









Multiparameter Approach to Dynamic Quantum Phase Estimation [†]

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Abstract: We have applied techniques of quantum phase estimation to the dynamical tracking of the optical activity of a solution of sucrose undergoing acid hydrolysis. We adopt a multiparameter approach that makes the estimation reliable and robust against setup instabilities.

Keywords: metrology; multiparameter; phase estimation

Quantum metrology techniques have been extensively adopted to perform static measurements in many scenarios [1,2]. Phase estimation has been particularly studied [3–5], and it has been shown that the precision of such measurements can be drastically improved in multiparameter scenarios when exploiting correlations [6,7].

Due to the requirements of the measurement techniques, current quantum technologies are limited to sub-second time scales. These time scale however are quite interesting as they encompass a wide range of chemical and biological reactions, and more specifically those which can be monitored by a chirality alteration [8]. Furthermore, while intense illumination can be detrimental to samples especially of biological nature, the so called optocution [9,10], measurements performed with quantum light could be the natural go-to for biological dynamical reactions.

Here we report on a preliminary experiment on dynamical multiparameter estimation of the optical activity in the acid hydrolysis of sucrose [11,12]. When an aqueous solution of sucrose is mixed with hydrochloric acid (HCl) the latter acts as a catalyst for the hydrolysis of sucrose. This results in a solution of the sugar monomers glucose and fructose. While sucrose and glucose are dexorotatory, fructose is levorotatory and its optical power is greater than that of glucose (for Glucose, $[\alpha]_D^{20} = 52.7^\circ$, while for Fructose $[\alpha]_D^{20} = -92.3^\circ$, with a similar ratio in the near-IR wavelength range). Hence, we expect a change in the chirality of the solution from dexorotatory before the reaction to an overall levorotatory behaviour when the reaction is completed.

To monitor the dynamic of the rotatory activity we use the multiparameter strategy proposed in [13]. The experimental setup is depicted in Figure 1: a photon pair is generated via Type I parametric down conversion (SPDC) from a CW laser with 80 mW power. The two photons with orthogonal polarizations are combined on a polarising beam splitter so that a N00N state in the circular polarisation with $N = 2$ is obtained through Hong-Ou-Mandel interference [14]. The N00N state in the left and right polarisation mode reads:

$$|\psi\rangle = a_H^\dagger a_V^\dagger |0\rangle = \frac{1}{\sqrt{2}}(|2_R, 0_L\rangle + |0_R, 2_L\rangle) \tag{1}$$

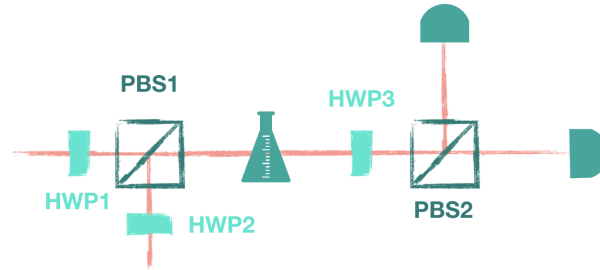


Figure 1. Experimental setup: single photons at 810 nm, generated via type-I SPDC from a β -barium-borate (BBO, 3mm length) nonlinear crystal excited via a continuous-wave pump laser are sent through a half-wave plate (HWP1 at 0° and HWP2 at 45°) before interfering on a beam splitter (PBS1) and the N00N state generated is sent on the chiral sample. A wave plate (HWP3) and a second polarizer (PBS2) project the outgoing photons onto different polarizations.

The photons are then sent on the chiral sample which will impart a phase ϕ on the R polarisation, so that the state becomes:

$$|\psi\rangle = \cos \phi (a_H^\dagger a_V^\dagger |0\rangle) - \sin \phi \left(\frac{(a_H^\dagger)^2 - (a_V^\dagger)^2}{2} |0\rangle \right) \tag{2}$$

The outgoing photons are then projected onto different polarisation to provide the measurements needed for the phase estimation. In a realistic case, the measured probabilities will also depend on the visibility of the modulations of Equation (2), which, if not correctly accounted for, would provide a bias to the phase estimation. In a dynamic scenario, where the visibility can change in time due to instabilities both of the sample and of the setup itself, monitoring the visibility thus becomes of paramount importance for reliably tracking the phase evolution of the sample.

We perform measurements over a time span of 6 hours. Each measurement (corresponding to four different settings with 2 s acquisition time) takes approximately 30 s to be performed, including delays due to the measurement process. The results are reported in Figure 2. The upper panel shows the phase in function of the number of subsequent measurements, i.e., in function of time. As expected the behaviour of the optical activity of the sample changes from dextrorotatory to levorotatory after the reaction is completed. Note that the visibility is indeed varying during the reaction, hence a single-parameter measurement would have led to a biased estimation over time.

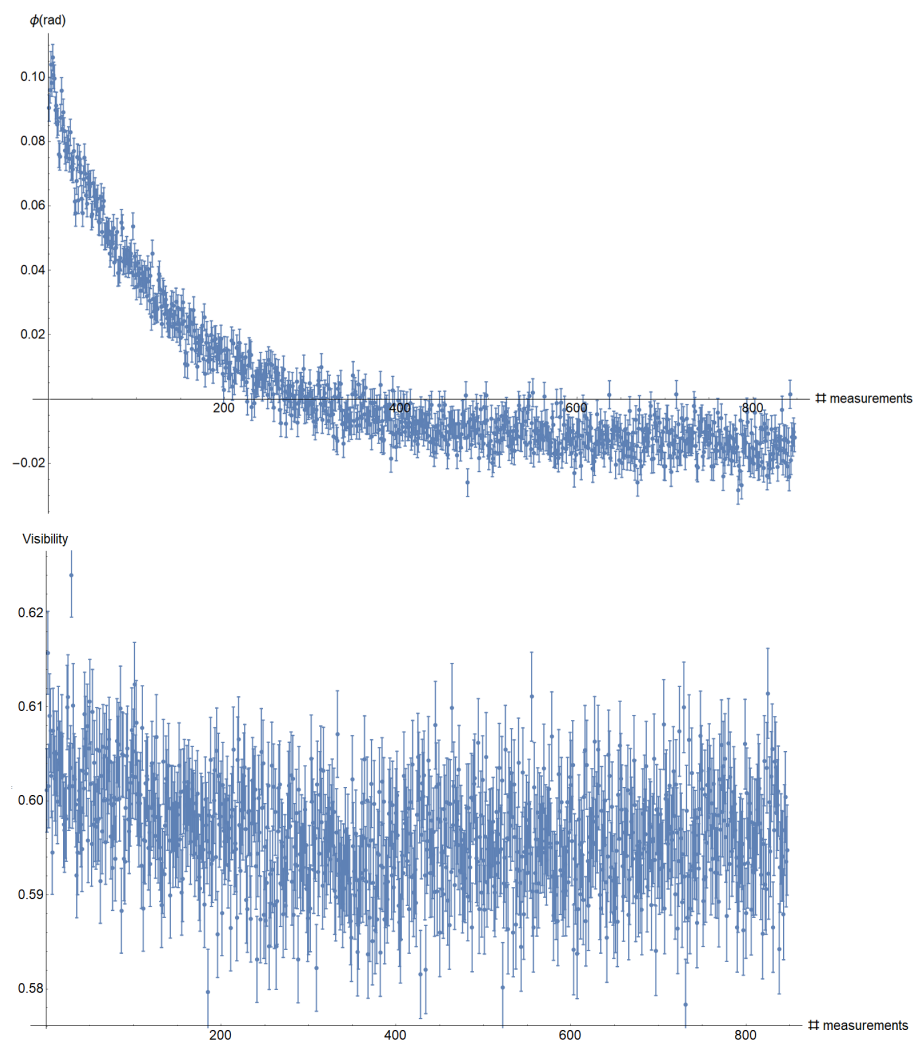


Figure 2. Experimental results: (**upper panel**) behaviour of the optical activity of the sucrose solution in time. (**lower panel**) visibility behaviour in time. The error bars are obtained with standard deviation.

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References

1. Giovannetti, V.; Lloyd, S.; Maccone, L. Quantum-Enhanced Measurements: Beating the Standard Quantum Limit. *Science* **2004**, *306*, 1330–1336, doi:10.1126/science.1104149.
2. Giovannetti, V.; Lloyd, S.; Maccone, L. Quantum Metrology. *Phys. Rev. Lett.* **2006**, *96*, 010401, doi:10.1103/PhysRevLett.96.010401.
3. Paris, M.G.A. Quantum estimation for quantum technology. *Int. J. Quantum Inf.* **2009**, *7*, 125–137, doi:10.1142/S0219749909004839.
4. Giovannetti, V.; Lloyd, S.; Maccone, L. Advances in quantum metrology. *Nat. Photonics* **2011**, *5*, 222–229, doi:10.1038/nphoton.2011.35.
5. Demkowicz-Dobrzanski, R.; Jarzyna, M.; Kolodynski, J. Quantum limits in optical interferometry. *Prog. Opt.* **2015**, *60*, 345–435, doi:10.1016/bs.po.2015.02.003.
6. Vidrighin, M.D.; Donati, G.; Genoni, M.G.; Jin, X.M.; Kolthammer, W.S.; Kim, M.S.; Datta, A.; Barbieri, M.; Walmsley, I.A. Joint estimation of phase and phase diffusion for quantum metrology. *Nat. Commun.* **2014**, *5*, doi:10.1038/ncomms4532.

7. Roccia, E.; Gianani, I.; Mancino, L.; Sbroscia, M.; Somma, F.; Genoni, M.G.; Barbieri, M. Entangling measurements for multiparameter estimation with two qubits. *Quantum Sci. Technol.* **2018**, *3*, doi:10.1088/2058-9565/aa9212.
8. Tischler, N.; Krenn, M.; Fickler, R.; Vidal, X.; Zeilinger, A.; Molina-Terriza, G. Quantum optical rotatory dispersion. *Sci Adv.* **2016**, *2*, e1601306.
9. Ashkin, A.; Dziedzic, J.; Yamane, T. Optical trapping and manipulation of single cells using infrared laser beams. *Nature* **1987**, *330*, 769–771.
10. Da silva, N.S.; Portich, W. Effect of GaAlAs laser irradiation on enzyme activity. *Photomed. Laser Surg.* **2010**, *28*, 3, doi:10.1089/pho.2008.2410.
11. Schoebel, T.; Labuza, S.R.T.T.P. Reaction at Limited Water Concentration 1. Sucrose Hydrolysis. *J. Food Sci.* **1969**, *34*, 4, doi:10.1111/j.1365-2621.1969.tb10355.x.
12. Tombari, E.; Salvetti, G.; Ferrari, C.; Johari, G.P. Kinetics and Thermodynamics of Sucrose Hydrolysis from Real-Time Enthalpy and Heat Capacity Measurements. *J. Phys. Chem. B* **2007**, *111*, 496–501, doi:10.1021/jp067061p.
13. Roccia, E.; Cimini, V.; Sbroscia, M.; Gianani, I.; Ruggiero, L.; Mancino, L.; Genoni, M.G.; Ricci, M.A.; Barbieri, M. Multiparameter approach to quantum phase estimation with limited visibility. *Optica* **2018**, *5*, 1171–1176.
14. Hong, C.K.; Ou, Z.Y.; Mandel, L. Measurement of subpicosecond time intervals between two photons by interference. *Phys. Rev. Lett.* **1987**, *59*, 2044–2046.



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