applied system innovation

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A Performance Study of the Impact of Different Perturbation Methods on the Efficiency of GVNS for Solving TSP

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Received: 18 August 2019; Accepted: 18 September 2019; Published: 20 September 2019

Abstract: The purpose of this paper is to assess how three shaking procedures affect the performance of a metaheuristic GVNS algorithm. The first shaking procedure is generally known in the literature as intensified shaking method. The second is a quantum-inspired perturbation method, and the third is a shuffle method. The GVNS schemes are evaluated using a search strategy for both First and Best improvement and a time limit of one and two minutes. The formed GVNS schemes were applied on Traveling Salesman Problem (sTSP, nTSP) benchmark instances from the well-known TSPLib. To examine the potential advantage of any of the three metaheuristic schemes, extensive statistical analysis was performed on the reported results. The experimental data shows that for aTSP instances the first two methods perform roughly equivalently and, in any case, much better than the shuffle approach. In addition, the first method performs better than the other two when using the First Improvement strategy, while the second method gives results quite similar to the third. However, no significant deviations were observed when different methods of perturbation were used for Symmetric TSP instances (sTSP, nTSP).

Keywords: variable neighborhood search; experimental comparison; statistical analysis; traveling salesman problem; soft computing

1. Introduction

Variable Neighborhood Search (VNS) is a metaheuristic approach proposed by Mladenovic and Hansen to solve combinatorial and global optimization problems [1,2]. This framework is primarily designed to systematically modify the neighborhood structure, to reach an optimal (or near-optimal) solution [3]. VNS and its extensions have demonstrated their effectiveness in solving many problems in the combinatorial and global optimization field [4,5].

Each VNS heuristic consists of three parts. The first is a process of shaking (phase of diversification) used to escape from local optimal solutions. The next one is changing the neighborhood, where the next neighborhood structure to be searched will be determined; an approval or rejection criterion will also be applied to the last solution found during this part. The third part is the phase of improvement (intensification) achieved by exploring neighborhood structures by applying various local search moves. This exploration is carried out primarily through one of the following steps to change the neighborhood:

- Cyclic neighborhood change step: Whether there is an improvement in some neighborhood or not, the search continues in the next neighborhood structure in the list.
- Pipe neighborhood change step: If the current solution is improved in some neighborhood, exploration in that neighborhood will continue.

- Skewed neighborhood change step: Accept as new incumbent alternatives that not only improve solutions, but also some that are worse than the current incumbent solution. Such a neighborhood change step is intended to allow valley exploration away from the incumbent solution. A trial solution is evaluated taking into consideration not only the trial's objective values and the incumbent solution, but also their distance.

Variable neighborhood search variants. Many VNS variants have already been developed and used to solve hard optimization problems [6,7]. The most commonly used variants are the Basic VNS (BVNS), the Variable Neighborhood Descent (VND), and the General VNS (GVNS) and the Reduced VNS (RVNS). In the BVNS a method of diversification is alternated with a local search operator. VND consists of an improvement procedure in which neighborhood structures are systematically explored and a neighborhood change step. According to their neighborhood change step, there are different variants of VND. The pipe-VND, which uses the pipe neighborhood change step, appears to be the most efficient way to solve computational problems [6]. General Variable Neighborhood Search (GVNS) is a VNS variant that uses a VND method to improve. In many applications, GVNS has been successfully tested, as several recent works have shown [8,9].

The efficiency of metaheuristics depends on the efficiency of their components. Performance studies are a prerequisite for evaluating different metaheuristics [10] or different components of a metaheuristic algorithm [11]. In this direction and based on the VNS, Huber and Geiger (2017) [12] examined the impact of different order of local search operators in the improvement component of a VNS algorithm. There are similar studies for the impact of the initial solution [13] or the use of different neighborhood change strategies [2] to the overall performance of a VNS algorithm. However, there is a lack of contributions on studying the impact of the shaking components to the overall performance of a VNS algorithm. Papalitsas et al. (2019) [14] attempted an initial study on the impact of diversification methods on the performance of GVNS by focusing on asymmetric TSP instances.

This work is a substantial extension of our recent conference paper [14] in which we investigated the impact of three shaking methods on a GVNS metaheuristic, applied on asymmetric Traveling Salesman Problem (TSP) instances from the TSPLib. In an effort to build a comprehensive view related to that potential impact of diversification methods, the findings of the previous work are integrated with further analysis on the obtained solutions of symmetric and world TSP instances from TSPLib. To examine this potential impact of the different perturbation strategies, the three shaking methods were examined within the same improvement step. Moreover, the resulting GVNS schemes were executed both with First and Best improvement search strategies, and two different time limits were used as the main stopping criteria: 60 s and 120 s. The obtained experimental results were analyzed statistically to establish whether the use of different perturbation methods affects the performance of the GVNS algorithm. Our findings demonstrate that the use of different perturbation strategies clearly affect the solution quality in aTSP instances, while no significant differences were observed for the case of sTSP instances, with the exception of the experiments conducted using Best Improvement and 120 s run time limit. Moreover, to examine the efficiency of the formed methods, a comparison is performed between the obtained results and other recent metaheuristic solution approaches for the TSP in the literature. As it can be confirmed by our experimental results, the proposed GVNS schemes produce better solutions than the other metaheuristics.

Organization

This paper is organized as follows. In Section 2 the proposed GVNS solution methods and their technical components are explained. Section 3 contains the experimental results of our performance analysis, while the statistical tests applied to our numerical results are presented in Section 4. Section 5 provides a comparative study between our algorithms and other metaheuristic solution approaches in the recent literature. Finally, conclusions and ideas for future work are given in Section 6.

2. GVNS Heuristics

The formed GVNS methods use the pipe-VND scheme, which means that the search is taking place in the same neighborhood where the improvement occurs, as their improvement phase.

2.1. Neighborhood Structures

Three local search operators are considered for exploring different solutions:

- **1-0 Relocate**. This move removes node i from its current position in the route and re-inserts it after a selected node b.
- **2-Opt**. The 2-Opt move breaks two arcs in the current solution and reconnects them in a different way.
- **1-1 Exchange**. This move swaps two nodes in the current route.

All three neighborhood structures are incorporated in a pipe-VND scheme, as illustrated in Algorithm 1, where $l_{max} = 3$ denotes the number of neighborhood structures.

Algorithm 1 pipe-VND.

```
procedure PVND(N, l_max)
    l = 1
    while l <= l_max do
        select case(l)
            case(1) : S' ← 1-0 Relocate(S)
            case(2) : S' ← 2-Opt(S)
            case(3) : S' ← 1-1 Exchange(S)
        end select
        if f(S') < f(S) then
            S ← S'
        else
            l = l + 1
        end if
    end while
    return S
end procedure
```

2.2. Shaking Methods

To avoid local optimum traps, three different shaking procedures are examined. These perturbation methods are the following:

Shake_1 . This diversification method randomly selects one of the predefined neighborhood structures and applies it k times ($1 < k < k_{max}$, where k_{max} is the maximum number of shaking iterations) in the current solution. The method is summarized in Algorithm 2.

Shake_2 [15]. The scientific community seems to tend to revolve around new unconventional computing methods. Overall, unconventional computing is a wide range of proposed new or unusual computing models. Part of these computing models is natural computing [16]. Nature-inspired computing has emerged as an efficient paradigm for designing and simulating innovative computational models inspired by natural phenomena to solve complex nonlinear, dynamic specific problems. Some of the well-known nature-inspired computational systems and algorithms are [17]:

1. Evolutionary, biological-inspired algorithms.
2. Swarm intelligence algorithms inspired by swarm/agent group behavior.
3. Social and cultural algorithms inspired by society's interactions and beliefs.
4. Inspired by quantum physics, Quantum-inspired algorithms.

Algorithm 2 Shake_1.

```
procedure SHAKE_1(S, k, l_max)
    l = random_integer(1, l_max)
    for i ← 1, k do
        select case(l)
        case(1)
        S' ← 1-0 Relocate(S)
        case(2)
        S' ← 2-Opt(S)
        case(3)
        S' ← 1-1 Exchange(S)
        end select
    end for
    return S'
end procedure
```

Quantum Computing Principles

Quantum inspired methods imitate the fundamental principles of quantum computing. Quantum computing, a natural computing subsection and a field recently introduced by Feynman (1980s). Feynman realized that an effective simulation of an actual quantum system using a standard computer is not possible because the simulation of actual quantum processes would be exponentially slowed down [18,19]. Quantum computing is an important addition to the existing standard computing models. A general concept which considers the process as a quantum phenomenon. Quantum computing combines, apart from computer science, definitions, mathematical abstractions, and physics. Mathematics, such as linear algebra, and physics, such as quantum mechanics, are mainly involved.

The *qubit* is the quantum analogue of the classical bit. Similarly, the *quantum register*, which is a collection of qubits, is the quantum analogue of the classical processor register. In each call of this shaking method, a simulated quantum n-qubit register generates a normalized complex n-dimensional unit vector. In this context, normalized means that if $(z_1, \ldots, z_n)$ is the complex vector, then $|z_1|^2 + \ldots + |z_n|^2 = 1$. The dimension n of the complex unit vector is greater than or equal to the dimension of the problem. The complex n-dimensional vector is converted into a real n-dimensional vector, the components of which are real numbers in the interval $[0, 1]$. If z_i and r_i are the ith components of the complex and real vectors respectively, then $r_i = |z_i|^2$, i.e., r_i is equal to the modulus squared of z_i. Moreover, each of the real vector's selected components corresponds to a current solution node. For each node of the incumbent solution, the components are used as a flag. Sorting the first vector affects the order in the solution vector due to the correspondence between components and nodes in a tour and thus drives the exploration effort to another point in the search space. This shaking procedure's pseudocode is given in Algorithm 3.

Algorithm 3 Shake_2.

```
procedure SHAKE_2(S, n)
    NQubits ← QuantumRegister(n)
    Compute the components based on the qubits.
    Save the n components in the vector QCompVector.
    Matching each element in the QCompVector with a node in S.
    Descending sorting on QCompVector produces S'.
    Recalculate the cost of the new S'.
    return S'
end procedure
```

Shake_3. This shaking method is a shuffle method, where in each iteration the customers are placed in a random order. The method is shown in Algorithm 4.

Algorithm 4 Shake_3.

```
procedure SHAKE_3(S)
    S' ← Shuffle(S)
    return S'
end procedure
```

2.3. GVNS Schemes

For each perturbation method a GVNS scheme is formed. Specifically, the GVNS_1 contains Shake_1 as its shaking method, GVNS_2 uses Shake_2 to diversify solutions, and GVNS_3 adopts the Shake_3 perturbation method. The initial solution is produced by the Nearest Neighbor heuristic in all GVNS schemes. The pseudocode for three GVNS approaches is given in Algorithms 5–7, respectively.

Algorithm 5 GVNS_1.

```
procedure GVNS_1(S, k_max, max_time, l_max)
    while time ≤ max_time do
        for k ← 1, k_max do
            S* = Shake_1(S, k, l_max)
            S' = pVND(S*)
            if f(S') < f(S) then
                S ← S'
            end if
        end for
    end while
    return S
end procedure
```

Algorithm 6 GVNS_2.

```
procedure GVNS_2(S, n, max_time)
    while time ≤ max_time do
        S* = Shake_2(S, n)
        S' = pVND(S*)
        if f(S') < f(S) then
            S ← S'
        end if
    end while
    return S
end procedure
```

Algorithm 7 GVNS_3.

```
procedure GVNS_3(S, max_time)
    while time ≤ max_time do
        S* = Shake_3(S)
        S' = pVND(S*)
        if f(S') < f(S) then
            S ← S'
        end if
    end while
    return S
end procedure
```

It should be mentioned that in all three GVNS methods the neighborhoods are searched with both the First and Best improvement search strategy.

3. Computational Analysis

3.1. Computing Environment & Parameter Settings

The aforementioned methods were implemented in Fortran and were executed in a PC running Windows 64-bit on an Intel Core i7-6700 CPU at 2.6 GHz with 16 GB RAM. The compilation of the code was done using the Intel Fortran 64 compiler XE with the optimization option /O3. The maximum execution time limit was set to $max\_time = 60$ s and $max\_time = 120$ s and the maximum number of shaking iterations in the Shake_1 was experimentally set to $k_{max} = 12$.

3.2. Computational Results

This section presents the computational results of the different perturbation strategies for each class of experiments. The GVNS schemes with the different shaking methods were applied on TSPLIB instances. The TSP is one of the most famous NP-hard combinatorial optimization problems. Solving the TSP means finding the minimum cost route so that the salesman starts from a particular node and returns to that node after passing from all the other nodes once.

All experiments were executed 5 times and the average value of all runs was computed. Tables 1 and 2 contain the aggregated experimental results. Specifically, they show the benchmark name, the optimal value (zOpt), the cost of the three GVNS schemes (GVNS_1, GVNS_2 and GVNS_3) and their corresponding gaps from the optimal value. The results depicted in Table 1 were obtained using the First Improvement search strategy and an execution time limit of 1 min, whereas the results in Table 2 were obtained using the Best Improvement search strategy and the same execution time of 1 min. As mentioned earlier, the cost of each GVNS scheme is the average of 5 runs for each problem. The reported gap is computed as follows: given the outcome x, its gap from the optimal value OV is given by the formula $\frac{100 \times (x - OV)}{OV}$.

Table 1. The results shown here were obtained using the First Improvement search strategy and an execution time limit of 1 min [14].

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1 (%)	GAP_2 (%)	GAP_3 (%)
br17.atsp	39	39	39	39	0.00	0.00	0.00
ft53.atsp	6905	7189	7328	7737	4.11	6.13	12.05
ft70.atsp	38673	39782	40691	40537	2.87	5.22	4.82
ftv33.atsp	1286	1318	1339	1450	2.49	4.12	12.75
ftv35.atsp	1473	1484	1499	1596	0.75	1.77	8.35
ftv38.atsp	1530	1546	1585	1579	1.05	3.59	3.20
ftv44.atsp	1613	1651	1760	1797	2.36	9.11	11.41
ftv47.atsp	1778	1821	1992	2101	2.42	12.04	18.17
ftv55.atsp	1608	1666	1985	1912	3.61	23.45	18.91
ftv64.atsp	1839	1961	2382	2395	6.63	29.53	30.23
ftv70.atsp	1950	2136	2557	2484	9.54	31.13	27.38
ftv170.atsp	2755	3487	3923	3923	26.57	42.40	42.40
kro124p.atsp	36230	39024	43187	40259	7.71	19.20	11.12
p43.atsp	5620	5620	5623	5658	0.00	0.05	0.68
rbg323.atsp	1326	1516	1563	1626	14.32	17.87	22.62
rbg358.atsp	1163	1347	1437	1404	15.82	23.55	20.72
rbg403.atsp	2465	2535	2587	2565	9.78	4.42	11.76
rbg443.atsp	2720	2814	2859	2814	3.46	5.11	3.46
ry48p.atsp	14422	14549	14901	14738	0.88	3.32	2.19
Average	**6599.74**	**6920.26**	**7328.37**	**7190.21**	**12.33**	**12.64**	**24.10**

The results in Table 1 indicate a definite pattern, namely that both GVNS_1 and GVNS_2 outperform GVNS_3 in most cases. Recall that GVNS_3 is a shuffle perturbation strategy. For example, consider benchmark ftv47; we can see that the cost of GVNS_1 is 1821, the cost of GVNS_2 is 1992 and of GVNS_3 is 2101. GVNS_1 and GVNS_2 both outperform GVNS_3 and are also relatively close to the optimal value (1778).

Table 2. The results depicted here were obtained using the Best Improvement search strategy and an execution time limit of 1 min [14].

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1 (%)	GAP_2 (%)	GAP_3 (%)
br17.atsp	39	39	39	39	0.00	0.00	0.00
ft53.atsp	6905	7043	7135	7674	2.00	3.33	11.14
ft70.atsp	38673	39507	40206	40539	2.16	3.96	4.83
ftv33.atsp	1286	1289	1286	1379	0.23	0.00	7.23
ftv35.atsp	1473	1476	1473	1533	0.20	0.00	4.07
ftv38.atsp	1530	1538	1541	1599	0.52	0.72	4.51
ftv44.atsp	1613	1632	1644	1728	1.18	1.92	7.13
ftv47.atsp	1778	1792	1816	1940	0.79	2.14	9.11
ftv55.atsp	1608	1642	1665	2012	2.11	3.54	25.12
ftv64.atsp	1839	1908	1986	2193	3.75	7.99	19.25
ftv70.atsp	1950	2110	2157	2346	8.21	10.62	20.31
ftv170.atsp	2755	3341	3852	3923	21.27	39.82	42.40
kro124p.atsp	36230	36501	37076	38195	0.75	2.34	5.42
p43.atsp	5620	5620	5620	5627	0.00	0.00	0.12
rbg323.atsp	1326	1486	1539	1633	12.06	16.06	23.15
rbg358.atsp	1163	1307	1409	1437	12.38	21.15	23.55
rbg403.atsp	2465	2510	2547	2554	11.76	11.76	11.76
rbg443.atsp	2720	2765	2824	2844	1.65	3.16	4.56
ry48p.atsp	14422	14480	14498	14659	0.40	0.12	1.64
Average	**6599.74**	**6736.11**	**6858.58**	**7044.95**	**15.88**	**17.69**	**22.28**

In addition, the provided results in Table 2 lead to the same statement that both GVNS_1 and GVNS_2 produce better results than GVNS_3 in most cases. Table 3 shows the results of the GVNS schemes within a 2 min run time limit and the First Improvement as search strategy. The results of Table 3 mention that GVNS_1 outperform GVNS_2 and GVNS_3 in most cases. However, the main difference from the results of Tables 1 and 2 is that now the behavior of GVNS_2 is closer to that of GVNS_3 solution approach. Table 4 shows the results achieved by the GVNS schemes within a 2 min time limit and the Best Improvement search strategy. The results of Table 4 corroborate the conclusion of Tables 1 and 2 that both GVNS_1 and GVNS_2 outperform GVNS_3 in most cases.

Table 3. Results using the First Improvement search strategy and an execution time limit of 2 min [14].

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1 (%)	GAP_2 (%)	GAP_3 (%)
br17.atsp	39	39	39	39	0.00	0.00	0.00
ft53.atsp	6905	7024	7498	7752	1.72	8.59	12.27
ft70.atsp	38673	39615	40827	40505	2.44	5.57	4.74
ftv33.atsp	1286	1330	1370	1454	3.42	6.53	13.06
ftv35.atsp	1473	1482	1519	1604	0.61	3.12	8.89
ftv38.atsp	1530	1547	1618	1576	1.11	5.75	3.01
ftv44.atsp	1613	1628	1839	1812	0.93	14.01	12.34
ftv47.atsp	1778	1787	2020	2097	0.51	13.61	17.94
ftv55.atsp	1608	1668	2012	1912	3.73	25.12	18.91
ftv64.atsp	1839	1951	2484	2476	6.09	35.07	34.64
ftv70.atsp	1950	2165	2571	2484	11.03	31.85	27.38
ftv170.atsp	2755	3412	3923	3923	23.85	42.40	42.40
kro124p.atsp	36230	39344	44243	40849	8.60	22.12	12.75
p43.atsp	5620	5620	5628	5657	0.00	0.14	0.66
rbg323.atsp	1326	1499	1576	1586	13.04	18.85	19.60
rbg358.atsp	1163	1329	1410	1406	14.27	21.23	20.89
rbg403.atsp	2465	2509	2586	2547	2.27	4.10	11.76
rbg443.atsp	2720	2808	2849	2811	3.24	4.74	3.35
ry48p.atsp	14422	14475	14936	14708	0.37	3.56	1.98
Average	**6599.74**	**6906.95**	**7418.32**	**7220.95**	**5.29**	**13.96**	**24.78**

Table 4. Results using the Best Improvement search strategy and an execution time limit of 2 min [14].

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1 (%)	GAP_2 (%)	GAP_3 (%)
br17.atsp	39	39	39	39	0.00	0.00	0.00
ft53.atsp	6905	7043	7207	7773	2.00	4.37	12.57
ft70.atsp	38673	39358	40230	40588	1.77	4.03	4.95
ftv33.atsp	1286	1286	1290	1370	0.00	0.31	6.53
ftv35.atsp	1473	1474	1475	1509	0.07	0.14	2.44
ftv38.atsp	1530	1538	1555	1599	0.52	1.63	4.51
ftv44.atsp	1613	1636	1664	1731	1.43	3.16	7.32
ftv47.atsp	1778	1787	1837	1903	0.51	3.32	7.03
ftv55.atsp	1608	1640	1686	2012	1.99	4.85	25.12
ftv64.atsp	1839	1914	2032	2217	4.08	10.49	20.55
ftv70.atsp	1950	2038	2189	2342	4.51	12.26	20.10
ftv170.atsp	2755	3351	3918	3923	21.63	42.21	42.40
kro124p.atsp	36230	36379	37378	37915	0.41	3.17	4.65
p43.atsp	5620	5620	5620	5625	0.00	0.00	0.09
rbg323.atsp	1326	1473	1531	1610	11.08	15.46	21.41
rbg358.atsp	1163	1292	1405	1435	11.09	20.80	23.38
rbg403.atsp	2465	2498	2547	2553	1.30	3.25	11.76
rbg443.atsp	2720	2771	2822	2842	1.88	3.75	4.49
ry48p.atsp	14422	14468	14464	14678	0.32	0.29	1.78
Average	**6599.74**	**6716.05**	**6888.89**	**7034.95**	**3.28**	**7.05**	**22.16**

Tables 5–8 contain the aggregated experimental results for Symmetric TSP instances. Specifically, they contain the benchmark name, the optimal value (zOpt), the cost of the three GVNS variations (GVNS_1, GVNS_2 and GVNS_3) and their corresponding gaps from the optimal value. Table 5 depicts GVNS using the First Improvement search strategy and an execution time limit of 1 min. Table 6 shows GVNS using the Best Improvement as search strategy and an execution time of 1 min. Tables 7 and 8 are executed for 120 s within First and Best improvement search strategy respectively. The reported results show that the GVNS_3 produce better solutions than the other two algorithms, and that GVNS_1 and GVNS_2 do not have significant differences.

Table 5. Results using the First Improvement search strategy and an execution time limit of 1 min.

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
a280.tsp	2579	2683	2739	2745	4.03	6.20	6.44
att48.tsp	10628	10628	10628	10635	0.00	0.00	0.07
bayg29.tsp	1610	1610	1610	1610	0.00	0.00	0.00
bays29.tsp	2020	2020	2020	2020	0.00	0.00	0.00
bier127.tsp	118282	118636	120066	119966	0.30	1.51	1.42
kroA100.tsp	21282	21296	21375	21398	0.07	0.44	0.55
burma14.tsp	3323	3323	3323	3323	0.00	0.00	0.00
ch130.tsp	6110	6156	6239	6235	0.75	2.11	2.05
ch150.tsp	6528	6583	6720	6723	0.84	2.94	2.99
d493.tsp	35002	36928	37307	37166	5.50	6.59	6.18
kroB100.tsp	22141	22187	22339	22362	0.21	0.89	1.00
kroC100.tsp	20749	20759	20864	20834	0.05	0.55	0.41
kroD100.tsp	21294	21370	21573	21667	0.36	1.31	1.75
kroE100.tsp	22068	22114	22291	22360	0.21	1.01	1.32
kroA150.tsp	26524	26792	27247	27206	1.01	2.73	2.57
kroB150.tsp	26130	26382	26680	26767	0.96	2.10	2.44
kroA200.tsp	29368	29753	30420	30392	1.31	3.58	3.49
kroB200.tsp	29437	30164	30727	30711	2.47	4.38	4.33
d198.tsp	15780	15908	16116	16147	0.81	2.13	2.33
brg180.tsp	1950	1960	2024	2040	0.51	3.79	4.62
berlin52.tsp	7542	7542	7542	7542	0.00	0.00	0.00

Table 5. *Cont.*

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
dantzig42.tsp	699	699	699	699	0.00	0.00	0.00
eil51.tsp	426	426	426	428	0.00	0.00	0.47
eil76.tsp	538	539	544	545	0.19	1.12	1.30
eil101.tsp	629	630	645	647	0.16	2.54	2.86
fri26.tsp	937	937	937	937	0.00	0.00	0.00
gil262.tsp	2378	2460	2509	2509	3.45	5.51	5.51
gr17.tsp	2085	2085	2085	2085	0.00	0.00	0.00
gr21.tsp	2707	2707	2707	2707	0.00	0.00	0.00
gr24.tsp	1272	1272	1272	1272	0.00	0.00	0.00
gr48.tsp	5046	5046	5046	5048	0.00	0.00	0.04
gr96.tsp	55209	55285	55635	55713	0.14	0.77	0.91
gr120.tsp	6942	6979	7085	7103	0.53	2.06	2.32
gr137.tsp	69853	70207	71158	71330	0.51	1.87	2.11
gr202.tsp	40160	41232	41752	41850	2.67	3.96	4.21
gr229.tsp	134602	137642	139570	140144	2.26	3.69	4.12
gr431.tsp	171414	179950	182365	182884	4.98	6.39	6.69
hk48.tsp	11461	11461	11461	11470	0.00	0.00	0.08
lin105.tsp	14379	14386	14433	14458	0.05	0.38	0.55
lin318.tsp	42029	43641	44175	44183	3.84	5.11	5.13
pcb442.tsp	50778	53301	54176	54486	4.97	6.69	7.30
pr76.tsp	108159	108168	108411	108621	0.01	0.23	0.43
pr107.tsp	44303	44428	44695	44708	0.28	0.88	0.91
pr124.tsp	59030	59045	59169	59222	0.03	0.24	0.33
pr136.tsp	96772	97875	99118	99300	1.14	2.42	2.61
pr144.tsp	58537	58538	58629	58627	0.00	0.16	0.15
pr152.tsp	73682	74016	74379	74299	0.45	0.95	0.84
pr226.tsp	80369	80605	81007	81267	0.29	0.79	1.12
pr264.tsp	49135	50237	50883	50847	2.24	3.56	3.48
pr299.tsp	48191	50331	50917	50883	4.44	5.66	5.59
pr439.tsp	107217	112633	114121	114191	5.05	6.44	6.50
rat99.tsp	1211	1215	1240	1243	0.33	2.39	2.64
rat195.tsp	2323	2371	2456	2453	2.07	5.73	5.60
rd100.tsp	7910	7925	8011	8051	0.19	1.28	1.78
rd400.tsp	15281	15951	16281	16269	4.38	6.54	6.47
si175.tsp	21407	21430	21482	21486	0.11	0.35	0.37
st70.tsp	675	677	675	677	0.30	0.00	0.30
swiss42.tsp	1273	1273	1273	1273	0.00	0.00	0.00
ts225.tsp	126643	126858	128223	128359	0.17	1.25	1.35
tsp225.tsp	3916	4014	4116	4122	2.50	5.11	5.26
u159.tsp	42080	42556	43373	43374	1.13	3.07	3.08
ulysses16.tsp	6859	6859	6859	6859	0.00	0.00	0.00
ulysses22.tsp	7013	7013	7013	7013	0.00	0.00	0.00
ali535.tsp	202339	218486	221688	217544	7.98	9.56	7.51
att532.tsp	27686	29351	29747	28818	6.01	7.44	4.09
brazil58.tsp	25395	33181	25395	25396	30.66	0.00	0.00
brg180.tsp	1950	1959	2040	2158	0.46	4.62	10.67
d657.tsp	48912	52126	53015	51059	6.57	8.39	4.39
d1291.tsp	50801	59103	60214	55243	16.34	18.53	8.74
d1655.tsp	62128	73791	74028	73982	18.77	19.15	19.08
d2103.tsp	80450	86653	86653	86653	7.71	7.71	7.71
dsj1000.tsp	18659688	24056781	24631467	20034159	20.28	23.16	0.17
fl417.tsp	11861	12366	12232	12227	4.26	3.13	3.09
fl1400.tsp	20127	27242	27447	25980	35.35	36.37	29.08
fl1577.tsp	22249	27941	27996	27996	25.58	25.83	25.83
fl3795.tsp	28772	35262	35285	35285	22.56	22.64	22.64
fnl4461.tsp	182566	229963	229963	229963	25.96	25.96	25.96

Table 5. *Cont.*

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
gr666.tsp	294358	317446	324339	309556	7.84	10.19	5.16
nrw1379.tsp	56638	67679	68964	61769	19.49	21.76	9.06
p654.tsp	34643	36502	36558	35569	5.37	5.53	2.67
pa561.tsp	2763	2928	3053	3003	5.97	10.49	8.69
pcb1173.tsp	56892	70520	71978	61273	23.95	26.52	7.70
pcb3038.tsp	137694	175926	176310	176310	27.77	28.04	28.04
pr1002.tsp	259045	323543	331103	277196	24.90	27.82	7.01
pr2392.tsp	378032	460547	461170	461170	21.83	21.99	21.99
rat575.tsp	6773	7179	7190	7153	5.99	6.15	5.61
rat783.tsp	8806	9634	9610	9341	9.40	9.13	6.08
rl1304.tsp	252948	330540	335779	277603	30.68	32.75	9.75
rl1323.tsp	270199	331586	332103	293133	22.72	22.91	8.49
rl1889.tsp	316536	388695	389270	389270	22.80	22.98	22.98
rl5915.tsp	565530	695602	695602	695602	23.00	23.00	23.00
rl5934.tsp	556045	672412	672412	672412	20.93	20.93	20.93
si535.tsp	48450	48697	48848	48807	0.51	0.82	0.74
si1032.tsp	92650	92883	94571	92909	0.25	2.07	0.28
u574.tsp	36905	40206	40020	39488	8.94	8.44	7.00
u724.tsp	41910	45583	45988	44646	8.76	9.73	6.53
u1060.tsp	224094	297757	308980	242181	32.87	37.88	8.07
u1432.tsp	152970	185839	188807	166714	21.49	23.43	8.98
u1817.tsp	57201	71999	72030	72030	25.87	25.92	25.92
u2152.tsp	64253	78870	79260	79260	22.75	23.36	23.36
u2319.tsp	234256	275453	278765	278765	17.59	19.00	19.00
vm1084.tsp	239297	295088	301477	258248	23.31	25.98	7.92
vm1748.tsp	336556	406536	408102	408102	20.79	21.26	21.26
Average	**266956.86**	**317607.30**	**323886.60**	**276033.63**	**7.31**	**8.06**	**6.03**

Table 6. Results using the Best Improvement search strategy and an execution time limit of 1 min.

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
a280.tsp	2579	2632	2738	2734	2.06	6.17	6.01
att48.tsp	10628	10628	10628	10631	0.00	0.00	0.03
bayg29.tsp	1610	1610	1610	1610	0.00	0.00	0.00
bays29.tsp	2020	2020	2020	2020	0.00	0.00	0.00
bier127.tsp	118282	118411	119593	119962	0.11	1.11	1.42
kroA100.tsp	21282	21282	21332	21413	0.00	0.23	0.62
burma14.tsp	3323	3323	3323	3323	0.00	0.00	0.00
ch130.tsp	6110	6147	6219	6237	0.61	1.78	2.08
ch150.tsp	6528	6571	6704	6725	0.66	2.70	3.02
d493.tsp	35002	36559	37182	37119	4.45	6.23	6.05
kroB100.tsp	22141	22162	22282	22362	0.09	0.64	1.00
kroC100.tsp	20749	20749	20837	20880	0.00	0.42	0.63
kroD100.tsp	21294	21346	21507	21581	0.24	1.00	1.35
kroE100.tsp	22068	22129	22241	22264	0.28	0.78	0.89
kroA150.tsp	26524	26696	27169	27220	0.65	2.43	2.62
kroB150.tsp	26130	26233	26631	26696	0.39	1.92	2.17
kroA200.tsp	29368	29549	30416	30381	0.62	3.57	3.45
kroB200.tsp	29437	29888	30618	30688	1.53	4.01	4.25
d198.tsp	15780	15845	16077	16090	0.41	1.88	1.96
brg180.tsp	1950	1963	2024	2035	0.67	3.79	4.36
berlin52.tsp	7542	7542	7542	7563	0.00	0.00	0.28
dantzig42.tsp	699	699	699	699	0.00	0.00	0.00
eil51.tsp	426	426	426	428	0.00	0.00	0.47
eil76.tsp	538	538	544	547	0.00	1.12	1.67

Table 6. *Cont.*

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
eil101.tsp	629	630	645	648	0.16	2.54	3.02
fri26.tsp	937	937	937	937	0.00	0.00	0.00
gil262.tsp	2378	2437	2515	2518	2.48	5.76	5.89
gr17.tsp	2085	2085	2085	2085	0.00	0.00	0.00
gr21.tsp	2707	2707	2707	2707	0.00	0.00	0.00
gr24.tsp	1272	1272	1272	1272	0.00	0.00	0.00
gr48.tsp	5046	5046	5046	5049	0.00	0.00	0.06
gr96.tsp	55209	55293	55521	55582	0.15	0.57	0.68
gr120.tsp	6942	6977	7085	7120	0.50	2.06	2.56
gr137.tsp	69853	69948	70964	71412	0.14	1.59	2.23
gr202.tsp	40160	41079	41693	41720	2.29	3.82	3.88
gr229.tsp	134602	136416	139729	140377	1.35	3.81	4.29
gr431.tsp	171414	177946	181693	182205	3.81	6.00	6.30
hk48.tsp	11461	11461	11461	11471	0.00	0.00	0.09
lin105.tsp	14379	14382	14396	14437	0.02	0.12	0.40
lin318.tsp	42029	43094	44070	44309	2.53	4.86	5.42
pcb442.tsp	50778	52584	54456	54581	3.56	7.24	7.49
pr76.tsp	108159	108159	108278	108513	0.00	0.11	0.33
pr107.tsp	44303	44396	44539	44607	0.21	0.53	0.69
pr124.tsp	59030	59030	59058	59081	0.00	0.05	0.09
pr136.tsp	96772	97262	98966	98879	0.51	2.27	2.18
pr144.tsp	58537	58537	58561	58561	0.00	0.04	0.04
pr152.tsp	73682	73781	74027	73966	0.13	0.47	0.39
pr226.tsp	80369	80462	80861	80834	0.12	0.61	0.58
pr264.tsp	49135	49670	50905	50886	1.09	3.60	3.56
pr299.tsp	48191	49245	50614	50646	2.19	5.03	5.09
pr439.tsp	107217	111621	113347	113038	4.11	5.72	5.43
rat99.tsp	1211	1213	1234	1240	0.17	1.90	2.39
rat195.tsp	2323	2356	2451	2457	1.42	5.51	5.77
rd100.tsp	7910	7927	7963	8022	0.21	0.67	1.42
rd400.tsp	15281	15802	16312	16311	3.41	6.75	6.74
si175.tsp	21407	21420	21472	21476	0.06	0.30	0.32
st70.tsp	675	676	675	676	0.15	0.00	0.15
swiss42.tsp	1273	1273	1273	1273	0.00	0.00	0.00
ts225.tsp	126643	126721	127690	127849	0.06	0.83	0.95
tsp225.tsp	3916	3987	4115	4119	1.81	5.08	5.18
u159.tsp	42080	42329	42969	43024	0.59	2.11	2.24
ulysses16.tsp	6859	6859	6859	6859	0.00	0.00	0.00
ulysses22.tsp	7013	7013	7013	7013	0.00	0.00	0.00
ali535.tsp	202339	213387	218429	218205	5.46	7.95	7.84
att532.tsp	27686	28764	29614	29525	3.89	6.96	6.64
brazil58.tsp	25395	33181	25395	25412	30.66	0.00	0.07
brg180.tsp	1950	1963	2019	2198	0.67	3.54	12.72
d657.tsp	48912	51497	52986	51934	5.29	8.33	6.18
d1291.tsp	50801	54927	57302	55431	8.12	12.80	9.11
d1655.tsp	62128	67236	70399	67683	8.22	13.31	8.94
d2103.tsp	80450	83240	86653	83486	3.47	7.71	3.77
dsj1000.tsp	18659688	20201248	20449409	20063920	8.26	9.59	7.53
fl417.tsp	11861	12023	12119	12161	1.37	2.18	2.53
fl1400.tsp	20127	21244	21198	21166	5.55	5.32	5.16
fl1577.tsp	22249	23721	24355	23736	6.62	9.47	6.68
fl3795.tsp	28772	34663	33535	35214	20.47	16.55	22.39
fnl4461.tsp	182566	217998	204703	199441	19.41	12.13	9.24
gr666.tsp	294358	313338	321656	315223	6.45	9.27	7.09
nrw1379.tsp	56638	60516	62459	60983	6.85	10.28	7.67
p654.tsp	34643	36083	35544	36741	4.16	2.60	6.06
pa561.tsp	2763	2893	2896	3058	4.71	4.81	10.68
pcb1173.tsp	56892	61883	63335	62161	8.77	11.32	9.26
pcb3038.tsp	137694	153475	156692	149788	11.46	13.80	8.78
pr1002.tsp	259045	278408	285203	279922	7.47	10.10	8.06

Table 6. *Cont.*

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
pr2392.tsp	378032	410784	430379	408360	8.66	13.85	8.02
rat575.tsp	6773	7195	7090	7224	6.23	4.68	6.66
rat783.tsp	8806	9373	9391	9391	6.44	6.64	6.64
rl1304.tsp	252948	282487	282839	274566	11.68	11.82	8.55
rl1323.tsp	270199	293350	300601	285668	8.57	11.25	5.73
rl1889.tsp	316536	344218	356697	342893	8.75	12.69	8.33
rl5915.tsp	565530	680825	695602	695602	20.39	23.00	23.00
rl5934.tsp	556045	664895	672412	661012	19.58	20.93	18.88
si535.tsp	48450	48622	48783	48847	0.36	0.69	0.82
si1032.tsp	92650	92918	93397	92908	0.29	0.81	0.28
u574.tsp	36905	40374	40022	39792	9.40	8.45	7.82
u724.tsp	41910	44662	45765	45273	6.57	9.20	8.02
u1060.tsp	224094	242630	246175	243291	8.27	9.85	8.57
u1432.tsp	152970	165304	170578	165833	8.06	11.51	8.41
u1817.tsp	57201	62782	66243	62050	9.76	15.81	8.48
u2152.tsp	64253	70205	74581	70787	9.26	16.07	10.17
u2319.tsp	234256	243928	249738	245475	4.13	6.61	4.79
vm1084.tsp	239297	256431	262955	255369	7.16	9.89	6.72
vm1748.tsp	336556	362026	373926	360551	7.57	11.10	7.13
Average	**253944.13**	**274974.75**	**278630.04**	**273507.26**	**13.05**	**4.88**	**4.40**

Table 7. Results using the First Improvement search strategy and an execution time limit of 2 min.

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
a280.tsp	2579	2672	2728	2706	3.61	5.78	4.92
att48.tsp	10628	10628	10628	10724	0.00	0.00	0.90
bayg29.tsp	1610	1610	1610	1615	0.00	0.00	0.31
bays29.tsp	2020	2020	2020	2028	0.00	0.00	0.40
bier127.tsp	118282	118518	119737	119860	0.20	1.23	1.33
kroA100.tsp	21282	21283	21337	21699	0.00	0.26	1.96
burma14.tsp	3323	3323	3323	3323	0.00	0.00	0.00
ch130.tsp	6110	6157	6218	6289	0.77	1.77	2.93
ch150.tsp	6528	6583	6680	6649	0.84	2.33	1.85
d493.tsp	35002	36659	37040	36557	4.73	5.82	4.44
kroB100.tsp	22141	22168	22317	22557	0.12	0.79	1.88
kroC100.tsp	20749	20757	20806	21321	0.04	0.27	2.76
kroD100.tsp	21294	21329	21567	21904	0.16	1.28	2.86
kroE100.tsp	22068	22144	22250	22407	0.34	0.82	1.54
kroA150.tsp	26524	26630	27050	27738	0.40	1.98	4.58
kroB150.tsp	26130	26232	26701	26742	0.39	2.19	2.34
kroA200.tsp	29368	29570	30177	29718	0.69	2.75	1.19
kroB200.tsp	29437	29801	30459	30205	1.24	3.47	2.61
d198.tsp	15780	15871	16077	15964	0.58	1.88	1.17
brg180.tsp	1950	1956	2026	2153	0.31	3.90	10.41
berlin52.tsp	7542	7542	7542	7591	0.00	0.00	0.65
dantzig42.tsp	699	699	699	706	0.00	0.00	1.00
eil51.tsp	426	426	426	430	0.00	0.00	0.94
eil76.tsp	538	538	543	544	0.00	0.93	1.12
eil101.tsp	629	629	642	649	0.00	2.07	3.18
fri26.tsp	937	937	937	948	0.00	0.00	1.17
gil262.tsp	2378	2444	2505	2542	2.78	5.34	6.90
gr17.tsp	2085	2085	2085	2085	0.00	0.00	0.00
gr21.tsp	2707	2707	2707	2707	0.00	0.00	0.00
gr24.tsp	1272	1272	1272	1272	0.00	0.00	0.00
gr48.tsp	5046	5046	5046	5063	0.00	0.00	0.34
gr96.tsp	55209	55247	55549	56109	0.07	0.62	1.63

Table 7. *Cont.*

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
gr120.tsp	6942	6960	7063	7103	0.26	1.74	2.32
gr137.tsp	69853	70005	71002	71579	0.22	1.64	2.47
gr202.tsp	40160	40973	41598	41672	2.02	3.58	3.76
gr229.tsp	134602	136884	139544	138929	1.70	3.67	3.21
gr431.tsp	171414	178310	181839	180800	4.02	6.08	5.48
hk48.tsp	11461	11461	11461	11664	0.00	0.00	1.77
lin105.tsp	14379	14394	14413	14672	0.10	0.24	2.04
lin318.tsp	42029	43463	44016	44024	3.41	4.73	4.75
pcb442.tsp	50778	52867	53957	52362	4.11	6.26	3.12
pr76.tsp	108159	108159	108373	109446	0.00	0.20	1.19
pr107.tsp	44303	44400	44687	44769	0.22	0.87	1.05
pr124.tsp	59030	59033	59108	59444	0.01	0.13	0.70
pr136.tsp	96772	97158	98762	101877	0.40	2.06	5.28
pr144.tsp	58537	58537	58605	60768	0.00	0.12	3.81
pr152.tsp	73682	73801	74185	75515	0.16	0.68	2.49
pr226.tsp	80369	80533	80840	81472	0.20	0.59	1.37
pr264.tsp	49135	49633	50391	50951	1.01	2.56	3.70
pr299.tsp	48191	49595	50642	50023	2.91	5.09	3.80
pr439.tsp	107217	111921	113644	113269	4.39	5.99	5.64
rat99.tsp	1211	1212	1236	1266	0.08	2.06	4.54
rat195.tsp	2323	2365	2448	2404	1.81	5.38	3.49
rd100.tsp	7910	7925	7998	8122	0.19	1.11	2.68
rd400.tsp	15281	15847	16203	15936	3.70	6.03	4.29
si175.tsp	21407	21428	21475	21514	0.10	0.32	0.50
st70.tsp	675	676	675	691	0.15	0.00	2.37
swiss42.tsp	1273	1273	1273	1273	0.00	0.00	0.00
ts225.tsp	126643	126815	128183	127374	0.14	1.22	0.58
tsp225.tsp	3916	3996	4101	4093	2.04	4.72	4.52
u159.tsp	42080	42341	43196	44203	0.62	2.65	5.05
ulysses16.tsp	6859	6859	6859	6860	0.00	0.00	0.01
ulysses22.tsp	7013	7013	7013	7013	0.00	0.00	0.00
ali535.tsp	202339	217245	219075	216606	7.37	8.27	7.05
att532.tsp	27686	29167	29615	28916	5.35	6.97	4.44
brazil58.tsp	25395	36518	25395	25395	43.80	0.00	0.00
brg180.tsp	1950	1956	2023	2165	0.31	3.74	11.03
d657.tsp	48912	51769	52769	50997	5.84	7.89	4.26
d1291.tsp	50801	56007	60214	55060	10.25	18.53	8.38
d1655.tsp	62128	73181	74028	66384	17.79	19.15	6.85
d2103.tsp	80450	86582	86653	83454	7.62	7.71	3.73
dsj1000.tsp	18659688	22538673	24631467	19905772	20.79	32.00	6.68
fl417.tsp	11861	12118	12233	12190	2.17	3.14	2.77
fl1400.tsp	20127	27026	27447	21220	34.28	36.37	5.43
fl1577.tsp	22249	27461	27996	23500	23.43	25.83	5.62
fl3795.tsp	28772	35731	35285	35285	24.19	22.64	22.64
fnl4461.tsp	182566	229761	229963	229963	25.85	25.96	25.96
gr666.tsp	294358	316182	321701	308908	7.41	9.29	4.94
nrw1379.tsp	56638	67094	68964	60193	18.46	21.76	6.28
p654.tsp	34643	36022	36019	35708	3.98	3.97	3.07
pa561.tsp	2763	2905	3001	3008	5.14	8.61	8.87
pcb1173.tsp	56892	65023	70731	61249	14.29	24.33	7.66
pcb3038.tsp	137694	175799	176310	176310	27.67	28.04	28.04
pr1002.tsp	259045	279694	296142	275098	7.97	14.32	6.20
pr2392.tsp	378032	459687	461170	461170	21.60	21.99	21.99
rat575.tsp	6773	7136	7182	7158	5.36	6.03	5.68
rat783.tsp	8806	9501	9620	9352	7.89	9.24	6.20
rl1304.tsp	252948	322802	335779	274942	27.62	32.75	8.70
rl1323.tsp	270199	316844	332103	291235	17.26	22.91	7.79
rl1889.tsp	316536	388400	389270	389270	22.70	22.98	22.98

Table 7. *Cont.*

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
rl5915.tsp	565530	695466	695602	695602	22.98	23.00	23.00
rl5934.tsp	556045	672290	672412	672412	20.91	20.93	20.93
si535.tsp	48450	48648	48803	48765	0.41	0.73	0.65
si1032.tsp	92650	92864	93285	92925	0.23	0.69	0.30
u574.tsp	36905	40248	39803	39467	9.06	7.85	6.94
u724.tsp	41910	44972	45492	44598	7.31	8.55	6.41
u1060.tsp	224094	286667	251451	240193	27.92	12.21	7.18
u1432.tsp	152970	181206	188807	164045	18.46	23.43	7.24
u1817.tsp	57201	71024	72030	63539	24.17	25.92	11.08
u2152.tsp	64253	78581	79260	79217	22.30	23.36	23.29
u2319.tsp	234256	274542	278765	266890	17.20	19.00	13.93
vm1084.tsp	239297	265725	279047	255832	11.04	16.61	6.91
vm1748.tsp	336556	407007	408102	361567	20.93	21.26	7.43
Average	**253944.13**	**301717.44**	**322626.30**	**273781.10**	**14.50**	**7.41**	**5.26**

In some cases, GVNS_3 with a time limit of 1 min produced better results than using the 2 min time limit in solving sTSP instances. This might be happened due to the use of pure random diversification method such as the shuffle operator. More precisely, by executing more times GVNS_3, the shuffle operator is also executed more times and consequently it can shift the search into not so promising areas. Thus, the search may be trapped into low quality local optima.

Table 8. Results using the Best Improvement search strategy and an execution time limit of 2 min.

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
a280.tsp	2579	2630	2725	2706	1.98	5.66	4.92
att48.tsp	10628	10628	10628	10820	0.00	0.00	1.81
bayg29.tsp	1610	1610	1610	1613	0.00	0.00	0.19
bays29.tsp	2020	2020	2020	2029	0.00	0.00	0.45
bier127.tsp	118282	118421	119583	119527	0.12	1.10	1.05
kroA100.tsp	21282	21282	21312	21631	0.00	0.14	1.64
burma14.tsp	3323	3323	3323	3323	0.00	0.00	0.00
ch130.tsp	6110	6137	6208	6337	0.44	1.60	3.72
ch150.tsp	6528	6564	6680	6749	0.55	2.33	3.39
d493.tsp	35002	36232	37168	36822	3.51	6.19	5.20
kroB100.tsp	22141	22168	22249	22517	0.12	0.49	1.70
kroC100.tsp	20749	20749	20804	21402	0.00	0.27	3.15
kroD100.tsp	21294	21317	21461	21935	0.11	0.78	3.01
kroE100.tsp	22068	22122	22201	22450	0.24	0.60	1.73
kroA150.tsp	26524	26656	27119	27400	0.50	2.24	3.30
kroB150.tsp	26130	26232	26605	26816	0.39	1.82	2.63
kroA200.tsp	29368	29494	30276	30135	0.43	3.09	2.61
kroB200.tsp	29437	29732	30481	30843	1.00	3.55	4.78
d198.tsp	15780	15832	16036	16036	0.33	1.62	1.62
brg180.tsp	1950	1955	2014	2166	0.26	3.28	11.08
berlin52.tsp	7542	7542	7542	7737	0.00	0.00	2.59
dantzig42.tsp	699	699	699	703	0.00	0.00	0.57
eil51.tsp	426	426	426	432	0.00	0.00	1.41
eil76.tsp	538	538	542	543	0.00	0.74	0.93
eil101.tsp	629	630	644	648	0.16	2.38	3.02
fri26.tsp	937	937	937	940	0.00	0.00	0.32
gil262.tsp	2378	2432	2505	2497	2.27	5.34	5.00
gr17.tsp	2085	2085	2085	2086	0.00	0.00	0.05
gr21.tsp	2707	2707	2707	2707	0.00	0.00	0.00
Average	**253944.1262**	**272756.94**	**277586.89**	**273027.78**	**3.55**	**4.49**	**4.24**

Table 8. *Cont.*

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
gr24.tsp	1272	1272	1272	1273	0.00	0.00	0.08
gr48.tsp	5046	5046	5046	5089	0.00	0.00	0.85
gr96.tsp	55209	55259	55431	56269	0.09	0.40	1.92
gr120.tsp	6942	6975	7072	7130	0.48	1.87	2.71
gr137.tsp	69853	69869	70885	71215	0.02	1.48	1.95
gr202.tsp	40160	40860	41588	41762	1.74	3.56	3.99
gr229.tsp	134602	136101	139315	138770	1.11	3.50	3.10
gr431.tsp	171414	176629	181296	178732	3.04	5.76	4.27
hk48.tsp	11461	11461	11461	11519	0.00	0.00	0.51
lin105.tsp	14379	14379	14398	14505	0.00	0.13	0.88
lin318.tsp	42029	42989	43941	43768	2.28	4.55	4.14
pcb442.tsp	50778	52381	54108	52870	3.16	6.56	4.12
pr76.tsp	108159	108159	108227	109317	0.00	0.06	1.07
pr107.tsp	44303	44384	44473	44502	0.18	0.38	0.45
pr124.tsp	59030	59030	59039	59460	0.00	0.02	0.73
pr136.tsp	96772	97202	98613	100072	0.44	1.90	3.41
pr144.tsp	58537	58537	58544	60558	0.00	0.01	3.45
pr152.tsp	73682	73808	73884	74209	0.17	0.27	0.72
pr226.tsp	80369	80411	80677	81071	0.05	0.38	0.87
pr264.tsp	49135	49324	50715	52468	0.38	3.22	6.78
pr299.tsp	48191	48906	50363	51424	1.48	4.51	6.71
pr439.tsp	107217	110910	112735	114367	3.44	5.15	6.67
rat99.tsp	1211	1211	1232	1251	0.00	1.73	3.30
rat195.tsp	2323	2349	2448	2395	1.12	5.38	3.10
rd100.tsp	7910	7912	7943	8190	0.03	0.42	3.54
rd400.tsp	15281	15684	16272	16102	2.64	6.49	5.37
si175.tsp	21407	21422	21463	21510	0.07	0.26	0.48
st70.tsp	675	676	675	690	0.15	0.00	2.22
swiss42.tsp	1273	1273	1273	1273	0.00	0.00	0.00
ts225.tsp	126643	126654	127458	128716	0.01	0.64	1.64
tsp225.tsp	3916	3976	4107	4044	1.53	4.88	3.27
u159.tsp	42080	42282	42941	44205	0.48	2.05	5.05
ulysses16.tsp	6859	6859	6859	6860	0.00	0.00	0.01
ulysses22.tsp	7013	7013	7013	7041	0.00	0.00	0.40
ali535.tsp	202339	211954	217611	217607	4.75	7.55	7.55
att532.tsp	27686	28736	29545	29247	3.79	6.71	5.64
brazil58.tsp	25395	36518	25395	25395	43.80	0.00	0.00
brg180.tsp	1950	1964	2012	2178	0.72	3.18	11.69
d657.tsp	48912	51393	52852	51827	5.07	8.06	5.96
d1291.tsp	50801	54485	57344	55645	7.25	12.88	9.54
d1655.tsp	62128	67025	70119	67295	7.88	12.86	8.32
d2103.tsp	80450	83138	86653	83452	3.34	7.71	3.73
dsj1000.tsp	18659688	20039179	20433714	20142317	7.39	9.51	7.95
fl417.tsp	11861	11987	12111	12128	1.06	2.11	2.25
fl1400.tsp	20127	21201	21064	21204	5.34	4.66	5.35
fl1577.tsp	22249	23558	24252	23746	5.88	9.00	6.73
fl3795.tsp	28772	34390	32152	30584	19.53	11.75	6.30
fnl4461.tsp	182566	199718	204450	196648	9.39	11.99	7.71
gr666.tsp	294358	312215	319791	314260	6.07	8.64	6.76
nrw1379.tsp	56638	60460	62354	60951	6.75	10.09	7.62
p654.tsp	34643	37951	35465	36421	9.55	2.37	5.13
pa561.tsp	2763	2880	2886	3067	4.23	4.45	11.00
pcb1173.tsp	56892	61479	63387	61469	8.06	11.42	8.05
pcb3038.tsp	137694	151130	155733	149676	9.76	13.10	8.70
pr1002.tsp	259045	275588	284850	279390	6.39	9.96	7.85
pr2392.tsp	378032	410246	428532	408360	8.52	13.36	8.02
rat575.tsp	6773	7133	7153	7245	5.32	5.61	6.97
rat783.tsp	8806	9344	9250	9390	6.11	5.04	6.63
rl1304.tsp	252948	278657	280329	275084	10.16	10.82	8.75
Average	**253944.1262**	**272756.94**	**277586.89**	**273027.78**	**3.55**	**4.49**	**4.24**

Table 8. *Cont.*

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1	GAP_2	GAP_3
rl1323.tsp	270199	291236	298519	285864	7.79	10.48	5.80
rl1889.tsp	316536	343698	353960	341771	8.58	11.82	7.97
rl5915.tsp	565530	677844	654919	633460	19.86	15.81	12.01
rl5934.tsp	556045	661867	644987	604596	19.03	16.00	8.73
si535.tsp	48450	48588	48769	48812	0.28	0.66	0.75
si1032.tsp	92650	92889	93382	92896	0.26	0.79	0.27
u574.tsp	36905	41429	39874	39865	12.26	8.04	8.02
u724.tsp	41910	44377	45604	44785	5.89	8.81	6.86
u1060.tsp	224094	241290	245211	243817	7.67	9.42	8.80
u1432.tsp	152970	164667	170476	165505	7.65	11.44	8.19
u1817.tsp	57201	62417	65840	61861	9.12	15.10	8.15
u2152.tsp	64253	69701	74338	70656	8.48	15.70	9.97
u2319.tsp	234256	243207	249416	245493	3.82	6.47	4.80
vm1084.tsp	239297	254315	262020	256838	6.28	9.50	7.33
vm1748.tsp	336556	359808	373774	356880	6.91	11.06	6.04
Average	**253944.1262**	**272756.94**	**277586.89**	**273027.78**	**3.55**	**4.49**	**4.24**

Tables 9–12 contain the aggregated experimental results for the National TSP instances. They contain the benchmark name, the optimal value (zOpt), the cost of the three GVNS algorithms (GVNS_1, GVNS_2 and GVNS_3) and the solution gaps from the optimal value for each method. Table 9 depicts GVNS using the First Improvement search strategy and an execution time limit of 1 min. Table 10 shows GVNS using the Best Improvement search strategy and an execution time limit of 1 min. Tables 11 and 12 provide the results achieved by the developed GVNS algorithms with a 2 min time limit within the First and Best improvement search strategy respectively. A notable observation is that in general there are not any significant differences between different methods. However, we notice that on First Improvement for both 1 and 2 min GVNS_3 outperforms GVNS_1 and GVNS_2 implementations. Contrariwise, on Best Improvement all three methods perform better in general than on First Improvement. However, there is no significant difference between them.

Table 9. Results using the First Improvement search strategy and an execution time limit of 1 min.

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1 (%)	GAP_2 (%)	GAP_3 (%)
ar9152.tsp	837479	1648442	1648596	1648596	96.83	96.85	96.85
gr9882.tsp	300899	388910	388944	388944	29.25	29.26	29.26
eg7146.tsp	172387	220232	220315	220315	27.75	27.80	27.80
fi10639.tsp	520527	649604	649604	649604	24.80	24.80	24.80
ho14473.tsp	177105	484571	484812	484812	173.61	173.74	173.74
ei8246.tsp	206171	258851	258889	258889	25.55	25.57	25.57
ja9847.tsp	491924	612157	612304	612304	24.44	24.47	24.47
kz9976.tsp	1061882	1358247	1358247	1358247	27.91	27.91	27.91
lu980.tsp	11340	18708	23688	12236	64.97	108.89	7.90
mo14185.tsp	427377	529729	529729	529729	23.95	23.95	23.95
nu3496.tsp	96132	220359	221920	221920	129.23	130.85	130.85
mu1979.tsp	86891	119104	120908	120908	37.07	39.15	39.15
qa194.tsp	9352	9501	9727	9706	1.59	4.01	3.79
rw1621.tsp	26051	53343	58148	58052	104.76	123.21	122.84
tz6117.tsp	394718	500936	501184	501184	26.91	26.97	26.97
uy734.tsp	79114	84131	86022	83282	6.34	8.73	5.27
wi29.tsp	27603	27603	27603	27681	0.00	0.00	0.28
ym7663.tsp	238314	308477	308747	308747	29.44	29.55	29.55
zi929.tsp	95345	101541	112775	100572	6.50	18.28	5.48
ca4663.tsp	1290319	1646429	1646889	1646889	27.60	27.63	27.63
Average	**327546.50**	**462043.75**	**463452.55**	**462130.85**	**44.43**	**48.58**	**42.70**

Table 10. Results using the Best Improvement search strategy and an execution time limit of 1 min.

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1 (%)	GAP_2 (%)	GAP_3 (%)
ar9152.tsp	837479	1317886	1648596	1648596	57.36	96.85	96.85
gr9882.tsp	300899	368968	388944	388944	22.62	29.26	29.26
eg7146.tsp	172387	206756	220315	220315	19.94	27.80	27.80
fi10639.tsp	520527	625547	649604	649604	20.18	24.80	24.80
ho14473.tsp	177105	362827	484812	459459	104.87	173.74	159.43
ei8246.tsp	206171	246249	258889	258889	19.44	25.57	25.57
ja9847.tsp	491924	595913	612304	612304	21.14	24.47	24.47
kz9976.tsp	1061882	1296253	1358247	1358247	22.07	27.91	27.91
lu980.tsp	11340	12052	12388	12530	6.28	9.24	10.49
mo14185.tsp	427377	514995	529729	529729	20.50	23.95	23.95
nu3496.tsp	96132	108839	108639	108685	13.22	13.01	13.06
mu1979.tsp	86891	93453	95413	93433	7.55	9.81	7.53
qa194.tsp	9352	9468	9717	9791	1.24	3.90	4.69
rw1621.tsp	26051	28360	28866	29127	8.86	10.81	11.81
tz6117.tsp	394718	478074	483082	445901	21.12	22.39	12.97
uy734.tsp	79114	83595	86201	84628	5.66	8.96	6.97
wi29.tsp	27603	27603	27603	27603	0.00	0.00	0.00
ym7663.tsp	238314	291939	308747	308747	22.50	29.55	29.55
zi929.tsp	95345	100461	103557	100474	5.37	8.61	5.38
ca4663.tsp	1290319	1532650	1463565	1428402	18.78	13.43	10.70
Average	**327546.50**	**415094.40**	**443960.90**	**438770.40**	**20.93**	**29.20**	**27.66**

Table 11. Results using the First Improvement search strategy and an execution time limit of 2 min.

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1 (%)	GAP_2 (%)	GAP_3 (%)
ar9152.tsp	837479	1648304	1648596	1648596	96.82	96.85	96.85
gr9882.tsp	300899	388916	388944	388944	29.25	29.26	29.26
eg7146.tsp	172387	220290	220315	220315	27.79	27.80	27.80
fi10639.tsp	520527	649604	649604	649604	24.80	24.80	24.80
ho14473.tsp	177105	484793	484812	484812	173.73	173.74	173.74
ei8246.tsp	206171	258867	258889	258889	25.56	25.57	25.57
ja9847.tsp	491924	612304	612304	612304	24.47	24.47	24.47
kz9976.tsp	1061882	1358246	1358247	1358247	27.91	27.91	27.91
lu980.tsp	11340	20909	23688	14382	84.38	108.89	26.83
mo14185.tsp	427377	529699	529729	529729	23.94	23.95	23.95
nu3496.tsp	96132	221466	221920	221920	130.38	130.85	130.85
mu1979.tsp	86891	119586	120908	120908	37.63	39.15	39.15
qa194.tsp	9352	9543	9811	9731	2.04	4.91	4.05
rw1621.tsp	26051	56293	58148	58148	116.09	123.21	123.21
tz6117.tsp	394718	501137	501184	501184	26.96	26.97	26.97
uy734.tsp	79114	85036	99005	83383	7.49	25.14	5.40
wi29.tsp	27603	27603	27603	27701	0.00	0.00	0.36
ym7663.tsp	238314	308713	308747	308747	29.54	29.55	29.55
zi929.tsp	95345	104572	113927	101795	9.68	19.49	6.76
ca4663.tsp	1290319	1645854	1646889	1646889	27.55	27.63	27.63
Average	**327546.50**	**462586.75**	**464163.50**	**462311.40**	**46.30**	**49.51**	**43.76**

Table 12. Results using the Best Improvement search strategy and an execution time limit of 2 min.

Instance	zOpt	GVNS_1	GVNS_2	GVNS_3	GAP_1 (%)	GAP_2 (%)	GAP_3 (%)
ar9152.tsp	837479	1372411	1648596	1648596	63.87	96.85	96.85
gr9882.tsp	300899	377898	388944	388944	25.59	29.26	29.26
eg7146.tsp	172387	209515	220315	220315	21.54	27.80	27.80
fi10639.tsp	520527	635913	649604	649604	22.17	24.80	24.80
ho14473.tsp	177105	449416	484812	474609	153.76	173.74	167.98
ei8246.tsp	206171	248441	258889	258889	20.50	25.57	25.57
ja9847.tsp	491924	602674	612304	612304	22.51	24.47	24.47
kz9976.tsp	1061882	1327647	1358247	1358247	25.03	27.91	27.91
lu980.tsp	11340	12151	12458	12559	7.15	9.86	10.75
mo14185.tsp	427377	527335	529729	529729	23.39	23.95	23.95
nu3496.tsp	96132	115162	108803	109804	19.80	13.18	14.22
mu1979.tsp	86891	94072	95874	94358	8.26	10.34	8.59
qa194.tsp	9352	9525	9757	9759	1.85	4.33	4.35
rw1621.tsp	26051	28784	29118	29131	10.49	11.77	11.82
tz6117.tsp	394718	475863	501184	501184	20.56	26.97	26.97
uy734.tsp	79114	84585	86358	84686	6.92	9.16	7.04
wi29.tsp	27603	27603	27603	27612	0.00	0.00	0.03
ym7663.tsp	238314	295414	308747	308747	23.96	29.55	29.55
zi929.tsp	95345	101161	104327	100881	6.10	9.42	5.81
ca4663.tsp	1290319	1573320	1646889	1470076	21.93	27.63	13.93
Average	**327546.50**	**428444.50**	**454127.90**	**444501.70**	**25.27**	**30.33**	**29.08**

4. Statistical Analysis on Computational Results

This section presents the statistical tests which were performed on the computational results, to evaluate the performance of the three different GVNS methods. Different statistical tests are applied to different data structures. In particular, statistical analysis methods can be divided on parametric and non-parametric tests. The first category examines normal variables whereas the other methods concern non-normal variables [20].

Initially, the application of a normality test showed that the numerical data does not follow the normal distribution. Consequently, we applied the Kruskal–Wallis test for checking the existence of a statistically significant difference between the methods. In this test receiving a p-value less than 0.05 means that there is a statistically significant difference.

At the present point, it should be mentioned that the statistical analysis was performed on median values to eliminate potential extreme deviation on the average values based on extreme values. Also, the related to the aTSP analysis is taken from our previous work [14] and it is presented here for building a more comprehensive view.

4.1. Statistical Analysis on aTSP Results

In Table 13 we can see that for all cases p-value is less than 0.05, which means that there is a statistically significant difference between the three methods in all cases. For further examination, pairwise Wilcoxon tests were performed.

Table 13. Kruskal–Wallis rank sum test.

	X^2	df	p-Value
FI_1min	6.8689	2	0.0322
FI_2mins	9.0314	2	0.0109
BI_1min	9.2739	2	0.0097
BI_2mins	9.6658	2	0.008

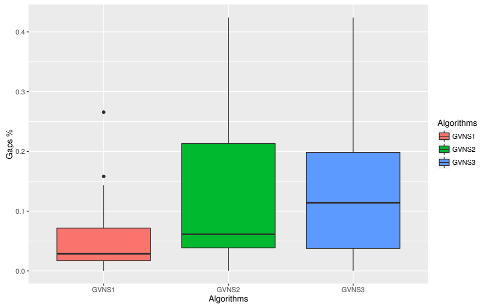
In accordance with the pairwise tests which are summarized in Table 14, it is clear that the GVNS_1 has significant differences with the other two schemes in all cases. Both GVNS_2 and GVNS_3 perform equivalently using the First Improvement search strategy independently of the execution time limit, while using the Best Improvement strategy they have significant reported differences with both time limits [14].

Table 14. KPairwise comparisons using Wilcoxon signed rank test.

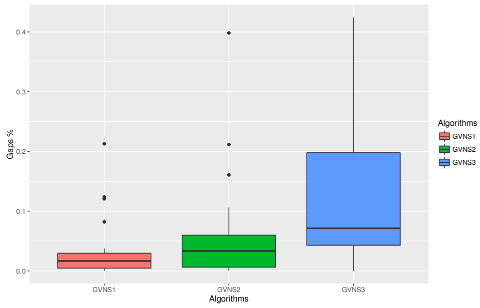
FI_1min		
	GVNS1	GVNS2
GVNS2	0.00064	
GVNS3	0.00064	0.6701
FI_2mins		
	GVNS1	GVNS2
GVNS2	0.00064	
GVNS3	0.00064	0.4488
BI_1min		
	GVNS1	GVNS2
GVNS2	0.00109	
GVNS3	0.00064	0.00064
BI_2mins		
GVNS2	0.00109	
GVNS3	0.00064	0.00064

In respect to this Kruskal–Wallis statistical analysis, we have four box-plots illustrated in Figures 1a,b and 2a,b four box plots. Each one depicts either First Improvement or Best Improvement for one minute, as well as for two minutes runs.

Moreover, based on the corresponding of the previous statistical summary, box plots, it is confirmed that the GVNS_1 produces much better results than the other two algorithms in all cases for the aTSP, while the GVNS_2 outperform the GVNS_3 in also all cases. Also, by checking the medians at the box plots it can be seen that the GVNS_2 performs significantly better on Best Improvement, as it produces results that are “close" to the results of GVNS_1.

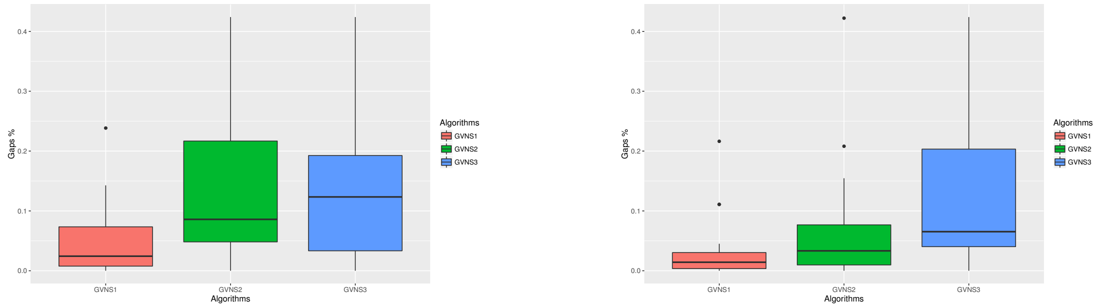


(**a**)



(**b**)

Figure 1. Statistical test for aTSP (1/2). (**a**) Statistical test box plots for aTSP 1min FI; (**b**) Statistical test box plots for aTSP 1min BI.



(**a**) (**b**)

Figure 2. Statistical test for aTSP (2/2). (**a**) Statistical test box plots for aTSP 2mins FI; (**b**) Statistical test box plots for aTSP 2mins BI.

4.2. Statistical Analysis on sTSP

In this subsection the statistical analysis on the results achieved by the three GVNS schemes on sTSP instances is provided.

According to the values in Table 15 we can see that only using the Best Improvement search strategy within a 2 min execution time limit, there are statistically significant differences.

Table 15. Kruskal–Wallis rank sum test.

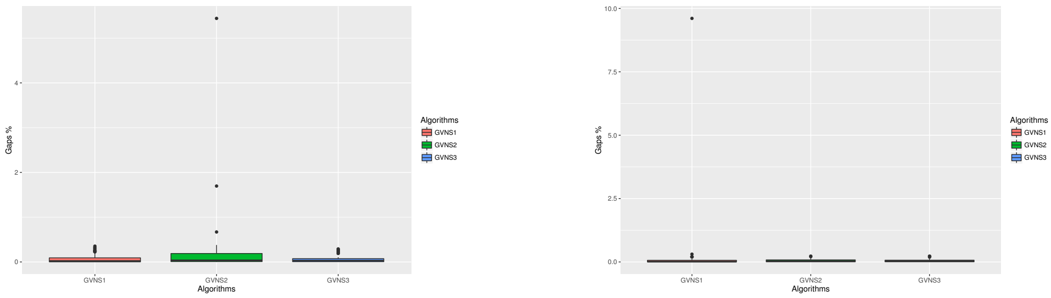
	X^2	df	*p*-Value
FI_1min	2.4392	2	0.2954
FI_2min	4.5181	2	0.1045
BI_1min	5.5397	2	0.06267
BI_2min	11.677	2	0.002913

More specifically, the values in Table 16 highlights that there is a difference between the GVNS_1 and the other two GVNS algorithms. In particular, based on the following box plots, the GVNS_1 is slightly better than the GVNS_2 and GVNS_3, which they perform almost equivalently.

Table 16. KPairwise comparisons using Wilcoxon signed rank test.

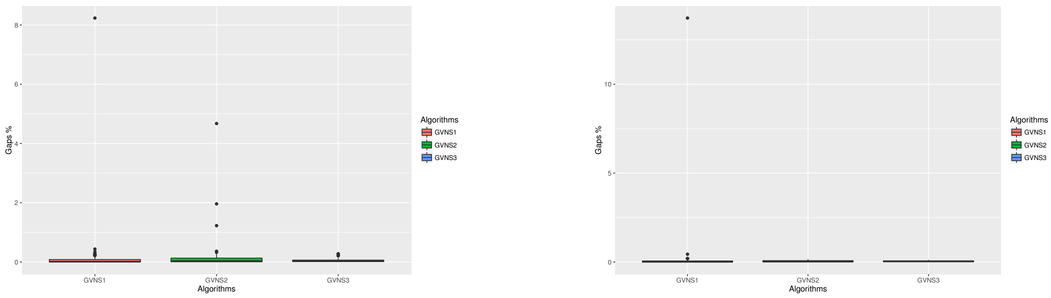
BI_2min		
	GVNS1	GVNS2
GVNS2	0.0000000990	
GVNS3	0.0000002300	0.6800000000

Subsequently of this Kruskal–Wallis statistical analysis, we have four box-plots illustrated in Figures 3a,b and 4a,b four box plots. Each one depicts either First Improvement or Best Improvement for one minute, as well as for two minutes runs.



(**a**) (**b**)

Figure 3. Statistical test for sTSP (1/2). (**a**) Statistical test box plots for sTSP 1min FI; (**b**) Statistical test box plots for sTSP 1min BI.



(**a**) (**b**)

Figure 4. Statistical test for sTSP (2/2). (**a**) Statistical test box plots for sTSP 2min FI; (**b**) Statistical test box plots for sTSP 2min BI.

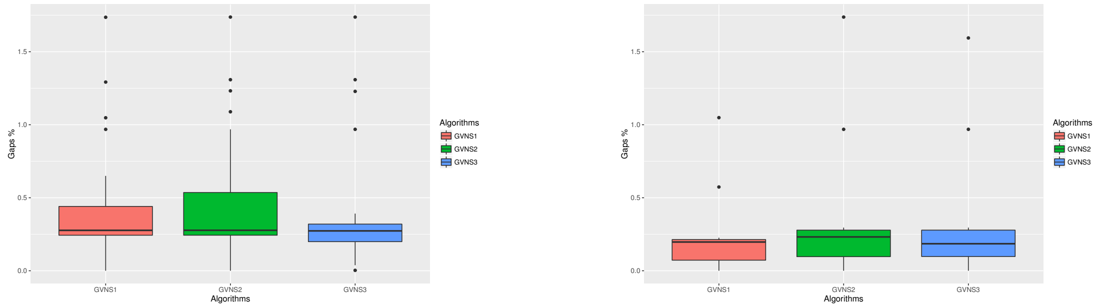
4.3. Statistical Analysis on nTSP

In the case of the National TSP instances and based on the values given in Table 17 we can see that there is no significant statistical difference between the three methods.

Table 17. Kruskal–Wallis rank sum test.

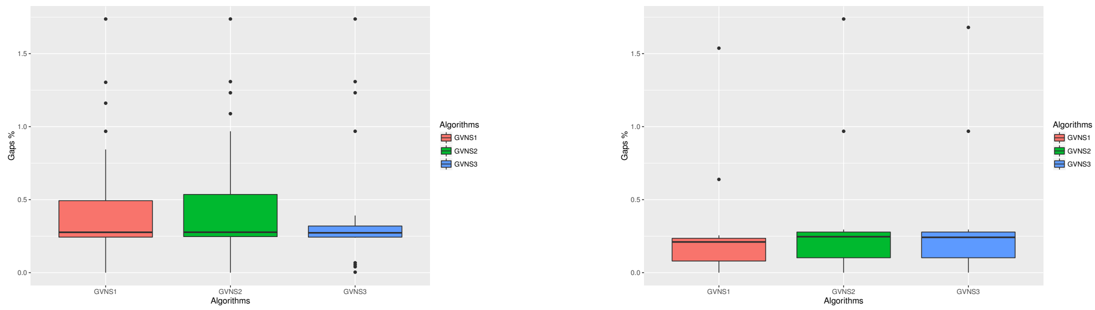
	X^2	df	p-Value
FI_1min	0.22253	2	0.8947
FI_2min	0.18068	2	0.9136
BI_1min	1.825	2	0.4015
BI_2min	1.9646	2	0.3744

As a result of this Kruskal–Wallis statistical analysis, we have four box-plots illustrated in Figures 5a,b and 6a,b four box plots. Each one depicts either First Improvement or Best Improvement for one minute, as well as for two minutes runs. By checking the median value of each method, it is clear that the three algorithms perform equivalently.



(**a**) (**b**)

Figure 5. Statistical test for nTSP (1/2). (**a**) Statistical test box plots for nTSP 1min FI; (**b**) Statistical test box plots for nTSP 1min BI.



(**a**) (**b**)

Figure 6. Statistical test for nTSP (2/2). (**a**) Statistical test box plots for nTSP 2min FI; (**b**) Statistical test box plots for nTSP 2min BI

5. Comparison with Recent Similar Works

In their recent work, Halim et al. have presented an extensive analysis over the performance of different heuristic and metaheuristic algorithms on some TSPLib instances [21]. In addition, Hore et al. [22] proposed an improved hybrid VNS algorithm for solving TSP instances. In this section, a comparison between our proposed GVNS schemes (GVNS_1 and GVNS_2) and those algorithms presented on papers [21,22] is performed.

Table 18 shows the comparison between our GVNS_1 and GVNS_2, within the Best Improvement search strategy and a time limit of 120 s, with all metaheuristic solution approaches presented in the work of Halim et al. [21]. The results show that our methods produce better results than the previously mentioned metaheuristics, except the case of instance rat195.tsp in which the GA and the TS perform better than GVNS_2.

Table 18. Comparisons between GVNS_1 and GVNS_2 with BI for 2mins with the recent work of Halim et al.

Instance	OV	GVNS_1	GVNS_2	GA	SA	TS	ACO	TPO
eil51.tsp	426	426	426	454.1	439.13	439.1	467.46	437.26
berlin52.tsp	7542	7542	7542	7946.4	7960.67	7740.1	7922.32	7705.8
st70.tsp	675	676	675	700.72	696.33	690.27	756.55	697.12
kroA100.tsp	21282	21282	21312	22726.2	22277.5	22521.64	22941.68	22463.6
ch130.tsp	6110	6137	6208	6610.8	6558.7	6717.06	6913.99	6515.28
rat195.tsp	2323	2349	2448	2414.52	2537.99	2373.94	2465.11	2573.47
a280.tsp	2579	2630	2725	2789.83	2830.18	2800.79	2867.85	2790.54
rd400.tsp	15281	15684	16272	16567.29	16816.65	20723.56	19259.06	18190.84
pcb442.tsp	50778	52381	54108	55718.9	57421.04	83123.01	63436.7	60750.43

Table 19 shows the comparison between our GVNS_1 and GVNS_2, within the Best Improvement search strategy and a time limit of 120 s with the results obtained by a hybrid VNS in the second mentioned work [22]. It is clearly observed that our methods outperform the hybrid VNS. More specifically, the GVNS_1 outperforms the hybrid VNS algorithm in all tested instances, while the GNVS_2 produce better results than those achieved by Hore et al. approach in seven out of 10 problem instances.

Table 19. Comparisons between GVNS_1 and GVNS_2 with BI for 2 mins with the recent VNS work of Hore et al.

Instance	OV	GVNS_1	GVNS_2	Average	Average Time
eil51.tsp	426	426	426	428.98	454.1
berlin52.tsp	7542	7542	7542	7544.36	7946.4
st70.tsp	675	676	675	677.11	700.72
kroA100.tsp	21282	21282	21312	21695.79	22726.2
ch130.tsp	6110	6137	6208	6153.72	6610.8
rat195.tsp	2323	2349	2448	2453.81	1382.34
rd400.tsp	15281	15684	16272	16250.21	1953.49
pcb1173.tsp	56892	61479	63387	63435.95	9531.54
pcb442.tsp	50778	52381	54108	50800.24	2183.27

Table 20 shows the abbreviations regarding the metaheuristic algorithms presented in [21].

Table 20. Abbreviations.

GA	Genetic Algorithm
SA	Simulated Annealing
TS	Tabu Search
ACO	Ant Colony Optimization
TPO	Tree Physiology Optimization

6. Conclusions and Future Work

A thorough and comprehensive performance analysis on the efficiency of three GVNS algorithms has been presented in this work. The main difference lies on the perturbation strategy used. Our comparative performance analysis involves problems that are modelled as asymmetric and Symmetric TSP instances that are resolved using GVNS. The well-known TSP benchmarks from the TSPLIB are used for extensive experimental testing. We believe that the experimental results are quite conclusive, as they confirm that for asymmetric TSP instances GVNS_1 outperforms the other two methods and GVNS_2 consistently provides better solutions in all cases compared to GVNS_3. Simultaneously, the perturbation strategy does not seem to critically affect the solutions of Symmetric TSP instances.

It is also worth emphasizing that even though the improvement stage of the GVNS schemes has been given limited attention, the present paper shows that the tested approaches to the solution, can be quite promising. This is justified by the solutions produced, which are significantly better than other metaheuristic approaches in recent literature.

The investigation of alternative neighborhood structures and neighborhood change movements in VND under the GVNS framework could be a possible direction for future work. In the same vein, one might potentially study modifications or specific combinations with more than one perturbation strategy during the perturbation phase in an effort to determine whether optimal solutions can be achieved even closer, especially on large asymmetric benchmarks.

Author Contributions: All of the authors have contributed extensively to this work. C.P. and P.K. conceived the initial algorithm and worked on the first prototypes. P.K. and C.P. thoroughly analyzed the current literature

gathering all the necessary material. T.A. assisted C.P. in designing the methods used in the main part. T.A. was responsible for supervising the construction of this work. C.P. was responsible for the interlinking between the theoretic model and the actual application. C.P. contributed to the appropriate typing of the formal definitions and the math used in the paper.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

TSP	Traveling Salesman Problem
VNS	Variable Neighborhood Search
GVNS	General Variable Neighborhood Search
GA	Genetic Algorithm
SA	Simulated Annealing
TS	Tabu Search
ACO	Ant Colony Optimization
TPO	Tree Physiology Optimization

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