plasma

MDPI

Article

Effects of Spiralling Trajectories on White Dwarf Spectra: Remarks on Different Calculations

Spiros Alexiou

Hellenic Army Academy, Varis-Koropiou Avenue, 16673 Vari, Greece; moka1@otenet.gr

Abstract: The purpose of this paper is to address conflicting results regarding a simple criterion that has been proposed as decisive in determining whether accounting for spiralling electron trajectories increases or decreases the widths of hydrogen lines in a parameter range relevant to the spectral lines of white dwarfs. We analyse the claims in detail and also provide explicit calculations. It is shown that the recent attempts to justify a simple theory are erroneous and miss important physics.

Keywords: Stark broadening; external magnetic field; quadratic Zeeman effect; line narrowing

1. Introduction

The purpose of this paper is to correct some confusing remarks [1,2] regarding the effects of accounting for spiralling trajectories in calculations of the Stark broadening of hydrogen lines in conditions relevant to white dwarf spectra. Stark broadening has been used for white dwarf diagnostics, modelling the complex Stark and Zeeman effect to infer plasma and magnetic field parameters, for example, in [3,4]. In [1], it was suggested that "for the range of plasma parameters typical for DA white dwarfs (i.e., for white dwarfs emitting hydrogen lines), the neglect for the actual, helical trajectories of perturbing electrons can lead to the following:the overestimation of the Stark width by up to one order of magnitude for the alpha- and beta-lines of the Lyman and Balmer series; the underestimation of the Stark width by several times for the delta- and higher-lines of the Balmer series". However, calculations [5] using the Collision-time statistics method, i.e., ensuring that *all* collisions that become relevant during the autocorrelation function lifetime are correctly accounted for, did not verify this assertion. It was subsequently suggested [2] that this was due to the calculations performed at a rather low (but reasonable for white dwarf spectra) electron density. In this paper, we first present calculations for the parameters claimed in [2] to be applicable and then review [1] and point out the assumptions that are problematic. To keep the analysis on a level compatible with the other assumptions in [1,2], we consider hydrogen atoms without fine structure, without strong magnetic field effects, with only dipole interactions (excluding penetrating collisions and Debye screening), and unaffected by the magnetic field or the spiralling trajectories. We also do not consider effects that can be important for white dwarfs, such as the modification of the atomic structure in strong B-fields or the motional Stark effect. Although references [1,2] discuss modifications to *electron* trajectories, they make predictions about line widths, which are also influenced by ions. Ions dominate lines such as H_β and H_δ at $B = 0$. As explained in detail in [5], when Zeeman splitting is taken into account, ions have a much smaller effect for nonzero B if spiralling is *not* taken into account, and are typically negligible if spiralling *is* taken into account. This is illustrated again in Figure 1, which shows that the reduction in *electron* broadening between the nonspiralling and spiralling results is actually *smaller* than the

check for updates

Academic Editor: Andrey Starikovskiy

Received: 3 December 2024
Revised: 23 December 2024
Accepted: 27 December 2024
Published: 3 January 2025

Citation: Alexiou, S. Effects of Spiralling Trajectories on White Dwarf Spectra: Remarks on Different Calculations. *Plasma* **2025**, *8*, 2. https://doi.org/10.3390/plasma8010002

reduction in the *ionic* contribution. Specifically, Figure 1 shows the lines for the σ and π profiles: (a) the profile with electrons and ions without spiralling (solid for π and dotted for σ), (b) the profile with electrons only without spiralling (blue for π and green for σ), and (c) the profile with electrons and ions with spiralling (dashed for π and red for σ). The profiles with electrons only and spiralling are not displayed, as they coincide with the electron plus ion broadening with spiralling.

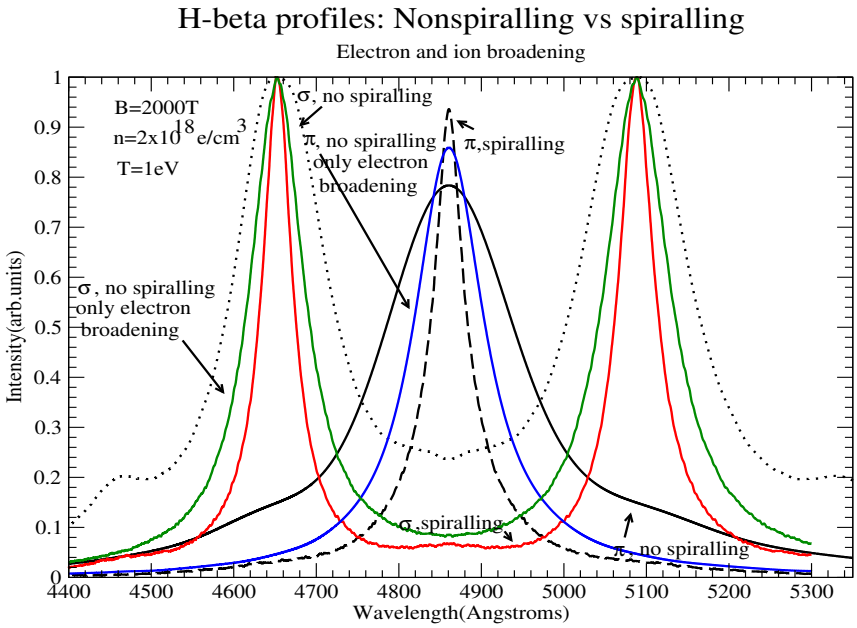

Figure 1. H_β calculation with and without account of spiralling and with and without account of ion broadening for a hydrogen plasma of electron density 2×10^{18} e/cm^3, temperature 1 eV, and magnetic field of 2000 T. Shown are the following profiles: (a) π profile with electrons and ions and no spiralling (solid); (b) σ profile with electrons and ions and no spiralling (dotted); (c) π profile with electrons and ions and spiralling (dashed); (d) σ profile with electrons and ions and spiralling (red); (e) π profile with electron broadening only and no spiralling (blue); and (f) σ profile with electron broadening only and no spiralling (green). The spiralling profiles with electron broadening only and spiralling are not shown, as they practically coincide with the spiralling results with both electrons and ions participating in broadening. This illustrates that spiralling ions are quite ineffective in broadening.

2. An Example

Ref. [1] introduces the (magnetic field-independent) dimensionless parameter

$$D = 5.57 \times 10^{-11} |X_{\alpha\beta}| [N_e(\mathrm{cm}^{-3})]^{1/2} / T_e(\mathrm{eV}) \quad (1)$$

and claim a significant increase in broadening for the spiralling calculations compared to the nonspiralling ones for $D > 0.44$.

To illustrate, we show calculations described in [5] for the H_ϵ line for an electron density of 4×10^{18} e/cm^3, a temperature of 1 eV, and a magnetic field of 2000 T. At these parameters, the dimensionless parameter D for the strongest Stark component $|X_{\alpha\beta}| = 14$ [2] is $D = 1.55 > 0.44$, so it is well above the limit of $D = 0.44$. $X_{\alpha\beta}$ is defined in terms of the principal (n) and parabolic quantum numbers n_1 and n_2 of the upper (α) and lower (β) levels as follows:

$$X_{\alpha\beta} = n_\alpha (n_1 - n_2)_\alpha - n_\beta (n_1 - n_2)_b eta \quad (2)$$

According to [1,2], the spiralling results should display a significant increase in width compared to the corresponding nonspiralling calculations. Figure 2 shows that this is not the case: the spiralling profiles are markedly *narrower* than the nonspiralling ones.

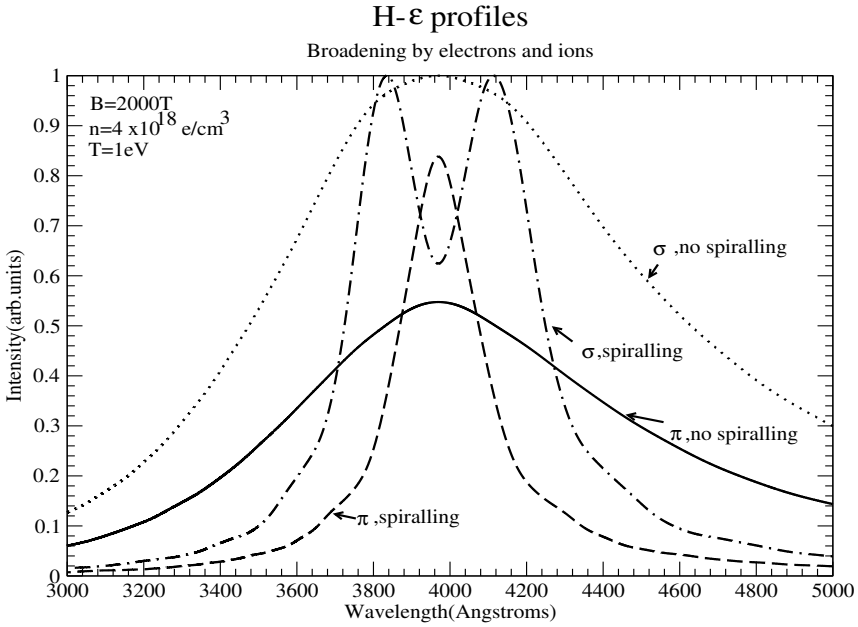

Figure 2. H_ϵ calculation with and without account of spiralling with account of both electron and ion broadening for a hydrogen plasma of electron density 4×10^{18} e/cm^3, temperature 1 eV, and magnetic field of 2000 T. Shown are the following profiles: (a) π profile with no spiralling (solid); (b) σ profile with no spiralling (dotted); (c) π profile with spiralling (dashed); and (d) σ profile with and spiralling (dash-dotted).

As discussed in the introduction, we can also compute the profiles without accounting for ionic broadening. The results are in Figure 3 with nonspiralling trajectories and Figure 4 with spiralling trajectories. As already seen in Figure 1, the ion contribution is very small in the spiralling trajectory calculation, but substantial and comparable to the electronic contribution in the nonspiralling case.

For clarity, we also show the electron-only profiles with and without spiralling. It is clear that spiralling causes a significant *reduction* in the (electron-only) widths, in contrast to the assertions in [1,2]. Figure 5 compares the nonspiralling and spiralling calculations, in both cases neglecting ions altogether, in order to compare the pure electron results, as refs. [1,2] deal with electrons alone.

Thus, both for the total profile and the electron-only contributions, spiralling results in a much *narrower* profile for D = 1.55, in contrast to [1,2], which predict a marked *increase* in the width. Note that we deliberately did *not* account for penetrating collisions in order to be at the same level as refs. [1,2]. In the following section, we analyse the development of the prediction of [1].

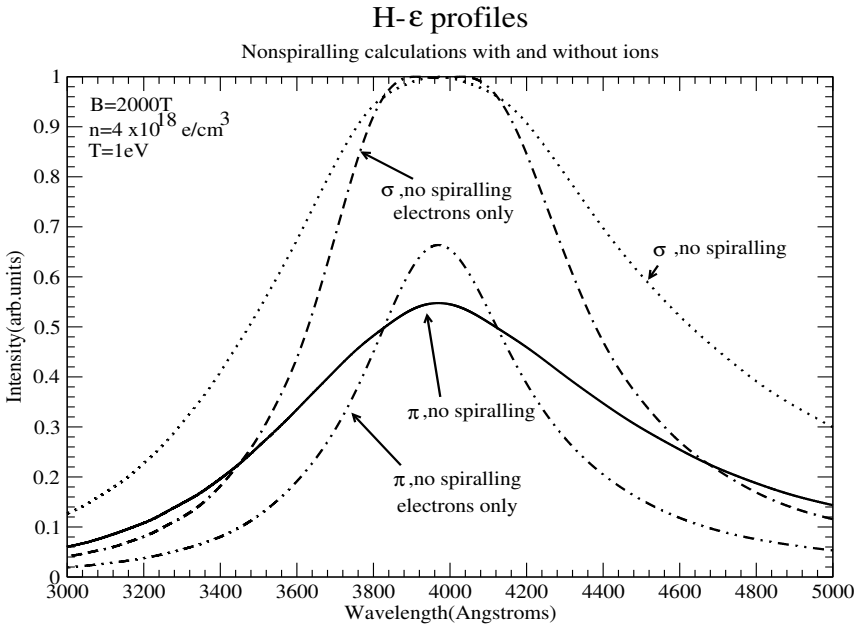

Figure 3. H_ϵ calculation with nonspiralling trajectories and with or without account of ion broadening for a hydrogen plasma of electron density 4×10^{18} e/cm^3, temperature 1 eV, and magnetic field of 2000 T. Shown are the following profiles: (a) π profile with electrons and ions (solid); (b) σ profile with electrons and ions (dotted); (c) π profile with electrons only (dashed); and (d) σ profile with electrons only (dash-dotted).

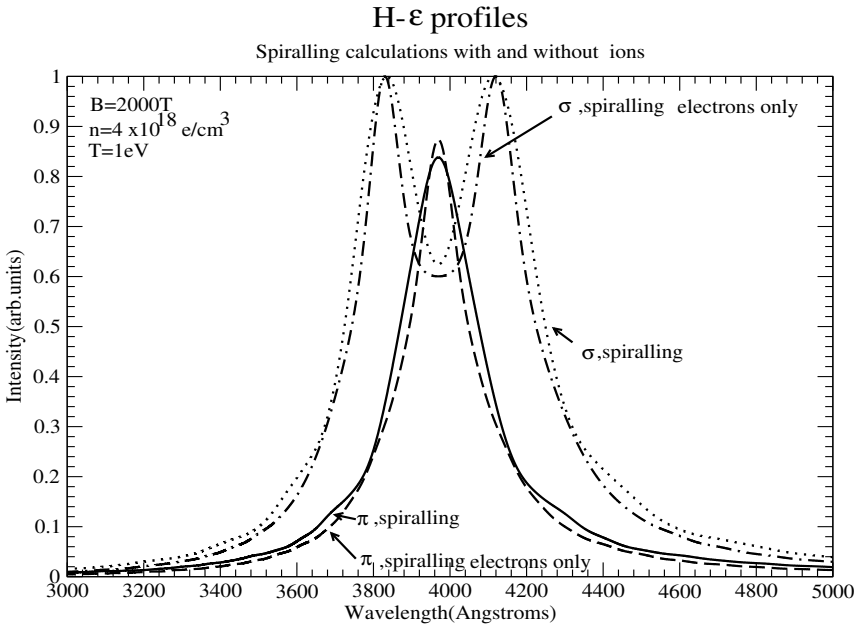

Figure 4. H_ϵ calculation with spiralling trajectories and with or without account of ion broadening for a hydrogen plasma of electron density 4×10^{18} e/cm^3, temperature 1 eV, and magnetic field of 2000 T. Shown are the following profiles: (a) π profile with electrons and ions (solid); (b) σ profile with electrons and ions (dotted); (c) π profile with electrons only (dashed); and (d) σ profile with electrons only (dash-dotted).

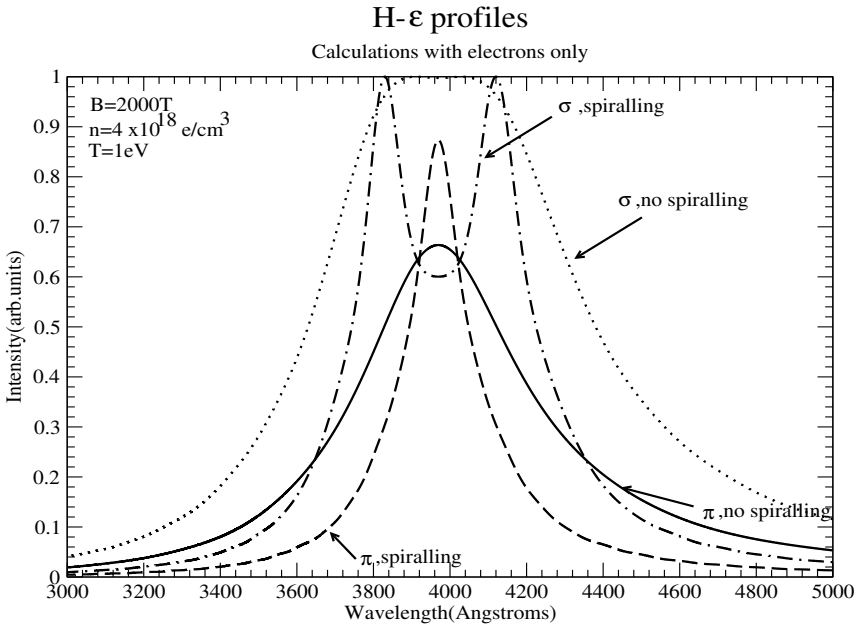

Figure 5. H_ϵ calculation without account of ion broadening with and without spiralling trajectories for a hydrogen plasma of electron density 4×10^{18} e/cm^3, temperature 1 eV, and magnetic field of 2000 T. Shown are the following profiles: (a) π profile without spiralling (solid); (b) σ profile without spiralling (dotted); (c) π profile with spiralling (dashed); and (d) σ profile with spiralling (dash-dotted).

3. Analysis of Ref. [1]

Before we embark on an analysis of ref. [1], we point out that, as already shown in [5] and also in the example calculation above, ions are also affected by spiralling and their effect on the final width differences between spiralling and nonspiralling calculations can be at least as large as that of the electrons. This point was discussed at length in [5], but is not addressed in either [1] or [2].

Ref. [1] starts by reviewing the Standard Theory of Impact broadening (which is relabelled "Conventional" (CT)). Starting with the impact parameter integral, ref. [1] correctly points out the need for cutoffs: at the lower impact parameter, in order to satisfy unitarity, if a perturbative expression is used. Perturbation theory is used in [1], but not in [5]. Next, the upper cutoff is taken up. It is well known that a cutoff corresponding to the shielding length (v/ω_p) with v the electron velocity and ω_p the electron plasma frequency is used. The reason for this upper cutoff is plasma screening. In standard Stark broadening calculations, the maximum impact parameter is chosen with an *unscreened* interaction. However, the picture is different when spiralling is taken into account [6,7]. In the example shown, the Debye length is about 3.7 nm, with the Larmor radius being 1.69 nm. An electron at an impact parameter of say 2 nm will spend a substantial amount of time outside the screening length. This fact is not accounted for in the treatment of [1,2] and results in a smaller width. The screening cutoff is augmented in [1] by the adiabatic cutoff $v/\Delta\omega$, with $\Delta\omega$ the energy difference between the states connected by collisionally induced transitions. The point is that, because of the imaginary exponentials in the perturbative expansion for the time evolution, (U-matrix) experiences significant cancellations from impact parameters larger than $v/\Delta\omega$ and negligible contributions. So the nonadiabatic (perpendicular to the magnetic field) part of the interaction becomes ineffective, as it involves $\Delta\omega$ of the Zeeman splittings, while the adiabatic (parallel to the magnetic field) component of the interaction involves $\Delta\omega = 0$ and hence suffers no such damping [5]. So [1] concentrates on the differences in the adiabatic contribution between spiralling and nonspiralling calculations. We recall one of the results in [5]:

"Furthermore, the nonadiabatic contribution is typically negligible; however, strong B effects may produce components with small energy separations from perturbing states, for which the nonadiabatic contribution may be quite important". This is also *not* addressed in either [1] or [2], even though fields up to 10^5 T are considered.

4. Conclusions

This work set out to clarify the situation and showed the following:

1. Ion broadening is significantly reduced due to linear Zeeman splitting, resulting in electron broadening dominating these lines. Furthermore, the nonadiabatic contribution is typically negligible; however, strong B effects may produce components with small energy separations from perturbing states, for which the nonadiabatic contribution may be quite important [5], an issue ignored in [1,2].
2. Spiralling further reduces the ionic contribution, typically drastically.
3. Spiralling reduces the line widths. Refs. [1,2] fail to address the issue of spiralling trajectories which spend a significant time outside the screening (Debye) length.
4. For the lines and parameter ranges considered in [1,2], both penetrating collisions and nonimpact electron effects can be significant.
5. Exact calculations taking into account screening for $D > 0.44$ show a marked decrease in broadening, in contrast to the claims of [1,2].

The physics responsible for the width reduction due to spiralling trajectories was discussed in ref. [6]: Briefly, the two main differences due to spiralling are, first, that more electron perturbers contribute, e.g., electrons with impact parameters up to the shielding length R_{max} plus a Larmor radius r_L, but on the other hand, this contribution is only partial and accompanied by a reduction in the contribution of perturbers between $R_{max} - r_L$ and R_{max}. Second, fewer electrons participate because the axial dimension of the collision volume is smaller, as the average of a two-dimensional Maxwellian distribution is smaller than the average of a three-dimensional distribution. Refs. [1,2] acknowledge a Debye shielding, as standard broadening theories do, in order to avoid a divergence at large impact parameters, and consider collisions with impact parameters smaller than the Debye length and an unshielded interaction. However, when trajectories spiral, such electrons can find themselves interacting beyond the Debye length with—according to refs. [1,2]—an *unshielded* interaction. This, of course, overestimates the broadening.

Again, we note the assumption spelled out in the introduction, introduced to make sure that predictions are compared under the same conditions. In summary, in contrast to the statement in [2] that "This situation is yet another demonstration of the superiority of analytical results over simulations: the analytical results are valid for a broad range of the electron density, while the simulations from paper [43] (Ref. [5] of the present paper) were performed for only one value of the electron density", the reality is that the "analytical" results both miss important physics and would have a very limited range of validity (such as strong B fields enabling nonadiabatic contributions). Ref. [5] instead uses a consistent, analytic modelling of spiralling [6,7] and relies on the analytic modelling of spiralling, including all relevant particles and interactions. We emphasize again that the purpose of this work was only to correct some errors in the literature and not to present a full discussion of all effects relevant to broadening in the presence of a magnetic field. Ref. [5] considers much more, while even there, effects such as the motional Stark effect or quantum effect [8] are left out.

Funding: This research received no external funding.

Data Availability Statement: Data available upon request.

Conflicts of Interest: The author declares no conflicts of interest.

References

1. Oks, E. Effect of Helical Trajectories of Electrons in Strongly Magnetized Plasmas on the Width of Hydrogen/Deuterium Spectral Lines: Analytical Results and Applications to White Dwarfs. *Int. Rev. At. Mol. Phys.* **2017**, *8*, 61–72.
2. Oks, E. O4 -Symmetry-Based Non-Perturbative Analytical Calculations of the Effect of the Helical Trajectories of Electrons in Strongly Magnetized Plasmas on the Width of Hydrogen/Deuterium Spectral Lines. *Symmetry* **2024**, *16*, 1009. [CrossRef]
3. Rosato, J.; Kieu, N.; Meireni, M.; Koubiti, M.; Marandet, Y.; Stamm, R.; Kovacević-Dojcinovic, J.; Dimitrijevic, M.S.; Popovic, L.C.; Simic, Z. A new analysis of spectral line shapes in white dwarf atmospheres. *J. Phys. Conf. Ser.* **2019**, *1289*, 012006–012008. [CrossRef]
4. Rosato, J.; Kieu, N.; Hannachi, I.; Koubiti, M.; Marandet, Y.; Stamm, R.; Dimitrijevic, M.S.; Simic, Z. Stark-Zeeman Line Shape Modeling for Magnetic White Dwarf and Tokamak Edge. *Atoms* **2017**, *5*, 36. [CrossRef]
5. Alexiou, S. Effects of Spiralling Trajectories on White Dwarf Spectra: High Rydberg States. *Atoms* **2023**, *11*, 141. [CrossRef]
6. Alexiou, S. Line Shapes in a Magnetic Field: Trajectory Modifications I: Electrons. *Atoms* **2019**, *7*, 52. [CrossRef]
7. Alexiou, S. Line Shapes in a Magnetic Field: Trajectory Modifictions II: Full Collision-Time Statistics. *Atoms* **2019**, *7*, 94. [CrossRef]
8. Gomez, T.; Zammit, M.C.; Fontes, C.J.; White, J.R. A Quantum-mechanical Treatment of Electron Broadening in Strong Magnetic Fields. *Astrophys. J.* **2023**, *951*, 143. [CrossRef]