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Dynamic Event-Triggered Prescribed-Time Consensus Tracking of Nonlinear Time-Delay Multiagent Systems by Output Feedback

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Abstract: Event-triggering mechanisms reported in the existing prescribed-time (PT) control do not adequately capture the dynamic nature of network environments, and are not applied to distributed consensus tracking problems with unknown time delays. Therefore, designing a dynamic event-triggering mechanism is crucial to ensuring PT stability, even in the presence of unknown time delays. This article focuses on developing a dynamic event-triggering mechanism to achieve adaptive practical PT output-feedback consensus tracking for nonlinear uncertain multiagent systems with unknown time delays. Firstly, a delay-independent PT state observer using a time-varying gain function is designed to estimate the immeasurable states. Following this, a novel distributed delay-independent PT consensus tracking scheme is constructed, incorporating a dynamic event-triggered mechanism through the command-filtered backstepping approach. In this design, dynamic variables based on a time-varying gain function are developed to support the event-triggering mechanism, ensuring practical stability within the prescribed settling time. Consequently, the proposed output-feedback control protocol can achieve practical PT stability, meaning that consensus tracking errors are constrained to a neighborhood around zero within a pre-specified time, regardless of the initial states of the agents or design parameters, while also avoiding the Zeno phenomenon. Finally, the effectiveness of the proposed strategy is validated through an illustrative example, which includes a comparative analysis.

Keywords: dynamic event-triggered mechanism; output-feedback design; practical prescribed-time (PT) convergence; PT consensus tracking; unknown time-varying delays

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1. Introduction

Prescribed-time (PT) control, initially proposed by Song et al. [1], has attracted substantial attention in recent years (see [1–5] and the references within). In [1], the PT control strategy using explicit time-varying feedback was proposed to regulate the finite PT, even with nonvanishing uncertain nonlinearities matched to the control input. The prescribed performance control problems have been addressed to ensure that control errors remain within predefined performance bounds [6–8]. The convergence time of the steady-state performance bound—rather than the actual convergence time of the tracking error—can be specified in advance. Compared to finite-time, fixed-time, and prescribed performance control, the principal advantage of this approach is that the true convergence time can be predetermined, independent of other design parameters and the initial agent states. In addition, fractional-order control methods and systems have been studied for various nonlinear systems [9–12]. Researchers have explored PT stabilization problems for systems with unmatched and known nonlinearities [13,14]. In [15,16], innovative combinations of the adaptive backstepping technique and PT stabilization design were introduced for uncertain strict-feedback nonlinear systems with unknown parameters. In addition, Ye et al. [17] proposed a temporal scaling-based adaptive approach for PT control in strict-feedback nonlinear systems with unknown time-varying parameters. In [18], a PT output-feedback control problem was studied for stochastic nonlinear strict-feedback systems, where sensor uncertainties were considered. Moreover, recent studies have yielded promising results in addressing PT tracking problems. Further, PT control approaches have been developed for strict-feedback nonlinear systems without model uncertainties [19] and nonvanishing uncertainties [20]. The time-varying finite-time gain functions employed in these studies exhibit unbounded growth as time approaches the preassigned settling time, potentially leading to singularity problems while implementing the designed controllers after the PT. Practical PT tracking approaches using saturated time-varying gain functions have been explored for nonlinear uncertain systems to address this concern [21,22]. However, the time-delay problems of nonlinear systems have not been addressed in the mentioned PT control research. A PT stabilization problem for nonlinear time-delay systems was recently considered in [23]. In [24], a double time-varying gain method was presented to design an adaptive PT stabilizer for time-delay nonlinear systems. Despite these PT stabilization efforts, the PT tracking problem of uncertain systems with unmatched time-delay nonlinearities remains an open challenge. Furthermore, to broaden the applicability of PT control in various scenarios, the PT output-feedback consensus-tracking problem must be investigated in time-delay nonlinear multiagent systems under a direct network. This is the first motivation of this paper.

In recent decades, the concept of distributed consensus control, characterized by information exchange solely between neighboring agents, has emerged as a crucial strategy for networked nonlinear multiagent systems [25,26] and nonlinear time-delay multiagent systems [27–32]. The PT control concept using time-varying gain functions was integrated with distributed control algorithms to preset the convergence time of distributed consensus control systems precisely [33]. In [34], PT consensus and containment control methods were explored for networked multiagent systems with first-order integrator dynamics. Addressing PT formation tracking, Ding et al. [35] considered the problem for networked second-order integrators with directed graphs. Furthermore, in [36], a practical PT output-feedback consensus-tracking strategy was developed for multiagent systems with nonlinearities satisfying input-matching conditions. A practical PT containment control design employing a distributed observer was investigated for strict-feedback nonlinear multiagent systems [37]. In [38], the quantized interagent communication problem was addressed in the practical PT formation tracking framework. However, to our knowledge, no work has been devoted to the distributed adaptive PT consensus tracker design with unknown time-varying state delays of strict-feedback nonlinear uncertain multiagent systems. This is the second motivation of this paper.

Event-triggered control has been explored in networked control, driven by its efficient use of network resources on band-limited communication channels (see [39,40] and the references within). Notably, lower-triangular uncertain nonlinear systems, versatile in describing numerous practical systems [41], have been a focal point for developing event-triggered control protocols [42–45]. Notable contributions have been made in addressing distributed event-triggered consensus control for uncertain multiagent systems encompassing unmatched nonlinearities [46,47]. Recently, efforts to enhance communication efficiency in implementing network-based PT control have led to the design of event-triggered mechanisms ensuring PT stability in control systems. In [48], a bipartite consensus problem was resolved by designing an event-triggered PT controller for multiple first-order integrators. For uncertain nonlinear systems, event-triggered PT stabilization [49] and neuroadaptive tracking methods [50] were developed in a strict-feedback form for nonlinear uncertain systems.

Despite these remarkable achievements in event-triggered PT control, the existing results [48–50] rely on static triggering conditions. Given the dynamic nature of the practical network environment with varying data transmission rates, dynamic event-triggering laws are more pragmatic and efficient than static conditions [51]. To date, no dynamic event-triggered PT control results have been reported with unknown time delays. This is the third motivation of this paper.

Based on these observations, a dynamic event-triggered PT output-feedback consensus-tracking problem of strict-feedback nonlinear uncertain time-delay multiagent systems must be considered. The following three primary challenges exist in solving this problem:

(i) The existing PT control methods [33–38,48–50,52–54] did not consider unknown time delays in multiagent systems, which influence the performance and stability of the control system. Thus, a distributed adaptive PT consensus-tracking problem must be studied with unknown time-varying delays. Accordingly, the first challenge is developing delay-independent local PT observers and distributed consensus trackers using only output information with unknown time-varying delays.

(ii) The existing PT event-triggered control approaches [48–50,52–54] do not adequately capture the dynamic nature of practical network environments with unknown time delays. The dynamic event-triggering mechanism must be designed to ensure PT stability of nonlinear uncertain time-delay multiagent systems. Thus, the second challenge is determining how to design the dynamics of the triggering variables and triggering mechanism to ensure PT stability.

(iii) The third challenging is establishing the PT stability of the proposed total closed-loop system while avoiding the Zeno phenomenon.

To address these challenges, we propose a memoryless PT output-feedback design strategy incorporating a dynamic event-triggered mechanism for adaptive consensus tracking of strict-feedback nonlinear uncertain time-delay multiagent systems in a fully distributed manner. The contributions of this work are as follows:

(1) Based on a continuous time-varying gain function for $t \in [0, \infty)$, local delay-independent PT observer and consensus trackers using only output information are designed for uncertain time-delay multiagent systems with unknown time delays. Compared to existing PT cooperative control results [33–38], this study derives a command-filtered backstepping design approach for the PT output-feedback consensus tracker to compensate for unknown time delays. The adaptive neural-network-based PT compensating variables are designed using the time-varying gain function in a distributed manner. In the proposed design and analysis, we employ a novel Lyapunov–Krasovskii function based on the design parameter of the time-varying gain function to ensure practical PT stability of the delay-independent PT consensus-tracking system.

(2) In contrast to the event-triggered PT control results [48–50,52–54], this study presents a dynamic event-triggered mechanism for PT consensus tracking with unknown time delays. The differential equations for the dynamic variables in this mechanism are designed via the time-varying gain function. Based on these dynamic variables, the proposed dynamic event-triggered PT consensus-tracking scheme can guarantee practical PT stability and avoid the Zeno phenomenon.

The remainder of this article is organized as follows. Section 2 introduces preliminaries and the problem formulation. Next, Section 3 presents the distributed dynamic event-triggered PT output-feedback consensus-tracking design.Section 4 provides a simulation example, and conclusions are drawn in Section 5.

2. Preliminaries and Problem Formulation

2.1. Preliminary on Graph Theory

The interaction graph of nonlinear time-delay multiagent systems is described by a directed graph $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} \triangleq \{0, 1, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ stand for the node set and the edge set, respectively. Node 0 corresponds to the leader, while nodes

$1,\dots,N$ represent followers. An edge $(j,f) \in \mathcal{E}$ signifies that agent j gives its information to agent f, but not vice versa. The set of follower nodes is defined as $\bar{\mathcal{V}} \triangleq \{1,\dots,N\}$. $\mathcal{N}_f = \{j|(j,f) \in \mathcal{E}\}$ designates the set of all neighbors of agent f. For the interaction subgraph of followers, the Laplacian matrix is $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \mathrm{diag}(d_1,\dots,d_N)$ with $d_f = \sum_{j=1}^{N} a_{fj}$, and $\mathcal{A}$ stands for the adjacency matrix defined as $\mathcal{A} = [a_{fj}]$. Here, $a_{fj} > 0$ if $j \in \mathcal{N}_f$ and $a_{fj} = 0$ otherwise, where $f, j \in \bar{\mathcal{V}}$ and $f \neq j$. The pining matrix is $\mathcal{B} = \mathrm{diag}(b_1,\dots,b_N)$, where $b_f > 0$ if $0 \in \mathcal{N}_f$ and $b_f = 0$ otherwise, and $f \in \bar{\mathcal{V}}$. Assuming a spanning tree in the directed graph $\mathcal{G}$ with the leader as the root, the matrix $\mathcal{W} = \mathcal{L} + \mathcal{B}$ is nonsingular [55,56].

2.2. Preliminary on Function Approximation Using Neural Networks

We employ the universal approximation property of radial basis function neural networks (RBFNNs) [57] to approximate unknown nonlinear functions induced from the output-feedback control design steps. The unidentified nonlinear functions $\Gamma_{f,k}(\varsigma_{f,k})$: $\mathbb{R}^{q_{f,k}} \mapsto \mathbb{R}$ in the compact set $\Pi_{\varsigma_{f,k}} \subset \mathbb{R}^{q_{f,k}}$ can be represented by employing the function approximation method, as follows:

$$\Gamma_{f,k}(\varsigma_{f,k}) = \lambda_{f,k}^{\top}\Delta_{f,k}(\varsigma_{f,k}) + \delta_{f,k}(\varsigma_{f,k}) \tag{1}$$

where $f \in \bar{\mathcal{V}}$, $\varsigma_{f,k} \in \Pi_{\varsigma_{f,k}}$ is the input of RBFNN, $\lambda_{f,k} \in \mathbb{R}^{m_{f,k}}$ represents the optimal weighting vector, defined as $\lambda_{f,k} = \arg\min_{\hat{\lambda}_{f,k}}[\sup_{\varsigma_{f,k}\in\Pi_{\varsigma_{f,k}}} |\Gamma_{f,k}(\varsigma_{f,k}) - \hat{\lambda}_{f,k}^{\top}\Delta_{f,k}(\varsigma_{f,k})|]$, $m_{f,k}$ is the node number, $\Delta_{f,k}(\varsigma_{f,k}) \in \mathbb{R}^{m_{f,k}}$ is the Gaussian function, $\hat{\lambda}_{f,k}$ is an estimate of $\lambda_{f,k}$ satisfying $\|\lambda_{f,k}\| \leq \bar{\lambda}_{f,k}$, and $\delta_{f,k}$ denotes the reconstruction error satisfying $|\delta_{f,k}| \leq \bar{\delta}_{f,k}$. Here, $\bar{\lambda}_{f,k}$ and $\bar{\delta}_{f,k}$ are constants.

Lemma 1 ([58])**.** *$\|\Delta_{f,k}(\varsigma_{f,k})\| \leq \bar{m}_{f,k}$ is satisfied, where $\bar{m}_{f,k}$ is a known constant.*

2.3. Control Problem

We consider a class of networked uncertain multiagent systems composed of N nonlinear time-delay systems as follows:

$$\begin{aligned} &\dot{x}_f = A_f x_f + g_f(x_f) + B_f u_f + \mu_f(x_{f,\tau(t)}) \\ &y_f = C_f x_f \\ &x_f(q) = \omega_f(q), \quad q \in [-\iota_f, 0] \end{aligned} \tag{2}$$

where $f \in \bar{\mathcal{V}}$, $x_f = [x_{f,1},\dots,x_{f,n_f}]^{\top}$ denotes the state vector of agent f, $A_f = \begin{bmatrix} 0 & & \\ \vdots & I_{n_f-1} & \\ 0 & \cdots & 0 \end{bmatrix}$, $B_f = [0,\dots,0,1]^{\top} \in \mathbb{R}^{n_f}$, $C_f = [1,0,\dots,0] \in \mathbb{R}^{1\times n_f}$, $u_f \in \mathbb{R}$ is the control input of agent f, $y_f \in \mathbb{R}$ is the output of agent f, $g_f(x_f) = [g_{f,1}(x_{f,1}), g_{f,2}(\bar{x}_{f,2}),\dots,g_{f,n_f}(\bar{x}_{f,n_f})]^{\top}$; $g_{f,k}(\bar{x}_{f,k}) : \mathbb{R}^k \mapsto \mathbb{R}$ with $\bar{x}_{f,k} = [x_{f,1},\dots,x_{f,k}]^{\top} \in \mathbb{R}^k$, $k = 1,\dots,n_f$ are unknown C^1 nonlinear functions of agent f, $\mu_f(x_{f,\tau(t)}) = [\mu_{f,1}(x_{f,1,\tau(t)}),\dots,\mu_{f,n_f}(\bar{x}_{f,n_f,\tau(t)})]^{\top}$; $\mu_{f,k}(\bar{x}_{f,k,\tau(t)}) : \mathbb{R}^k \mapsto \mathbb{R}$, $k = 1,\dots,n_f$ denote unknown C^1 nonlinear time-delay functions, $\bar{x}_{f,k,\tau(t)} = [x_{f,1}(t-\tau_{f,1}(t)),\dots,x_{f,k}(t-\tau_{f,k}(t))]^{\top}$ is the delayed state vector of agent f, $\tau_{f,k}(t), k = 1,\dots,n_f$ denote the unknown delays of agent f, such that $0 < \tau_{f,k}(t) \leq \iota_{f,k} < \infty$ and $\dot{\tau}_{f,k}(t) \leq \bar{\iota}_{f,k} < 1$ with unknown constants $\iota_{f,k} > 0$ and $\bar{\iota}_{f,k} > 0$, $\iota_f = \max_{k=1,\dots,n_f}\{\iota_{f,k}\}$, and ω_f is a continuous function vector indicating initial conditions of agent f.

Assumption 1. *The output signals y_f are only measurable for the consensus tracking design.*

Assumption 2 ([25])**.** *The follower f with edge $(0,f) \in \mathcal{E}$ can only receive the leader output r information, where r, $\dot{r}$, and $\ddot{r}$ are continuous and bounded.*

Assumption 3 ([59]). Unknown *nonlinearities* $\zeta_{f,k,l}(\cdot) \geq 0$ *exist, such that*

$$\mu_{f,k}^2(\bar{x}_{f,k,\tau(t)}) \leq \sum_{l=1}^{k} \zeta_{f,k,l}(x_{f,l}(t - \tau_{f,l}(t))).$$

Definition 1. *Consider the follower agents* $\dot{x}_f = g_f(x_f, x_{f,\tau}, u_f)$, *where* $f = 1, \ldots, N$, $x_f(q) = \omega(q), \forall q \in [-\iota_f, 0]$, $x_f \in \mathbb{R}$ *is the state variable of agent* f, $x_{f,\tau} \in \mathbb{R}$ *is the delayed state variable of agent* f, $u_f \in \mathbb{R}$ *denotes a control input of agent* f, *and* $g_f(x_f, x_{f,\tau}, u_f) : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ *is a continuous nonlinear function of agent* f. *For system* $\dot{x}_f = g_f(x_f, x_{f,\tau}, u_f)$, *practical PT consensus tracking is achieved if there exist a controller* u_f *and a user-assignable time* $T > 0$ *such that* $\lim_{t\to T} |x_f(t) - r(t)| \leq \epsilon$ *and* $|x_f(t) - r(t)| \leq \epsilon, \forall t \geq T$, *where* $r(t)$ *is the leader signal.*

Control problem: The control problem of this study is to design dynamic event-triggered PT output-feedback control laws u_f for systems (2), such that all closed-loop signals are bounded and $|y_f(t) - r(t)| \leq \epsilon, \forall t \geq T$, where $\epsilon > 0$ is an adjustable small constant and $T > 0$ is the PT selected regardless of the initial system conditions and design parameters.

Remark 1. *It is worth mentioning that there are no published results, even for the dynamic event-triggered PT state-feedback tracking problem of single strict-feedback nonlinear time-delay systems. In contrast to the existing PT control results [33–38,48–50,52–54], we establish a dynamic event-triggered PT output-feedback consensus tracking strategy in the command-filtered backstepping design framework. The design incorporates distributed adaptive compensating variables and dynamic variables for event-triggering, both based on a continuous time-varying gain function. The practical PT stability of the delay-independent consensus tracking system is analyzed using a novel Lyapunov–Krasovskii function.*

Remark 2. *Practical applications such as chemical reactors, rolling mill systems, and metallurgical processing systems can be typically described as strict-feedback systems [60,61]. Therefore, we consider models such as System* (2) *to account for unknown nonlinearities* g_f *and time-delayed nonlinearities* μ_f *that are unmatched by the control input* u_f. *Thus, the purpose of this paper is to provide a fundamental solution to the dynamic event-triggered PT output-feedback consensus tracking problem for the class of system* (2).

3. Distributed Dynamic Event-Triggered PT Consensus Tracking by Output Feedback

This section first introduces a time-varying gain function and its properties to achieve dynamic event-triggered PT output-feedback consensus tracking. Second, a local delay-independent PT observer for each follower is designed to estimate the immeasurable states $x_{f,k}$, $k = 2, \ldots, n_f$. Third, a distributed delay-independent PT consensus-tracking scheme with a dynamic event-triggered mechanism is constructed in a recursive design manner. Finally, the PT stability of the output-feedback control system is analyzed based on the Lyapunov stability theorem.

3.1. Time-Varying Gain Function

For the practical PT observer and controller design, the time-varying gain function is defined as

$$\chi(t) = \begin{cases} \frac{cT}{(1-c\rho)(T-t)+c\rho T}, & \forall t \in [0, T) \\ 1/\rho, & \forall t \in [T, \infty) \end{cases} \tag{3}$$

where $T > 0$ is the prescribed time, $c \geq 1$ is a constant, and ρ denotes a performance tuning constant, which satisfies $0 < \rho < 1/c$.

Remark 3. *To design the dynamic event-triggered PT output-feedback consensus tracking system, we employ the continuous time-varying gain function $\chi(t)$ with the following properties: (i) $\chi(t)$ is the monotonically increasing continuous function during the time interval $t \in [0, T)$, $\lim_{t\to T} \chi(t) = 1/\rho$, and $\chi(t) = 1/\rho$, $t \geq T$; (ii) $\chi(t)$ is continuously differentiable; and (iii) $\chi(0) = c$ and $\chi(t)$ is bounded as $c \leq \chi(t) \leq 1/\rho$, where the constant c offers flexibility in selecting the initial gain values of the PT observer and consensus tracker and ρ is a design parameter to adjust the PT convergence bound of the proposed PT consensus tracking system.*

3.2. Delay-Independent PT Observer

The delay-independent PT observer is designed as follows:

$$\begin{aligned} \dot{\hat{x}}_{f,k} &= \hat{x}_{f,k+1} + \phi_f^k \theta_{f,k}(y_f - \hat{y}_f) \\ \dot{\hat{x}}_{f,n_f} &= u_f + \phi_f^{n_f} \theta_{f,n_f}(y_f - \hat{y}_f) \\ \hat{y}_f &= \hat{x}_{f,1} \end{aligned} \tag{4}$$

where $f \in \bar{\mathcal{V}}$, $k = 1, \ldots, n_f - 1$, $\hat{x}_{f,l}$, $l = 1, \ldots, n_f$ are estimates of $x_{f,l}$, $\phi_f = \chi + \psi_f$ with a constant $\psi_f > 0$, the design constants $\theta_{f,l}$, $l = 1, \ldots, n_f$ are chosen to make the matrix $A_f - \Theta_f C_f$ be Hurwitz, and $\Theta_f = [\theta_{f,1}, \ldots, \theta_{f,n_f}]^\top \in \mathbb{R}^{n_f}$.

Defining the observer error $s_{f,k} = (x_{f,k} - \hat{x}_{f,k})/\phi_f^k$, $k = 1, \ldots, n_f$, it follows from (2) and (4) that

$$\dot{s}_f = \begin{cases} \phi_f \bar{A}_f s_f - \frac{1-c\rho}{cT} \frac{\chi^2}{\phi_f} D_f s_f + G_f + M_f, \ \forall t \in [0, T) \\ \phi_f \bar{A}_f s_f + G_f + M_f, \quad \forall t \in [T, \infty) \end{cases} \tag{5}$$

where $s_f = [s_{f,1}, \ldots, s_{f,n_f}]^\top$, $\bar{A}_f = A_f - \Theta_f C_f$, $D_f = \mathrm{diag}(1, 2, \ldots, n_f) \in \mathbb{R}^{n_f \times n_f}$, $M_f = [\mu_{f,1}/\phi_f, \mu_{f,2}/\phi_f^2, \ldots, \mu_{f,n_f}/\phi_f^{n_f}]^\top$, and $G_f = [g_{f,1}(x_{f,1})/\phi_f, g_{f,2}(\bar{x}_{f,2})/\phi_f^2, \ldots, g_{f,n_f}(\bar{x}_{f,n_f}) /\phi_f^{n_f}]^\top$.

Choose the Lyapunov function $V_{f,o} = s_f^\top P_f s_f$ with a symmetric matrix $P_f > 0$. From (5), we have

$$\dot{V}_{f,o} = \begin{cases} \phi_f s_f^\top (P_f \bar{A}_f + \bar{A}_f^\top P_f) s_f - \frac{1-c\rho}{cT} \frac{\chi^2}{\phi_f} s_f^\top (D_f P_f + P_f D_f) s_f \\ +2 s_f^\top P_f G_f + 2 s_f^\top P_f M_f, \ \forall t \in [0, T) \\ \phi_f s_f^\top (P_f \bar{A}_f + \bar{A}_f^\top P_f) s_f + 2 s_f^\top P_f G_f + 2 s_f^\top P_f M_f, \ \forall t \in [T, \infty). \end{cases} \tag{6}$$

Since $\bar{A}_f$ is the Hurwitz matrix, there exists a symmetric matrix $P_f > 0$ such that $P_f \bar{A}_f + \bar{A}_f^\top P_f \leq -I_{n_f}$ and $D_f P_f + P_f D_f \geq 0$, where $I_{n_f} \in \mathbb{R}^{n_f \times n_f}$ is the identify matrix. Thus, (6) becomes

$$\dot{V}_{f,o} \leq -\phi_f s_f^\top s_f + 2 s_f^\top P_f G_f + 2 s_f^\top P_f M_f, \ \forall t \geq 0. \tag{7}$$

By Young's inequality, we obtain

$$2 s_f^\top P_f G_f \leq \|P_f\|^2 \|s_f\|^2 + \|g_f(x_f)\|^2 \tag{8}$$

$$2 s_f^\top P_f M_f \leq \|P_f\|^2 \|s_f\|^2 + \|\mu_f(x_{f,\tau(t)})\|^2. \tag{9}$$

Choosing $\psi_f = 2\|P_f\|^2$, we obtain $\phi_f = \chi + 2\|P_f\|^2$. Then, using (8) and (9), we have

$$\dot{V}_{f,o} \leq -\chi s_f^\top s_f + \|g_f(x_f)\|^2 + \|\mu_f(x_{f,\tau(t)})\|^2. \tag{10}$$

Remark 4. *Compared with the distributed practical PT cooperative control results [37,38] for strict-feedback nonlinear multiagent systems without time delays, a delay-independent PT observer (4) using the time-varying gain function χ is designed in this paper.*

3.3. Distributed Delay-Independent Dynamic Event-Triggered PT Output-Feedback Tracker

In this section, employing the adaptive command-filter backstepping design methodology, we construct a distributed dynamic event-triggered PT output-feedback tracker.

Compared with the existing PT control methods [33–38,48–50,52–54], the primary difficulties in this design stem from (i) addressing the unknown time-delay nonlinearities and (ii) formulating the dynamic event-triggered mechanism to ensure the practical PT stability of the delay-independent event-triggered PT consensus tracking system. To overcome difficulty (i), we propose a novel Lyapunov–Krasovskii function based on the design parameter of the time-varying gain function. Using this function, we design an adaptive event-triggered PT tracker using only output information in a fully distributed manner. Moreover, to deal with difficulty (ii), we derive triggering conditions with dynamic variables based on the time-varying gain function χ.

First, we define the error variables as follows:

$$e_{f,1} = \sum_{j=1}^{N} a_{fj}(y_f - y_j) + b_f(y_f - r) \tag{11}$$

$$e_{f,k} = \hat{x}_{f,k} - \bar{v}_{f,k-1}, \quad k = 2, \dots, n_f \tag{12}$$

where $f \in \bar{\mathcal{V}}$ and $\bar{v}_{f,k-1}$ is the first-order filtered signal, determined by

$$\dot{\bar{v}}_{f,k-1} = \chi\sigma_{f,k-1}(v_{f,k-1} - \bar{v}_{f,k-1}) \tag{13}$$

with the virtual control laws $v_{f,k-1}$ and the constant $\sigma_{f,k-1} > 0$.

Then, for the command-filter backstepping design, the compensating errors are constructed as

$$z_{f,k} = e_{f,k} - \alpha_{f,k}, \quad k = 1, \dots, n_f \tag{14}$$

where $\alpha_{f,k}$ are the compensating signals which are provided to compensate for unknown terms and filtering errors in the design steps, and $\alpha_{f,n_f} = 0$. Based on these error variables, the detailed steps are as follows.

Step 1: We first design the local virtual control law $v_{f,1}$. Defining the vector $z_1 = [z_{1,1}, \dots, z_{N,1}]^\top$ and using $x_{f,2} = s_{f,2}\phi_f^2 + z_{f,2} + \alpha_{f,2} + \bar{v}_{f,1}$, the time derivative of z_1 along (2), (12), and (14) is given by

$$\begin{aligned}\dot{z}_1 &= \mathcal{W}(\dot{y} - 1_N\dot{r}) - \dot{\alpha}_1 \\ &= \mathcal{W}(\Phi\check{s}_2 + z_2 + \alpha_2 + \bar{v}_1 + \check{g}_1(\check{x}_1) + \check{\mu}_1(\check{x}_{1,\tau(t)}) - 1_N\dot{r}) - \dot{\alpha}_1\end{aligned} \tag{15}$$

where $\mathcal{W}$ is defined in Section 2.1, $y = [y_1, \dots, y_N]^\top$, $\Phi = \text{diag}(\phi_1^2, \dots, \phi_N^2)$, $\check{s}_2 = [s_{1,2}, \dots, s_{N,2}]^\top$, $z_2 = [z_{1,2}, \dots, z_{N,2}]^\top$, $\alpha_2 = [\alpha_{1,2}, \dots, \alpha_{N,2}]^\top$, $\bar{v}_1 = [\bar{v}_{1,1}, \dots, \bar{v}_{N,1}]^\top$, $\check{x}_1 = [x_{1,1}, \dots, x_{N,1}]^\top$, $\check{g}_1(\check{x}_1) = [g_{1,1}(x_{1,1}), \dots, g_{N,1}(x_{N,1})]^\top$, $\check{\mu}_1(\check{x}_{1,\tau(t)}) = [\mu_{1,1}(x_{1,1}(t - \tau_{1,1})), \dots, \mu_{N,1}(x_{N,1}(t - \tau_{N,1}))]^\top$, $1_N = [1, \dots, 1]^\top \in \mathbb{R}^N$, and $\alpha_1 = [\alpha_{1,1}, \dots, \alpha_{N,1}]^\top$.

Choose the Lyapunov function

$$V_1 = \frac{1}{2}z_1^\top z_1 + \sum_{f=1}^{N}\left(V_{f,o} + \frac{1}{2\beta_{f,1}}\tilde{\lambda}_{f,1}^\top\tilde{\lambda}_{f,1}\right) \tag{16}$$

where $\tilde{\lambda}_{f,1} = \lambda_{f,1} - \hat{\lambda}_{f,1}$ and $\beta_{f,1} > 0$ is a design constant.

Substituting (10) and (15) into the time derivative of V_1, we have

$$\begin{aligned}\dot{V}_1 \leq\ & \sum_{f=1}^{N}\left(-\chi s_f^\top s_f + \|g_f(x_f)\|^2 + \|\mu_f(x_{f,\tau(t)})\|^2 - \frac{1}{\beta_{f,1}}\tilde{\lambda}_{f,1}^\top\dot{\hat{\lambda}}_{f,1}\right) \\ & + z_1^\top(\mathcal{W}(\Phi\check{s}_2 + z_2 + \alpha_2 + \bar{v}_1 - v_1 + v_1 + \check{g}_1(\check{x}_1) + \check{\mu}_1(\check{x}_{1,\tau(t)}) - 1_N\dot{r}) - \dot{\alpha}_1)\end{aligned} \tag{17}$$

where $v_1 = [v_{1,1}, \ldots, v_{N,1}]^\top$.

From Assumption 3, we have

$$\begin{aligned} \|\mu_f(x_{f,\tau(t)})\|^2 &\leq \sum_{k=1}^{n_f} \sum_{l=1}^{k} \zeta_{f,k,l}(x_{f,l}(t-\tau_{f,l})) \\ &= \sum_{k=1}^{n_f} \sum_{l=k}^{n_f} \zeta_{f,l,k}(x_{f,k}(t-\tau_{f,k})) \end{aligned} \tag{18}$$

$$z_1^\top \mathcal{W} \breve{\mu}_1(\breve{x}_{1,\tau(t)}) \leq \frac{1}{4} z_1^\top z_1 + \|\mathcal{W}\|^2 \sum_{f=1}^{N} \zeta_{f,1,1}(x_{f,1}(t-\tau_{f,1})). \tag{19}$$

Using (18) and (19) and choosing the distributed delay-independent PT virtual control law $v_{f,1}$ as

$$v_{f,1} = -\chi \eta_{f,1} e_{f,1}, \tag{20}$$

we obtain

$$\begin{aligned} \dot{V}_1 \leq & \sum_{f=1}^{N} \Big(-\chi s_f^\top s_f + \|g_f(x_f)\|^2 - \tfrac{1}{\beta_{f,1}} \tilde{\lambda}_{f,1}^\top \dot{\hat{\lambda}}_{f,1} \Big) \\ & + \sum_{f=1}^{N} \Lambda_f(\bar{x}_{f,n_f,\tau(t)}) + z_1^\top \Big(\mathcal{W}(\Phi \breve{s}_2 + z_2 - \chi \eta_1 e_1 \\ & + \alpha_2 + \bar{v}_1 - v_1 + \breve{g}_1(\breve{x}_1) - 1_N \dot{r}) + \tfrac{1}{4} z_1 - \dot{\alpha}_1 \Big) \end{aligned} \tag{21}$$

where $\eta_1 = \mathrm{diag}(\eta_{1,1}, \ldots, \eta_{N,1})$, $\eta_{f,1} > 0$ is a constant, $e_1 = [e_{1,1}, \ldots, e_{N,1}]^\top$, and $\Lambda_f = \sum_{k=1}^{n_f} \sum_{l=k}^{n_f} \zeta_{f,l,k}(x_{f,k}\,(t-\tau_{f,k})) + \|\mathcal{W}\|^2 \zeta_{f,1,1}(x_{f,1}(t-\tau_{f,1}))$.

Using the inequality $z_1^\top \mathcal{W} \Phi \breve{s}_2 \leq z_1^\top z_1 \|\mathcal{W}\|^2 \|\Phi\|^2 + \sum_{f=1}^{N} \chi s_f^\top s_f / 4$ in (21) yields

$$\begin{aligned} \dot{V}_1 \leq & \sum_{f=1}^{N} \Big(-\tfrac{3}{4} \chi s_f^\top s_f + \|g_f(x_f)\|^2 - \tfrac{1}{\beta_{f,1}} \tilde{\lambda}_{f,1}^\top \dot{\hat{\lambda}}_{f,1} \Big) \\ & + z_1^\top (\mathcal{W}(z_2 - \chi \eta_1 e_1 - 1_N \dot{r}) + \Gamma_1 - \kappa_\eta z_1 - \dot{\alpha}_1) + \sum_{f=1}^{N} \Lambda_f(\bar{x}_{f,n_f,\tau(t)}) \end{aligned} \tag{22}$$

where $\Gamma_1 = [\Gamma_{1,1}(\varsigma_{1,1}), \ldots, \Gamma_{N,1}(\varsigma_{N,1})]^\top = \mathcal{W}(\alpha_2 + \bar{v}_1 - v_1 + \breve{g}_1(\breve{x}_1)) + z_1 \|\mathcal{W}\|^2 \|\Phi\|^2 + z_1/4 + \kappa_\eta z_1$ with $\varsigma_{f,1} = [\alpha_{f,2}, \alpha_{j,2}, \bar{v}_{f,1}, \bar{v}_{j,1}, y_f, y_j, b_f r, z_{f,1}, \chi]^\top$, $j \in \mathcal{N}_f$ and κ_η is a constant to be determined later.

Using $\Gamma_{f,1}(\varsigma_{f,1}) = \lambda_{f,1}^\top \Delta_{f,1}(\varsigma_{f,1}) + \delta_{f,1}(\varsigma_{f,1})$ from the RBFNN approximation (1), we obtain

$$\begin{aligned} \dot{V}_1 \leq & \sum_{f=1}^{N} \Big(-\tfrac{3}{4} \chi s_f^\top s_f + \|g_f(x_f)\|^2 - \tfrac{1}{\beta_{f,1}} \tilde{\lambda}_{f,1}^\top \dot{\hat{\lambda}}_{f,1} \Big) \\ & + z_1^\top (\mathcal{W}(z_2 - \chi \eta_1 e_1) + \Delta_1 + \varpi_1 - \kappa_\eta z_1 - \dot{\alpha}_1) + \sum_{f=1}^{N} \Lambda_f(\bar{x}_{f,n_f,\tau(t)}) \end{aligned} \tag{23}$$

where $\Delta_1 = [\lambda_{1,1}^\top \Delta_{1,1}(\varsigma_{1,1}), \ldots, \lambda_{N,1}^\top \Delta_{N,1}(\varsigma_{N,1})]^\top$, $\varpi_1 = -\mathcal{W} 1_N \dot{r} + \delta_1$, and $\delta_1 = [\delta_{1,1}, \ldots, \delta_{N,1}]^\top$.

The neural-network-based PT compensating variable $\alpha_{f,1}$ is provided in a distributed manner as follows:

$$\dot{\alpha}_{f,1} = \chi \sum_{j=1}^{N} a_{fj}(\eta_{j,1} \alpha_{j,1} - \eta_{f,1} \alpha_{f,1}) - \chi b_f \eta_{f,1} \alpha_{f,1} + \hat{\lambda}_{f,1}^\top \Delta_{f,1}(\varsigma_{f,1}) \tag{24}$$

where $\alpha_{f,1}(0) = 0$ and $\hat{\lambda}_{f,1}$, $f \in \bar{\mathcal{V}}$ is tuned by the following adaptive law

$$\dot{\hat{\lambda}}_{f,1} = \beta_{f,1}(\Delta_{f,1}(\varsigma_{f,1}) z_{f,1} - \vartheta_{f,1} \hat{\lambda}_{f,1}) \tag{25}$$

with a constant $\vartheta_{f,1} > 0$.

Substituting (24) and (25) into (23) gives

$$\begin{aligned}\dot{V}_1 \leq & \sum_{f=1}^{N}\Big(-\tfrac{3}{4}\chi s_f^\top s_f + \|g_f(x_f)\|^2 + \vartheta_{f,1}\tilde{\lambda}_{f,1}^\top\hat{\lambda}_{f,1}\Big) \\ & + z_1^\top(\mathcal{W}(z_2 - \chi\eta_1 z_1) - \kappa_\eta z_1 + \varpi_1) + \sum_{f=1}^{N}\Lambda_f(\bar{x}_{f,n_f,\tau(t)}).\end{aligned} \tag{26}$$

Remark 5. *For the fully distributed PT tracker design, the delay-independent virtual controller (20) and the dynamics (24) of the adaptive compensating variable $\alpha_{f,1}$ are designed using the time-varying gain function χ and adaptive tuning vector $\hat{\lambda}_{f,1}$ in a distributed manner. In (24), the adaptive function approximation approach using neural networks is employed to compensate for the distributed terms, including filtering errors.*

Step 2*:* For the design of the k-th local adaptive virtual tracking controller $v_{f,k}$, we choose the Lyapunov function

$$V_2 = \sum_{f=1}^{N}\left[\sum_{k=2}^{n_f-1}\left(\frac{1}{2}z_{f,k}^2 + \frac{1}{2\beta_{f,k}}\tilde{\lambda}_{f,k}^\top\tilde{\lambda}_{f,k}\right)\right] \tag{27}$$

where $\beta_{f,k} > 0$ is the design constant and $\tilde{\lambda}_{f,k} = \lambda_{f,k} - \hat{\lambda}_{f,k}$.

Using (4), (12), and (14), $\dot{V}_2$ is given by

$$\begin{aligned}\dot{V}_2 \leq & \sum_{f=1}^{N}\Bigg[\sum_{k=2}^{n_f-1}\Big(z_{f,k}(z_{f,k+1} + \alpha_{f,k+1} + \bar{v}_{f,k} - v_{f,k} + v_{f,k} \\ & + \phi_f^k\theta_{f,k}(y_f - \hat{y}_f) - \dot{\bar{v}}_{f,k-1} - \dot{\alpha}_{f,k}) - \tfrac{1}{\beta_{f,k}}\tilde{\lambda}_{f,k}^\top\dot{\hat{\lambda}}_{f,k}\Big)\Bigg].\end{aligned} \tag{28}$$

Using the virtual control law

$$v_{f,k} = -\chi\eta_{f,k}e_{f,k} - \phi_f^k\theta_{f,k}(y_f - \hat{y}_f) + \chi\sigma_{f,k-1}(v_{f,k-1} - \bar{v}_{f,k-1}), \tag{29}$$

we have

$$\begin{aligned}\dot{V}_2 & \leq \sum_{f=1}^{N}\Bigg[\sum_{k=2}^{n_f-1}\Big(z_{f,k}(z_{f,k+1} - \chi\eta_{f,k}e_{f,k} + \alpha_{f,k+1} + \bar{v}_{f,k} - v_{f,k} - \dot{\alpha}_{f,k}) - \tfrac{1}{\beta_{f,k}}\tilde{\lambda}_{f,k}^\top\dot{\hat{\lambda}}_{f,k}\Big)\Bigg] \\ & \leq \sum_{f=1}^{N}\Bigg[\sum_{k=2}^{n_f-1}\Big(z_{f,k}(z_{f,k+1} - \chi\eta_{f,k}e_{f,k} + \Gamma_{f,k}(x_{f,k}) - \dot{\alpha}_{f,k}) - \tfrac{1}{\beta_{f,k}}\tilde{\lambda}_{f,k}^\top\dot{\hat{\lambda}}_{f,k}\Big) - \|\mathcal{W}\|^2 z_{f,2}^2\Bigg]\end{aligned} \tag{30}$$

where $\Gamma_{f,2}(\varsigma_{f,2}) = \alpha_{f,3} + \bar{v}_{f,2} - v_{f,2} + \|\mathcal{W}\|^2 z_{f,2}$ and $\Gamma_{f,l}(\varsigma_{f,l}) = \alpha_{f,l+1} + \bar{v}_{f,l} - v_{f,l}$, $l = 3, \ldots, n_f - 1$ with $\varsigma_{f,k} = [\alpha_{f,k+1}, \bar{v}_{f,k} - v_{f,k}, z_{f,k}, x_{f,k}]^\top$, $k = 2, \ldots, n_f - 1$.

Using the function approximation (1), it holds that $\Gamma_{f,k}(\varsigma_{f,k}) = \lambda_{f,k}^\top\Delta_{f,k}(\varsigma_{f,k}) + \delta_{f,k}(\varsigma_{f,k})$. Then, the adaptive compensating variable $\alpha_{f,k}$ is constructed by

$$\dot{\alpha}_{f,k} = -\chi\eta_{f,k}\alpha_{f,k} + \hat{\lambda}_{f,k}^\top\Delta_{f,k}(\varsigma_{f,k}) \tag{31}$$

where $\alpha_{f,k}(0) = 0$ and $\hat{\lambda}_{f,k}$ is updated using

$$\dot{\hat{\lambda}}_{f,k} = \beta_{f,k}(\Delta_{f,k}(\varsigma_{f,k})z_{f,k} - \vartheta_{f,k}\hat{\lambda}_{f,k}) \tag{32}$$

with a constant $\vartheta_{f,k} > 0$.

Substituting (31) and (32) into (30) results in

$$\dot{V}_2 \le \sum_{f=1}^{N}\left[\sum_{k=2}^{n_f-1}\left(z_{f,k}(z_{f,k+1}-\chi\eta_{f,k}z_{f,k}+\delta_{f,k})+\vartheta_{f,k}\tilde{\lambda}_{f,k}^{\top}\hat{\lambda}_{f,k}\right)-\|\mathcal{W}\|^2 z_{f,2}^2\right]. \quad (33)$$

Step 3: For the design of the event-triggered PT tracking law u_f, we choose the Lyapunov function

$$V_3 = \sum_{f=1}^{N}\left[\frac{1}{2}z_{f,n_f}^2+\sum_{k=1}^{n_f}L_{f,k}+\frac{1}{2\beta_{f,n_f}}\tilde{\bar{\lambda}}_{f,n_f}^2\right] \quad (34)$$

where $\tilde{\bar{\lambda}}_{f,n_f}=\bar{\lambda}_{f,n_f}-\hat{\bar{\lambda}}_{f,n_f}$, $\beta_{f,n_f}>0$ is a design constant, $\bar{\lambda}_{f,n_f}>0$ is an unknown constant determined to be later, $\hat{\bar{\lambda}}_{f,n_f}$ is an estimate of $\bar{\lambda}_{f,n_f}$, and $L_{f,k}$ is the Lyapunov–Krasovskii function, defined as

$$L_{f,k}=\frac{e^{\iota_{f,k}/\rho}}{1-\bar{\iota}_{f,k}}\int_{t-\tau_{f,k}(t)}^{t}e^{-(t-\nu)/\rho}\bar{\Lambda}_{f,k}(x_{f,k})d\nu \quad (35)$$

with $\bar{\Lambda}_{f,1}(x_{f,1})=\|\mathcal{W}\|^2\zeta_{f,1,1}(x_{f,1})+\sum_{l=1}^{n_f}\zeta_{f,l,1}(x_{f,1})$, $\bar{\Lambda}_{f,m}(x_{f,m})=\sum_{l=k}^{n_f}\zeta_{f,l,k}(x_{f,k})$, $m=2,\dots,n_f-1$, and $\bar{\Lambda}_{f,n_f}(x_{f,n_f})=\zeta_{f,n_f,n_f}(x_{f,n_f})$.

Using (4), the time derivative of V_3 is obtained as

$$\begin{aligned}\dot{V}_3 \le & \sum_{f=1}^{N}\Big[z_{f,n_f}(u_f+\phi_f^{n_f}\theta_{f,n_f}(y_f-\hat{y}_f)-\dot{\bar{v}}_{f,n_f-1})\\ &-\frac{1}{\rho}\sum_{k=1}^{n_f}L_{f,k}+\rho\Gamma_{f,n_f}(\varsigma_{f,n_f})-\|g_f(x_f)\|^2\\ &-\sum_{k=1}^{n_f}\bar{\Lambda}_{f,k}(x_{f,k}(t-\tau_{f,k}))-\frac{1}{\beta_{f,n_f}}\tilde{\bar{\lambda}}_{f,n_f}\dot{\hat{\bar{\lambda}}}_{f,n_f}\Big]\end{aligned} \quad (36)$$

where $\Gamma_{f,n_f}(\varsigma_{f,n_f}) = \sum_{k=1}^{n_f}e^{\iota_{f,k}/\rho}\bar{\Lambda}_{f,k}(x_{f,k}(t))/(\rho(1-\bar{\iota}_{f,k}))+\|g_f(x_f)\|^2/\rho$ with $\varsigma_{f,n_f}=\bar{x}_{f,n_f}$.

Using the neural network approximation $\Gamma_{f,n_f}(\varsigma_{f,n_f})=\lambda_{f,n_f}^{\top}\Delta_{f,n_f}(\varsigma_{f,n_f})+\delta_{f,n_f}(\varsigma_{f,n_f})$, we have

$$\rho\Gamma_{f,n_f}(\varsigma_{f,n_f})\le\rho\bar{\lambda}_{f,n_f}\|\Delta_{f,n_f}(\varsigma_{f,n_f})\|+\rho\bar{\delta}_{f,n_f} \quad (37)$$

where $\bar{\lambda}_{f,n_f}$ is a constant satisfying $\|\lambda_{f,n_f}\|\le\bar{\lambda}_{f,n_f}$.

The dynamic event-triggered PT actual control law and triggered condition are proposed as

$$u_f(t)=\hat{u}_f(t_{f,h}),\quad \forall t\in[t_{f,h},t_{f,h+1}) \quad (38)$$

$$t_{f,h+1}=\inf\{t>t_{f,h}|\frac{1}{\chi}\tilde{u}_f^2-\chi\sum_{k=1}^{n_f}z_{f,k}^2>\frac{\chi}{\varphi_f}\varrho_f\} \quad (39)$$

where $\tilde{u}_f=u_f-\hat{u}_f$ is the input triggering error, φ_f is a positive constant, $t_{f,h}$, $h\in\mathbb{Z}^+$ is the update time of the control input, $t_{f,1}=0$, the dynamic variable ϱ_f is provided by the following differential equation

$$\dot{\varrho}_f=-\chi\kappa_f\varrho_f+\chi\sum_{k=1}^{n_f}z_{f,k}^2-\frac{1}{\chi}\tilde{u}_f^2 \quad (40)$$

with $\varrho_f(0) > 0$, a positive constant κ_f, and the adaptive control signal $\hat{u}_f$ is designed as

$$\begin{aligned}\hat{u}_f =& -\chi\eta_{f,n_f} z_{f,n_f} - \phi_f^{n_f}\theta_{f,n_f}(y_f - \hat{y}_f) + \chi\sigma_{f,n_f-1}(v_{f,n_f-1} - \bar{v}_{f,n_f-1}) \\ & - \frac{z_{f,n_f}}{z_{f,n_f}^2 + \bar{\epsilon}_f}\hat{\bar{\lambda}}_{f,n_f}\|\Delta_{f,n_f}(\varsigma_{f,n_f})\|\end{aligned} \tag{41}$$

$$\dot{\hat{\bar{\lambda}}}_{f,n_f} = \beta_{f,n_f}\left(\|\Delta_{f,n_f}(\varsigma_{f,n_f})\|\frac{z_{f,n_f}^2}{z_{f,n_f}^2 + \bar{\epsilon}_f} - \vartheta_{f,n_f}\hat{\bar{\lambda}}_{f,n_f}\right) \tag{42}$$

with constants $\eta_{f,n_f} > 0$, $\vartheta_{f,n_f} > 0$, and $\bar{\epsilon}_f > 0$. From (38) and (39), the control input $u_f(t)$ has a constant value $\hat{u}_f(t_{f,h})$ during the time interval $[t_{f,h}, t_{f,h+1})$. Then, the control input $u_f(t)$ is updated to $\hat{u}_f(t_{f,h+1})$ when (39) is satisfied.

Substituting $u_f = \hat{u}_f + \tilde{u}_f$, (37), (41), and (42) into (36), we obtain

$$\begin{aligned}\dot{V}_3 \leq \; & \sum_{f=1}^{N}\Bigg[z_{f,n_f}(\tilde{u}_f - \chi\eta_{f,n_f} z_{f,n_f}) - \frac{1}{\rho}\sum_{k=1}^{n_f} L_{f,k} + \left(\rho - \frac{z_{f,n_f}^2}{z_{f,n_f}^2 + \bar{\epsilon}_f}\right)\bar{\lambda}_{f,n_f}\|\Delta_{f,n_f}(\varsigma_{f,n_f})\| \\ & + \rho\bar{\delta}_{f,n_f} - \sum_{k=1}^{n_f}\bar{\Lambda}_{f,k}(x_{f,k}(t - \tau_{f,k})) - \|g_f(x_f)\|^2 + \vartheta_{f,n_f}\tilde{\bar{\lambda}}_{f,n_f}\hat{\bar{\lambda}}_{f,n_f}\Bigg].\end{aligned} \tag{43}$$

Remark 6. *Different from the existing event-triggered PT control results utilizing static mechanisms [48–50], we design the dynamic event-triggered mechanism consisting of (39) and (40). In this design, the time-varying gain function χ plays a crucial role in ensuring the PT stability of the event-triggered consensus tracking system and maintaining the positivity of the dynamic variable ϱ_f. Notably, when χ is not considered (i.e., $\chi = 1$), the proposed dynamic event-triggered mechanism aligns with similar ones reported in [39,40]. Therefore, the design approach presented for event-triggered PT tracking can be regarded as a more general form.*

3.4. Practical PT Stability Analysis

In this subsection, we analyze the practical PT stability of the proposed dynamic event-triggered PT control system.

Lemma 2. *For $\varrho_f(0) > 0$, $\varrho_f(t) > 0$ is ensured for all $t \geq 0$.*

Proof. Using (39), $\tilde{u}_f^2/\chi - \chi\sum_{k=1}^{n_f} z_{f,k}^2 \leq \chi\varrho_f/\varphi_f$ is satisfied for the time intervals $[t_{f,h}, t_{f,h+1})$, $\forall h \in \mathbb{Z}^+$. Thus, from (40), we obtain $\dot{\varrho}_f \geq -\chi(\kappa_f + (1/\varphi_f))\varrho_f$. The solution is given by

$$\varrho_f(t) \geq e^{-(\kappa_f + \frac{1}{\varphi_f})\int_0^t \chi(w)dw}\varrho_f(0) > 0. \tag{44}$$

This completes the proof of this lemma. □

Remark 7. *To establish the non-existence of Zeno behavior, it is crucial to show the positivity of the dynamic variable ϱ_f based on differential Equation (40) incorporating χ. In Lemma 2, we establish the positivity of ϱ_f by utilizing the event-triggered condition (39) and the dynamics (40), regardless of the use of χ. This outcome will be employed in proving the non-existence of Zeno behavior, as outlined in Theorem 1.*

The main theorem of this study is as follows.

Theorem 1. *Consider the time-delay multiagent system (2). The dynamic event-triggered PT tracking scheme (4) and (38)-(42) ensures that the practical PT stability of the consensus tracking errors $y_f - r$ is ensured while all closed-loop signals are uniformly bounded. Furthermore, Zeno behavior is avoided.*

Proof. Defining the overall Lyapunov function $V = \sum_{i=1}^{3} V_i + \sum_{f=1}^{N} \varrho_f$ and using the equality $\sum_{f=1}^{N}\sum_{k=1}^{n_f} \bar{\Lambda}_{f,k}\left(x_{f,k}(t-\tau_{f,k})\right) = \sum_{f=1}^{N} \Lambda_f(\bar{x}_{f,n_f,\tau(t)})$, we have

$$\begin{aligned}\dot{V} \le\ & \sum_{f=1}^{N}\Bigg[-\tfrac{3}{4}\chi s_f^\top s_f - \tfrac{1}{\rho}\sum_{k=1}^{n_f} L_{f,k} - \sum_{k=2}^{n_f}\chi\eta_{f,k}z_{f,k}^2 + \sum_{k=2}^{n_f-1} z_{f,k}z_{f,k+1} + z_{f,n_f}\tilde{u}_f - \chi\kappa_f\varrho_f \\ & + \sum_{k=2}^{n_f-1} z_{f,k}\delta_{f,k} + \sum_{k=1}^{n_f-1}\vartheta_{f,k}\tilde{\lambda}_{f,k}^\top\hat{\lambda}_{f,k} + \vartheta_{f,n_f}\tilde{\bar{\lambda}}_{f,n_f}\hat{\bar{\lambda}}_{f,n_f} - \|\mathcal{W}\|^2 z_{f,2}^2 \\ & + \tfrac{(\rho-1)z_{f,n_f}^2+\rho\bar{\epsilon}_f}{z_{f,n_f}^2+\bar{\epsilon}_f}\bar{\lambda}_{f,n_f}\|\Delta_{f,n_f}(\varsigma_{f,n_f})\| + \rho\bar{\delta}_{f,n_f} + \chi\sum_{k=1}^{n_f} z_{f,k}^2 - \tfrac{1}{\chi}\tilde{u}_f^2\Bigg] \\ & - \tfrac{1}{2}\chi z_1^\top W_\eta z_1 - \kappa_\eta z_1^\top z_1 + z_1^\top(\mathcal{W}z_2 + \varpi_1)\end{aligned} \tag{45}$$

where $W_\eta = \mathcal{W}\eta_1 + \eta_1\mathcal{W}^\top$.

Using Young's inequality, $1 \le \chi \le 1/\rho$, and $\bar{\epsilon}_f/(z_{f,n_f}^2 + \bar{\epsilon}_f) \le 1$, the following inequalities are obtained as

$$z_{f,k}z_{f,k+1} \le \frac{\chi}{2}z_{f,k}^2 + \frac{\chi}{2}z_{f,k+1}^2 \tag{46}$$

$$z_{f,n_f}\tilde{u}_f \le \frac{\chi}{4}z_{f,n_f}^2 + \frac{1}{\chi}\tilde{u}_f^2 \tag{47}$$

$$z_{f,k}\delta_{f,k} \le \frac{\chi}{4}z_{f,k}^2 + \bar{\delta}_{f,k}^2 \tag{48}$$

$$\tilde{\lambda}_{f,k}^\top\hat{\lambda}_{f,k} \le -\frac{1}{2}\tilde{\lambda}_{f,k}^\top\tilde{\lambda}_{f,k} + \frac{1}{2}\lambda_{f,k}^\top\lambda_{f,k} \tag{49}$$

$$\tilde{\bar{\lambda}}_{f,n_f}\hat{\bar{\lambda}}_{f,n_f} \le -\frac{1}{2}\tilde{\bar{\lambda}}_{f,n_f}^2 + \frac{1}{2}\bar{\lambda}_{f,n_f}^2 \tag{50}$$

$$\begin{aligned}\tfrac{(\rho-1)z_{f,n_f}^2+\rho\bar{\epsilon}_f}{z_{f,n_f}^2+\bar{\epsilon}_f}\quad & \bar{\lambda}_{f,n_f}\|\Delta_{f,n_f}(\varsigma_{f,n_f})\| \\ & \le \tfrac{\rho\bar{\epsilon}_f}{z_{f,n_f}^2+\bar{\epsilon}_f}\bar{\lambda}_{f,n_f}\|\Delta_{f,n_f}(\varsigma_{f,n_f})\| \\ & \le \rho\bar{\lambda}_{f,n_f}\bar{m}_{f,n_f}\end{aligned} \tag{51}$$

$$z_1^\top\mathcal{W}z_2 \le \sum_{f=1}^{N}\left(\frac{1}{4}z_{f,1}^2 + \|\mathcal{W}\|z_{f,2}^2\right) \tag{52}$$

$$z_1^\top\varpi_1 \le \sum_{f=1}^{N}\frac{1}{4}z_{f,1}^2 + \bar{\varpi}_1^2 \tag{53}$$

where $\bar{\varpi}_1 = \|\mathcal{W}\|N\bar{r} + \sum_{f=1}^{N}\bar{\delta}_{f,1}$ with a constant $\bar{r}$ satisfying $|\dot{r}| \le \bar{r}$.

Substituting (46)-(51) into (45), using $1 \le \chi \le 1/\rho$, and selecting $\kappa_\eta = 1/2$, $\eta_{f,2} = 7/4 + \eta_{f,2}^*$, $\eta_{f,m} = 9/4 + \eta_{f,m}^*$, $m = 3,\ldots,n_f-1$, and $\eta_{f,n_f} = 7/4 + \eta_{f,n_f}^*$ yields

$$\begin{aligned}\dot{V} \le\ & \sum_{f=1}^{N}\Bigg[-\tfrac{3}{4}\chi s_f^\top s_f - \sum_{k=1}^{n_f}\chi L_{f,k} - \tfrac{\lambda_{W_\eta}}{2}\chi z_{f,1}^2 - \sum_{k=2}^{n_f}\chi\eta_{f,k}^* z_{f,k}^2 - \chi\kappa_f\varrho_f - \sum_{k=1}^{n_f-1}\tfrac{\varrho}{2}\chi\vartheta_{f,k}\tilde{\lambda}_{f,k}^\top\tilde{\lambda}_{f,k} \\ & -\tfrac{\varrho}{2}\chi\vartheta_{f,n_f}\tilde{\bar{\lambda}}_{f,n_f}^2\Bigg] + \xi_1 \\ \le\ & -\chi\xi_2 V + \xi_1\end{aligned} \tag{54}$$

where $\eta_{f,k}^* > 0$ is a constant, λ_{W_η} denotes the minimum eigenvalue of W_η, $\xi_1 = \bar{\varpi}_1^2 + \sum_{f=1}^{N}[\sum_{k=2}^{n_f-1}(\bar{\delta}_{f,k}^2 + \vartheta_{f,k}\lambda_{f,k}^\top\lambda_{f,k}/2) + \rho\bar{\delta}_{f,n_f} + \vartheta_{f,n_f}\bar{\lambda}_{f,n_f}^2/2 + \rho\bar{\lambda}_{f,n_f}\bar{m}_{f,n_f}]$, and $\xi_2 = \min_{\forall f,k}\{3/(4\bar{p}_f), \lambda_{W_\eta}, 2\eta_{f,k}^*, \beta_{f,k}\vartheta_{f,k}\rho, \kappa_f\}$; $\bar{p}_f$ is the maximum eigenvalue of P_f.

Practical PT stability: From (54), the solution is given by

$$V(t) \leq e^{-M(t)}V(0) + \xi_1 e^{-M(t)} \int_0^t e^{M(w)}dw \tag{55}$$

where $M(t) = \xi_2 \int_0^t \chi(w)dw$. For all $t \in [0, T)$, it is satisfied that

$$\int_0^t \chi(w)dw = cT\frac{\ln((c\rho-1)t+T)}{c\rho-1} - cT\frac{\ln(T)}{c\rho-1}. \tag{56}$$

Using (56), it holds that

$$\lim_{t\to T}\int_0^t \chi(w)dw = cT\frac{\ln c\rho}{c\rho-1} \triangleq M^*. \tag{57}$$

From (57), we have

$$\begin{aligned} \lim_{t\to T} \xi_1 e^{-M(t)} \int_0^t e^{M(w)}dw &= \xi_1 \frac{\int_0^T e^{M(w)}dw}{\lim_{t\to T} e^{M(t)}} \\ &= \xi_1 \frac{\int_0^T e^{M(w)}dw}{e^{\xi_2 M^*}}. \end{aligned} \tag{58}$$

Using (56), we obtain

$$\begin{aligned} \int_0^T e^{M(w)}dw &= \int_0^T \left(\frac{(c\rho-1)w+T}{T}\right)^{\frac{\xi_2 cT}{c\rho-1}} dw \\ &= \frac{T}{\xi_2 cT+(c\rho-1)}\left\{(c\rho)^{\frac{\xi_2 cT}{c\rho-1}+1} - 1\right\}. \end{aligned} \tag{59}$$

Using (57)–(59) yields

$$\lim_{t\to T} V(t) \leq e^{-\xi_2 M^*}V(0) + \frac{\xi_1 T}{\xi_2 cT + (c\rho-1)}\left\{c\rho - (c\rho)^{-\frac{\xi_2 cT}{c\rho-1}}\right\} \triangleq \bar{V} \tag{60}$$

where $\bar{V}$ is a constant. Therefore, s_f, $z_{f,k}$, $\tilde{\lambda}_{f,k}$, and $\tilde{\bar{\lambda}}_{f,n_f}$ are bounded within a pre-specified time T. From the boundedness of $\lambda_{f,k}$, $\hat{\lambda}_{f,k}$ is bounded within a pre-specified time T. Then, using Lemma 1, there exists a constant $\Lambda_{\Delta_{f,k}}$ such that $|\hat{\lambda}_{f,k}^{\top}\Delta_{f,k}(\varsigma_{f,k})| \leq \Lambda_{\Delta_{f,k}}$. Using (24) and (31), and defining the Lyapunov function $V_\alpha = \alpha_1^{\top}\alpha_1/2 + \sum_{f=1}^{N}\sum_{k=2}^{n_f}\alpha_{f,k}^2/2$, we have

$$\begin{aligned} \dot{V}_\alpha \leq & \sum_{f=1}^{N}\left[\sum_{k=2}^{n_f}\left(-\chi\eta_{f,k}\alpha_{f,k}^2 + \frac{\chi}{2}\alpha_{f,k}^2 + \frac{1}{2}\Lambda_{\Delta_{f,k}}^2\right)\right. \\ & \left. -\frac{\lambda_{W_\eta}}{2}\chi\alpha_{f,1}^2 + \frac{\lambda_{W_\eta}\chi}{4}\alpha_{f,1}^2 + \frac{\Lambda_{\Delta_{f,1}}^2}{\lambda_{W_\eta}}\right] \\ \leq & -\chi\xi_{\alpha_2}V_\alpha + \xi_{\alpha_1} \end{aligned} \tag{61}$$

where $\xi_{\alpha_2} = \min_{\forall f, m=3,\ldots,n_f-1}\{5/2 + 2\eta_{f,2}^*, 7/2 + 2\eta_{f,m}^*, 5/2 + 2\eta_{f,n_f}^*, \lambda_{W_\eta}/2\}$ and $\xi_{\alpha_1} = \sum_{f=1}^{N}(\sum_{k=2}^{n_f}\Lambda_{\Delta_{f,k}}^2/2 + \Lambda_{\Delta_{f,1}}^2/\lambda_{W_\eta})$. By a procedure similar to that in (55)–(60), we obtain

$$\lim_{t\to T} V_\alpha(t) \leq e^{-\xi_{\alpha_2}M^*}V_\alpha(0) + \frac{\xi_{\alpha_1}T}{\xi_{\alpha_2}cT + (c\rho-1)}\left\{c\rho - (c\rho)^{-\frac{\xi_{\alpha_2}cT}{c\rho-1}}\right\} \triangleq \bar{V}_\alpha \tag{62}$$

with a constant $\bar{V}_\alpha$. As the design constant ρ decreases, M^* in (57) increases. Thus, using $c\rho - 1 < 0$, $\bar{V}$ and $\bar{V}_\alpha$ can be reduced by decreasing ρ. Based on $z_1^{\top}z_1/2 \leq V$, $\alpha_1^{\top}\alpha_1/2 \leq V_\alpha$, and (14), the consensus tracking error vector e_1 is practically PT stable, as follows:

$$\lim_{t\to T}\|e_1(t)\| \le \sqrt{2\bar{V}} + \sqrt{2\bar{V}_\alpha} \triangleq \bar{\epsilon} \tag{63}$$

$$\|e_1(t)\| \le \bar{\epsilon}, \quad \forall t \ge T. \tag{64}$$

Using $e_1 = \mathcal{W}(y - 1_N r)$, $\|y(t) - 1_N r(t)\| \le \bar{\epsilon}/W_m \triangleq \epsilon, \forall t \ge T$ is guaranteed with a minimum singular value W_m of $\mathcal{W}$. Additionally, the bound ϵ/W_m can be kept arbitrarily small by decreasing ρ.

Exclusion of Zeno behavior: Let us prove that the Zeno behavior does not exist under the dynamic event-triggered condition (39). We define $\bar{t}_{f,h} = t_{f,h+1} - t_{f,h}$. By assuming $\bar{t}_{f,h} \to 0$ and using $u_f(t) = \hat{u}_f(t_{f,h}), \forall t \in [t_{f,h}, t_{f,h+1})$ and the continuity of $\hat{u}_f$, we have

$$\lim_{\bar{t}_{f,h}\to 0} |\tilde{u}_f(t_{f,h} + \bar{t}_{f,h})| = \lim_{\bar{t}_{f,h}\to 0} |\hat{u}_f(t_{f,h}) - \hat{u}_f(t_{f,h} + \bar{t}_{f,h})| = 0. \tag{65}$$

Contrarily, from (39), (44), $1 \le \chi(t) \le 1/\rho$, and $\chi \sum_{k=1}^{n_f} z_{f,k}^2 \ge 0$, we have

$$|\tilde{u}_f(t_{f,h+1})| > \sqrt{\frac{\chi}{\varphi_f} e^{-(\kappa_f + \frac{1}{\varphi_f})\int_0^t \chi(w)dw} \varrho_f(0)} > 0. \tag{66}$$

This implies that (65) and (66) are contradictory. Thus, $\bar{t}_{f,h} \to 0$ is not satisfied, and there exists a constant $\hat{t}_{f,h} > 0$, such that $\bar{t}_{f,h} \ge \hat{t}_{f,h}$. Namely, the Zeno behavior does not exist. □

Remark 8. *The Lyapunov–Krasovskii function (35) is selected using the design parameter ρ of the time-varying gain function χ to deal with the time delay term $\sum_{f=1}^{N} \Lambda_f\,(\bar{x}_{f,n_f,\tau(t)})$ in (26) in the PT stability sense. Then, the adaptive function approximation term based on the normalized error $-\frac{z_{f,n_f}}{z_{f,n_f}^2 + \bar{\epsilon}_f}\hat{\bar{\lambda}}_{f,n_f}\|\Delta_{f,n_f}(\varsigma_{f,n_f})\|$ is designed in (41). Using the inequality (51), the bound $\rho\bar{\lambda}_{f,n_f}\bar{m}_{f,n_f}$ of the nonlinear compensating term can be reduced by decreasing on the design parameter ρ.*

Remark 9. *The following design guidelines are provided based on the practical PT stability result.*

(i) The practical PT convergence bound $\bar{\epsilon}$ in (63) and (64) can be reduced by decreasing $\bar{V}$ and $\bar{V}_\alpha$. From (60) and (62), reducing ρ leads to a decrease in $\bar{V}$ and $\bar{V}_\alpha$. Therefore, the practical PT convergence bound $\bar{\epsilon}$ can be adjusted using ρ.

(ii) As indicated in (60) and (62), increasing ξ_2 and ξ_{α_2} also decreases $\bar{V}$ and $\bar{V}_\alpha$. Thus, selecting appropriate design parameters to increase ξ_2 and ξ_{α_2} can reduce the practical PT convergence bound $\bar{\epsilon}$.

4. Simulation Results

To demonstrate the effectiveness of the proposed method in achieving consensus tracking within a prescribed time, even in the presence of unknown time-varying delays and uncertainties in nonlinear multiagent systems, the following multiple two-stage chemical reactors with delayed recycle streams are considered:

$$\begin{aligned} \dot{x}_{f,1} &= \tfrac{1-R_{f,a}}{V_{f,a}} x_{f,2} + g_{f,1}(x_{f,1}) + \mu_{f,1}(x_{f,1,\tau(t)}) \\ \dot{x}_{f,2} &= \tfrac{F_{f,b}}{V_{f,b}} u_f + g_{f,2}(\bar{x}_{f,2}) + \mu_{f,2}(\bar{x}_{f,2,\tau(t)}) \end{aligned} \tag{67}$$

where $f = 1, \ldots, 4$, $g_{f,1} = -(1/G_{f,a})x_{f,1} - K_{f,a}x_{f,1}$, $g_{f,2} = -(1/G_{f,b})x_{f,2} - K_{f,b}x_{f,2}$, $\mu_{f,1} = 1.5\sin(x_{f,1}(t - \tau_{f,1}))$, and $\mu_{f,2} = (R_{f,a}/V_{f,b})x_{f,1}(t - \tau_{f,1}) + (R_{f,b}/V_{f,b})x_{f,2}(t - \tau_{f,2}) + 0.5e^{-0.5x_{f,1}(t-\tau_{f,1})}\cos(x_{f,2}^2(t - \tau_{f,2}))$. Here, $V_{f,a}$ and $V_{f,b}$ are the reactor volumes, $F_{f,b}$ denotes the feed rate, $R_{f,a}$ and $R_{f,b}$ denote the recycle flow rates, $G_{f,a}$ and $G_{f,b}$ are the reactor residence times, and $K_{f,a}$ and $K_{f,b}$ denote the reaction constants. The system parameters of the proposed approach are chosen as $V_{f,a} = V_{f,b} = 0.5$, $F_{f,b} = 0.1$, $K_{1,a} = K_{1,b} = K_{3,a} = K_{3,b} = 0.3$, $K_{2,a} = K_{2,b} = K_{4,a} = K_{4,b} = 0.5$, $R_{f,a} = R_{f,b} = 0.5$, and $G_{f,a} = G_{f,b} = 2$. The time-varying delays are set to $\tau_{f,1}(t) = 0.2(4 + \sin(t))$ and $\tau_{f,2}(t) = 0.2(4 + \cos(t))$, and the leader signal r is defined as $r = \cos(1.5t)$. The network graph topology for agents is illustrated in Figure 1, where $b_3 = 2$, $a_{13} = a_{23} = a_{43} = a_{41} = 2$, and $a_{fj} = 0$ and $b_f = 0$ otherwise. The sampling time is set to 0.002 s.

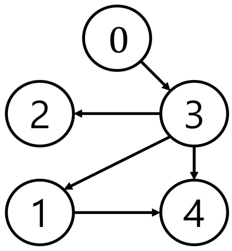


Figure 1. Directed graph.

We compare the consensus-tracking performance of the proposed event-triggered PT command-filtered backstepping approach with the existing consensus-tracking approach (CA) [27] and dynamic surface consensus-tracking approach (DSCA) [32] for nonlinear multiagent systems with unknown time delays. The design parameters for the PT gain function of the proposed approach are $\rho = 0.05$, $c = 2$, and $T = 1$ s. The PT observers and controllers of the proposed approach are parameterized with $\theta_{f,k} = 4$, $\eta_{f,1} = 1$, $\eta_{f,2} = 2$, $\beta_{f,k} = 4$, $\vartheta_{f,k} = 0.02$, $\sigma_{f,1} = 5$, $\varphi_f = 0.25$, $\kappa_f = 0.0004$, $\bar{\epsilon}_f = 0.1$, and $\varrho_f(0) = 2$, where $f = 1, \ldots, 4$ and $k = 1, 2$. Using $\theta_{f,k} = 4$, P_f is set to $P_f = \begin{bmatrix} 0.1563 & 0.125 \\ 0.125 & 1.125 \end{bmatrix}$. This study sets the system conditions and convergence times similarly to previous studies. The design parameters of the controller reported in [27] are selected as follows: $\phi_{i,1} = \phi_{i,2} = 1$, $k_{i,1} = 15$, $k_{i,2} = 5$, $r_i = 1$, $\sigma_i = 0.02$, $\lambda_{i,1,1} = \lambda_{i,2,1} = 13$, and $\lambda_{i,1,2} = \lambda_{i,2,2} = 100$ with $i = 1, \ldots, 4$. The design parameters of the controller in [32] are selected as follows: $k_{i,1} = 50$, $k_{i,2} = 10$, $\lambda_{i,1} = 0.5$, $\lambda_{i,2} = 0.03$, $\vartheta_{i,m} = 0.001$, and $\sigma_{i,m} = 0.002$ with $i = 1, \ldots, 4$ and $m = 1, 2$.

The distributed delay-independent PT consensus-tracking results are compared in Figure 2, where the initial conditions of followers are set to $x_1(0) = [2, 0]^\top$, $x_2(0) = [-1, 0]^\top$, $x_3(0) = [4, 0]^\top$, and $x_4(0) = [0, 0]^\top$. In addition, Figure 2a demonstrates that the proposed approach allows presetting the PT synchronization time $T = 1$ of the output signals y_f to the leader signal r by designing the PT T in χ. In contrast, the synchronization times of the existing approaches [27,32] are determined by adjusting the design parameters through trial and error. The control performance of the proposed consensus-tracking scheme after the synchronization time surpasses that of the existing control schemes [27,32], as depicted in Figure 2b. The proposed approach can ensure the PT convergence of the consensus-tracking errors compared to existing approaches [27,32]. The control inputs are presented in Figure 2c,d. Table 1 compares the number of signal transmissions to implement the controllers. The proposed dynamic event-triggered control inputs demand fewer transmissions than the time-triggered control inputs of the existing approaches [27,32]. The proposed dynamic event-triggered control transmissions are about 5.9%, 5.8%, 6.1%, and 6.6% of the time-triggered control [27,32]. The cumulative number of events and time intervals between two consecutive events of the proposed approach are presented in Figure 3. Figure 4 displays the proposed dynamic variables ϱ_f for event-triggering and adaptive estimates, illustrating that $\varrho_f > 0$ is ensured, as proved in Lemma 2, and that the adaptive estimates are bounded. The proposed PT consensus-tracking performance under various PT and initial state conditions is demonstrated in Figures 5 and 6, without altering other design parameters. Figure 5 reveals that the distributed PT consensus tracking can be achieved under the given PT conditions $T = 0.5$ and 1.5 s. Despite the different initial conditions for state variables, Figure 6 demonstrates that the output signals of followers are synchronized to the leader signal at the PT $T = 1$ s.

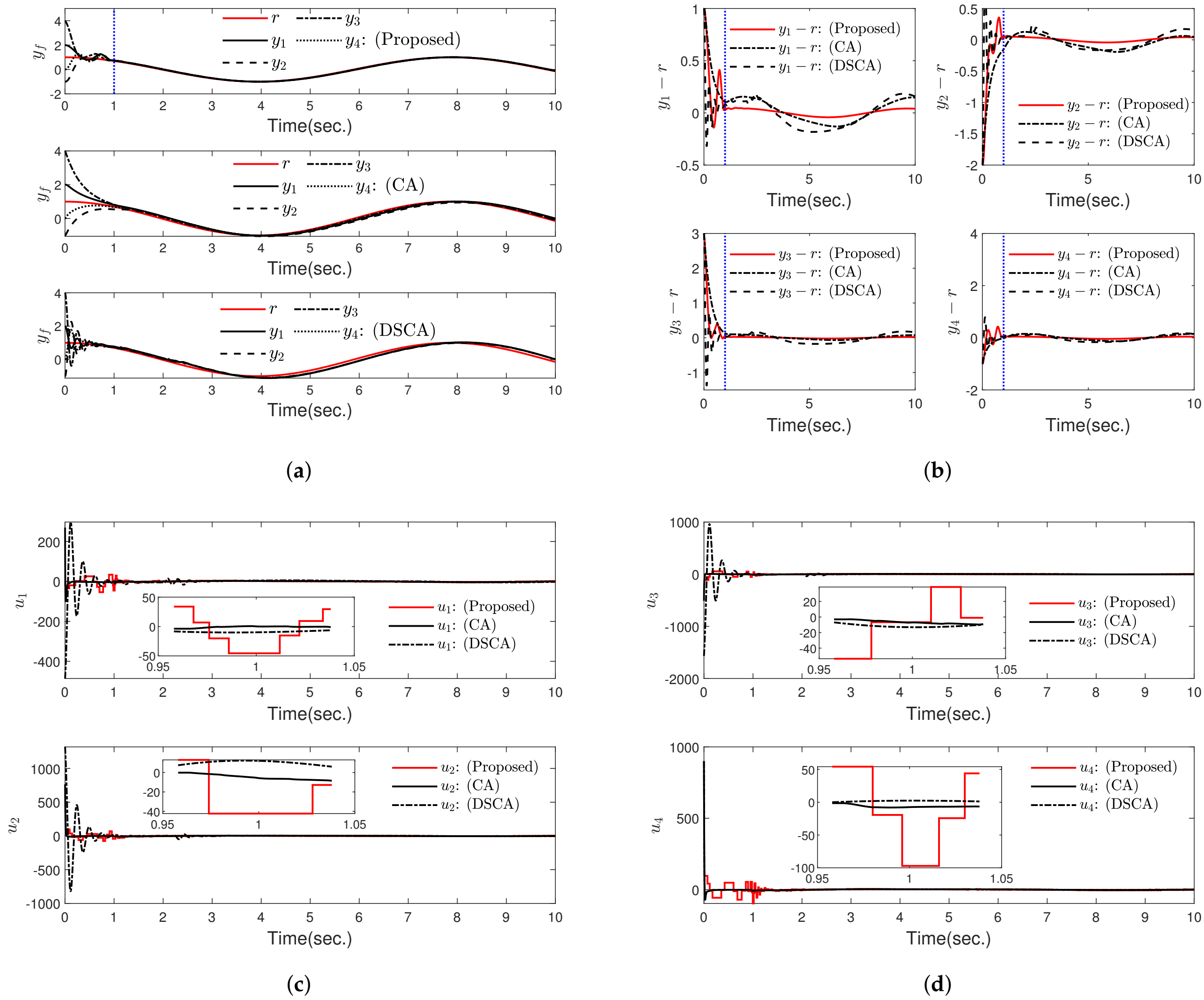


Figure 2. Comparison of PT consensus tracking results, errors, and control inputs (**a**) y_f and r (**b**) $y_f - r$ (**c**) u_1 and u_2 (**d**) u_3 and u_4.

Table 1. Comparison of the number of signal transmissions.

	Proposed	**[27,32]**
Follower 1	297	5000
Follower 2	289	5000
Follower 3	305	5000
Follower 4	331	5000

The external disturbances $d_1 = 2\sin(t)$ and $d_2 = 2\cos(t)$ are added to the first and second equations of the time-delay system (67), respectively, to demonstrate the robustness of the proposed method. Figure 7 presents the consensus-tracking results. Although the external disturbances influence the time-delay system (67), the consensus-tracking errors converge to nearly zero within the PT $T = 1$. Thus, the proposed approach is robust against external disturbances and time delays. This result highlights that the proposed delay-independent PT consensus-tracking approach ensures the practical PT convergence and robustness of the consensus-tracking errors, even when the output signals are only feedback and the time delays are unknown.

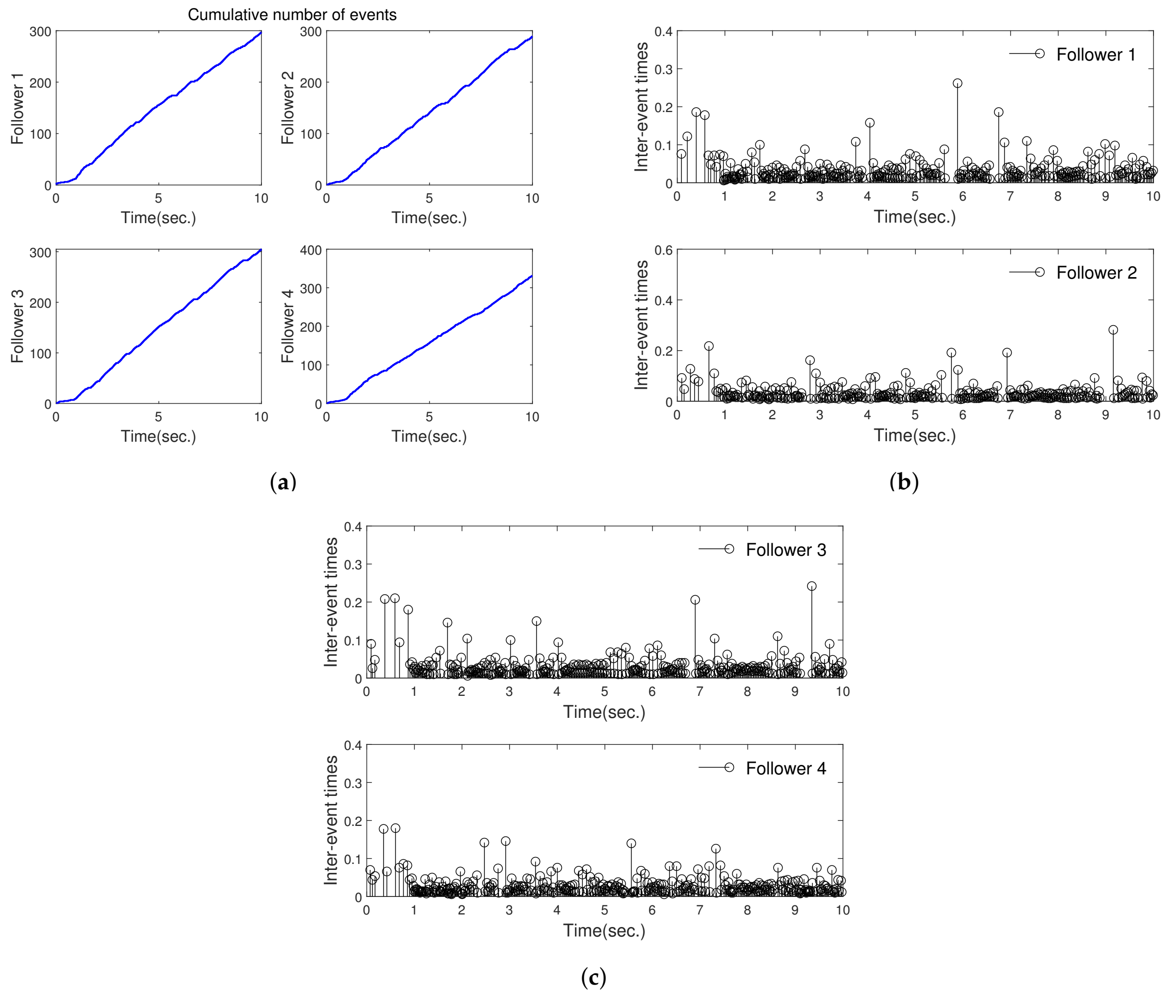


Figure 3. The cumulative number of events and inter-event times for the proposed approach: (**a**) the cumulative number of events; (**b**) inter-event times for followers 1 and 2; (**c**) inter-event times for followers 3 and 4.

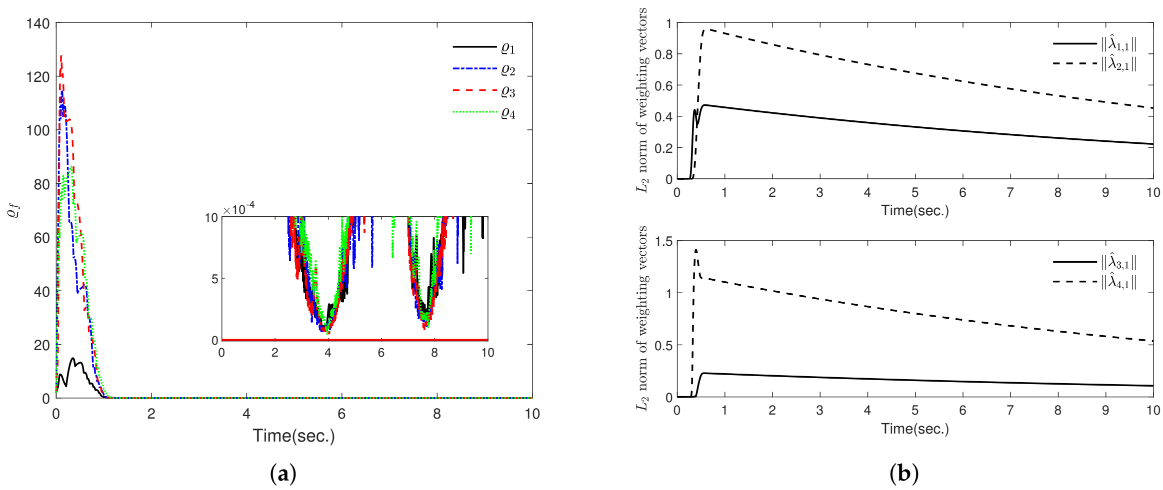
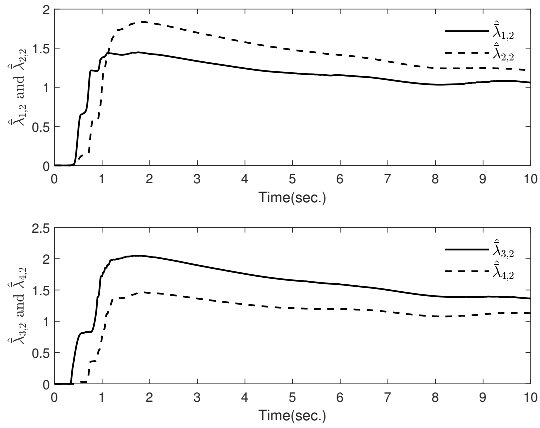
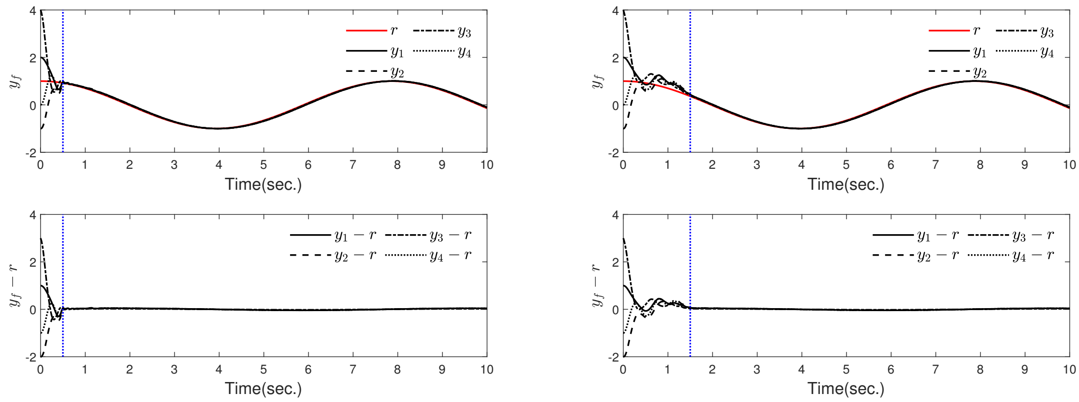


Figure 4. *Cont.*



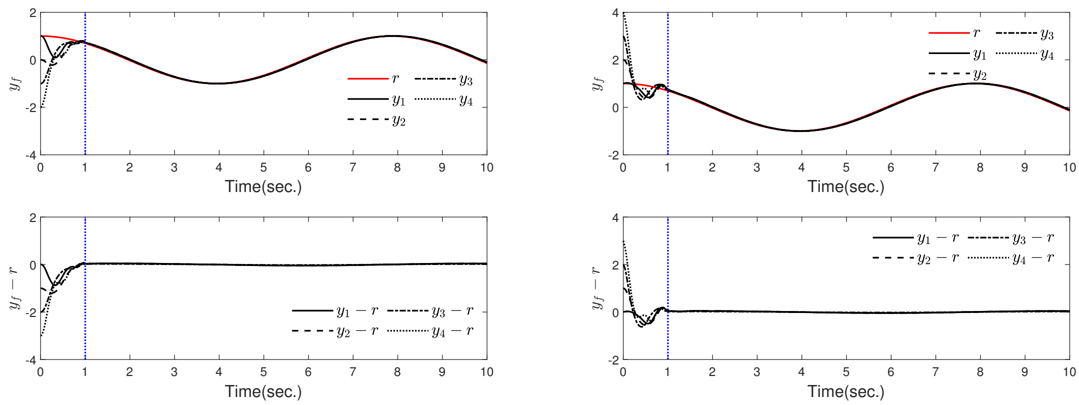
(**c**)

Figure 4. Dynamic variables and adaptive estimates for the proposed approach: (**a**) dynamic variables for event triggering; (**b**) $\|\hat{\lambda}_{f,1}\|$; (**c**) $\hat{\lambda}_{f,2}$.



(**a**) (**b**)

Figure 5. Distributed PT consensus tracking results of the proposed approach under different prescribed times: (**a**) $T = 0.5$ s; (**b**) $T = 1.5$ s.



(**a**) (**b**)

Figure 6. Distributed PT consensus tracking results of the proposed approach under different initial conditions: (**a**) $x_1(0) = [1,0]^\top$, $x_2(0) = [0,0]^\top$, $x_3(0) = [-1,0]^\top$, and $x_4(0) = [-2,0]^\top$; (**b**) $x_1(0) = [1,0]^\top$, $x_2(0) = [2,0]^\top$, $x_3(0) = [3,0]^\top$, and $x_4(0) = [4,0]^\top$.

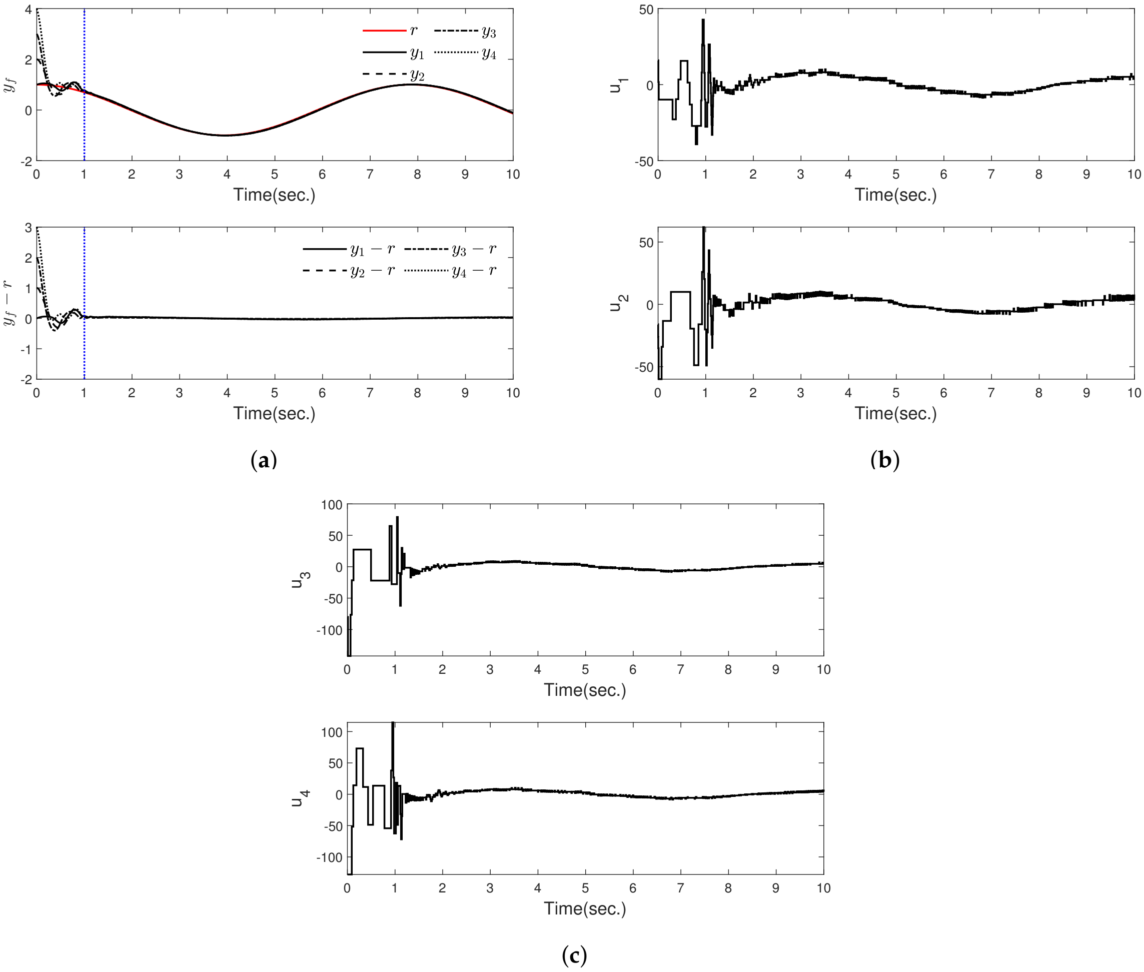


Figure 7. Robust tracking result of the proposed approach under external disturbances: (**a**) tracking results y_f and r, and errors $y_f - r$; (**b**) u_1 and u_2; (**c**) u_3 and u_4.

5. Conclusions

This article has addressed the dynamic event-triggered control problem for delay-independent PT output-feedback consensus tracking in uncertain nonlinear strict-feedback multiagent systems with unknown time-varying delays. The challenge is ensuring robust and efficient consensus tracking with uncertainties, time delays, and external disturbances while minimizing the frequency of data transmissions. This study has developed an observer-based adaptive PT command-filtered backstepping controller integrated with a dynamic event-triggered mechanism to address this problem. This approach has applied a time-varying gain function to construct distributed adaptive compensating variables and dynamic event-triggering variables. The critical innovation of the proposed method has been its ability to ensure PT convergence and robustness with reduced data transmission, even when dealing with unknown time delays. The proposed method has addressed the limitations of static event-triggering mechanisms, which do not adequately account for the dynamic nature of practical network environments. The proposed approach adapts to varying network conditions by incorporating dynamic event-triggering, improving the efficiency and reliability of the control system. This study has derived a Lyapunov–Krasovskii function that incorporates the design parameter of the time-varying gain function to analyze the PT stability of the proposed method. This analysis confirms that the PT stability is maintained, ensuring that consensus-tracking errors remain in a predetermined neighborhood around zero within a PT. The method avoids the Zeno phenomenon, which can occur with event-triggered control, enhancing its practical applicability.

The proposed method offers several critical benefits:

- Robustness to uncertainties and delays: the controller is robust against uncertainties, time delays, and external disturbances, maintaining PT stability under challenging conditions.
- Efficiency in data transmission: the method improves the control system efficiency without sacrificing performance by reducing the data transmission frequency.
- Adaptability to dynamic network conditions: the dynamic event-triggering mechanism enables the control strategy to adapt to varying network conditions, which is crucial for practical implementations in real-world systems.
- Prevention of the Zeno phenomenon: the proposed approach avoids the Zeno phenomenon, ensuring that the event-triggering mechanism operates effectively without causing problems in the system.

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