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A Study on the Mechanical Characteristics of String Planes of Badminton Racquets by Nonlinear Finite Element Analysis †

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† Presented at the 13th Conference of the International Sports Engineering Association, Online, 22–26 June 2020.

Published: 15 June 2020

Abstract: In this study, the finite element analysis of the string planes of badminton racquets was investigated to evaluate the effect of the mechanical characteristics of polymer strings. The nonlinear mechanical characteristics of commercially available polymer strings were obtained by the uniaxial loading tests experimentally. The effects of the strain rate on the mechanical characteristics of the polymer strings were also investigated to consider the dynamic effect on the numerical simulation. The numerical simulation code used to analyze the string planes of the badminton racquets was developed originally. A nonlinear elastic model (Yeoh model) was applied to the mechanical characteristics of the polymer string. Simulated results were compared with the experimental results. The effect of the mechanical characteristics of the polymer strings and the geometrical shape of the badminton racquets on the out-of-plane stiffnesses were investigated.

Keywords: badminton; finite element analysis; string plane

1. Introduction

In this paper, the effect of the mechanical characteristics of the polymer strings and the geometrical shape of the badminton racquets on the out-of-plane stiffnesses of the string planes of badminton racquets were investigated. The performance of a badminton racquet is determined by the combination of the racquet's frame, shaft, grip and string plane [1–4]. The out-of-plane stiffness of the string planes affects the speed and the control of the impacted shuttle [5]. The mechanical characteristics of the strings and the geometrical shape of the badminton racquets are important parameters for determining the out-of-plane stiffness of the string planes [6]. Thus, the effect of the mechanical characteristics of the strings and the geometrical shape of the badminton racquets on the out-of-plane stiffness of the string planes is important for designing a badminton racquet with better performance. In this study, the out-of-plane stiffness of the string planes of the badminton racquets was calculated by the nonlinear finite element analysis program which was developed originally. The formula for the numerical simulation code was derived from the shape of the string planes and the mechanical characteristics of the polymer strings. The hyperelastic model was applied to the nonlinear stress-strain relationships of the nylon strings given by uniaxial tensile tests. The effect of the mechanical characteristics of the polymer strings and the geometrical shape of the badminton racquets on the out-of-plane stiffnesses of the string planes were investigated by comparing the out-of-plane stiffness of the analysis models.

2. Materials and Methods

In this study, the 3-dimensional Cartesian coordinate system (x_1,x_2,x_3) was applied to the string plane. The x_1-x_2 plane was defined as the in-plane direction and the x_3-direction was defined as the out-of-plane direction. x_1 and x_2 directions were defined as the orthogonal directions of the plane surface (Figure 1) of the string plane. In preparation for the computational simulation, nodes for the finite element analysis were prepared at all intersections of the main and cross strings. Additional nodes were prepared at grommets and their displacements were constrained. Beam elements were applied between the nodes to reproduce the badminton strings. A deformation behavior of a cross string is shown in Figure 2. The dotted lines and the solid lines are the strings before and after deformation. Black circles are the intersections of the vertical (main) and horizontal (cross) strings. $L^{(k,l)}$ is the length between the *k*th and *l*th nodes, and $\bar{L}^{(k,l)}$ is the length after deformation. T_1 and T_2 are the tension of the horizontal and vertical strings before deformation, and $\bar{T}_1(\bar{u})$ and $\bar{T}_2(\bar{u})$ are the tension after deformation. $\bar{u}$ is the displacement of the strings in the axial direction. The deformation behavior of the main string is shown in Figure 2 by replacing 1 with 2 in subscripts. The formulation of the simulation code in this study was based on simulation code to calculate the out-of-plane stiffness of the string bed of a tennis racquet [6]. In this study, the mechanical characteristics of strings and nonlinearity of geometrical shape of string planes were considered with the formulation of the simulation code. The force at the *k*th and *l*th nodes in the x_3-direction was given by the following equations:

$$f_3^{(k)} = \alpha\bar{T}(\bar{u})sin\theta = \alpha\frac{\bar{T}(\bar{u})}{\bar{L}^{(k,l)}}\left(u_3^{(k)} - u_3^{(l)}\right), f_3^{(l)} = -\alpha\bar{T}sin\theta = \alpha\frac{\bar{T}(\bar{u})}{\bar{L}^{(k,l)}}\left(u_3^{(l)} - u_3^{(k)}\right). \quad (1)$$

Here, k and l indicate the node numbers respectively. $u_3^{(k)}$ is the displacement in the x_3-direction of the *k*th node. θ is the angle between the strings before deformation and the strings after deformation. α is the adjusting coefficient of the out-of-plane stiffness. The coefficient α is provided because there were strings that contributed to the out-of-plane stiffness and strings that did not. In this study, the coefficient α is assumed to be 0.65 [6].

For the in-plane stiffness of the string planes, the Coulomb friction model was applied to the simulation code. The strings were assumed to be slipped if the in-plane force was greater than the maximum static friction force. The formulation of the friction force was based on the formulation to calculate out-of-plane stiffness of the string bed of a tennis racquet [6]. $\bar{T}(\bar{u})$ in Equation (1) corresponds to $\bar{T}_1(\bar{u})$ for horizontal strings and $\bar{T}_2(\bar{u})$ for vertical strings. When the string plane was deformed, the tension of the strings was increased by the elongation of the strings. Thus, the tension $\bar{T}_1(\bar{u})$ and $\bar{T}_2(\bar{u})$ of the strings was calculated from the elongation of the strings in the axial direction. Another 3-dimensional Cartesian coordinate system (x_1', x_2', x_3') was applied to represent the deformation of the strings. The x_1'-direction was defined as the axial direction of the horizontal and vertical strings, and the x_2' and x_3'-direction were defined as the lateral direction to the axial direction. The displacement $\bar{u}$ of the strings in the axial direction (x_1'-direction) was calculated by the following equation:

$$\bar{u} = \sqrt{(L^{(k,l)} + (u_1^{(k)} - u_1^{(l)}))^2 + (u_2^{(k)} - u_2^{(l)})^2 + (u_3^{(k)} - u_3^{(l)})^2} - L^{(k,l)}. \quad (2)$$

The stretch of the strings λ in the axial direction was calculated by the following equation:

$$\lambda = 1 + \frac{\bar{u}}{L^{(k,l)}}. \quad (3)$$

An incompressible hyperelasticity was applied to represent the mechanical characteristics of the strings. Strain energy function $W(\bar{\boldsymbol{C}})$ was applied the Yeoh Model [7]. The strain energy function $W(\bar{\boldsymbol{C}})$ was given by the following equation:

$$W(\bar{\boldsymbol{C}}) = C_{10}(\bar{I}_1 - 3) + \frac{C_{20}}{2}(\bar{I}_1 - 3)^2 + \frac{C_{30}}{3}(\bar{I}_1 - 3)^3. \quad (5)$$

Here, $\bar{I}_1$ is the modified first invariant of the modified right Cauchy–Green deformation tensor $\bar{\boldsymbol{C}}$, and C_{10}, C_{20} and C_{30} are the material parameters. The second Piola–Kirchhoff stress tensor $\boldsymbol{S}$ is given by the partial differentiation of the strain energy function $W(\bar{\boldsymbol{C}})$ with respect to the right Cauchy–Green deformation tensor $\boldsymbol{C}$ as follows:

$$\boldsymbol{S} = 2\frac{\partial W(\bar{\boldsymbol{C}})}{\partial \boldsymbol{C}} + p\boldsymbol{C}^{-1}. \quad (6)$$

Here, p is the hydrostatic pressure calculated from the boundary condition. The second Piola–Kirchhoff stress tensor $\boldsymbol{S}$ was converted to the Cauchy stress tensor $\boldsymbol{T}$ by the deformation gradient tensor $\boldsymbol{F}$. The Cauchy stress tensor $\boldsymbol{T}$ was given by the following equation:

$$\boldsymbol{T} = \boldsymbol{F}\boldsymbol{S}\boldsymbol{F}^{T} = 2\frac{\partial W(\bar{\boldsymbol{C}})}{\partial \bar{I}_1}\begin{pmatrix} \lambda^2 - \frac{1}{\lambda} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (7)$$

In the case of the nonlinear mechanical characteristics, the tension of the string element $\bar{T}$ was calculated as follows:

$$\bar{T} = T_{11}\mathrm{A} = 2\{C_{10} + C_{20}(\bar{I}_1 - 3) + C_{30}(\bar{I}_1 - 3)^2\}\left(\lambda^2 - \frac{1}{\lambda}\right)\mathrm{A}. \quad (8)$$

Here, A is the cross section area of polymer string. T_{11} is (11) a component of the Cauchy stress tensor $\boldsymbol{T}$. In the case of the linear mechanical characteristics, tension $\bar{T}$ was supposed as a constant value.

$$\bar{T} = 106.76\ \ \mathrm{N}. \quad (9)$$

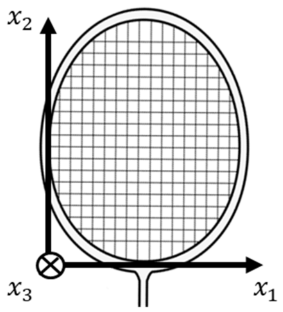


Figure 1. 3-Dimentional coordination system.

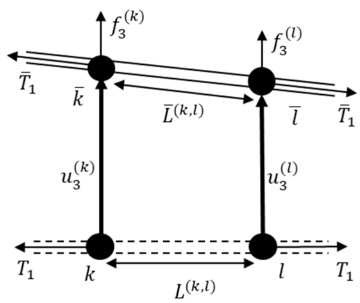


Figure 2. Deformation behavior of the string.

3. Results

3.1. Loading Test of Nylon Strings

For loading test of nylon strings, three specimens for the uniaxial tensile test were prepared. The total length of test part was 30 mm and the diameter of the string was 0.7 mm. The specimens are shown in Figure 3. Uniaxial tensile tests of the strings were conducted using a uniaxial loading machine (Autograph AG-20kNXplus, Shimadzu Corporation Kyoto, Kyoto 604-8511, Japan) shown in Figure 4. The uniaxial tensile tests are shown in Figure 5. The strain rates of the loading test were 1.0 %/s, 0.1 %/s and 0.01 %/s, respectively. The stress-strain relationships obtained by the uniaxial tensile tests are shown in Figure 6. From Figure 6, the strain rate effects on the stress-strain relationships of the nylon strings were not confirmed.

Equation (8) can be transform to the following form:

$$\frac{T_{11}}{2(\lambda^2-\frac{1}{\lambda})} = C_{10} + C_{20}(\bar{I}_1 - 3) + C_{30}(\bar{I}_1 - 3)^2. \quad (10)$$

The relationships between $(\bar{I}_1 - 3)$ and $\frac{T_{11}}{2(\lambda^2-\frac{1}{\lambda})}$ of the tensile loading test results are plotted in Figure 7. The material parameters C_{10}, C_{20} and C_{30} were identified as $C_{10} = 1.45 \times 10^2$, $C_{20} = 5.99 \times 10^3$ and $C_{30} = 2.53 \times 10^4$ by the least squares method as independent quantities from Figure 7.

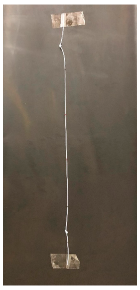


Figure 3. One of the nylon specimens.

Figure 4. Uniaxial loading testing machine.

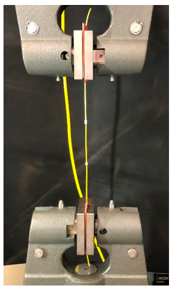


Figure 5. Uniaxial loading test.

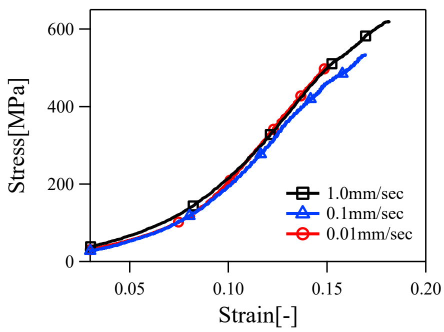


Figure 6. Stress-strain relationships of the nylon specimens.

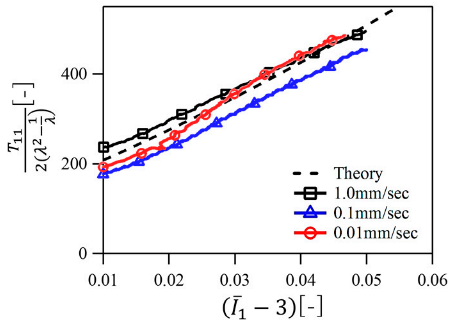


Figure 7. Relationship between $(\bar{I}_1 - 3)$ and $\frac{T_{11}}{2(\lambda^2-\frac{1}{\lambda})}$.

3.2. Analysis of the String Plane Models of Three Badminton Racquets

Three kinds of string planes of badminton racquets were prepared for numerical simulation. The string planes of the badminton racquets (Racquet A, Racquet B and Racquet C) were introduced to analysis models. The distances between the strings in Racquet A and Racquet B were almost the same. The distances between the strings of the lower parts of the string plane in Racquet C were wider than the upper parts. As for the shape of the string planes, Racquet A was the largest and Racquet C was the smallest in the three racquets. The three racquets are shown in Figure 8. Line elements were applied to the analysis models as the polymer strings. In addition, the line elements between the grommets were applied to the analysis models as the frames. The diameter and initial tension of the strings were set to be 0.7 mm and 106.76 N, respectively. The material parameters of the nylon strings identified in Section 3.1 were applied to the simulation code. The out-of-plane load of 10 N was applied to all the nodes in the string planes and the out-of-plane stiffness was recorded. The result is shown in Figure 9. The stiffness distribution of the string planes of Racquet A and Racquet B decreased from the edge of the string planes toward the center. The stiffness of the lower part of the string plane in Racquet C was lower than the upper parts.

(**a**) Racquet A (**b**) Racquet B (**c**) Racquet C

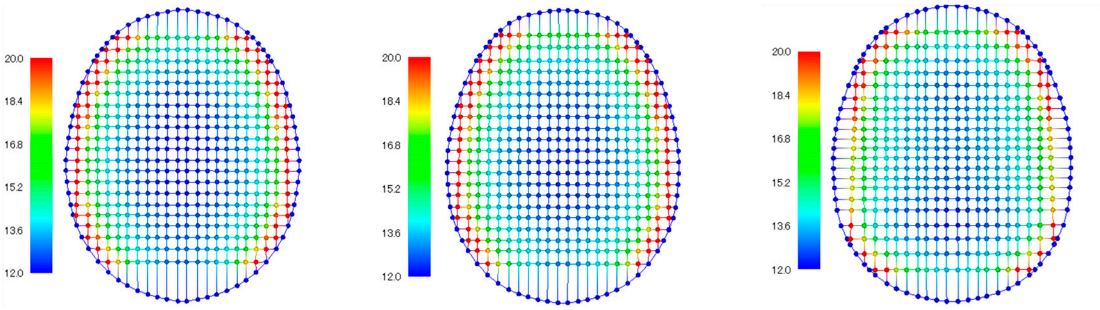
Figure 8. Three badminton racquets made by different companies, respectively

3.3. Effect of the Nonlinear Property of Polymer Strings

In order to investigate the effect of the nonlinear property of polymer strings on the out-of-plane stiffness of the string planes, the analysis model of the string planes of Racquet A was applied to the numerical calculation. The out-of-plane stiffness of the analysis model was calculated under two different conditions.

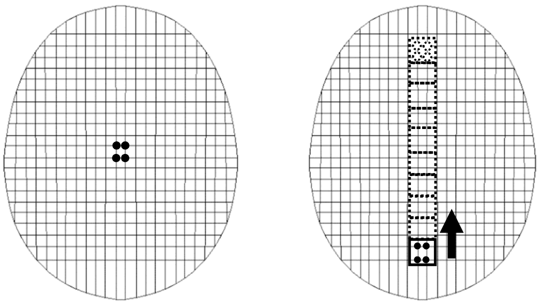
For simulation case 1, an out-of-plane load from 10 N to 50 N was applied to the four nodes shown in Figure 9a. The out-of-plane stiffness of the string planes of the badminton racquets was calculated considering the nonlinear properties of the polymer strings. For the nonlinear case, the hyperelastic model shown in Equation (6) was applied to the mechanical characteristics of the polymer strings. To investigate the effect of the nonlinear mechanical properties of the polymer strings, the calculated results were compared with the results of the linear case. The condition of the linear case was that the tension of the strings was constant at 106.76 N. The displacement of the model with the out-of-plane load of 50 N is shown in Figure 10. The relationships between the load and displacement of the most deformed node in the analysis models are shown in Figure 11. The out-of-plane stiffness calculated using the nonlinear mechanical characteristics of the polymer string was larger than using the linear condition.

For simulation case 2, the out-of-plane load of 50 N was applied to four neighboring nodes from the bottom to the top of the racquet shown in Figure 9b. The relationships between the stiffness calculated with and without the nonlinear effect and the position of x_2 is shown in Figure 12. The stiffness of the analysis model with the nonlinear effect was higher than the stiffness without the nonlinear effect.



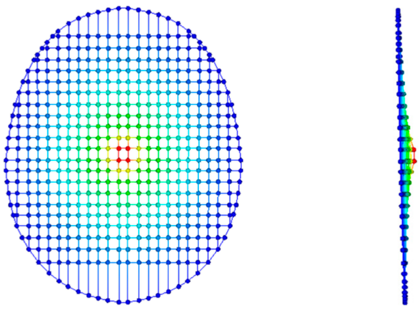
(**a**) Racquet A (**b**) Racquet B (**c**) Racquet C

Figure 9. Out-of-plane stiffness of the string planes of the three racquets.



(**a**) Case 1 (**b**) Case 2

Figure 10. The positions applied to the load in the analysis model of the badminton racquet.



(**a**) Front view (**b**) Side view

Figure 11. Deformations of the string plane when applying 50N to each node.

4. Discussion

From the simulated results of the string plane models of the three badminton racquets in Figure 12, the effect of the geometrical shape of the badminton racquets was shown. The stiffness distribution was determined by the shape of the string planes with the same distances between strings. In addition, the wide distances between the strings made the stiffness of a part of the string planes lower.

From the analysis results of the string plane model under two different conditions in Figure 13, the effect of the mechanical characteristics of the polymer strings was performed. The nonlinear effect of the polymer strings made the calculated sweet spot of the string planes smaller. The effect of the material characteristics of the nylon strings on the stiffness of the string planes was small for a weak shot and large for a strong shot.

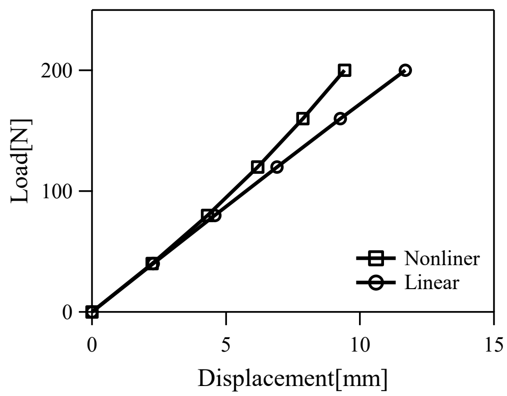


Figure 12. Relationships between load and displacement in the center of the racquet.

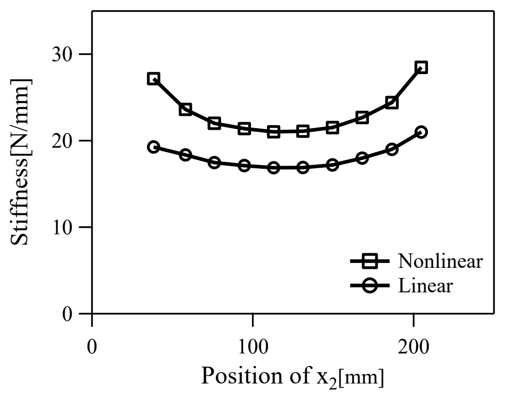


Figure 13. Relationships between stiffness and position of x_2 of the string plane.

5. Conclusions

In this study, the finite element analysis considering the nonlinear mechanical characteristics of polymer strings was performed to predict the out-of-plane stiffness of the string planes of badminton racquets. The material parameters which were obtained by uniaxial tensile tests of nylon strings were applied to the numerical simulation. The effect of the geometrical shape of the badminton racquets and the mechanical characteristics of the polymer strings on the out-of-plane stiffness of the string planes was performed by comparing the stiffness of the three badminton racquets. As a result, design optimization of the string plane of the badminton racquets was applicable using the simulation code which is proposed in this paper.

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