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Optimizing the Hub-and-Spoke Network with Drone-Based Traveling Salesman Problem

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Abstract: The hub-and-spoke network (HSN) design generally assumes direct transportation between a spoke node and its assigned hub, while the spoke's demand may be far less than a truckload. Therefore, the total number of trucks on the network increases unnecessarily. We form a drone-based traveling salesman problem (TSP-D) for the cluster of spokes assigned to a hub. A truck starts from the hub, visiting each spoke node of the hub in turn and finally returning to the hub. We propose a three-stage decomposition model to solve the HSN with TSPD (HSNTSP-D). The corresponding three-stage decomposition algorithm is developed, including cooperation among variable neighborhood search (VNA) heuristics and nearest neighbor algorithm (NNA), and then the spoke-to-hub assignment algorithm through the reassignment strategy (RA) method. The performance of the three-stage decomposition algorithm is tested and compared on standard datasets (CAB, AP, and TR). The numerical analysis of the scenarios shows that whether it is trunk hub-level transportation or drone spoke-level transportation, it integrates resources to form a scale effect, which can reduce transport devices significantly, as well as decreasing the investment and operating costs.

Keywords: hub and spoke network; traveling salesman problem; drone routing problem; variable neighborhood search; nearest neighbor algorithm

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1. Introduction

The hub and spoke network (HSN) has applications in many fields, such as postal systems, railway express [1], ultra-cold COVID-19 vaccine distribution [2], and intermodal transportation [3]. For example, in less-than-truckload (LTL) industries, an HSN can combine demands from many origins, consolidate them at hubs, and then transport them to their destinations. In parcel delivery, drones have great potential, particularly in rural areas. In these areas, drones operate faster than trucks. As drones are limited in their payload, a combination of trucks and drones can be beneficial in reducing last-mile delivery costs. In recent years, the study of parcel delivery using the collaboration of trucks and drones has attracted the interest of many researchers. Karak and Abdelghany [1] studied a truck–drone cooperative pickup and delivery services problem with a simplified drone endurance model and without precedence constraints, demonstrating the effective cost reduction achieved with the cooperative model. This paper investigates an HSN with a drone-based traveling salesman problem (HSNTSP-D), a new variant of the location routing problem with drones (LRP-D). We considered using drones in an HSN for first- and last-mile pickup and delivery tasks.

Most studies on HSN designs use a star structure network, and every direct link requires a dedicated vehicle [4]. Such HSNs generally minimize the cost of the hub-to-hub, spoke-to-hub, and hub-to-spoke transportation flows. Nonetheless, the routing decision is not considered. Kartal et al. [5] proposed that when the spokes do not have enough demand to use the trucks fully, HSN will significantly increase the number of vehicles. In addition, Karimi-Mamaghan et al. [6] consider that this direct link between each spoke node and hub may cause traffic congestion. In this case, we should consider the location and routing decisions jointly. If each vehicle can access multiple spokes, much fewer vehicles are needed, significantly reducing the total investment cost of the vehicle. In addition, the number of vehicles entering and leaving the hub is reduced, avoiding traffic congestion in the hubs.

Therefore, this paper proposes a model for truck–drone cooperation. Trucks are responsible for hub-to-hub transportation, taking advantage of economies of scale. Drones consolidate packages from and distribute packages to spokes, giving full play to high efficiency and low-cost benefits. The literature on coordinating the logistics operations using a truck–drone combination focused predominantly on extending classical routing problems, a variant of traveling salesman problems with drones (TSP-D), and a generalization of vehicle routing problems (VRP) to include drones (VRP-D). Gómez-Lagos et al. [7] propose the well-known pickup and delivery problem (PDP) related to the problem defined here. In the PDP, two types of customers are defined, one part of them asks for a pickup operation, and the other asks for a delivery operation. Ham [8] extended the parallel drone scheduling TSP (PDSTSP) with both pickup and delivery operations to be performed by the drone.

Truck–drone pickup and delivery problem is NP-hard in nature. Previous studies have focused on exact and heuristic methods to solve these NP-hard problems. For example, Ziaei and Jabbarzadeh [9] studied green location-routing planning of multi-modal transportation systems. Sun et al. [10] studied green road–rail intermodal routing problem with improved pickup and delivery services. These studies demonstrated that exact solution methods and heuristic algorithms are feasible. Mixed-integer programs (MIP) and dynamic programming exhaustively explore the search space to find the optimal solution. However, they can generally solve small-scale instances. Due to the combinatorial nature of the truck–drone pickup and delivery problem, the performance of the conventional approaches can be significantly affected by the size of the problem. Therefore, the current challenge is to develop a scalable and efficient method for truck–drone pickup and delivery. This paper proposed a new variant of the location routing problem with Drones (LRP-D), considering using a truck fleet consisting of trucks and drones for last-mile pickup and delivery tasks. We seek to construct an HSN design with TSPP-D (HSNTSP-D), which consists of determining the number of hubs to constitute a hub-level network and assigning each spoke node to a single hub to form a spoke-level route. For a hub-level network, it is the interconnection structure between p hubs. While for the spoke-level route, the hub and all the spokes assigned to it form a directed circle route, completed by drones using TSP.

The study contributes to the literature in the following three aspects. First, a variant of the HSN design problem is introduced that encompasses hub location and routing decisions. We combine the advantages of both drones and trucks in parcel delivery operations. Second, we propose a three-stage decomposition model to solve this problem. Third, we developed a three-stage decomposition algorithm to solve practical large-scale instances. The rest of this paper consists of the following parts. Section 2 presents a systematic literature review summarizing the research background and published results. Section 3 offers a three-stage decomposition model for HSNTSP-D. In Section 4, we give details of the proposed three-stage decomposition algorithm. First, we proposed a VNS for the p-hub location and spoke node allocation problem. The nearest neighbor algorithm for the VRP is presented in the second stage. In the third stage, the node allocation reassignment strategy is proposed. Section 5 analyzes computational results. We conclude and describe the direction of future research in Section 6.

2. Literature Review

The HSN design problem is often referred to as hub location and allocation problems [11]. Kartal, Krishnamoorthy, and Ernst [5] divided the HSN design problems into p-hub median, hub covering, and hub center problems. When HSN design problems consider route optimization, it becomes hub location and routing problem (HLRP), which identifies hubs and assigns non-hubs to hubs to form vehicle routes. Nagy and Salhi [12] first raised the many-to-many HLRP. In their study, the demands are collected, aggregated at the hub center, and delivered to the demand nodes. Wasner and Zäpfel [13] develop a generalized hub location and vehicle routing model to optimal design of depot and hub transportation networks for parcel service providers. Çetiner et al. [14] described an iterative two-stage heuristics for a combined multiple allocation HLRP. The first stage is the implementation of hub location and distribution of spokes. Next stage, the transportation between a hub and its spokes are formatted as VRPs. They considered simultaneous pickup and delivery at the spokes. For many-to-many HLRP, Camargo et al. [15] proposed a new formulation to solve. They also provided pickup and delivery services on the same vehicle route, and each hub assigned only one vehicle. Rodríguez-Martín et al. [16] presented a hub network with direct connections between hubs, and each hub has a circular spoke-level route. Lopes et al. [17] addressed a variant of the MMHLRP wherein they proposed an integer programming model and three heuristics algorithms to solve it. Kartal et al. [18] studied to minimize transferring cost between hubs and routing cost in the local route, developed mixed integer programming models, and proposed a multi-start simulated annealing heuristic and ant colony system algorithm to solve it. Karimi [19] proposed Tabu search-based heuristics for hub location and VRP with simultaneous pickup and delivery (VRPPD), in which the nodes assigned to a hub form a tour. Fontes and Goncalves [20] studied deep sea service through the interconnection between hubs and short sea service by a circular local route. XiaoYang et al. [21] studied the capacitated single allocation HLRP where collection and delivery served on distinct routes. Danach et al. [22] located a predetermined number of hubs and allocated the spokes to the hubs. Wu et al. [23] investigated a multi-allocation hub location routing problem (MAHLRP) to design an intra-city express service system in which mail and parcels are exchanged among the branch offices of the service provider via local tours and hubs.

Table 1 summarizes the pioneering studies on HSNs. The distinct features compared with basic HSNs include tasks of spokes to hubs, the routing constraints of consolidation and distribution, and time and capacity constraints. The HSNs are generally formulated as MILPs.

Table 1. Features, models, and algorithms of HSNs.

Study	Features	Model	Algorithm
[12]	(1) Many-to-many HLRP; (2) Different pickup and delivery routes; (3) Unfixed hub number.	-	H
[13]	Many-to-many HLRP.	MINP	H
[14]	(1) Many-to-many HLRP; (2) Simultaneous pickup and delivery; (3) Multiple allocations of spokes to hubs.	-	H
[15]	(1) Many-to-many HLRP; (2) Single allocation.	MILP	MH
[16]	(1) Unfixed hub number; (2) Simultaneous pickup and delivery; (3) One vehicle per route; (4) Most q nodes per cycle.	MINP	MH
[19]	(1) Capacitated hub covering LRP; (2) Spokes assigned to a hub form a tour.	MILP	TS
[20]	(1) Many-to-many LRP; (2) Multiple allocations of spokes to hubs.	MILP	VNS
[21]	Capacitated single allocation HLRP.	MILP	EA
[22]	Single allocation p-hubs.	MILP	MH + H
[23]	Multi-allocation HLRP.	MILP	ALNS

Note: ALNS = adaptive large neighborhood search; EA = evolutionary algorithm; H = heuristic; HLPP = hub location routing problem; LPP = location routing problem; MH = math heuristics; MILP = mixed-integer linear program; MINP = mixed-integer non-linear program; TS = Tabu search; VNS = variable neighborhood search.

This work studies the HSNTSP-D and focuses on collaborating the trucks and drones. For the truck–drone routing problem, Murray and Chu [24] proposed the deployment of trucks and drones together for package delivery in logistics distribution, naming the problem the Flying Sidekick TSP (FSTSP). Agatz et al. [25] proposed a similar problem called the TSP with Drones (TSP-D), which was solved using a truck-first-drone-second heuristic and a dynamic programming algorithm. A MILP formulation for the problem is detailed and the math heuristic proposed is a Random Restart Local Search (RRLS). Ha et al. [26] minimized the total operational cost of a drone-truck delivery system. They offered a MILP formulation for the problem and presented a GRASP heuristic. Moshref-Javadi et al. [27] studied the multi-trip TSP-D (MTSP-D), which assumes that a truck can stop at a discrete set of customer locations and launch one or multiple UAVs to serve other customers. They developed a hybrid algorithm based on Simulated Annealing (SA) and Tabu Search (TS). Dell'Amico et al. [28] approached the Parallel Drone Scheduling TSP (PDSTSP), proposing a MILP and several math heuristics. Wang et al. [29] studied the VRP with Drones (VRP-D) from a worst-case point of view. Murray and Raj [30] introduced the multiple-flying sidekicks TSP (mFSTSP), considering a delivery truck operating in coordination with a fleet of drones. The drones are launched from the truck to deliver a single package, then return to the truck where it can be loaded again.

Karak and Abdelghany [1] present a mathematical formulation and solution methodology for the hybrid vehicle–drone routing problem (HVDRP) for pickup and delivery services. A novel solution methodology is developed, which extends the classic Clarke and Wright algorithm (HCWH) to solve the HVDRP. Luo et al. [31] proposed the Multi-Visit TSP with Multiple Drones (MTSP-MD), which considers energy consumption during hovering when a drone waits for the truck to arrive. Gu et al. [32] proposed a general VRP with Drones and Multiple Visits (VRPD-MV) where the drones can serve multiple customers per trip. They offered a VNS algorithm. Montaña et al. [33] introduce a new variant of the Location Routing Problem with Drones (LRP-D), in which multiple trucks and drones are deployed to make deliveries. The main idea is to combine the advantages of both drones and trucks in parcel delivery operations. Arishi et al. [34] introduced the Parking Location and TSP with Homogeneous Drones (PLTSPHD). This paper proposes a two-phase machine learning (ML) approach for the PLTSPHD, which minimizes the total operational cost of the last-mile problem. The proposed ML approach for PLTSPHD consists of clustering and routing phases. Luo et al. [35] investigate a novel and pioneering problem, the one-to-one pickup and delivery problem with multi-trucks and multi-visit drones (OPDP-MTMV). This problem involves multiple trucks equipped with a single drone for pickup and delivery services. The drones are capable of carrying several packages per trip.

Table 2 presents the studies on drone-based location and routing problems. Using drones to extend the TSP and VRP studies is popular. Because the fundamental problems of TSP and VRP are difficult to solve, coordinating with drones makes them more challenging. Although they can be formatted as MILPs, various basic, meta- and math-heuristics are developed for solving medium- and large-scale instances.

Truck–drone collaboration for parcel delivery can open several research paths due to the evolution of e-commerce and vehicle technology in logistics. The research in this paper adopts the HSN structure, and the transportation route between hub nodes is called hub-level transportation, and the transportation route connecting non-hub nodes is called spoke-level transportation. In hub-level transportation, the OD cargo flow of non-hub nodes within the radiation area is gathered, and the use of truck transportation at the hub will form significant economies of scale and reduce transportation costs. Hub-level transportation generally has a long distance, and due to the limited distance of drone transportation, hub-level lines can only be transported by truck. However, spoke-level transportation usually has a short distance, which is suitable for using drones to complete the last mile of transportation and can fully use the advantages of drone transportation. Montaña, Malagon-Alvarado, Miranda, Arboleda, L.Solano-Charris, and A.Vega-Mejía [33] combine the benefits of drones and trucks in parcel delivery operations. However, they studied only one central depot, not considering the transportation of the hub trunk line or the need for pickup. Wu, Qureshi, and Yamada [23] studied the issue of express services in the city and considered the need for simultaneous pickup and delivery, but the cost of spoke-level transportation only depends on the distance between nodes and does not consider the actual flow of goods flowing between spoke-level route nodes. Since the spoke-level transportation in this paper is limited by the carrying capacity of the drone, it is necessary to select the appropriate drone type according to the actual cargo flow of each section of the spoke-level route.

Table 2. Studies on drone-based location and routing problems.

Study	Problem Features	Model	Algorithm
[24]	TSP with drones	MILP	H
[25]	TSP with drones	MILP	H + DP
[26]	TSP with drones	MILP	GRASP
[27]	TSP with drones	-	SA + TS
[28]	TSP with drones	MILP	LS
[29]	VRP with drones		RO
[30]	TSP with drones	MILP	H
[1]	Truck–drone routing	MILP	H
[31]	TSP with drones	-	-
[32]	VRP with drones	-	ILS + VNS
[33]	LRP with drones	MILP	-
[34]	Location and TSP with drones	MILP	ML
[35]	VRPPD and drones	MILP	ILS

Note: DP = dynamic programming; GRASP = greedy randomized adaptive search procedures; H = heuristics algorithm; ILS = iterated local search; LRP = location routing problem; LS = local search; MILP = mixed-integer linear program; ML = machine learning; RO = robust optimization; SA = simulated annealing; TS = tabu search; VNS = variable neighborhood search; VRP = vehicle routing problem; VRPPD = VRP with pickup and delivery.

3. Problem Description

This research analyzes a new variant of the location routing problem with drones (LRP-D). We considered using drones in an HSN for first- and last-mile pickup and delivery tasks. First, we describe the research status of HSN network; second, we describe the research status of the truck–drone routing problem. Finally, according to the existing research on the collaboration of trucks and drones, we designed an HSN with a drone-based traveling salesman problem. By adopting the HSNTSP-D structure, both in hub-level transportation and spoke-level transportation, resources are integrated to form a scale effect, which can reduce transport devices significantly reduced, as well as the investment and operating costs.

The HSN consists of four parts: hubs, spokes, transportation among hubs, and transportation between spokes and hubs. The origin-to-destination (OD) requirements for cargo transportation are defined between nodes. An HSN aims to determine the hub's location among all nodes and the allocation of spokes according to OD requirements between nodes.

The decomposition strategy is prevalent in HSNs, which includes the decomposition of the model, dividing a model into multiple models for the solution. Some literature decomposes the problem into the hub and distribution network construction stages. Based on the above analysis, this paper constructs a three-stage decomposition model. The model of the first stage solves the location problem of hub facilities and obtains the hub selection and node allocation scheme. In the second stage, according to the node allocation relationship, starting from the hub, then visiting each node assigned to the same hub in turn, and finally return to the hub to complete truck route planning. The third stage model is to adjust the distribution scheme of spokes, optimize truck routes, and make the total cost of the network minimal.

Recently, the application of drones in delivery logistics has emerged prominently. Companies are competing to achieve cost-efficient and faster last-mile delivery operations. Integrating emerging technology, such as unmanned aerial trucks or drones, in the last-mile network design can overcome these challenges and provide a competitive advantage.

Parcel delivery with drones has the potential to revolutionize classic parcel delivery since drone delivery can reduce costs and increase the speed of last-mile logistics. In a truck-and-drone coordinated HSN, a truck carries a set of drones and customer orders to a hub located. Then the drones can transport multiple low-weight orders simultaneously within the flight range. In recent research and practice, a new drone that can make three delivery drop-offs on a single flight has proposed as an emerging technology.

This study investigates a novel and pioneering last-mile delivery problem named HSNTSP-D. Each customer order requires a package to be transported from the origin node to the destination node. The origin-to-destination pickup and delivery problem with multi-truck and multi-visit drone deploys multiple truck–drone pairs to serve customers. Both the trucks and the drones can perform pickup and delivery operations. A drone picks up packages from a truck and then drops them into the spokes. A drone picks up packages at the spokes and unloads them onto a truck for consolidation.

The transportation between hubs can improve the load ratio of trucks through hub-level transportation. When the traffic volume of a spoke is less than a drone load, the direct transportation mode between spokes and hubs will not fully utilize the drones' loading capacity and depends on too many drones, as shown in Figure 1. Solid lines represent the trunk transportation between hubs, and dashed lines represent the drone transportation between spoke nodes. We can use drones for all the spokes assigned to the same hub, as shown in Figure 2. This HSN design with a drone has the dual advantage of HSN planning and traveling salesman problem (TSP). It affects economies of scale in the hub- and spoke-level transportation, improves resource utilization, and reduces resource waste.

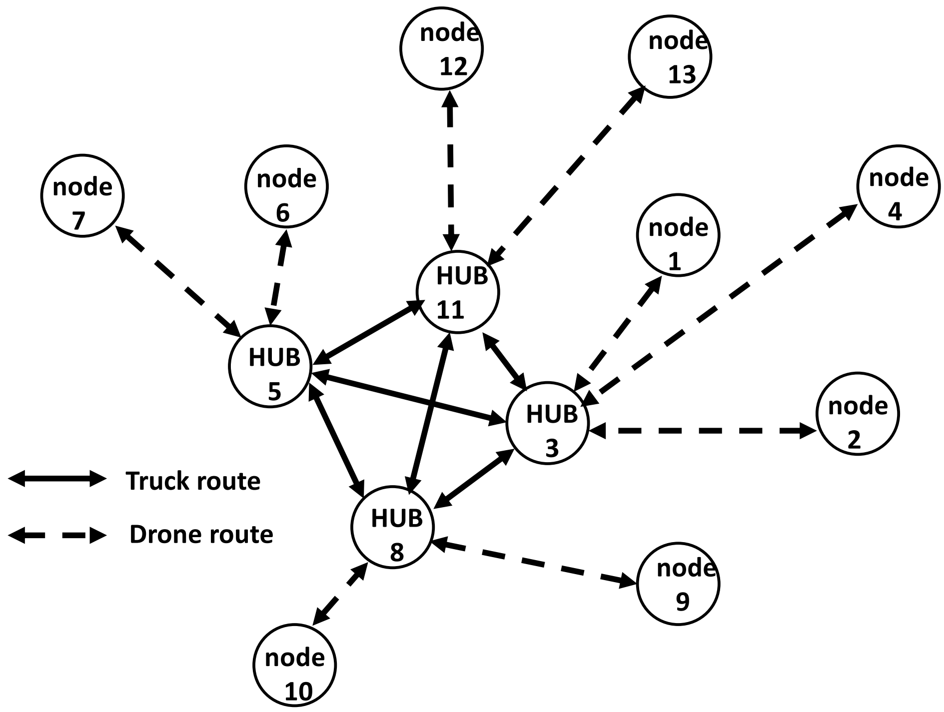

Figure 1. A star structure HSN with drones.

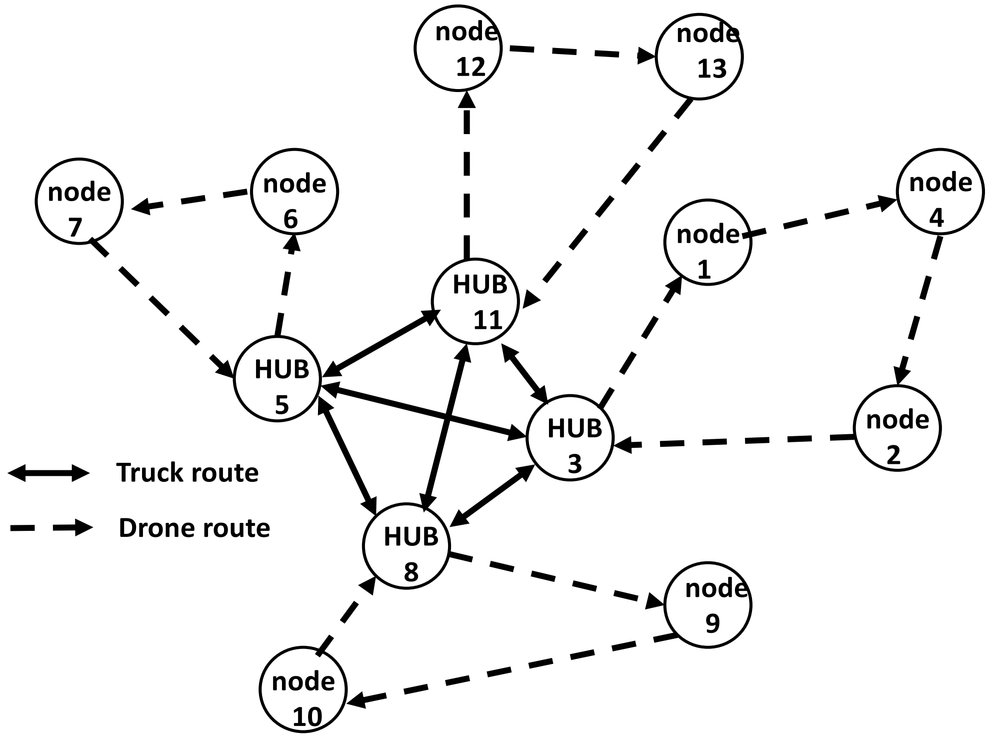

Figure 2. An HSN with a drone-based traveling salesman problem (HSNTSP-D).

4. Formulation

4.1. Baseline Model

The baseline model [M1] formulates the p-hub median location problem. [M1] aims to choose p nodes from N node as hubs, and the rest nodes are spokes, then allocate each spoke to exactly one hub by minimizing the total HSN cost. The notations and definitions are summarized in Table 3.

Table 3. Description of symbols.

Set	Description
N	Set of nodes, $N = \{1, 2, \ldots, \lvert N \rvert\}$, indexed by i, k, l, j
Data	
C_{ik}	The distance transferred from node i to node k. It is assumed that $C_{ii} = 0$ and that the distances satisfy the triangle inequality
W_{ij}	The cargo flow transferred from node i to node j
α	The discount factor of transport cost efficiency over inter-hub links
O_i	Total flow originating from node i, $O_i = \sum_j W_{ij}$
D_i	Total flow destining to node i, $D_i = \sum_j W_{ji}$
p	The number of hubs
Variable	
x_{ik}	$x_{ik} = 1$ if node i is allocated to hub k; zero otherwise
y_{ikl}	It is defined as flow originating from node $i \in N$ and traversed from hub $k \in N$ to hub $l \in N$.

The HSN is formulated as [M1].

$$[\text{M1}]\min f = \sum_{ik} (O_i + D_i) C_{ik} x_{ik} + \alpha \sum_{ikl} C_{kl} y_{ikl} \tag{1}$$

Subject to:

$$\sum_{k \in N} x_{kk} = p \tag{2}$$

$$\sum_{k \in N} x_{ik} = 1, \forall i \in N \tag{3}$$

$$\sum_{i \in N} x_{ik} \geq 2 \cdot x_{kk}, \forall k \in N \tag{4}$$

$$x_{ik} \leq x_{kk}, \forall i, k \in N \tag{5}$$

$$\sum_{l \in N} y_{ikl} - \sum_{l \in N} y_{ilk} = O_i x_{ik} - \sum_{j \in N} W_{ij} x_{jk}, \forall i, k \in N \tag{6}$$

$$\sum_{l \neq k \in N} y_{ikl} \leq O_i x_{ik}, \forall i, k \in N \tag{7}$$

$$x_{ik} \in \{0, 1\},\ \forall i, k \in N \tag{8}$$

$$y_{ikl} \geq 0,\ \forall i, k, l \in N \tag{9}$$

In (1), the objective of [M1] is to minimize total transportation costs, including collection and distribution costs between hubs and spokes and transshipment costs between hubs. In Constraint (2), p hubs are chosen from the nodes. Constraint (3) satisfies the single allocation constraint. In (4), ensure that the hub is assigned at least one spoke node in the model. In (5), each node is assigned to the hub rather than to other nodes. The flows among hubs and spokes are calculated in (6) and (7). The binary variable and integer are defined in Constraints (8) and (9).

4.2. Extended Model

4.2.1. Vehicle Routing Optimization Model

In this stage, hubs and spokes distribution schemes are obtained according to the first model, and the spoke-level transportation route is optimized for the spokes assigned to the same hub. The drones start from the hub, pass through each node served by the hub in turn, which allows simultaneous pickup and delivery on those nodes, and finally return to the hub so that the HSN spoke-level transportation could also integrate cargo transportation, replace the direct transportation mode in the previous stage, and reduce the number of drones. Determining the hub set first and then optimizing the drone route can reduce the solution scale of the problem. The certificate is as follows:

Lemma 1. *Determine the hub set first and then optimize the drone route, which can reduce the complexity of network optimization.*

Prove as follows, the hub set is not determined, and the selection range of the hub is all nodes. The optional quantity is N, i,k,h comes from N. Hence, the solution quantity of variable y_{ikh} is $|N|^3$. When the hub set H is determined, and then k,h comes from H. Simultaneously, the number of solutions for variable y_{ikh} becomes $|N| \times \overline{H}^2$. Because $\overline{H} \ll |N|$, $|N| \times \overline{H}^2 \ll |N|^3$, the solution scale of the model is significantly reduced.

According to the first model solution results, when $x_{ii} = 1$, add node i to the hub set, then obtain hub set H. When $x_{ih} = 1$, represents hub h serving node i. The set and decision variables are summarized in Table 4.

Table 4. Description of symbols.

Set	Description
H	Set of hubs, $H = \{h_1, h_2, \ldots, h_p\}$, indexed by h
Variable	
v_{hij}	$v_{hij} = 1$ if the arc (i, j) is served by hub $h \in H$; zero otherwise.
f_{ikl}	It is defined as the flow originating from node $i \in N$ and traversed spoke − level route arc (k, l).

The HSNTSP-D is formulated as [M2].

$$[\text{M2}] \min f^{HR} = \sum_{ikl} C_{kl} f_{ikl} + \alpha \sum_{ikl} C_{kl} y_{ikl} \tag{10}$$

Subject to:

$$\sum_{j \neq i \in N} v_{hij} \geq x_{ih}, \forall i \in N, h \in H \tag{11}$$

$$\sum_{h \in H} \sum_{j \neq i \in N} v_{hij} = 1, \forall i \in N \tag{12}$$

$$\sum_{j \neq i \in N} v_{hij} - \sum_{j \neq i \in N} v_{hji} = 0, \forall i \in N, h \in H \tag{13}$$

$$\sum_{l \neq i \in N} (y_{iil} + f_{iil}) = O_i, \forall i \in N \tag{14}$$

$$\sum_{k \in N} (y_{ikl} + f_{ikl}) = \sum_{k \in N} (y_{ilk} + f_{ilk}) + W_{il}, \forall i, l \neq i \in N \tag{15}$$

$$f_{ikl} \leq O_i \sum_{h \in H} v_{hkl},\ \forall i, k, l \neq k \in N \tag{16}$$

$$\sum_{l \in N} (y_{ikl} + y_{ilk}) = O_i \cdot x_{kk}, \forall i \in N, k \in H \tag{17}$$

$$v_{hij} \in \{0, 1\},\ \forall i, j \in N, h \in H \tag{18}$$

$$f_{ikl} \geq 0,\ \forall i, k, l \in N \tag{19}$$

To minimize the HSNTSP-D network's total cost expressed as an objective function (10), we formulate the total cost as a sum of the transportation costs of the local routes within a closed-loop TSP and the costs of direct connections between hubs. Constraint (11) denotes that if node i is assigned the hub h, it will serve the links arc (i, j). Constraint (12) ensures that each link can be served only by one hub in a spoke-level route. Constraint (13) ensures each node has a drone arriving and leaving from hub h. Constraint (14) provides that the flow originating at a node is equal to the outgoing flow of this node. In Constraint (15), the transport demand W_{il} between nodes i and l is satisfied. Constraint (16) the flow originating at node i routed via nodes k and l can only occur if the links arc (k, l) is allocated to hub h. Constraint (17) is known as the inter-hub flow conservation constraint. Constraint (18) defines the decision variables v_{hij} is binary. Constraint (19) implies that the decision variables f_{ikj} is non-negative.

4.2.2. Node Allocation Reassignment Model

According to the solution result of the first model, when $x_{ii} = 1$, i is a hub. Carrying out TSP optimization for all spokes assigned to the same hub, the allocation scheme with the lowest cost of the HSN will be obtained. A new decision variable z_{hi} is introduced into [M3]. When it is 1, the drone departing from hub $h \in H$ visits node $i \in N$, otherwise it is zero. Then we can convert the HSNTSP-D model into [M3].

$$[\text{M3}] \min f^{HR} = \sum_{ikl} C_{kl} f_{ikl} + \alpha \sum_{ikl} C_{kl} y_{ikl}$$

Subject to:
Constraints (12)–(19)

$$\sum_{h \in H} z_{hi} = x_{ii}, \forall i \in N \tag{20}$$

$$\sum_{i \in N} z_{hi} = 1, \forall h \in H \tag{21}$$

$$z_{hi} \leq \sum_{j \neq i \in N} v_{hji}, \forall i \in N, h \in H \tag{22}$$

$$z_{hi} \in \{0, 1\},\ \forall i \in N, h \in H \tag{23}$$

Constraint (20) expresses that if the hub is node i, only one truck is assigned to this hub. Constraint (21) ensures that only one drone is assigned to each node. Constraint (22) denotes that if node i is assigned the hub h, it will serve the links arc (j, i). Constraint (23) defines the decision variable z_{hi} as binary.

4.2.3. Solution Procedure Integrating M2 and M3

The optimization process consists of three stages from the perspective of the problem. The first stage model is to select the hubs and assign spokes to hubs; the second stage model routes the drones; and the third stage model reallocates the spokes to hubs, as shown below.

[M1]	Description
Variables	x_{kk}, x_{ik}
Objective	Construct an HSN, determine the hubs, and allocate the spokes to the hubs.
[M2]	**Description**
Variables	v_{hij}
Objective	Based on [M1], construct an HSNTSP-D to obtain the shortest paths for drone delivery.
[M3]	**Description**
Variables	z_{hi}, v_{hij}
Objective	Based on [M1], change spoke-hub assignments, and reconfigure and optimize the existing HSN foundation. Use the hubs, and reassign the spokes to obtain a solution with a lower total transportation cost.

5. Proposed Method

In this section, a three-stage decomposition algorithm is described to solve the optimization problem of HSNTSP-D, which consists of a p-hub location and spoke node allocation stage, routing optimization stage, and spoke node reassignment strategy stage. For a p-hub location and spoke node allocation stage (Section 5.2), we developed a VNS algorithm to select hubs and allocate spokes. The second stage proposed a nearest-neighbor algorithm for drone routing optimization (Section 5.3). The third stage offers the reassignment strategy to adjust node allocation, and change spoke-hub assignments, then it returns to stage 2 for solving, and after finite iterations of stage 2 and stage 3, the local optimal solution is output. (Section 5.4). Figure 3 depicts a system scheme of the solution method.

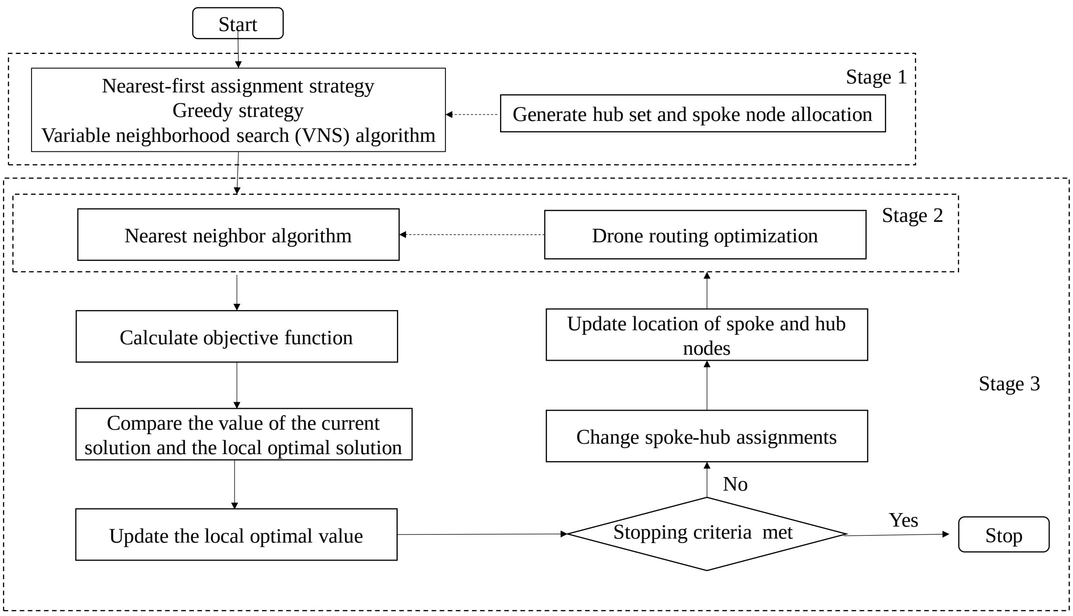

Figure 3. A system scheme of the solution method.

5.1. Nearest-First Assignment Strategy

A solution of HSN design is to identify a set of hubs, and once the hub is determined, assign spokes to the nearest hub based on distance. A solution can be expressed as $S = (H, A)$, where H is the set of hubs and A is the assignment relationship between spokes and hubs. For example, $A[i] = h$ represents hub h service node i, $A[h] = h$ represents the node h is a hub. In this article, we use O'kelly's [36] nearest allocation heuristic to generate a spoke allocation set A. When a hub set H is obtained, the distance from each spoke node to each hub is calculated, and the spokes are assigned to the nearest hub with the shortest distance; hereby the spoke node allocation set A is obtained, which is called the nearest allocation strategy. The detailed procedure of the nearest-first assignment strategy of Algorithm 1 is as follows: given a hub set H in N, the NA strategy allocates each spoke in $N \backslash H$ to the nearest hub in H (according to the cost matrix C).

Algorithm 1: Nearest-first assignment strategy [NA]

Input	N; H; C_{ij}
Output	A: a set of assignments of spokes to hubs
Step 1	Initialize $A = \varnothing$
Step 2	For i in N:
Step 3	$C_{ih} \leftarrow \min\{C_{ik}, \forall k \in H\}$
Step 4	Assign node i to the nearest hub h in A: $A[i] \leftarrow h$
Step 5	End For
Step 6	Output A

After obtaining a solution $S = (H, A)$, we combine a hub and its assigned nodes to form a subset expressed as $A^h(h \in H)$, where $A^h = \{i \in N : A[i] = h\}$ indicates that the subset contains all spokes assigned to hub h. To evaluation of a solution $S = (H, A)$, there are three types of cargo flow according to the method mentioned in Corberán et al. [37]. Mikić et al. [38] proposed three types of flows to evaluate feasible solutions $S = (H, A)$. The first one, the traffic flow between hubs—hub-to-hub flow, called HHF—represents the flow sent between two clusters. For example, the total flow on the inter-hub arc $(k, l), k, l \in H$ is obtained as

$$\textstyle\sum_{i \in A^k, j \in A^l} \left(W_{ij} + W_{ji}\right).$$

The second spoke node to the hub traffic flow—node-to-hub flow, called NHF—represents the sent flow from a spoke node to the hub. For instance, the total flow on the node $i \left(i \in A^k\right)$ to hub $k(k \in H)$ is obtained as

$$O_i = \sum_{j \in N} W_{ij}$$

The third traffic flow from the hub to the spoke node—hub-to-node flow, called HNF—represents hub-to-node traffic flow. For example, the total flow for hub $l(l \in H)$ to node $j\left(j \in A^l\right)$ is obtained as

$$D_j = \sum_{i \in N} W_{ij}.$$

5.2. First Stage: The p-Hub Location and Spoke Node Allocation Problem Stage

Inspired by the successful application of the general VNS heuristic to the hub location problem [39], we proposed a VNS algorithm to seek p number of nodes as hub set, and then assign spoke node to a hub. At this stage, a greedy strategy is used to generate the initial solution and then shake procedure to explore different hub set neighborhood structures. A local search is implemented based on the best improvement search strategy to improve the solution.

5.2.1. An Initialization Algorithm

In this algorithm, select $\overline{H}$ hubs, and the nearest spokes are assigned to the hub until all spokes are assigned. The initialization algorithm (Algorithm 2) generates the initial solution according to the greedy strategy. Aiming to make the total cost minimal, select a hub, assign nodes according to the nearest-first assignment strategy, and then add a hub, comparing with the previous scheme's cost. If the cost is reduced, the new hub will continue to be added.

Algorithm 2: Initialization algorithm	
Input	$N; \overline{H}:$ number of hubs, $H \leftarrow \varnothing$, $f \leftarrow +\infty$
Output	H, A, f
Step 1	Initialize $H \leftarrow \varnothing$, $f \leftarrow +\infty$
Step 2	While the hub number is not satisfied, do:
Step 3	For i in $N \backslash H$:
Step 4	Set $H_t \leftarrow H$
Step 5	Add i to a temporary set of hubs H_t : $H_t \leftarrow H_t \cup \{i\}$
Step 6	Assign $N \backslash H_t$ to H_t to obtain A_t
Step 7	Use H_t, A_t to calculate the cos t f_t
Step 8	If $f_t < f$
Step 9	$f \leftarrow f_t$, $H \leftarrow H_t$, $A \leftarrow A_t$
Step 10	End If
Step 11	End For
Step 12	End While

5.2.2. Shake Procedure

The shake procedure consists in selecting one hub i from the neighborhood of the hub set, then take it as an alternative hub. Algorithm 3 shows the critical steps of the shaking process. In the iterative process, each time the first node i is selected from the hub set H, the node i is moved out of the hub set, adding node i to the spoke node set S, to construct new neighborhoods H and S.

Algorithm 3: Shaking procedure	
Input	N, H
Output	H, S, i
Step 1	Get the spokes set by $S \leftarrow N - H$
Step 2	Pick out the first hub i in H ($H \leftarrow H - \{i\}$)
Step 3	Add node i to the spokes set S ($S \leftarrow S \cup \{i\}$)
Step 4	Return H, S, i

5.2.3. Local Search

A local search heuristic starts after the shaking procedure, the search strategy based on the best improvement search strategy. Each iteration chooses a solution that is better than the previous solution and sets it as the new current solution. After iteration, the new current solution is the local optimal solution and the best solution among all improved solutions.

In Algorithm 4, first, traverse the remaining node set S, successively insert the current node h_i into the hub set H_t through steps 4.2 to 4.4. The node allocation set A_t is obtained by using the nearest allocation strategy and calculating the current cost. Then, in steps 4.7 to 4.9, compare the current cost with the previous cost. If $f_t < f$, replace f, A, i with f_t, A_t, h_i. Finally, the node i is inserted into the original hub set H, the hub set is updated, and the node allocation scheme is output.

Algorithm 4: Local search based on the best improvement search strategy

Input	$S:$ a set of nodes, $S \leftarrow N - H$;
Output	$f,\ A$
Step 1	Initialize $A = \varnothing$
Step 2	For h_i in S:
Step 3	Set $H_t \leftarrow H$
Step 4	Add h_i to H_t: $H_t \leftarrow H_t \cup \{h_i\}$
Step 5	Assign $N \backslash H_t$ to H_t by [NA] : $A_t \leftarrow NA(N \backslash H_t, H_t)$
Step 6	Calculate the objective function value f_t by H_t, A_t
Step 7	If $f_t < f$
Step 8	$f \leftarrow f_t$, $A \leftarrow A_t$, $i \leftarrow h_i$
Step 9	End If
Step 10	End For
Step 11	Add i to H: $H \leftarrow H \cup \{i\}$
Step 12	Return $f,\ H,\ A$

5.2.4. Basic VNS

The basic VNS contains two main procedures: the shaking and local search procedures. The shaking procedure can jump out the optimal local solution, and the local search procedure can obtain an improved solution. In Algorithm 5, we executed alternately shaking and local search procedures until the limit on the number of iterations was reached. The criterion for VNS stopping the iteration is that the solution has not improved after k_{max} iterations.

Algorithm 5: Basic variable neighborhood search

Input	$N, \overline{H},\ k_{max}$
Output	H, A, f
Step 1	Initialize $(H, f, A) \leftarrow$ Initialization algorithm $\left(N, \overline{H}\right)$
Step 2	$k = 1$
Step 3	While $k \leq k_{max}$:
Step 4	Shaking procedure(f, H, A)
Step 5	Local search $(f_t, H_t, A_t) \leftarrow GH\ (N, i, H, f)$
Step 6	$k \leftarrow k + 1$
Step 7	If $f_t < f$
Step 8	$f \leftarrow f_t$, $H = H_t,\ A = A_t$
Step 9	$k = 1$
Step 10	End If
Step 11	End While
Step 12	Return $f,\ H, A$

Algorithm 5 is the detailed design process of basic VNS, which outputs the local optimal solution results in limited iterations. Firstly, the initial solution is generated through the Initialization algorithm to promote the execution of the algorithm. In the iterative process, the shaking procedure is used to delete a hub i from the hub set H as the replacement hub. In Step 5.5, the local search procedure is executed to search the remaining spoke node set S, and a spoke node is selected to exchange with the hub. After the exchanges are completed, the node with the lowest current total cost is found to replace the original hub and return to the new hub set H_t. When the objective function is not updated after k_{max} iterations, the algorithm stops, which means that no better solution can be found by exchanging any hub from H with other spokes, and the algorithm converges to the local optimal solution.

5.3. Second Stage: The Nearest Neighbor Algorithm for the Truck Routing Stage

In the second stage, hub set H and node assignment set A are obtained according to the first stage. The nodes assigned to the same hub are summarized into a subset $\left\{A^h, h \in H\right\}$. If there are p hubs, there are p subsets. The order of nodes in each subset represents the arrival order $V_i \in V$ of the truck route. The aim of the algorithm is to optimize the sequence order of each subset of nodes—that is, to construct the shortest path optimization problem. Using the nearest neighbor algorithm (NNA), trucks start from the hub, visit the closest node in turn, and return to the hub. Find the node sequence with the shortest path and complete the route optimization. The steps of the nearest neighbor algorithm (Algorithm 6) are as follows:

Algorithm 6: Nearest neighbor algorithm (NNA)	
Input	A: a set of nodes;
	H: a set of hubs;
	C_{ij} : distance-dependant transportation costs;
Output	The best node sequence vector for each hub V
Step 1	For each hub sequence vector $V_h = \varnothing,\ V_h \in V\ ,\forall h \in H$
Step 2	For h in H:
Step 3	Set $A_h \in A$
Step 4	Add h to the route V_h: $V_h \leftarrow V_h \cup \ \{h\}$
Step 5	Picking the last node i from the sequence vector of nodes at hub h Set V_h
Step 6	Visit the nearest unvisited node $\min\{C_{ij},\ \forall j \in A^h\}$
Step 7	Update the end node and add j to the Set $V_h \leftarrow V_h \cup \ \{j\}$
Step 8	Remove j of Set $A^h \leftarrow A^h - \{j\}$
Step 9	Is there any unvisited node left of Set A^h?
Step 10	If yes, go to step 6.5
Step 11	Add the hub sequence vector V_h to Set V
Step 12	End For
Step 13	Return V

To find a better sequence of nodes V_h by performing reordering in the same subset A^h, the nearest neighbor algorithm starts from the hub in the local route, then visits the unvisited node closest to the previous node and adds it to the route. After visiting all nodes, the algorithm ends and returns to the hub.

To evaluate the cost of this structure, we consider two types of expenses. First, the transportation between hubs still adopts the direct connection, so the cargo flow calculation method between hub k and hub l is to calculate the cargo flow from subset A^k to nodes in subset A^l through the hub-to-hub flow (HHF). The drones arrive at each spoke in turn on the same route according to the sequence of spokes. On the first trip, a truck departing from a hub can pick up any packages and deliver some cargo at those nodes. If the final destination node is on the route, another cargo is brought to the hub for processing. On the second trip, on the same route, the drone only delivers packages on each spoke. According to the above type of service, the packages between spokes on each route can be calculated. The calculation formula of transportation flow from node j to node l in the subset A^h represents

$$\sum_{i \in N} f_{ijl}, \forall j, l \in A^h.$$

Therefore, the route cost is computed as the arc length multiplied by the packages

$$C_{jl} \cdot \sum_{i \in N} f_{ijl}, \forall j, l \in A^h.$$

5.4. Third Stage: The Node Allocation Reassignment Strategy (RA) Stage

The reassignment strategy is to find a scheme to reduce the total transportation cost. Select a spoke from the spoke set and replace the hub. Namely, a hub is selected from the hub and is used to replace the initially allocated hub. After the replacement, the number of spokes in the subset served by the two hubs changes. The number of nodes served by the original hub decreases by one, and another hub-served node increases by one. For example, spoke node i assigned to hub k transitions to hub l, changes in cargo flow between hubs can be described as the following three cases:

(1) The change of cargo flow between hub k and other hub g is

$$HHF\{H_k\}\{H_g\} - NHF[i][g] + HHF\{H_g\}\{H_k\} - HNF[g][i].$$

(2) The change of cargo flow between hub l and other hub g is

$$HHF\{H_l\}\{H_g\} + NHF[i][g] + HHF\{H_g\}\{H_l\} + HNF[g][i].$$

(3) The change of cargo flow between hub k and hub l is

$$HHF\{H_k\}\{H_l\} - NHF[i][l] + HNF[k][i] + HHF\{H_l\}\{H_k\} - HNF[l][i] + NHF[i][k].$$

Due to the spokes assigned to hub k and hub l changes, the two routes of hub k and hub l be reoptimized, and the transportation costs of the two routes need to be recalculated according to the second stage method.

In brief, the design idea of the three-stage decomposition algorithm is as follows: Firstly, according to the first stage method, the hub set H and spoke node allocation set A are obtained through VNS. Then, according to the second stage method—for the truck's shortest path starting from each hub, using the nearest neighbor algorithm to optimize—the node sequence set V of each route is obtained. Finally, the spokes are redistributed through the third stage, resulting in the new subset A^h of the hub h, the nearest neighbor algorithm is used to optimize the truck route for hub h. A solution with a lower total network cost is finally obtained through the multiple iterations of the second and third stages.

The three-stage decomposition algorithm process is described in detail in Algorithm 7. In the first stage, p-hub location and spoke node allocations problem is solved by VNS, and then the hub set and node assignment set H, A are obtained. Stage 2 optimizes the truck route of each hub by using the nearest neighbor algorithm. The initial solution of truck route V is obtained and calculates the objective function f^{HT}. Steps 1 to 5 indicate that, from the spoke node set S, a spoke node is selected in turn and assigned to other hubs, change hub allocation scheme A, then call the second stage nearest neighbor algorithm for shortest path optimization, and calculate the total cost. If reducing the total cost, then update the local optimal solution. Finally, a better allocation scheme A_t is used to replace the original allocation scheme A and outputs the truck route set V.

Algorithm 7: The three-stage decomposition algorithm (TDA)

Input	$N, \overline{H}, k_{max}$
Output	f, H, A
Stage 1	Initialize $H \leftarrow \varnothing, \ f \leftarrow +\infty$
	Apply VSN to HSN : $H, A, f \leftarrow$ B_VSN(N, H, f)
Stage 2	The nearest neighbor algorithm is applied : $V \leftarrow$ NNA(H, A)
	Calculate the objective function value f^{HR} by H, V
Stage 3	Initialize spoke node set $S = N \backslash H$
Step 1	For i in S:
Step 2	Find the hub h_i of spoke i
Step 3	$H_t = H - \{h_i\}$
Step 4	For h in H_t:
Step 5	Replace h_i with h in set A, get A_t
Stage 2	Find the best node sequence for each hub $V_t \leftarrow$ NNA(H, A_t)
Step 6	Calculate the objective function value f_t^{HR} by H, V_t
Step 7	If $f_t^{HR} < f^{HR}$
Step 8	$f^{HR} \leftarrow f_t^{HR}$, $A \leftarrow A_t$, $V \leftarrow V_t$
Step 9	End If
Step 10	End For
Step 11	End For
Step 12	Return f^{HR}, H, A, V

6. Numerical Experiments

6.1. Datasets

We generated our computational experiments based on the Civil Aeronautics Board (CAB), Turkish network (TR), and Australia Post (AP) datasets. O'kelly [36] developed the CAB data between 25 US cities. The TR data from [40] includes 81 nodes. The AP dataset developed by Ernst and Krishnamoorthy [41] contains 200 postcodes.

6.2. Experimental Settings

To compare the mathematical models with the pHMLRPSPD model proposed by Kartal, Hasgul, and Ernst [18], we compare the results of operations on the CAB and TR cases with n = 10 and n = 15. The model was implemented in Python version 3.8 and solved using CPLEX version 12.6.1. We code the algorithms in Python and perform calculation experiments run by an Intel 2.6 GHz core i7 CPU and 16 GB RAM machine with Windows 10. Table 5 presents the purposes, datasets, and results of four groups of numerical experiments.

Table 5. Purposes, datasets, and results.

No.	Purpose	Dataset	Results
1	(1) Compare the objectives of [M1], [M2], [M3], pHMLRPSPD, and TDA. (2) Compare [M2], [M3], and pHMLRPSPD in optimality and computing time. (3) Influence of discount coefficients and number of hubs on spoke-level routes	CAB	Figure 4, Table 6, Table 7
2	(1) Compare the objectives of [M1], [M2], [M3], pHMLRPSPD, and TDA. (2) Compare [M2], [M3], and pHMLRPSPD in optimality and computing time.	TR	Figure 5, Table 8
3	Compare the three-stage algorithm with [M2] and [M3].	AP	Table 9, Figure 6, Figure 7, Figure 8, Figure 9
4	Compare TSD (stage1+stage2) and TDA (three stages) in solving large-scale instances.	TR+AP	Table 10

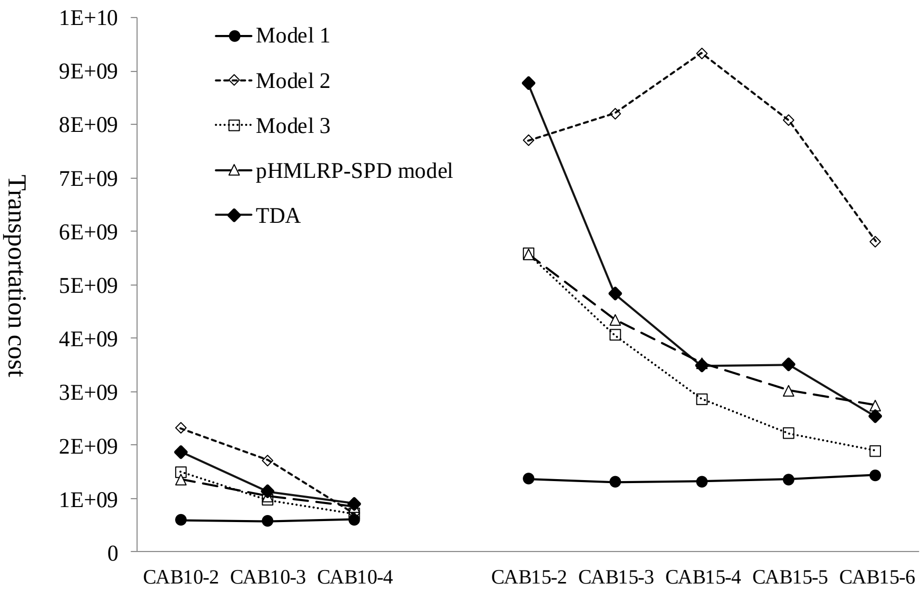

Figure 4. The change of objective function value in different scenarios.

Table 6. Results of solving the CAB datasets.

		pHMLRPSPD			[M2]		[M3]	
	Hubs	**Best**	**Dev/%**	**CT/s**	**Dev/%**	**CT/s**	**Dev/%**	**CT/s**
CAB-10	2	1351350000	0	332	71.5	0.37	10.47	220
	3	965164805	7.74	95	177.48	0.33	0	10
	4	710830792	17.54	33	0	0.20	0	
CAB-15	2	5577930000	0	7200	38.15	6897	0.19	7200
	3	4060215393	7.03	7200	102.27	241	0	7200
	4	2855341243	23.73	7200	227.11	2	0	7200
	5	2213149480	36.33	7200	265.02	1	0	1698
	6	1891936477	44.71	7200	207.05	1	0	58

Table 7. Influence of discount coefficient and number of hubs on spoke-level routes.

Dataset	pHMLRPSPD			[M3]			
CAB-10-	**Hubs**	**TSPs**	**CT/s**	**Hubs**	**TSPs**	**CT/s**	**Gap/%**
2-1.0	4,9	8-7-10-1; 5-2-3-6	332	5,10	4; 3-1-8-7-2-9	220	10.47
2-0.6	4,9	8-7-10-1; 5-2-3-6	450	5,10	4;9-2-7- 6-8-1-3	222	13.32
2-0.2	4,7	5-1-2-3-6-9; 8-10	922	5,10	8-1-6-7-3; 4-2-9	201	17.02
3-1.0	4,5,9	8;7-10-1;3-2-6	95	2,5,8	4;3-7; 9-10-6-1	10	−7.19
3-0.6	4,7,9	1-5;8-10; 6-2-3	77	2,5,8	4;7-3; 9-10-6-1	5	−3.95
3-0.2	4,7,9	5-1;8-10; 6-2-3	66	2,8,10	4;5-3-1; 9-7-6	6	−3.00
4-1.0	4,6,7,9	8;3-2;10;5-1	33	2,5,8,10	4;3-7;6-1;9	4	−14.93
4-0.6	4,6,7,9	8;3-2;10;5-1	27	2,5,8,10	4;7-3;1-6;9	1	−7.31
4-0.2	1,3,4,7	8;2;5-6-9;10	23	2,5,8,10	4;3-7;6-1;9	2	9.32

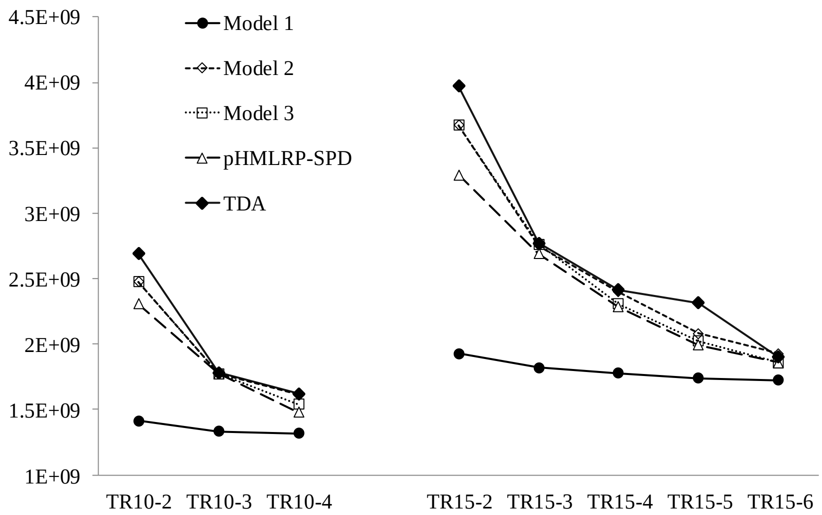

Figure 5. Changes of objective values in different scenarios of the TR dataset.

Table 8. Results of solving the TR datasets.

		pHMLRPSPD			[M2]		[M3]	
		Best	**Dev/%**	**CT/s**	**Dev/%**	**CT/s**	**Dev/%**	**CT/s**
TR-10	2	2308150000	0	121	7.21	0.28	7.21	22
	3	1774320000	0	15	0.00	0.29	0.00	5
	4	1479620000	0	7	8.84	0.22	3.83	2
TR-15	2	3293150000	0	7200	11.55	176.17	11.55	7200
	3	2690030000	0	7200	2.25	0.89	2.75	7200
	4	2282630000	0	7200	5.00	0.58	1.01	7200
	5	1991790000	0	2807	4.45	0.72	1.52	408
	6	1856130000	0	1401	3.76	0.68	0.00	39

Table 9. Solution results of the AP dataset.

			[M2]		[M3]		TDA	
Instance	p	Best	Dev/%	CT/s	Dev/	CT/s	Dev/%	CT/s
AP -15	3	143376472	4.68	5	0.19	7200	0.00	0.6
	4	99480179	9.99	0.41	0.00	3002	5.39	0.7
	5	86699082	6.05	0.34	0.00	186	0.46	0.7
	6	74048842	13.43	0.36	0.00	17	2.78	0.6
AP -20	3	156903693	0.00	1861	19.48	7200	5.94	0.9
	4	136808881	0.00	26	6.15	7200	2.81	0.9
	5	110978517	8.53	2	0.00	7200	1.58	0.9
	6	94750675	2.00	0.81	0.00	7200	1.72	0.8
AP -25	3	202657477	2.36	7200	23.97	7200	0.00	0.9
	4	144081680	0.00	7200	39.36	7200	6.26	1.1
	5	123553514	2.05	3413	20.11	7200	0.00	1.3
	6	105457701	6.58	9	11.63	7200	0.00	1.6

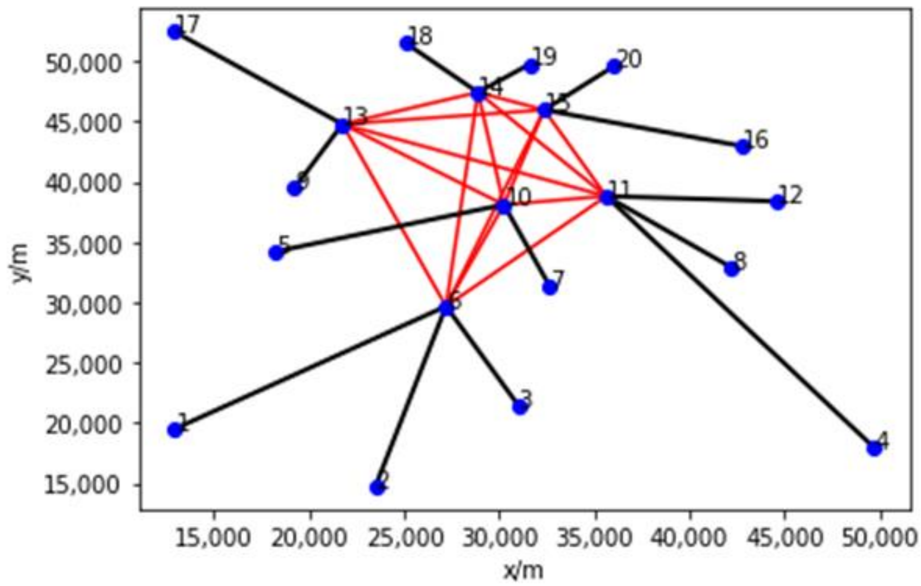

Figure 6. Results of solving [M1].

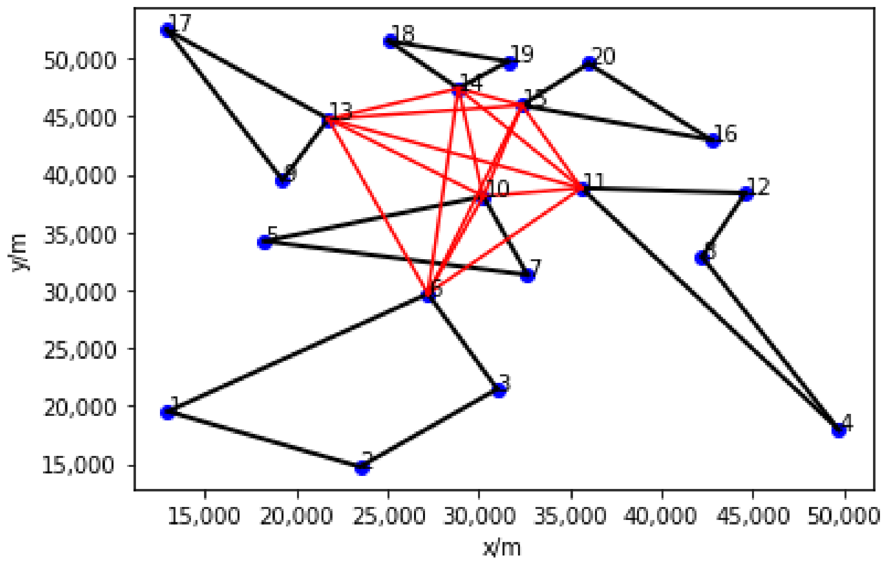

Figure 7. Results of solving [M2].

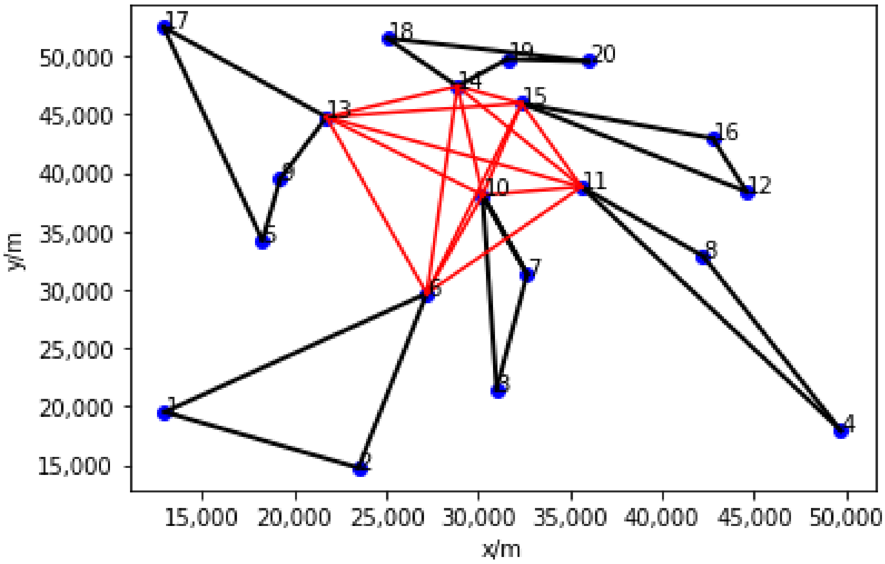

Figure 8. Results of solving [M3].

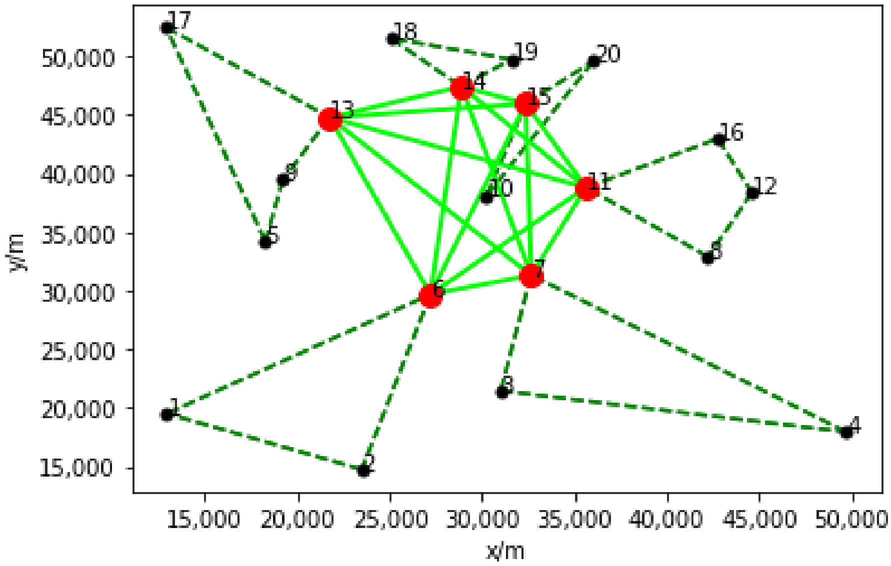

Figure 9. Results of performing TDA.

Table 10. Results of solving the large-scale instances.

		TSD			TDA		
Dataset	**P**	**Best**	**Dev%**	**CPU**	**Dev%**	**CPU**	**Ratio**
TR-81	5	1.71959×10^{11}	18.52	2.8	0.00	6.0	2.76
	10	1.05923×10^{11}	43.02	4.8	0.00	9.1	1.78
	15	81130618852	11.93	14.1	0.00	19.3	1.46
	20	66631297498	6.13	16.5	0.00	22.9	1.23
AP-200	5	444218560	19.67	44.4	0.00	118.1	5.49
	10	221374947	16.13	90.9	0.00	160.2	2.97
	15	154236703	24.39	192.1	0.00	274.6	2.14
	20	127360897	18.31	289.8	0.00	380.5	1.81

6.3. Results and Analysis

6.3.1. Comparative Analysis of Solution Results of the CAB Dataset

In Figure 4, the solution result of [M1] is a general HSN. [M2], [M3], pHMLRPSPD, and TDA can solve HSNTSP-D. The figure shows that as the number of hubs increases, the total transportation cost of HSNTSP-D tends to decrease. Secondly, as the number of hubs increases, the gap between the transportation cost of HSNTSP-D and HSN is narrowed. More hubs and spokes in the HSNTSP-D contributes to decreasing transportation cost. It can be seen from the figure that [M3] has good solution results and can obtain the optimal solution for the HSNTSP-D.

Table 6 compares the results of CPLEX solver solving [M2], [M3], and pHMLRPSPD for the HSNTSP-D. The hub discount coefficient is 1. However, the solver cannot find the optimal solution in an acceptable time due to the computational difficulty. Hence, we set the maximum computing time to 7200 s and obtain a solution for comparison. The column labeled 'Best' displays the optimal solution, and Dev(%) presents the percentage deviation from the optimal solution. CT(s) provides computational time in seconds employed by the CPLEX.

Among the eight small-scale examples of the CAB dataset, [M3] obtains six optimal solutions, while [M2] and pHMLRPSPD obtain one optimal solution, respectively. [M3] can determine the optimal solution many times for small-scale examples of the CAB dataset. In the computational time analysis, [M2] needs the minimum computational time, but the deviation between the solution result and the optimal solution is enormous. Compared with the pHMLRPSPD, the [M3] has less solution time and improved optimal solution.

The instance with 10 nodes in the CAB dataset is selected, the number of hubs takes 2, 3, and 4, respectively, and the hub discount coefficient is taken as 0.2, 0.6, and 1, constituting nine examples. The solution results of [M3] are compared with pHMLRPSPD proposed by Kartal et al. (2017), as shown in Table 7 below, in which the 'Hubs' column shows the location of the hubs in the optimal solution. Local routes mean spoke-level routes departing from the hub. The percent deviation between the solution results of [M3] and the solution results of pHMLRPSPD is presented in the last column Gap(%), calculated as

$$Gap = \frac{[M3] - pHMLRPSPD}{pHMLRPSPD}.$$

Comparative analysis shows that the hubs obtained by [M3] and the pHMLRPSPD model are different, and the constructed local routes are also different. However, changing the discount coefficient of transportation will result in the selection of hubs and spoke assignments to the hubs. For example, in the CAB-10-2 instance solved by pHMLRPSPD, when the discount coefficient decreases from 0.6 to 0.2, hub 7 is added as a hub, the original hub 9 is adjusted as spoke node, and the corresponding truck route from the hub is also optimized and adjusted. Although the hub location has not changed in some examples, the truck route has also been revised and optimized. For instance, in the CAB-10-2 solved by [M3], when the hub discount coefficient decreases from 1.0 to 0.6, the TSP route starting from hub 10 does not change, but the route direction changes. When the hub discount coefficient decreases to 0.2, although the hub does not change, the truck routes from hubs 5 and 10 change.

6.3.2. Comparative Analysis of Solution Results of the TR Dataset

As presented in Figure 5, in the TR dataset, as the number of hubs increases, the total transportation cost of the HSNTSP-D and the star-structure HSN transportation cost also show a gradually approaching trend. When the hubs are set reasonably and the spokes are evenly distributed, the network of HSNTSP-D can reduce the number of trucks. Moreover, the network transportation cost will not increase significantly. Similarly, through further analysis, [M2] and [M3] are compared with the pHMLRPSPD proposed by Kartala et al. (2017). Table 8 describes the results of solving the TR dataset.

In Table 8, among the eight small-scale examples of the TR dataset, the solution results of the pHMLRPSPD are the optimal solution, [M3] obtains the optimal solution twice, and [M2] only obtains the optimal solution once. For TR-10-3, the optimal solutions obtained by the three methods are the same, but in calculation time, [M2] takes the least time, and the pHMLRPSPD model takes the longest. Similarly, in TR-15-3, [M3] and pHMLRPSPD achieve the optimal solution. In the small-scale TR instance, the pHMLRPSPD model can achieve the optimal solution every time, which shows that this model has advantages in solving TR dataset problems. From the comparative analysis in Table 8, the solution deviations obtained by the above three methods are minor. The average variation of [M2] is 5.38%, and [M3] is 3.48%, but the computational time is quite different, especially since [M2] significantly reduces the computational time.

6.3.3. Analysis of the AP Dataset

The number of nodes in the AP dataset is 15, 20, and 25 in turn, and the number of hubs is 3, 4, 5, and 6, forming 12 instances. Table 9 below shows the comparison solution results among [M2], [M3], and the three-stage algorithm TDA.

In Table 9, [M3] obtains the optimal solution five times, the TDA model obtains it four times, and [M2] does so three times. Comparative analysis shows that [M3] can obtain the optimal solution many times in the AP-15 and AP-20 scenarios. For the AP-25, the solution complexity of the model increases, and it is challenging for [M3] to arrive at the optimal solution, but the TDA algorithm can obtain the optimal solution many times. Therefore, algorithms need to be used to solve large-scale problems. In addition, in the comparison between [M2] and [M3] in AP-20, [M2] is better than [M3] because the computational complexity of [M2] is lower than [M3]. When the problem scale increases, the [M2] solution result is easier to approach the optimal solution. Besides, with the increase in the number of hubs for the same scenario, [M3] can quickly obtain the optimal solution. For example, in the AP-20, when the number of hubs changes from 4 to 5, [M3] can determine the optimal solution. Because the complexity of the solution decreases with the number of hubs increases, [M3] is easier to solve and obtain the optimal solution.

It is found from Table 8 that, in the AP-20-6, the results calculated by the three methods have little difference. The differences between hubs and truck routes solved by the three methods are analyzed by comparing the network structure diagrams.

In Figure 6, the spokes of a hub are in a star structure, whereas Figure 7 presents that the spokes of a hub form a closed route. A drone starts from a hub, followed by a sequence of visits to the spokes, and finally returns to the hub. In Figure 8, the position of the hub is kept unchanged, and the spoke node allocation is reassignment. Figures 7 and 8 of the comparative analysis show that the spokes allocated to the hub have been adjusted. For example, the hub 10 allocated by node 5 has been adjusted to hub 13, the hub 6 allocated by node 3 has been adjusted to hub 10, the hub 11 allocated by node 12 has been adjusted to hub 15, and the hub 15 allocated by node 20 has been adjusted to hub 14. Through the reassignment of spokes, the solution obtained by [M3] is 2% lower than the solution obtained by [M2]. Figure 9 shows the results calculated by the TDA algorithm. Compared with Figure 8, the hub's location is adjusted, node 7 is added as a hub, hub 10 is adjusted as a spoke node, and other hubs remain unchanged. In addition, through the comparative analysis of Figures 8 and 9, it is found that the truck routes starting from hub 13 and hub 6 are the same in the two figures.

6.3.4. Solving Large-Scale Instances

According to the analysis in the previous stage, when the number of nodes exceeds 25, obtaining the optimal solution within 2 h is difficult for [M2] and [M3]. Therefore, a comparative study of the two-stage decomposition (TSD (stage1 + stage2)) algorithm and three-stage decomposition (TDA) algorithm is conducted to analyze the effect of spoke-node reassignment strategy on solving large-scale problems. The last column, 'Ratio', is calculated as (the HSNTSP-D problem optimal solution / the value obtained using the same HSN node allocation scheme to solve the p-hub median problem).

Table 10 shows that when the number of hubs increases, the HSNTSP-D transportation cost is gradually reduced. If the fixed cost of the hub settings is considered, adding hubs will increase the fixed cost of the hub setting. Therefore, this paper provides a solution for balancing the fixed cost of hub setup and truck transportation costs. In TR-81 and AP-200, several nodes are chosen as hubs frequently, although the number of hubs is different. For example, in the four scenarios in the AP-200 instance, node 151 and node 159 have always been hubs, indicating that these two nodes occupy a crucial geographical position, so they can be built into strategic hubs to give better play to the radiation effect of the hub.

The comparative study of two- and three-stage algorithms shows that the transportation cost can be significantly reduced after the reassignment strategy optimization of spokes. As shown in Table 10, the transportation cost of each instance shows a downward trend after reassignment strategy optimization. In the eight datasets, the transportation cost is reduced by an average of 15%. It shows that according to the hub set and spoke-node allocation scheme obtained from solving the p-hub medium problem, it is necessary to change the original spoke-hub assignments and then construct the local routes.

The ratio in the last column of Table 10 reflects the ratio of the transportation cost between the HSNTSP-D and general HSN. If the ratio is greater than 1, it indicates that the transportation cost of the HSNTSP-D is higher than HSN. In the HSNTSP-D, the drone's visited sequence is consistent with the order of the route, and at each spoke, the drone picks up and delivers cargo simultaneously and finally returns to the hub. However, the star structure is that a dedicated truck provides direct transportation services for each spoke node, and there is no accumulated cargo for transportation, so the transportation cost is low. In TR81 and AP200, with the increase in the number of hubs, the ratio of the transportation costs of the two structures showed a decreasing trend, and the ratio gradually approached 1. It shows that in large-scale problems, the transportation cost of HSNTSP-D is close to that of the HSN when a reasonable number of hubs are set. In addition, compared with HSN, in the HSNTSP-D, the drones pick up and deliver packages simultaneously, providing more direct routes between adjacent spokes. Therefore, the HSNTSP-D can reduce the number of transport devices significantly, as well as lower the investment and operating costs.

6.4. Managerial Implications

Based on the experimental results and analysis, we made the following generations for managing the operations mode using trucks and drones in HSNTSP-D.

(1) The cooperative truck–drone deployment model holds substantial potential for improving the operational efficiency of pickup and delivery scenarios in commercial applications—for example, in logistics and food delivery.

(2) When the spokes' demands are less than a truckload, these demands can be merged and serviced by a TSP solution conducted by several drones. In this study, we assume that the spokes of a hub are serviced by only one drone. In practice, the capacity constraint is an element parameter in the drone delivery problem. Hence, the diversified demands can be distributed by various capacity drones. We can extend the proposed models and algorithms to enable cargo delivery by several drones simultaneously at one hub. The spokes' demands can be met by several drones, which can be formulated as a truck routing problem with drones.

(3) The HSNTSP-D can be interpreted as a solution for 'light' HSN, where the demands are far less than a truck, and the consolidation, sorting, and distribution burdens are not severe at the hubs. For example, the weight or volume of a batch of demand is not enough to fill a truck, i.e., less-than-carload logistics (LCL). Hence, the models and algorithms in this paper can be used in LCL scenarios. Furthermore, the first- and last-mile distances should be limited. Drone-based logistics can apply to such conditions; therefore, it should be a promising solution for future city logistics.

7. Conclusions

This study investigated an extension of the classical hub location problem for HSN to reduce costs. When there is insufficient demand for a spoke node to support a direct connection to the hub, it must consider non-fully connected links between hubs and nodes. Therefore, we form a ring structure route for the hub, and the assigned spokes. Our method has three phases: first select p hubs from N nodes, then assign the remaining spokes to hubs and then form a cycle structure routing for each cluster of spokes assigned to hubs. This way, the demand from the origin to the destination on the spoke-level route can be delivered directly without being transported to the hub and then dispatched to the destination. Although this may increase travel distance and cost, it can decrease truck investment and operating costs.

To solve the HSNTSP-D, we formulated a three-stage decomposition model, and a three-stage decomposition algorithm was developed to solve large-sized instances. First, a VNS algorithm is proposed to seek hub set and spoke-to-hub assignments. Then, the drone routing problem constituted by hubs and distributed spokes is solved to obtain the optimal TSP routes using an NNP neighborhood search. Last, through the RA strategy to change spoke-hub assignments for improving feasible solutions. We compare the three-stage decomposition algorithm with the CPLEX running results on the CAB date, TR data, and AP data. As the problem size increases, the three-stage decomposition algorithm performs well compared to CPLEX.

Analysis of numerical results presents that our proposed three-stage decomposition method is competitive in obtaining optimal solutions. For large-scale instances, the CPU computing time is within 10 min. To compare the costs of HSN with the drone and star structure HSN, the solutions obtained by our proposed model are compared with the solutions obtained by solving the single allocation p-hub median location problem. The HSN with drone structure replaces the mode of dedicated direct links in spoke-level routes, reducing the fixed and operating costs of using drones. This kind of hub-and-spoke network is most suitable to be applied to the distribution chain of pharmaceutical products, such as the dispatch of small quantities of emergency pharmaceutical supplies in emergency logistics—for instance, traffic congestion or disruptions such as traffic accidents, trucks breaking down, or road closures. In these cases, we need to use drones for transportation.

We provide a theoretical study for HSN design decisions with drones. The mathematical models and heuristics we propose may be critical tools for decision-makers when the fixed and operating costs of drones and hubs are considered when making decisions. However, there are also limitations of the study, such as not considering the parameters/constraints of the drone and the limits on the number of packages it can carry. For future research, the HSNTSP-D structure will add more real-world applications, such as selecting the appropriate drone type (weight, maximum payload, and maximum flight distance).

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