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Semantic Numeration Systems as Information Tools †

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Abstract: We propose the concept of a semantic numeration system (SNS) as a certain class of context-based numeration methods. The development of the SNS concept required the introduction of fundamentally new concepts such as a cardinal abstract entity, a cardinal semantic operator, a cardinal abstract object and a numeration space. The focus is on the key elements of semantic numeration systems—cardinal semantic operators. A classification of semantic numeration systems and suggestions for their possible application are given.

Keywords: cardinal abstract entity; cardinal semantic operator; cardinal abstract object; semantic numeration system

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1. Introduction

A modern world is characterized by a constantly increasing volume of processed and stored information, as well as a growing variety and complexity of its processing algorithms. Information processing plays a key role in such areas as control systems and artificial intelligence, cryptography and biometrics, GIS, and image processing. Along with the development of new algorithms for information processing, a search is underway for effective ways to represent numerical data (in particular, numeration systems).

A numeration system is a symbolic method of representing numbers using signs. Despite a significant variety of works in the field of positional numeration systems [1–4], we can say that the overwhelming majority of them bear the "stamp of the game in the bases". Even works on abstract numeration systems [3,4] do not go beyond the traditionally linear (string) representation of numbers.

The semantics of the traditional place-value representation can be expressed as follows: *n* units of some abstract entity *i* are given the meaning (~>) of a unit of another abstract entity *j*: $\mathrm{n} \cdot 1_\mathrm{i} = \mathrm{n_i} \sim> 1_\mathrm{j}$. Assume that there are such abstract entities *i* whose *n* units n_i are given the meaning of both the unit of the abstract entity *j* (1_j) and the unit of the abstract entity *k* (1_k) simultaneously: $\mathrm{n_i} \sim> (1_\mathrm{j}, 1_\mathrm{k})$. Consider another situation: to form a unit of an abstract entity *k*, exactly *n* units of an abstract entity *I*, and *m* units of an abstract entity *j*, are required: $(\mathrm{n_i}, \mathrm{m_j}) \sim> 1_\mathrm{k}$.

What kinds of semantic numeration systems are fundamentally possible?

2. Preliminaries

The concepts of the abstract entity and an abstract object are actively discussed in modern science [5,6]. Considering the ambiguity and variety of interpretations of these concepts, definitions adapted to the semantics of numeration systems are given below.

Definition 1. *An abstract entity (Æ) is an entity of arbitrary nature provided with an identifier name that allows it to be distinguished from other entities.*

Definition 2. *A Multeity is the manifestation of something essentially uniform in various kinds and forms, as well as the quality or condition of being multiple or consisting of many parts.*

Since we will further deal with the transformation of meanings, we define the corresponding specific type of multeity as semantic.

Definition 3. *A Semantic multeity is an abstract space with no more than a countable set of abstract entities, semantically united by the unity of the goal of the description (context).*

Definition 4. *A Cardinal Semantic Multeity (CSM) is a semantic multeity, each element of which is equipped with a cardinal characteristic—the multiplicity of a given abstract entity represented in multeity.*

From a set-theoretic point of view, a cardinal semantic multeity is a multiset, the carrier of which is contextually conditioned. The elements of cardinal semantic multeity are called cardinal abstract entities.

Definition 5. *A Cardinal Abstract Entity (CÆ) is an abstract entity with a cardinal characteristic* $CÆ_i = (i; N_i)$*, where i is the name of the cardinal abstract entity,* $N_i = Card(CÆ_i) = \#(1_i, 1_i, ..., 1_i)$*,* $N_i \in N$*.*

3. Cardinal Semantic Operator

In essence, the action of the cardinal semantic operator is to give a certain number of units n_i of the cardinal abstract entity $CÆ_i$ the meaning of unit 1_j of a cardinal abstract entity $CÆ_j$, $(I \neq j)$: $n_i \sim> 1_j$. Basically, other options are also possible, for example, when the n_i of the abstract entity $CÆ_i$ is assigned not one but, simultaneously, several different semantic units of respectively different CÆs: $n_i \sim> (1_j, ..., 1_k)$.

Definition 6. *A Cardinal Semantic Operator is a multivalued mapping of the cardinal semantic multeity on itself, which associates a set of entity operands from the multeity with a set of entity images from the same multeity, transforming their cardinals using the operations defined by the operator signature:* $Signt(CSO) = (K, Form, \mid n >_w, \mid r >_v)$*, where K is a kind of operator, Form is a type of operator,* $\mid n>$ *is a radix vector, and* $\mid r>$ *is a conversion vector. The pair (W, V) is a valence of the Cardinal Semantic Operator.*

Let us define the main forms of cardinal semantic operators of the (↑#) kind (radix-multiplicity).

Definition 7. *L-operator (Line-operator):* $(\uparrow\#, L, n_i, r_{ij})$*—a cardinal semantic operator of valency (W, V) = (1, 1), which assigns (gives the meaning of)* r_{ij} *units of the transformant* q_j*, added to the cardinal* N_j *of the abstract entity* $CÆ_j$*, to each* n_i *of the cardinal abstract entity* $CÆ_i$*.*

Definition 8. *D-operator (Distribution operator):* $(\uparrow\#, D, n_i, (r_{ij}, \ldots, r_{ik}))$*—a cardinal semantic operator of valency (W, V) = (1, v), which assigns the following units to each* n_i *of the cardinal abstract entity* $CÆ_i$ *v transformants:* r_{ij} *units of j-transformants* q_j *for the cardinal abstract entity* $CÆ_j, \ldots$*, and* r_{ik} *units of k-transformants* q_k *for the cardinal abstract entity* $CÆ_k$*.*

Definition 9. *F-operator (Fusion operator):* $(\uparrow\#, F, (n_i, \ldots, n_j), r_{.k})$*—a cardinal semantic operator of valency (W, V) = (w, 1), which assigns* $r_{.k}$ *units of the transformant* q_k *to each w-tuple* $(n_i, \ldots, n_j)$ *of CÆ-operands for the cardinal abstract entity* $CÆ_k$*.*

Definition 10. *M-operator (Multi-operator):* $(\uparrow\#, M, (n_i, \ldots, n_j), (r_{.k}, \ldots, r_{.l}))$*—a cardinal semantic operator of valency (W, V) = (w, v), which assigns v-tuple conversion coefficients* $(r_{.k}, \ldots, r_{.l})$ *of transformants to the w-tuple* $(n_i, \ldots, n_j)$ *of CÆ-operands:* $r_{.k}$ *units of k-transformant* q_k *for the cardinal abstract entity* $CÆ_k, \ldots$*, and* $r_{.l}$ *units of l-transformants* q_l *for the cardinal abstract entity* $CÆ_l$*.*

These four cardinal semantic operators form the operator basis of any semantic numeration system.

4. Cardinal Abstract Object and Cardinal Semantic Transformation

To represent complex multistage semantic transformations, mono-operator transformations, as usual, are not enough. Let us introduce the concept of a numeration space, the elements of which are numeration methods. By a method of numeration, we mean a contextually conditioned method of transforming semantic units from a cardinal semantic multeity using cardinal semantic operators.

Definition 11. *A Cardinal Abstract Object (CAO) is a collection of cardinal abstract entities connected in a certain topology by cardinal semantic operators.*

The signature of CAO_I is Signt(CAO) = (I; CSM; CSO; STop), where I is a set of names denoting (naming) the methods of numeration, CSM is the cardinal semantic multeity, CSO is a set of cardinal semantic operators, and STop are the possible topologies of the semantic connectivity of cardinal abstract entities by cardinal semantic operators.

Definition 12. *A Numeration Space (NS) is an abstract space, the elements of which are cardinal abstract objects.*

A certain CAO_I implements a specific method of numeration I in a numeration space. Let us agree to call CSO "allowed" if the values of the cardinals of all its operands ensure the execution of the given operator.

Definition 13. *Cardinal Semantic Transformation (CST) consists in executing, for a given CAO_I, all "allowed" cardinal semantic operators. A CST step will mean a single execution of all "allowed" cardinal semantic operators for a given CAO_I.*

Definition 14. *After an arbitrary step τ of the cardinal semantic transformation the multiset of cardinals of all CÆs from CAO_I is called a multicardinal of CAO_I of the step τ, and denoted by $<A_I(\tau)>$ ($<A_I(\tau)> = [N_i(\tau), N_j(\tau), \ldots, N_k(\tau)]$).*

Definition 15. *The holistic structural-cardinal representation of CAO_I after the τ-th step of the cardinal semantic transformation is called the I-multinumber of the step τ, and is denoted by $A_I(\tau)$.*

We assume that the multicardinal determines precisely the *meaning* of the CAO after the τ-th step of the cardinal semantic transformation, and the multinumber is its *sense*.

5. Semantic Numeration Systems

Informally, the semantic numeration system can be defined as a collection of homogeneous numeration methods.

Definition 16. *By the Semantic Numeration System in the numeration space NS, we mean its subspace SNSφ with some given properties determined by the classification features. Here, φ is the name identifier of the semantic numeration system, due to a set of classification features.*

We propose the following classification of Semantic Numeration Systems:

(1) by influence on the operand cardinal: transforming or preserving;
(2) by type of uncertainty: deterministic, stochastic and fuzzy;
(3) by kind of transformation: radix-multiplicity, radix-excess value; radix-excess fact; arbitrary function;
(4) by kinds of number systems used in the numeration system: natural, integer or rational numbers;
(5) by controllability: autonomous or controlled;

(6) by variability of the operator parameters (radices and conversion rates): homogeneous (the same for all operators) or heterogeneous (different for different operators);
(7) by topology of operators' connectivity: linear (with not only L-operators), tree-like, lattice, cyclic, amorphous or of a special form. Regular structures can be either isotropic or anisotropic, and the latter can be homogeneous or heterogeneous.

Within the framework of the classification, most of the generally accepted "numeration systems", for example, binary or decimal, are particular methods of numeration of the transforming, homogeneous, deterministic, radix-multiplicity, natural, linear topology semantic numeration system.

As an example of a possible SNS application, consider the method of black–white image compression [7]. The main idea of the proposed method is to present the digital relief (matrix) of an image as a superposition of final multicardinal numbers in a homogeneous lattice SNS with radix-2. By the inverse cardinal semantic transformation of the final multicardinal number, we obtain the initial multicardinal number with a more compact set of cardinal abstract entities $\{CÆ_i, \ldots, CÆ_j\}$ intended for storage or transmission. Restoration of the image (decompression) consists of performing a procedure involving representation of the initial multicardinal numbers in the same SNS with their subsequent superposition. To compress halftone images, it is necessary to use a homogeneous lattice SNS with a radix-n equal to the number of gray gradations.

6. Conclusions

The theory of Semantic Numeration Systems is at the initial stage of its development. Nevertheless, even now, it can be assumed that SNS will be in demand in many areas related to information processing, such as:

- Cryptoprotection—the creation of fundamentally new cryptosystems to protect information of increased cryptographic strength;
- Databases—compact representation, efficient storage and fast data transfer (exchange);
- Geoinformation systems (GIS)—compact storage of digital terrain maps and their efficient transmission through communication channels;
- Biometrics—effective identification of a person by their fingerprints, the iris of the eye and photographs;
- Radar, sonar, and radio navigation—high-speed data processing.

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