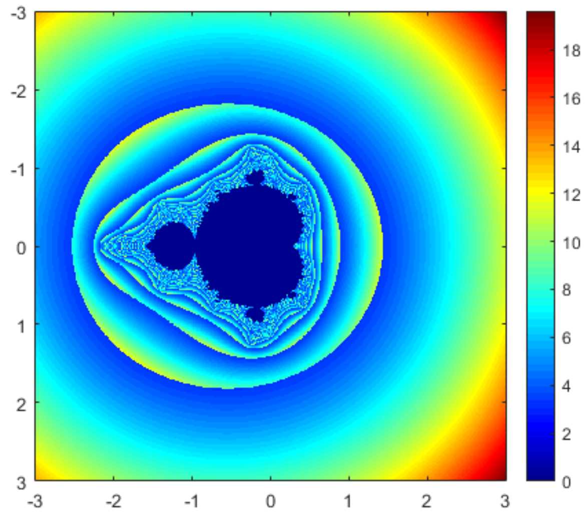

Figure 10. M-set using MIH for $\alpha = 0.3$.

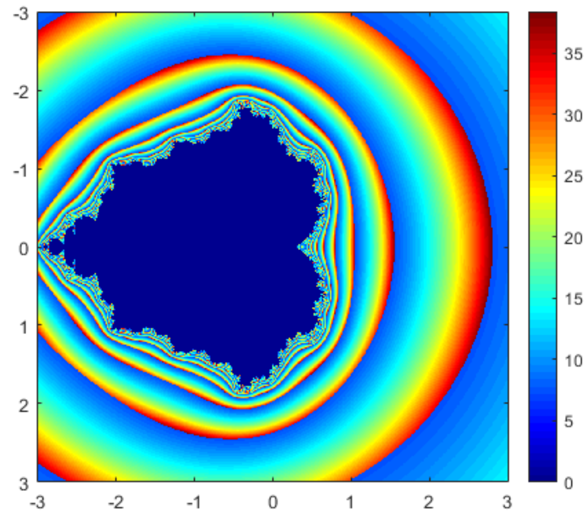

Figure 11. M-set using MI for $\alpha = 0.4$.

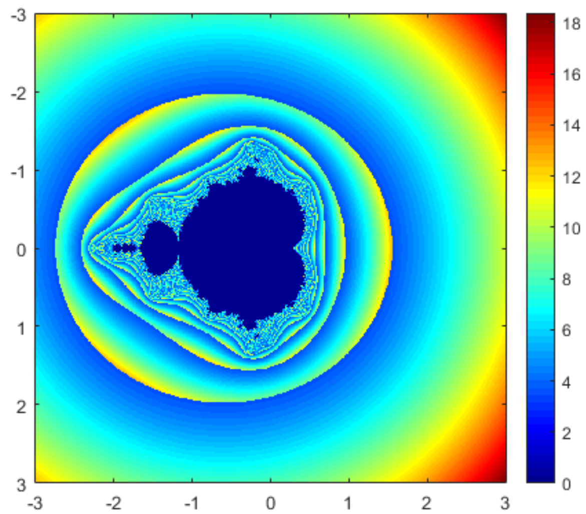

Figure 12. M-set using MIH for $\alpha = 0.4$.

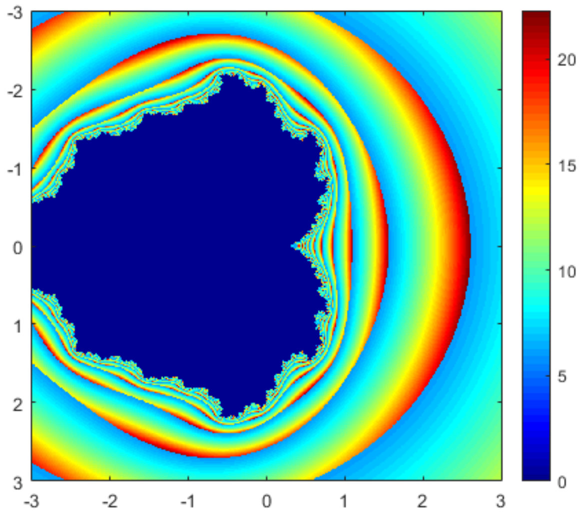

Figure 13. M-set using MI for $\alpha = 0.5$.

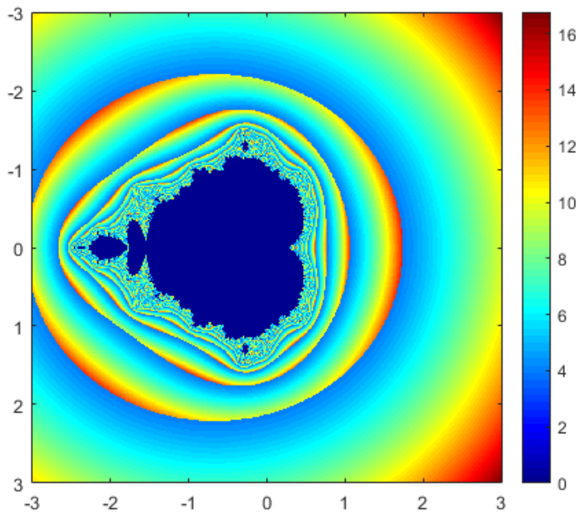

Figure 14. M-set using MIH for $\alpha = 0.5$.

Example 4. *In the next figures, we demonstrate how the Mandelbrot sets correspond to the function* $f(z) = x^n + c^t$ *with* $n = 3$. *This is shown in Figures 15–24, where we fix the parameter* $\alpha = 0.03$ *and vary the other parameter* t *to achieve visually appealing Mandelbrot sets. Again, from Table 2, we observe that the Mann iterative scheme with h-convexity outperforms the standard Mann scheme.*

Table 2. Comparison table when $\alpha = 0.03$ and $A = [-3, 3]^2$.

t	ANI for MI	ANI for MIH	Time (s) for MI	Time (s) for MIH
1.2	11,500	100	6.17	4.74
1.4	13,500	98	5.62	4.61
1.6	16,000	96	5.58	4.57
1.8	18,500	68	4.83	4.38
2	23,000	23	4.16	3.35

This table shows that the maximum number of iterations required for a simple Mann iteration is 23,000, which is significantly larger than 100 in the case of the proposed Mann iteration with h-convexity. Similarly, the time required to perform many iterations is also comparatively higher. Moreover, the images generated by our newly proposed Mann iteration with h-convexity at $n = 3$ *are far better than the usual ones.*

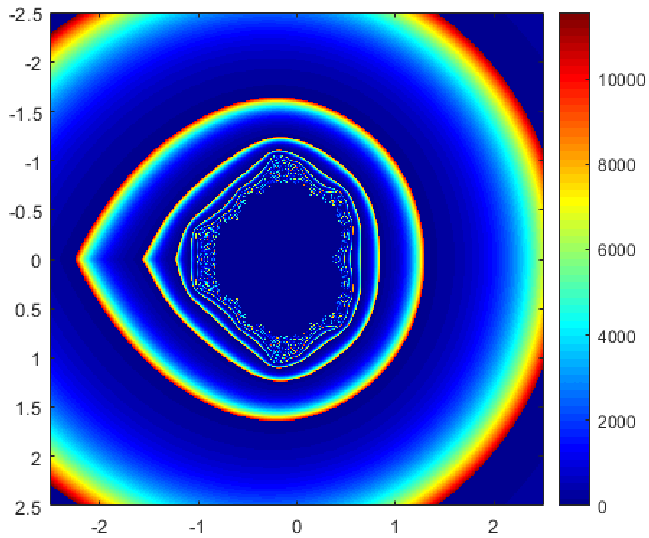

Figure 15. M-set by using MI for $t = 1.2$.

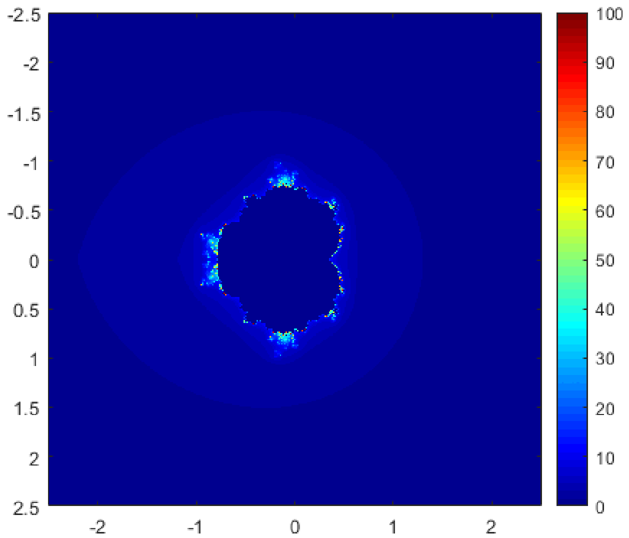

Figure 16. M-set using MIH for $t = 1.2$.

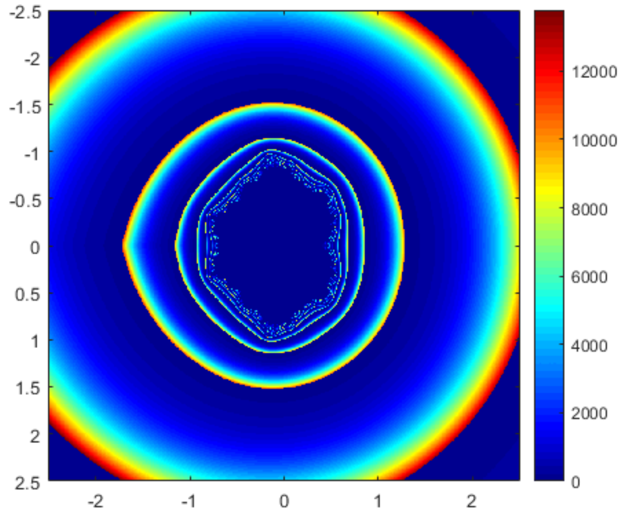

Figure 17. M-set using MI for $t = 1.4$.

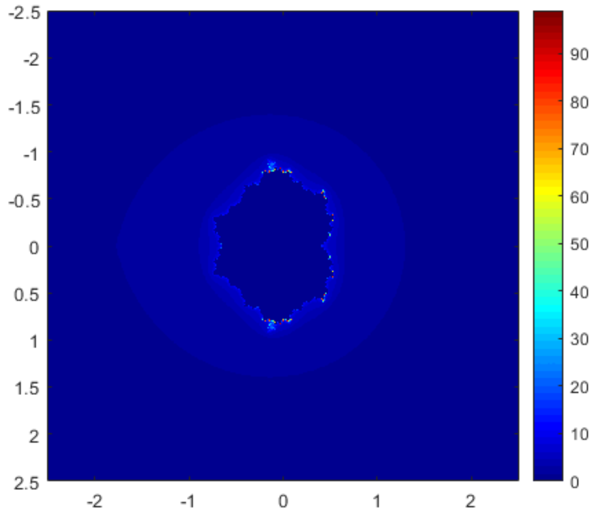

Figure 18. M-set using MIH for $t = 1.4$.

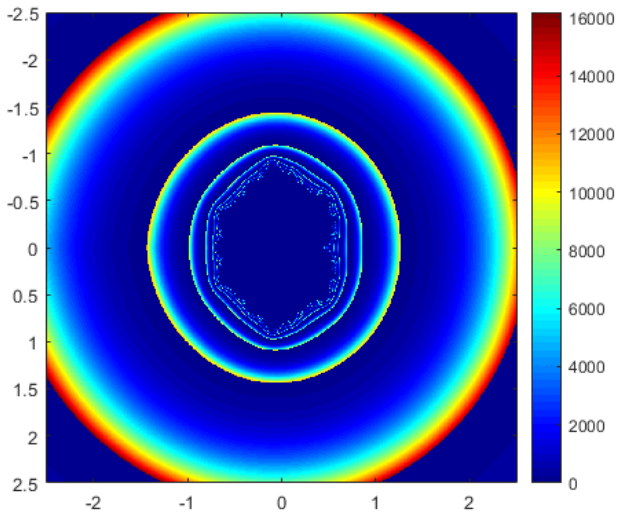

Figure 19. M-set using MI for $t = 1.6$.

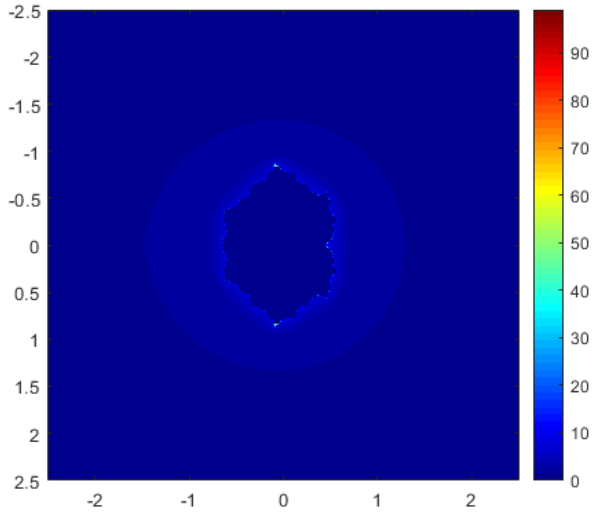

Figure 20. M-set using MIH for $t = 1.6$.

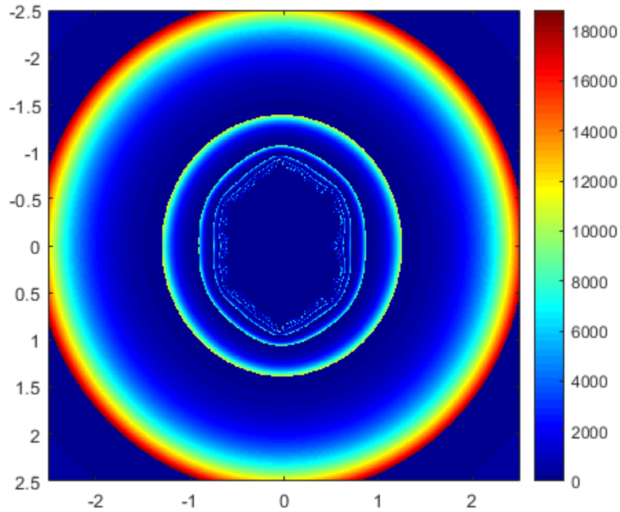

Figure 21. M-set using MI for $t = 1.8$.

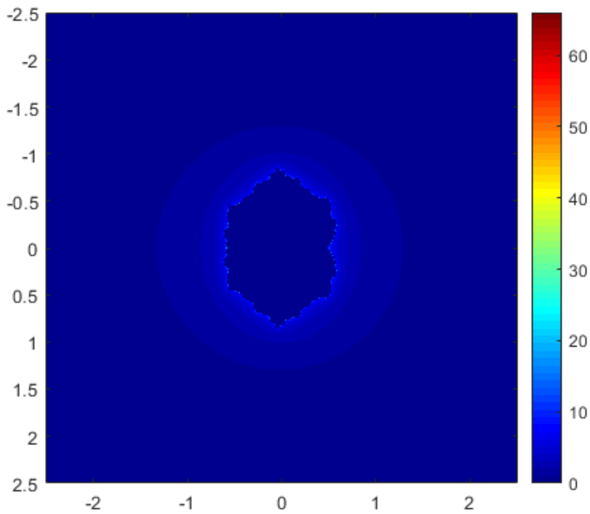

Figure 22. M-set using MIH for $t = 1.8$.

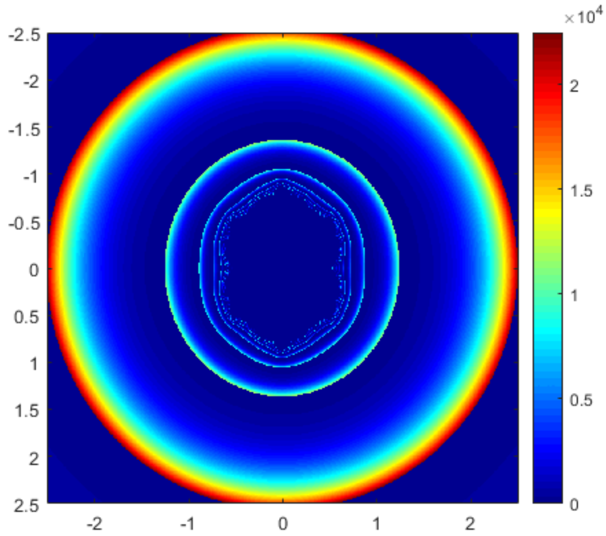

Figure 23. M-set using MI for $t = 2$.

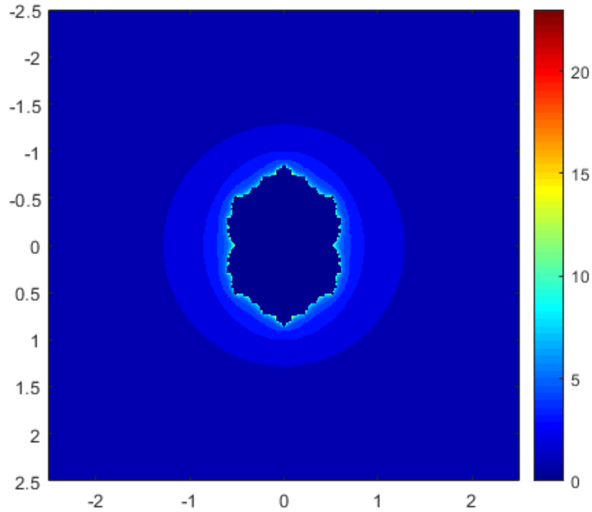

Figure 24. M-set using MIH for $t = 2$.

The Mandelbrot set, generated using the Mann-iterative scheme with h-convexity, holds significant potential across various scientific disciplines. Fractals, including the Mandelbrot set, are not merely mathematical curiosities; they provide profound insights and applications in several areas of science and technology, such as natural phenomena and geophysics, art and design, physics, biology and medicine, materials science and engineering, computer graphics and image processing, chaos theory and dynamical systems. Highlighting such significance, these findings will also be a road to quixotic research in diverse directions such as using fractal dimension analysis to describe the three-dimensional association of biomolecules. Such insights have incredible potential to transform the modeling and understanding of essential developments in biophysics [43].

5. Further Discussion and Conclusions

We introduced the Mann orbit with h-convexity and established a novel escape criterion for a complex function $f(x) = x^n + c^t$, where $n \geq 2$, c belongs to the set of complex numbers, and t is a real number. Our work resulted in Algorithm 1 for generating Mandelbrot sets. We provided detailed presentations of quadratic and cubic fractals, particularly Mandelbrot sets, with illustrative examples. Our comparative analysis, as demonstrated in the previous examples, consistently shows that the Mann iterative scheme with h-convexity outperforms the standard Mann iterative scheme because MIH takes less time and generates higher-quality fractals as compared to the MI.

Analyzing the relationship between the Mandelbrot set and the input parameters of iteration is extremely intricate. In this part, we visualized the graphs of the Mandelbrot set through the implementation of escape time algorithm for the Mann iterative scheme with h-convexity and we investigated the variation in the average visual time required for execution by adjusting two factors (α, t) for Mann iterative scheme with h-convexity. We expressed the parameters $t \in [1, 1.99]$ and $\alpha \in (0, 1]$ as intervals for further analysis. To obtain the desired outcomes, we perform calculations for both " α" and "t" using a step size of 0.01 within each interval. In total, we compute 100 values for " α" and 100 values for "t". It should be noted that all calculations and visual analyses were performed on a computer equipped with the following configuration: an Intel(R) Core(TM) i7-7500U CPU running at 2.70 GHz, 8 GB of RAM, and a 64-bit Windows 10 Pro Education operating system. To determine the mean image processing time, we employed Mathematica 10 software and executed the algorithm. Investigating the influence of input parameters t and α on the Mandelbrot set during the Mann-iteration with h-convexity orbit, we fixed the number of iterations at 15. The color map used to depict the time values was constrained within the interval of 4.287 to 43.858 s. The visualization uncovers intriguing insights into the execution time of the algorithm when applied to various parameter values in Figure 25.

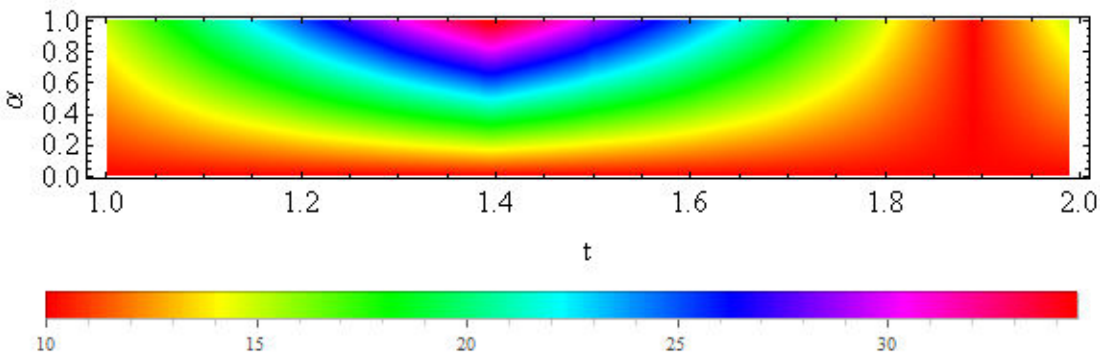

Figure 25. M-set for $f(x) = x^n + c^t$ via Mann-iterative scheme having image visual period.

Overall, our research lays the foundation for the further exploration and development of iterative schemes for fractal generation, with potential applications in various scientific, artistic, and computational fields. It is remarkable that usually, for h-convex functions, researchers use $h(x) = x$ or $h(x) = 1 - x$, but in this study, we use $h(x) = x^2$ to generalize the Mann-iterative scheme.

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Abbreviations

The following abbreviations are used in this manuscript:

M-Set	Mandelbrot set
J-Set	Julia set
ANI	Average number of iterations
MI	Mann-iterative scheme
MIH	Mann-iterative scheme with h-convexity

References

1. Barnsley, M. *Fractals Everywhere*; Academic: Boston, MA, USA, 1993.
2. Taylor, R.P. The potential of biophilic fractal designs to promote health and performance: A review of experiments and applications. *Sustainability* **2021**, *13*, 823. [CrossRef]
3. Smith, J.; Rowland, C.; Moslehi, S.; Taylor, R.; Lesjak, A.; Lesjak, M.; Stadlober, S.; Lee, L.; Dettmar, J.; Page, M.; et al. Relaxing floors: Fractal fluency in the built environment. *Nonlinear Dyn. Psychol. Life Sci.* **2020**, *24*, 127–141.
4. Fisher, Y. Fractal image compression. *Fractals* **1994**, *2*, 347–361. [CrossRef]
5. Kumar, S. Public key cryptographic system using Mandelbrot sets. In Proceedings of the MILCOM 2006—2006 IEEE Military Communications Conference, Washington, DC, USA, 23–25 October 2006; pp. 1–5.
6. Zhang, X.; Wang, L.; Zhou, Z.; Niu, Y. A chaos-based image encryption technique utilizing Hilbert curves and H-Fractals. *IEEE Access* **2019**, *7*, 74734–74746. [CrossRef]
7. Kharbanda, M.; Bajaj, N. An exploration of fractal art in fashion design. In Proceedings of the 2013 International Conference on Communication and Signal Processing, Melmaruvathur, India, 3–5 April 2013; pp. 226–230.
8. Cohen, N. Fractal antenna applications in wireless telecommunications. In Proceedings of the Professional Program Proceedings. Electronic Industries Forum of New England, Boston, MA, USA, 6–8 May 1997; pp. 43–49.
9. Mandelbrot, B.B. *The Fractal Geometry Nature*; Freeman: New York, NY, USA, 1982; Volume 2.
10. Lakhtakia, A.; Varadan, W.; Messier, R.; Varadan, V.K. On the symmetries of the Julia sets for the process z^p+c. *J. Phys. A Math. Gen.* **1987**, *20*, 3533–3535. [CrossRef]
11. Blanchard, P.; Devaney, R.L.; Garijo, A.; Russell, E.D. A generalized version of the Mcmullen domain. *Int. J. Bifurc. Chaos* **2008**, *18*, 2309–2318. [CrossRef]
12. Crowe, W.D.; Hasson, R.; Rippon, P.J.; Strain-Clark, P.E.D. On the structure of the Mandelbar set. *Nonlinearity* **1989**, *2*, 541. [CrossRef]
13. Kim, T. Quaternion Julia set shape optimization. *Comput. Graph. Forum* **2015**, *34*, 167–176. [CrossRef]
14. Drakopoulos, V.; Mimikou, N.; Theoharis, T. An overview of parallel visualisation methods for Mandelbrot and Julia sets. *Comput. Graph.* **2003**, *27*, 635–646. [CrossRef]
15. Sun, Y.; Chen, L.; Xu, R.; Kong, R. An image encryption algorithm utilizing Julia sets and Hilbert curves. *PLoS ONE* **2014**, *9*, e84655. [CrossRef]
16. Prasad, B.; Katiyar, K. Fractals via Ishikawa iteration. In Proceedings of the International Conference on Logic, Information, Control and Computation, Gandhigram, India, 25–27 February 2011; pp. 197–203.
17. Rani, M.; Agarwal, R. Effect of stochastic noise on superior Julia sets. *J. Math. Imag. Vis.* **2010**, *36*, 63. [CrossRef]
18. Ashish, M.R.; Chugh, R. Julia sets and mandelbrot sets in Noor orbit. *Appl. Math. Comput.* **2014**, *228*, 615–631. [CrossRef]
19. Kang, S.M.; Rafiq, A.; Latif, A.; Shahid, A.A.; Kwun, Y.C. Tricorns and Multi-corns of S-iteration scheme. *J. Funct. Spaces* **2015**, *2015*, 1–7.
20. Tassaddiq, A.; Tanveer, M.; Azhar, M.; Lakhani, F.; Nazeer, W.; Afzal, Z. Escape criterion for generating fractals using Picard–Thakur hybrid iteration. *Alex. Eng. J.* **2024**, *100*, 331–339. [CrossRef]
21. Tassaddiq, A.; Tanveer, M.; Azhar, M.; Nazeer, W.; Qureshi, S. A Four Step Feedback Iteration and Its Applications in Fractals. *Fractal Fract.* **2023**, *7*, 662. [CrossRef]
22. Goyal, K.; Prasad, B. Dynamics of iterative schemes for quadratic polynomial. *Proc. AIP Conf.* **2001**, *9*, 149–153.
23. Tassaddiq, A.; Tanveer, M.; Israr, K.; Arshad, M.; Shehzad, K.; Srivastava, R. Multicorn Sets of $(\bar{z})^k+c^m$ via S-Iteration with h-Convexity. *Fractal Fract.* **2023**, *7*, 486. [CrossRef]
24. Chugh, R.; Kumar, V.; Kumar, S. Strong convergence of a new three step iterative scheme in Banach spaces. *Am. J. Comput. Math.* **2012**, *2*, 345. [CrossRef]
25. Phuengrattana, W.; Suantai, S. On the rate of convergence of Mann, Ishikawa, Noor and SP-iterations for continuous functions on an arbitrary interval. *J. Comput. Appl. Math.* **2011**, *235*, 3006–3014. [CrossRef]
26. Tassaddiq, A.; Kalsoom, A.; Rashid, M.; Sehr, K.; Almutairi, D.K. Generating Geometric Patterns Using Complex Polynomials and Iterative Schemes. *Axioms* **2024**, *13*, 204. [CrossRef]
27. Srivastava, R.; Tassaddiq, A.; Kasmani, R.M. Escape Criteria Using Hybrid Picard S-Iteration Leading to a Comparative Analysis of Fractal Mandelbrot Sets Generated with S-Iteration. *Fractal Fract.* **2024**, *8*, 116. [CrossRef]

28. Li, D.; Tanveer, M.; Nazeer, W.; Guo, X. Boundaries of filled Julia sets in generalized Jungck-Mann orbit. *IEEE Access* **2019**, *7*, 76859–76867. [CrossRef]
29. Tassaddiq, A. General escape criteria for the generation of fractals in extended Jungck-Noor orbit. *Math. Comput. Simul.* **2022**, *196*, 1–14. [CrossRef]
30. Gdawiec, K.; Kotarski, W.; Lisowska, A. Biomorphs via modified iterations. *J. Nonlinear Sci. Appl.* **2016**, *9*, 2305–2315. [CrossRef]
31. Pickover, C.A. Biomorphs: Computer displays of biological forms generated from mathematical feedback loops. *Comput. Graph. Forum* **1986**, *5*, 313–316. [CrossRef]
32. Alonso-Sanz, R. Biomorphs with memory. *Int. J. Parallel Emergent Distrib. Syst.* **2018**, *33*, 1–11. [CrossRef]
33. Jakubska-Busse, A.; Janowicz, M.W.; Ochnio, L.; Ashbourn, J.M.A. Pickover biomorphs and non-standard complex numbers. *Chaos Solitons Fractals* **2018**, *113*, 46–52. [CrossRef]
34. Devaney, R. *A First Course in Chaotic Dynamical Systems: Theory and Experiment*; Addison-Wesley: New York, NY, USA, 1992.
35. Liu, X.; Zhu, Z.; Wang, G.; Zhu, W. Composed accelerated escape time algorithm to construct the general Mandelbrot sets. *Fractals* **2001**, *9*, 149–153. [CrossRef]
36. Mann, W.R. Mean value methods in iteration. *Proc. Amer. Math. Soc.* **1953**, *4*, 506–510. [CrossRef]
37. Ishikawa, S. Fixed points by a new iteration method. *Proc. Amer. Math.Soc.* **1974**, *44*, 147–150. [CrossRef]
38. Varoanec, S. On h-convexity. *J. Math. Anal. Appl.* **2007**, *326*, 303–311. [CrossRef]
39. Agarwal, R.; Regan, D.O.; Sahu, D. Iterative construction of fixed points of nearly asymptotically nonexpansive mappings. *J. Nonlinear Convex Anal.* **2007**, *8*, 61.
40. Strotov, V.V.; Smirnov, S.A.; Korepanov, S.E.; Cherpalkin, A.V. Object distance estimation algorithm for real-time fpga-based stereoscopic vision system. *High-Perform. Comput. Geosci. Remote Sens.* **2018**, *10792*, 71–78.
41. Khatib, O. Real-Time Obstacle Avoidance for Manipulators and Mobile Robots. In *Autonomous Robot Vehicles*; Springer: Berlin/Heidelberg, Germany, 1986; pp. 396–404.
42. Barrallo, J.; Jones, D.M. Coloring algorithms for dynamical systems in the complex plane. In *Visual Mathematics*; Mathematical Institute SASA: Belgrade, Serbia, 1999; Volume 1.
43. Singh, P.; Saxena, K.; Singhania, A.; Sahoo, P.; Ghosh, S.; Chhajed, R.; Ray, K.; Fujita, D.; Bandyopadhyay, A. A Self-Operating Time Crystal Model of the Human Brain: Can We Replace Entire Brain Hardware with a 3D Fractal Architecture of Clocks Alone? *Information* **2020**, *11*, 238. [CrossRef]