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All Lives Matter: A Model for Resource Allocation to Fire Departments in Portugal

Milad K. Eslamzadeh [1,*], António Grilo [1,2] and Pedro Espadinha-Cruz [1,2]

[1] UNIDEMI, Department of Mechanical and Industrial Engineering, NOVA School of Science and Technology, Universidade NOVA de Lisboa, 2829-516 Caparica, Portugal
[2] Laboratório Associado de Sistemas Inteligentes (LASI), 4800-058 Guimarães, Portugal
* Correspondence: s.eslamzade@campus.fct.unl.pt

Abstract: Optimizing Resource Allocation in Fire Departments (RAFD) is crucial for enhancing Fire Protection Services (FPS) and ultimately saving lives. Efficient RAFD ensures that fire departments have the necessary resources to respond effectively to emergencies. This paper presents a method for optimizing RAFD based on performance assessment results, examining its impact on Fire Department (FD) efficiency in Portugal. Evaluating data from 353 FDs, two RAFD optimization methods were assessed: one adhering to Portuguese regulations and constraints, such as budget allocation limitations, and another without such constraints. Integrating a slack-based data envelopment analysis model and mixed-integer linear programming, the study found that incorporating FD efficiency scores in RAFD improved overall efficiency at national, district, and FD levels. While adherence to Portuguese regulations led to balanced resource allocation and a 4% performance improvement at the national level, relaxing constraints yielded an 8% improvement, albeit with potential performance deterioration in some FDs. The detailed budget and efficiency metric analysis provided in this paper offers actionable insights for fire protection services enhancement. This underscores the importance of diverse optimization strategies to enhance FD efficiency, with implications for decision-makers at the Portuguese National Authority for Emergency and Civil Protection and similar organizations globally.

Keywords: resource allocation; performance assessment; fire protection services; agent-based modeling; mixed-integer linear programming

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1. Introduction

Providing efficient Fire Protection Services (FPS), whose main objectives are to reduce the number of fire incidents and casualties, has always been an important part of public management. It has been in the spotlight due to its indisputable importance to the safety of both people and the environment [1]. Hence, ensuring the perpetual assessment of Fire Department (FD) performance is a compelling imperative for nations and local governing bodies [2,3]. This responsibility assumes a formidable dimension for decision-makers within FPS, owing to the dynamic and evolving nature of societies and the limited financial and technical resources for FPS.

Concurrently, the challenge of providing efficient FPS is rooted in the critical role of Resource Allocation in Fire Departments (RAFD) and the strategic placement of fire stations [4,5]. Therefore, allocating limited resources among FDs to provide efficient FPS in minimum time and cover maximum demands becomes a pivotal goal for FDs, and achieving this goal is contingent upon the judicious selection of RAFD methods and variables [6–8].

Due to the essential role of RAFD in improving the performance of FDs in allocating the limited FPS resources (e.g., budget, firefighters, fire engines) among the FDs, the RAFD has been studied at the local or national level [9]. Some of the studies resulted in providing national RAFD models for different countries or cities (e.g., see research about the RAFD methods in Chile [10], Taiwan [11], the UK [12], China [13], India [14], the US [15], South Korea [16], Iran [7], Ukraine [17], and Canada [18]).

To obtain the state of the art in this field, some of the recent RAFD literature in different countries is reviewed in this section. A recent study in Chile [10] used Integer Programming (IP)—one of the most common RAFD models [9]—and Geographic Information System (GIS) analysis to optimize the number of FPS vehicles and their locations based on maximizing the coverage of expected FPS demand in FDs. Balancing the ratio of firefighters and populations of the cities in Thailand to provide more efficient FPS was the objective of a study [11] that applied an omit resource approach for human resource allocation and Data Envelopment Analysis (DEA) for FPS performance assessment

Like other European countries, the FPS authority in Portugal—the National Authority for Emergency and Civil Protection (ANEPC))—faces challenges in delivering efficient fire protection services promptly [4,19,20]. This task of ensuring timely FPS delivery is a common challenge for the ANEPC, reflecting broader concerns shared by nations dealing with the complexities of urban fire management; to address it, multiple studies have been conducted in Portugal, including on optimizing the locations of the Portuguese fire stations [20], a model for the Fire Department Performance Assessment (FDPA) in Portuguese FDs [8], analyzing the urban fire in Portugal [19], assessing and mitigating fire risk in the Portuguese cities [21,22], urban resilience measures in Portuguese districts [23], and characterizations on the urban fire incidents in Portuguese cities [24].

However, to the best of our knowledge, there is no research on Resource Allocation in Fire Departments (RAFD) in Portugal that provides a model for allocating constrained financial and technical resources of the ANEPC among the Portuguese FDs. Therefore, the objective of this research is to develop an RAFD model for Portugal that improves the performance of Portuguese FDs in urban and residential areas.

Portuguese Law for RAFD

According to the publicly published law N.94/2015 in Portugal [25], the referenced budget (RB) of the ANEPC in each year should be allocated to the PT FD_i ($i \in I = [1, \ldots, N]$) based on Formula (1):

$$\begin{aligned} \text{RB}_{FDi} = {} & 10\% RB\left(\frac{1}{N}\right) + 20\% RB\left(\frac{Covered\_Area_{FDi}}{Total\_Covered\_Areas}\right) + 20\% RB\left(\frac{Population_{FDi}}{Total\_Population}\right) \\ & + 20\% RB\left(\frac{Risk\_Index_{FDi}}{Total\_Risk\_Index}\right) + 10\% RB\left(\frac{Fire\_Incidents_{FDi}}{Total\_Fire\_Incidents}\right) \\ & + 10\% RB\left(\frac{Firefighters_{FDi}}{Total\_Firefighters}\right) \end{aligned} \quad (1)$$

While the Portuguese RAFD law aligns with the general RAFD framework by providing coefficients for the socioeconomic and spatiotemporal characteristics of PT FDs [9], it does not incorporate the performance of the fire departments into any aspect of the formula. Given the significance of FDPA within the RAFD context [2,4,13,14], the research question endeavors to ascertain whether integrating FDPA results into RAFD in Portugal will enhance the efficiency of the FDs.

While past research in the US has looked at RAFD on a statewide level [15], one of the contributions of this study is that it takes a broader perspective by exploring a nationwide FD-level RAFD and confirming its positive impact on FD performance. This sets the current study apart by offering a detailed understanding of RAFD dynamics at different levels. This study also significantly contributes to the management and distribution of limited resources among Portuguese FDs, particularly their financial resources. The aim is to enhance FD performance in delivering FPS. The RAFD model and findings of this study can assist FPS decision-makers in improving FD performance by facilitating efficient resource allocation. This, in turn, helps FDs more efficiently use financial resources to safeguard lives and properties. Additionally, the findings can inform investigations into optimizing human resources, vehicles, and the size and location of FDs and fire stations to improve overall performance.

Moreover, this study provides valuable insights for policymakers and analysts in the field of fire protection and safety, offering strategies to mitigate economic costs and safeguard civilian and firefighters' lives. The findings serve as valuable references for governments and governmental organizations, aiding them in making informed budgetary and policy decisions related to fire protection.

A significant contribution of this study is the adaptation of the general RAFD framework to the Portuguese context. This adaptation involves utilizing available and accessible data categories specific to Portugal and employing advanced methodologies such as Agent-Based Modeling (ABM) [26] to simulate an important input variable, response time. Furthermore, this study incorporates the value statistical life (VSL) [27] for accurately calculating the fire cost value, a key output variable essential for comprehensive analysis. Additionally, the RAFD framework employs Mixed-Integer Linear Programming (MILP) [28], enabling a thorough examination of RAFD dynamics within the context of Portugal. These methodological advancements ensure a robust and tailored approach to understanding and optimizing financial resource allocation for Portuguese FD.

Further explanation of these methodologies is provided in detail across the next four sections: Section 2 provides an overview of the RAFD literature; Section 3 outlines the research methodology and expounds upon the ABM and MILP models; Section 4 presents the empirical findings and associated recommendations specific to the Portuguese RAFD landscape. After this, Section 5 undertakes an in-depth examination and discussion of the research findings. Ultimately, this paper culminates with a comprehensive conclusion in Section 5. The abbreviations utilized within this paper are consolidated in Table A1 in the Appendix A.

2. The Literature Review

RAFD has been studied from various perspectives, including the location and allocation of facilities and vehicles, the placement and deployment of firefighters, and budget distribution. These studies employ diverse models, objective functions, and analytical components. Table 1 provides an overview of the reviewed literature in this section. All the papers study the RAFD, and in Table 1, they are categorized by key criteria: type of allocation (vehicle, firefighter, and budget), the FDPA and RAFD models used, the application of GIS and simulation methods, considerations of equity in the RAFD, the inclusion of socioeconomic and spatiotemporal variables, and the presence of case studies. According to the results of the literature review and aligned with the findings of previous studies [4,9], DEA and Linear Programming (LP)—specifically MILP—are the most commonly used methods for FDPA and RAFD, respectively.

Among the reviewed papers, six controlled the effect of RAFD on FD performance by conducting FDPA analysis; however, none of these studies integrated efficiency scores directly into the RAFD optimization process as recommended by the general RAFD framework [9]. Melolidakis (1993) [29] utilized game theory to provide fire stations with bargaining power for acquiring more vehicles using the Shapley–Shubik (S–S) power index. Lan et al. (2007) [2] applied a Multi-Stage Resource Allocation Approach to allocate a limited number of firefighters to stations while improving the stations' efficiencies, controlled by DEA. In their later study in 2011 [11], Lan et al. applied DEA to evaluate organizational performance and determine the production efficiency of fire services in Taiwan. They used a total efficiency-based scale approach to suggest an ideal human resource allocation model. Fang et al. (2008) [30] employed a two-stage DEA model to allocate firefighters and budget between fire stations in China to improve performance. They used the current number of firefighters and the fire stations' expenses as inputs, and the proportion of lives saved to lives at risk and the number of emergency calls as outputs, aiming to find the best combination of inputs that produce the best outputs. Recently, Lim et al. (2020) [16] used a revised two-stage DEA model with budget constraints for FDPA in South Korea, using firefighters and fire engines as inputs and damage reduction and rescued lives as outputs.

Table 1. Summary of reviewed RAFD papers, including methods and variables. ReTyp: Resource type, EqM: Equity Method, Sim: Simulation Model, SoEco: Socioeconomic variables, SpTe: Spatiotemporal variable, FF: Firefighters, Veh: Vehicles, Bud: Budget.

Paper	ReTyp	FDPA Model	RAFD Model	EqM	GIS	Sim	SoEco	SpTe	Case Study
Schilling et al., 1979 [31]	Veh	-	LP	-	-	-	Yes	Yes	US
Marianov et al., 1992 [32]	Veh	-	LP	-	-	-	Yes	Yes	-
Melolidakis, 1993 [29]	Veh	-	S–S index	Yes	-	-	-	-	Greece
Jayaraman et al., 1995 [33]	Veh	-	LP	-	-	-	-	Yes	-
Revelle et al., 1995 [34]	Veh	-	LP	-	-	-	Yes	Yes	-
Athanassopoulos, 1998 [35]	FF and Veh	DEA	TABRA	Yes	-	-	Yes	-	UK
Peace, 2001 [12]	Veh	-	Risk-based	-	-	-	Yes	Yes	UK
Araz et al., 2007 [36]	Veh	-	LP	-	-	-	Yes	Yes	-
Lan et al., 2007 [2]	FF	DEA	MSRAA	-	-	-	-	-	Taiwan
Huang et al., 2007 [28]	Veh	-	LP	-	Yes	Yes	-	Yes	Singapore
Cheu et al., 2008 [37]	Veh	-	LP	-	-	-	-	Yes	Singapore
Fang et al., 2008 [30]	Bud and FF	DEA	DEA	-	-	-	Yes	-	China
Lan et al., 2009 [38]	FF	DEA	MSRAA	-	-	-	-	-	Taiwan
Cheu et al., 2010 [39]	Veh	-	LP	-	-	-	-	Yes	US
Lan et al., 2011 [11]	FF	DEA	TEBSA	-	-	-	Yes	-	Taiwan
Chevalier et al., 2012 [40]	FF and Veh	-	LP	Yes	Yes	-	Yes	Yes	Belgium
Chalfant et al., 2016 [41]	Veh	-	Distance-based	-	-	-	Yes	Yes	US
Perez et al., 2016 [42]	Veh	-	LP	-	Yes	-	-	Yes	Chile
Wang et al., 2016 [43]	Veh	-	LP	-	-	-	-	Yes	China
Perez et al., 2016 [44]	Veh	-	LP	-	-	-	-	Yes	Chile
Alavi et al., 2018 [7]	Veh	-	LP	-	-	-	-	-	Iran
Yeboah & Park, 2018 [18]	Veh	-	Risk-based	-	-	-	-	Yes	Canada
Kumar et al., 2019 [14]	Veh	-	LP	-	-	-	Yes	Yes	India
Behrendt et al., 2019 [15]	Bud	-	LP	Yes	-	-	Yes	-	US
Kovalenko, 2019 [17]	Veh	-	LP	-	-	-	Yes	Yes	Ukraine
Lim et al., 2020 [16]	Budget	DEA	DEA	-	-	-	-	-	Republic of Korea
Maqbool et al., 2020 [45]	Veh	-	LP	-	Yes	Yes	-	Yes	Pakistan
Kumar et al., 2020 [46]	Veh	-	LP	-	-	-	Yes	Yes	India
Rodriguez et al., 2020 [10]	Veh	-	LP	-	Yes	-	Yes	Yes	Chile
Liu et al., 2021 [47]	Veh	-	Risk-based	-	Yes	-	-	Yes	China
Ghasemi et al., 2021 [48]	Veh	-	Simulation	-	-	Yes	-	Yes	Iran
Hajipour et al., 2022 [49]	Veh	-	LP	-	-	-	-	Yes	-
Ming et al., 2022 [13]	Veh	-	LP	-	-	Yes	-	Yes	China
Rodriguez et al., 2023 [50]	Veh	-	LP	-	Yes	Yes	Yes	Yes	Chile
Liu et al., 2023 [51]	Veh	-	Time-based	-	Yes	-	-	Yes	China
This Paper	Bud	DEA	LP	Yes	Yes	Yes	Yes	Yes	Portugal

The six aforementioned papers are the closest group of reviewed studies to this research since they utilize both FDPA and RAFD analyses in alignment with the general RAFD framework, which serves as the reference framework for this study. A detailed analysis of these papers has been conducted, controlling for other important characteristics based on the reference framework.

Spatiotemporal variables (e.g., traffic, response time, distances) are crucial for FDPA analysis [9], and while many reviewed papers include them, none of these six studies incorporated spatiotemporal variables into their FDPA or RAFD analyses. When important variables like spatiotemporal data are unavailable, some of the reviewed papers used simulation and GIS [28,40,44,50] to generate or collect the necessary information. However, none of the six papers utilized GIS or simulation to include spatiotemporal variables in their analyses. Another critical topic in RAFD studies is the use of equitable resource distribution among FDs to ensure a minimum level of FPS efficiency [15]. Portuguese law also mandates the ANEPC to incorporate equity in the RAFD process [25]. Despite this, only one of the six studies [35] incorporated equity considerations in its RAFD model.

As highlighted in the last row of Table 1, this research comprehensively addresses several key areas, marking significant novelties in the field. It utilizes MILP for RAFD, GIS, simulation for producing response time values, and DEA for FDPA. Moreover, it incorporates equity considerations and includes socioeconomic and spatiotemporal variables in its formulation, setting it apart from previous research by integrating these diverse elements into a cohesive analysis framework. Additionally, this study is grounded in a detailed case study conducted in Portugal.

3. Research Methodology

This study adheres to the RAFD method's structured four-stage approach proposed by Eslamzadeh et al. (2022, 2023) [8,9]. A pivotal facet of this methodology is the incorporation of the RAFD framework [5], serving as a guiding framework for the selection of input and output variables, as well as the methodology applied in FDPA. Illustrated in Figure 1, the research methodology unfolds through four sequential stages encompassing data gathering, processing, analysis, and reporting. Subsequent sections will delve into each of these stages in greater detail. To address the identified limitations in the RAFD framework's original implementation and enhance its applicability within the Portuguese context, this study proposes four key recommendations. These are important for improving data collection accuracy and completeness, integrating efficiency metrics into the RAFD process, and expanding resource allocation adjustments. By incorporating these recommendations, decision-makers at the ANEPC and other FPS authorities can optimize their resource allocation strategies, leading to better performance and increased effectiveness of fire departments. These recommendations are grounded in the need to refine data collection practices, ensure comprehensive assessments, and adopt a more flexible approach to resource distribution, ultimately fostering a more robust and efficient firefighting system in Portugal. The graphical structure illustrating the PT-RAFD framework is shown in Figure 1, which has been adapted from the original model presented in [9].

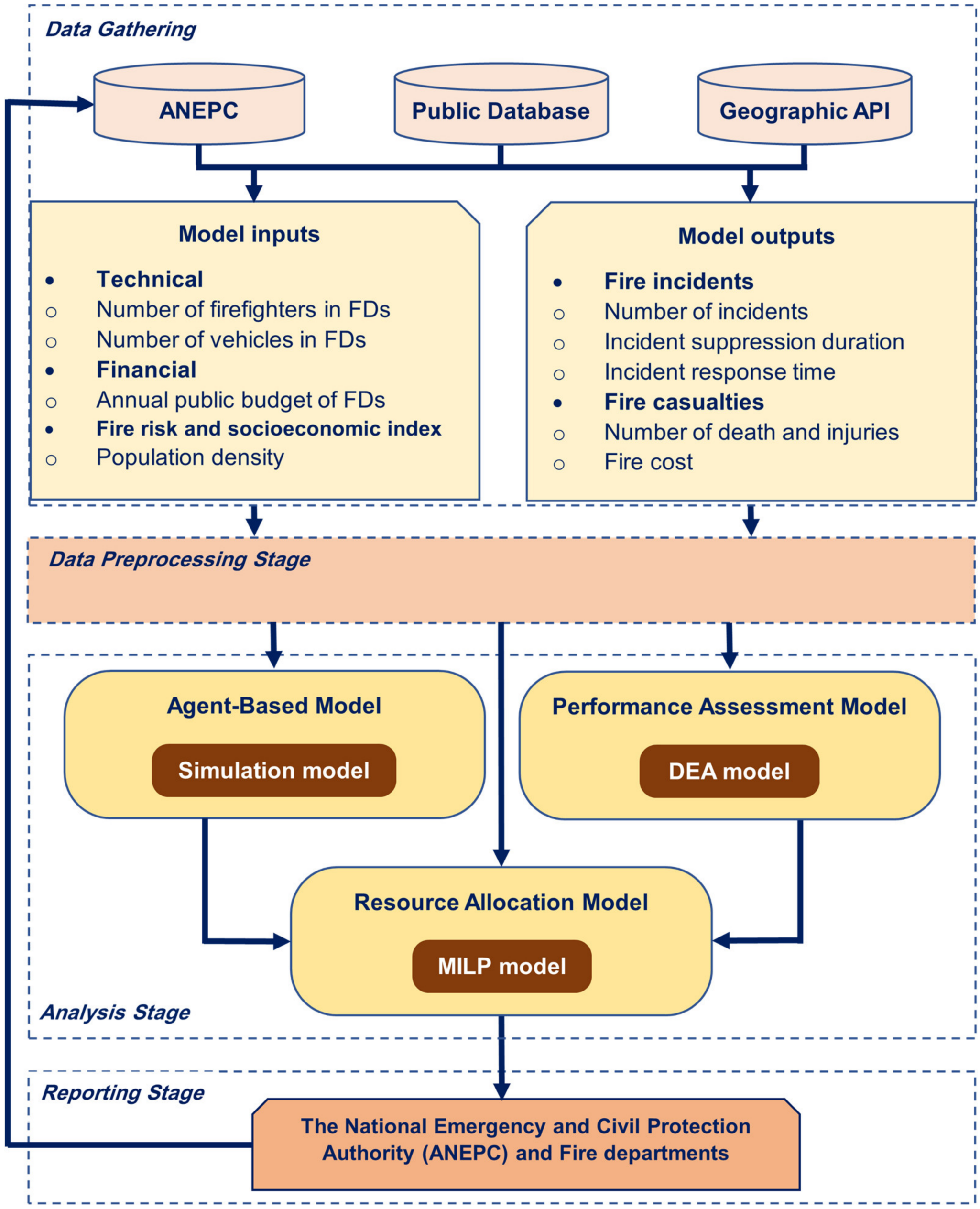


Figure 1. The PT-RAFD framework-2024, adapted from [9].

3.1. First Stage: Data Gathering

The initial phase of this study, data gathering, was dedicated to acquiring essential information from various public and private sources. Since urban (residential) fires require FPS approaches that are completely different from wildfires [15], the focus of this research is on urban fires and allocating resources to the FDs that are providing FPS to the residential areas of Portugal. According to the FDPA framework [9], four categories of data are required for RAFD analysis: incident, spatiotemporal, travel time, and socioeconomic.

- *Category 1, response and operation time data:* The duration values in the ANEPC dataset were the vehicle's idle time, not the incident response time. In other words, the provided duration was the time between a vehicle's departure from the station and its return, not until its arrival at the incident location. Therefore, an ABM has been used to simulate the interaction between PT-FDs, vehicles, and fire incidents to find the response time based on geographical data. Further details about the ABM are provided within the analysis stage section;

- *Category 2, PT census and economic data:* the public database of the National Institute of Statistics of Portugal [52] was utilized for accessing Portuguese data, including the Gross Reported Income (GRI), and population at district level in 2020, and the public database of the World Bank [53] for the Gross National Income (GNI) per capita of US and PT;
- *Categories 3 and 4, PT FDs, incidents, and spatiotemporal data:* The ANEPC played a crucial role in providing these two categories of information regarding PT-FDs and 72,176 urban incidents over the years 2012–2020. Considering the RAFD framework [9], the majority of the required data for RAFD analysis were included in the ANEPC's datasets, which are the FD's number of firefighters, vehicles, locations, annual governmental budget, covered area, and incidents' times, locations, durations, and number and severity of injuries. However, the fire cost, which is one of the important metrics for the RAFD [9,15], was not available in the ANEPC's databases at the time of this research.

The cost of fire is defined by the US National Fire Protection Association (NFPA) [54] as the "total cost of fire as the collective of all net expenditure on fire protection and all net losses due to fire incidents". As depicted in Figure 2, and according to the NFPA, the cost of fire is a mixed metric that consists of active and passive fire protection expenditures such as fire insurance, direct human loss and property damages, and indirect losses due to nonphysical damages and interruptions in production and service provision.

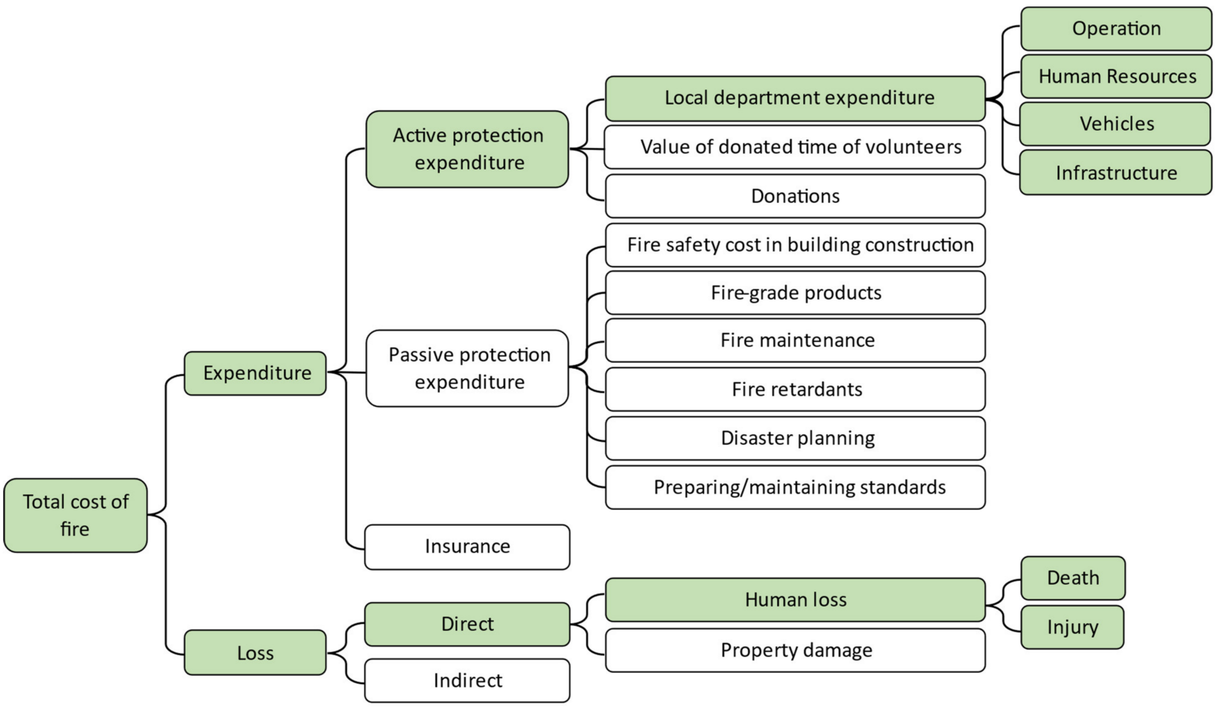


Figure 2. The accessible variables (in green) of the total cost of fire in Portugal.

Recommendation 1: The fire cost is one of the undesirable outcomes of the FPS and fire incidents that plays a key role in FDPA and RAFD analysis [9]. It is a mixed metric that uses the components in Figure 2. This study suggests that PT FDs record and update the components of the total fire cost.

Although the ANEPC's datasets provided some of the required metrics for the cost of fire (e.g., FD expenditure: operation, human resources, vehicles, and infrastructure, direct human loss: number of deaths and severity of injuries), the financial costs of human casualties are still the missing components of the fire cost in Portugal. A common approach for calculating the cost of human casualties is using the VSL [15]. As mortality risk decreases across the population, the incidence of fatalities diminishes, resulting in an overall reduction in deaths. The quantification of these risk reductions is commonly assessed through the metric known as VSL, which represents the monetary value attributed to each expected life saved [55]. Formula (2) can be used for calculating the VSL in different countries, here Portugal, based on the US-VSL [27]:

$$VSL_{PT} = VSL_{US} \times (Average.Income_{PT}/Average.Income_{US})^{Income.Elacticity} \quad (2)$$

The 2020 VSL_{US} value, according to the US Department of Transportation was USD 11.6 million. For the *Average.Income* of the US and PT, the GNI per capita has been used [27], and according to the World Bank data for 2020, the GNI per capita for the US was USD 64,650, and for PT, it was USD 21,850. Since the *Income.Elasticity* in Portugal was not publicly available, it was considered 1.00 as suggested for international countries [27]. Therefore, the 2020 VSL in Portugal was USD 3,920,495, and this value will be the basis for the further calculation of the cost of fire and potential loss in Portugal in this research.

3.2. Second Stage: Data Preprocessing

The datasets acquired from the ANEPC exhibited minor discrepancies, characterized by instances of incomplete, incorrect, missing, and outlier values. Following an in-depth analysis of FDs and incident data spanning from 2012 to 2020 and subsequent consultation with the ANEPC's experts, it was determined that the most comprehensive dataset was from the year 2020.

Although there are municipal, private, and voluntary FDs providing FPS in Portugal, the main incident dataset consisted of 7038 fire incidents that occurred in 410 voluntary FDs in Portugal because the financial data of the municipal and private FDs were not accessible. The incident and FD data were fragmented across multiple datasets, containing intricate details deemed unnecessary for the scope of this study. Therefore, a meticulous data preprocessing protocol was implemented, encompassing cleansing, integration, reduction, and transformation steps. These measures were undertaken to ensure the integrity and suitability of the data for subsequent evaluation and analysis processes [56,57]. After the data processing stage, the recorded data of 5698 incidents in 353 FDs with all the necessary details was aggregated into one dataset for the analysis stage. Further explanations of the preprocessing steps are provided in Table 2. This dataset served as the foundational basis for the RAFD analysis conducted in this research endeavor.

Table 2. The four steps of the data preprocessing stage.

Steps	Process Details
Cleansing	Recovering the correct values for the noisy (incomplete, incorrect, or missing) records from other ANEPC datasets or removing the irrecoverable noisy and outlier records (i.e., 20 incidents recorded as less than 10 or more than 1440 min in duration).
Integration	Collecting the required data about incidents and FDs from all provided datasets by the ANEPC and integrating them into one dataset.
Reduction	Removing: - Unnecessary, or irrelevant granular data for PT-RAFD (e.g., details of the suppression or FD maintenance cost, incident codes); - 57 FDs with one or more zero values in financial resources, number of incidents, and covered population fields.
Transformation	The processed dataset, which underwent cleaning, integration, and reduction, was converted into comma-separated formats to facilitate the subsequent analysis stages.

Recommendation 2: The reliability and validity of analyses like RAFD or FDPA are dependent on the accuracy and completeness of the referenced datasets. This underscores the importance for FD commanders and firefighters to diligently record incident and managerial information with the utmost precision and thoroughness. This is an important practice to prevent any potential loss of required data for future FDPA and RAFD analyses.

3.3. Third Stage: Analysis

The PT-RAFD model is based on the general RAFD framework [9], and uses the following three models:

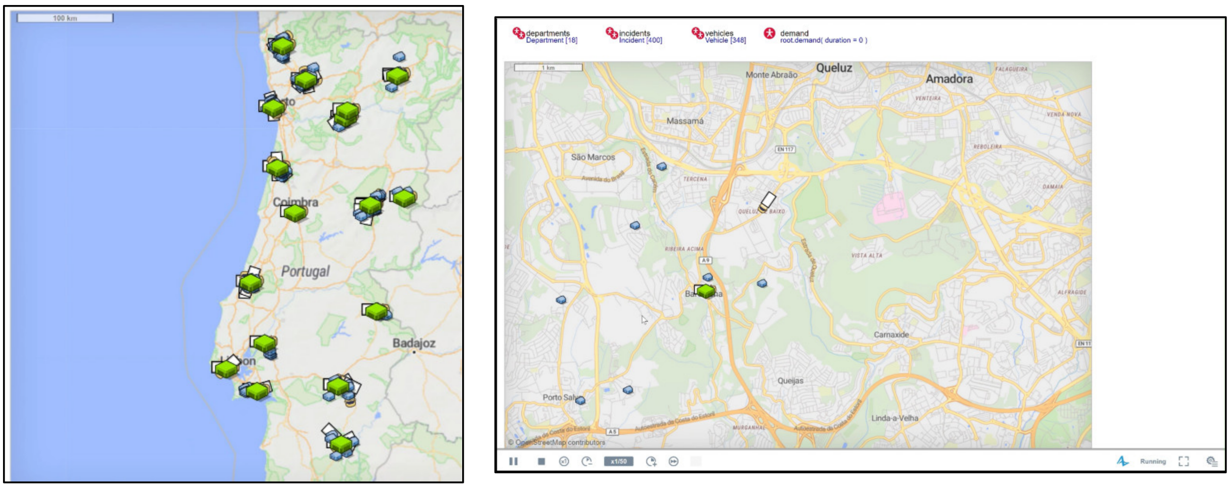
- The ABM for simulating the interactions between FDs and incidents and gathering the response time and suppression operation durations;
- The Data Envelopment Analysis (DEA) for conducting the PT-FDPA analysis and calculating the efficiencies of PT-FDs;
- The MILP for finding the optimized version of the RA that minimizes the cost of fire and improves the performance of PT-FDs;
- The ANEPC's experts and decision-makers had the responsibility of validating and confirming the reliability of the analytical process and findings of this study. This expert group consisted of the former director of the ANEPC and the current dean of Portugal's National School of Firefighters, the ANEPC's national senior chief technician, and two chief commanders of FDs.

3.3.1. Agent-Based Modeling (ABM)

ABM is a computational technique that models the behavior and outcomes of a complex system by simulating the autonomous agents, such as individuals or organizations, that act and interact within it. ABM can represent the diversity, adaptation, and emergence of the system through the agents' rules and behaviors. To find the travel time between PT-FDs and the incidents, a model with four agents has been created, which includes the FD, vehicle, incident, and demand.

Illustrated in Figure 3a,b, the travel time between the station and incident locations was obtained by simulating the interactions between PT-FDs and the incident location. The ABM consisted of four agents, i.e., the FD, vehicle, incident, and demand, and was implemented on actual road maps of Portugal within AnyLogic software (Version 8.8.1) [58].

The average speed of fire engines is set at 45 km/h [37]. Although the current computation does not incorporate specific traffic regulations or congestion scenarios due to computational constraints, the tool could be readily updated to accommodate such factors. The ABM ran with all the 2020 incidents, and the obtained response times were added to the incident record for further FDPA and RAFD analysis.



(**a**) (**b**)

Figure 3. (**a**) The agent-based model implemented in AnyLogic to simulate the travel and response time between PT-FDs and the incidents. (**b**) Expanded view of an FD (CBV Barcarena) in Lisbon, and utilization of roads by its fire engines.

3.3.2. Data Envelopment Analysis (DEA)

This stage began with analyzing the performance of the PT-FDs in accordance with the PT-FDPA model [8] using a slack-based DEA with the variable return to scale format that was output-oriented. The DEA model creates a frontier line from the best-performing FDs (decision-making units) and considers all the provided desired or undesired input and output variables. Then, it compares the FDs with the frontier group and provides the efficiency of each FD in comparison to the target FDs on the frontier line, and the slacks are the input excesses and output shortfalls of the FDs.

Tone (2001) [59] proposed the Slacks-Based Measure (SBM) version of the DEA to solve this deficiency. The SBM model differs from traditional radial efficiency models by considering all slack variables in the assessment of efficiency [59,60]. The SBM model allowed us to simultaneously conduct FDPA and calculate the slacks, representing excesses of the input and shortfalls of the output [8].

Let FD = {$FD_1, \ldots, FDn$} present a set of n FDs, each with i inputs and j outputs. X and Y denote the input and output variables of the reference set FD, respectively, with P defining the production possibility set for FD. For an FD with m inputs and s outputs—denoted by a pair of nonnegative vectors (x, y) where $x \in \mathbb{R}^m_+$ are the *inputs* vector and $y \in \mathbb{R}^s_+$ are the *outputs* vector—the SBM efficiency score can be defined as follows [8]:

$$f^*(x,y) = \underset{\lambda,s^-,s^+}{\text{MIN}}\, f\left(x,y,s^-,s^+\right)\left(1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_i}\right)\Big/\left(1-\frac{1}{s}\sum_{j=1}^{s}\frac{s_j^+}{y_j}\right)$$

$$s.t.x = X\lambda + s^- \tag{3}$$

$$y = Y\lambda + s^+$$

$$\lambda \in \mathbb{R}^n_+, s^- \in \mathbb{R}^m_+, s^+ \in \mathbb{R}^s_+,$$

where vectors s^+ and s^- represent inefficiency slack vectors [61]; $f^*(x,y)$ is the SBM efficiency score assigned to a new FD with input–output pair (x, y). According to Tone (2001) [59], an FD is deemed efficient if $f^*(x,y)$ equals one and the optimal slacks s^+* and s^-* are zero for every optimal solution [8]. The SBM DEA method has been used for PT-FDPA with multiple nonnegative desired inputs and multiple nonnegative undesired outputs; however, the FDPA evaluators have more alternatives according to their objectives [4]. The weighting of inputs and outputs was treated equally, as recommended by an expert group, and set to one to maintain their values. The nonnegative desirable inputs in the model encompassed the financial budget of the fire departments, along with the counts of vehicles and firefighters. Conversely, the nonnegative undesirable outputs included the total number of incidents attended by the fire departments, the total cost of fires, and the overall duration of fire incidents. Employing an output-oriented and slack-based Data Envelopment Analysis (DEA) model, this study utilized the DeaR-Shiny online solver [62], and the primary objective function of this model was to minimize the cost of fire. To ensure consistency and reliability, all inputs and outputs were assigned a uniform weight of one, maintaining the potency of their current values, as validated by the expert group. For further details about the slack-based DEA model and PT-FDPA, please refer to Eslamzadeh et al. (2023) [8]. The DEA results are provided in Section 3.

3.3.3. Mixed-Integer Linear Programming (MILP)

The MILP, as the most frequent RAFD method [9], has been implemented for RA optimization in PT FDs. Let us assume the following:

- n is the number of FDs that are evaluated with respect to one another;
- t is the reference year;
- V is the value statistical life;
- l_i is the ($\geq$0) value of the actual loss in FD_i ($i = 1, \ldots, n$);
- c_i is the ($\geq$0) value of the total cost of fire in FD_i;

- f_i is the efficiency of FD_i where ($1 \geq f_i > 0$);
- b_i is the ($\geq$0) value of the financial budget of FD_i.

The objective function of the model is to minimize the total cost of fire Q, and the Gurobi solver (http://www.gurobi.com, accessed on 15 September 2023) has been used to solve it. As discussed in the previous stage, Q consisted of the expenditures of active and passive protection and direct and indirect losses, and the VSL is a measure for converting the fire casualties to a financial metric.

Similarly, the value of statistical injuries [54] is a similar metric that provides coefficients for calculating the cost of injuries to fire based on the severity of the injuries (minor = 0.003 V, moderate = 0.047 V, serious = 0.105 V, severe = 0.266 V, critical = 0.593 V, and unsurvivable = 1 V). In the ANEPC's datasets, four categories of casualties were provided: number of deaths, critical, serious, and minor injuries. Therefore, as shown in Table 3, the direct or actual loss of fire in Portugal was calculated from the number of fire casualties in a particular FD_i $i \epsilon I$ during the year 2020 by using Formula (4):

$$l_i = (death_s \times \text{V}) + (critical\_injury_i \times 0.593\ \text{V}) + (serious\_injury_i \times 0.105\ \text{V}) + (minor\_injury_i \times 0.003\ \text{V}) \quad (4)$$

Table 3. The values of VSL and VSI in Portugal between the years 2015 and 2021.

Year	GNI Per Capita	VSL	VSI Ratios and Values					
			Minor 0.003	Moderate 0.047	Serious 0.105	Severe 0.266	Critical 0.593	Unsurvivable 1.0
2015	20,460	3,469,022	10,407	163,044	364,247	922,760	2,057,130	3,469,022
2016	19,940	3,454,778	10,364	162,375	362,752	918,971	2,048,683	3,454,778
2017	20,060	3,455,117	10,365	162,390	362,787	919,061	2,048,884	3,455,117
2018	22,060	3,650,016	10,950	171,551	383,252	970,904	2,164,459	3,650,016
2019	23,200	3,823,983	11,472	179,727	401,518	1,017,179	2,267,622	3,823,983
2020	21,850	3,920,495	11,761	184,263	411,652	1,042,852	2,324,854	3,920,495
2021	23,890	3,974,369	11,923	186,795	417,309	1,057,182	2,356,801	3,974,369

The total cost of fire in FD_i is the sum of the actual loss and the total expenditure of the FD_i provided by the ANEPC. However, considering the performance of the FDs, the degree of their efficiency has a direct effect on the RAFD efficiency [4,8]. Therefore, with the confirmation of the expert group, the inefficiency of the FDs ($1 - f_i$) calculated by the slack-based DEA model of FDPA [8] was added to the total cost of the FD_i. In this case, the budget of the efficient FDs will remain the same but the inefficient FDs will receive more budget to help them take corrective action in the next financial year. Therefore, the total cost of fire is $c_i = l_i + (1 - f_i).b_i$. After calculating the total cost of fire of the FDs, the MILP functions and constraints will be as follows:

$$\text{MINC} = \sum_{(i=1)}^{n} c_i = \sum_{(i=1)}^{n} (l_i + (1 - f_i).b_i) \quad (5)$$

$$s.t. \sum_{(i=1)}^{n} c_i = 80{,}000{,}000 \forall i = 1, \ldots, n;$$

The decision factor of MILP is the FD's financial budget (b_i), and its objective function model is to minimize the total cost of fire $C = \{c_1, \ldots, c_n\}$ by optimizing the financial resources $F = \{f_1, \ldots, f_n\}$. The following constraints limit the total allocatable resources to EUR 80 million, according to the ANEPC total budget for 2020. Therefore, the MILP will try to allocate the budget and will keep the total sum of the allocated budget equal to EUR 80 million.

$$0.95b_{i_t} \leq b_{i_{(t+1)}} \geq 1.1b_{i_t} (\text{C5})$$

Constraint 5, denoted as C5 for clarity throughout the paper, complies with the RAFD law in Portugal [25], where the total allocated budget to FD_i in the new year should be within 95% to 110% of its budget in the last year. Although the expert group confirmed the use of constraints and inefficiency values in the PT-RAFD model, the model will be run both with and without C5, incorporating both efficiency (f_i) and inefficiency ($1 - f_i$) to verify changes in FDs' efficiencies without limiting the optimization system to Portuguese legal allocation boundaries.

In summary, the PT-RAFD model consisted of three important steps: Firstly, leveraging ABM to determine travel times between FDs and incidents (i.e., response times) as one of the inputs of the FDPA analysis. Secondly, employing DEA for FDPA analysis to assess FD efficiencies, integrating response times alongside financial and technical inputs as well as undesirable outputs. Lastly, employing MILP to optimize the allocation of the ANEPC's resources among FDs and incorporating FD efficiencies into the actual cost of fire to minimize overall expenses. To validate the results, the previous financial budget of FDs was replaced with the recommended budget by the PT-RAFD model, and their performance with the new budget was re-evaluated using the FDPA model. All the results were controlled and verified by experts from the ANEPC, ensuring the reliability and applicability of the findings.

3.4. Fourth Stage: Reporting

In the final stage of the PT-RAFD framework, the findings of the research are synthesized and presented to ANEPC decision-makers. This pivotal stage serves as a conduit for communicating the results of the PT-RAFD model, allowing decision-makers to gain valuable insights into the performance of PT FDs and FPS. Section 4 of the research outlines the empirical findings derived from the PT-RAFD model, and Section 5 delves into detailed discussions surrounding these findings, providing nuanced insights and actionable recommendations for improving FD and FPS performance. The overarching goal of the reporting stage is to empower ANEPC decision-makers to undertake corrective actions within their strategic framework for RAFD. By leveraging the insights gleaned from the PT-RAFD model, decision-makers can enact informed strategies to enhance FDs and FPS performance, ultimately contributing to the safety and well-being of communities across Portugal.

4. Findings

This section provides and discusses the results of the analysis stage to provide an answer to the research question and shows that integrating FDPA results into RAFD in Portugal will enhance the efficiency of the FDs. The analysis stage commenced with the performance evaluation of 353 FDs in Portugal using the FDPA model [8]. Subsequently, three rounds of RAFD optimization were performed using MILP.

The first round integrated the inefficiencies of the FDs ($1 - f_i$) and C5, which stipulates the budget change threshold mandated by Portuguese law. The second round relaxed this constraint, allowing MILP to allocate resources without any limitations. In the third round, both C5 and the efficiency (f_i) were directly incorporated into the optimization process. In the RAFD methods, with and without C5, the inefficiencies from the previous year ($c_i = l_i + (1 - f_i).b_i$) were utilized in the RAFD objective function.

Following each round, the FDPA analysis was conducted to assess changes in the FDs' efficiency scores using different RAFD methods. Notably, the third RAFD method led to the deterioration of performance in 115 FDs and was thus excluded from further analysis and discussion. Table A2 in the Appendix A presents the performance scores for all Portuguese FDs based on three FDPA analyses: their current performance score, their performance score after budget reallocation using C5 in the RAFD optimization, and their performance score after budget reallocation without C5 in the RAFD optimization.

As illustrated in Figure 4, after conducting two runs of the PT-FDPA model—one with and one without C5—and subsequently verifying efficiency using the FDPA model, the results indicated that while optimizing budget allocation within the confines of Portuguese law improved the overall efficiency of FDs from 0.5037 to 0.5137, it did not change the number of efficient FDs. However, the second run of the model, which involved relaxing C5 and allowing the MILP to optimize budget allocation to minimize the total fire cost of FDs, yielded even more promising outcomes. Not only did the average efficiency of FDs improve to 0.5335 but there was also a substantial increase in the number of efficient FDs from 16 to 22.

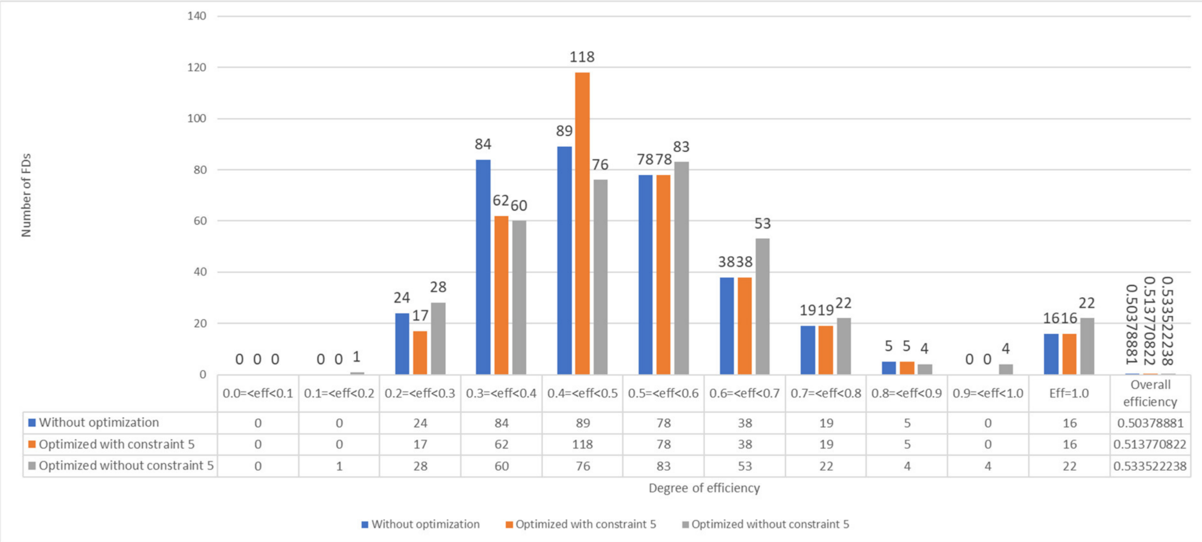


Figure 4. The degree of efficiency of PT FDs before and after using PT-RAFD model.

The results of the PT-FDPA model offer a clear response to the research question, demonstrating that incorporating FDPA results and efficiencies of PT-FDs in RAFD leads to enhanced performance of FDs in Portugal. These results indicate that utilizing FDs' efficiency for optimizing the RAFD notably enhances their performance. However, considering C5, which mandates keeping changes between 95% and 110% of their last-year budget, it is evident that the RAFD primarily enhances the performance of low-efficient FDs while maintaining the performance of FDs with higher degrees of efficiency unchanged. This suggests that the RAFD model effectively targets areas where improvements are most needed, ensuring efficient allocation of resources while preserving the performance of already efficient FDs.

On the contrary, relaxing C5 allowed the PT-RAFD model to optimize the budget primarily based on last year's efficiencies, allocating a larger share of the budget to low-performing FDs and less to more efficient ones, aiming to maximize overall efficiency; consequently, improvements were observed across almost all FDs. However, it is worth noting that while the differences between FDs' last-year and optimized budgets were generally within ±30% of the last year's budget, there were instances with higher percentage changes.

The results of the analysis stage highlight efficient FDs as target points for other FDs to improve their performance. The slack-based Data Envelopment Analysis (DEA) within the FDPA model identifies the variables and degree of improvement necessary for each non-efficient FD to reach the target point. Figure 5 illustrates the frequency with which a specific efficient FD is selected as a target for a non-efficient FD in both runs of the RAFD and FDPA models.

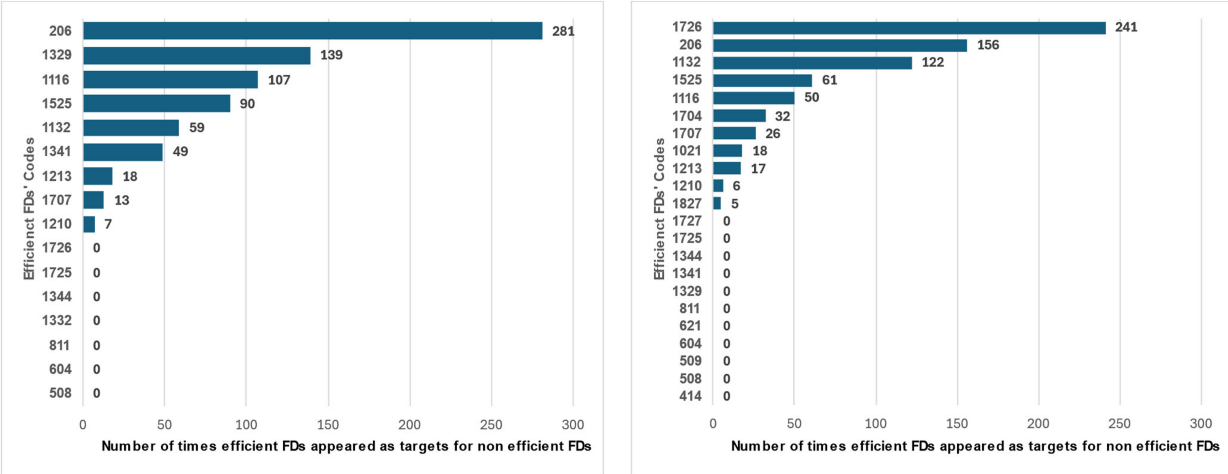


Figure 5. Number of times efficient FDs appeared as targets for non-efficient FDs in the final dataset, (**left**): RAFD with using C5; (**right**): RAFD with relaxing C5.

In the next section, the findings will be thoroughly investigated and discussed, providing detailed insights into the results obtained. Additionally, recommendations will be provided for FPS decision-makers based on the analysis conducted.

5. Discussion

This section will explain the implications of the findings and offer actionable suggestions for improving the performance of PT FDs. As depicted in Figure 6 and detailed in Table 4, analysis reveals that incorporating C5 in the PT-RAFD model—limiting RAFD changes between 95% and 110% of FDs' last-year resources—does not degrade the efficiency of any FDs while enhancing the performance of 47% of FDs (165 out of 353). Conversely, relaxation of C5 improves the efficiency of over 60% of FDs (213 out of 353), though it is accompanied by deterioration in 125 FDs. Moreover, direct comparison shows that the relaxation method yields higher improvement in the efficiency of 181 FDs but lower results for 157 FDs compared to including C5. These findings suggest that while optimized RA can enhance performance for many FDs, it may also incur cost deterioration for some. Given that Portuguese RAFD law emphasizes socioeconomic and spatiotemporal characteristics of FDs, relaxing C5 may optimize RAFD without due consideration of these factors, leading to improved performance for most FDs at the expense of efficiency for some.

Recommendation 3: Incorporate efficiency as a variable in the national RAFD Formula (1) within Portuguese Law. Conducting RAFD optimization based on FDPA results will enable FPS decision-makers to comprehensively assess different RAFD strategies and select the most suitable option based on its impact on FD efficiency.

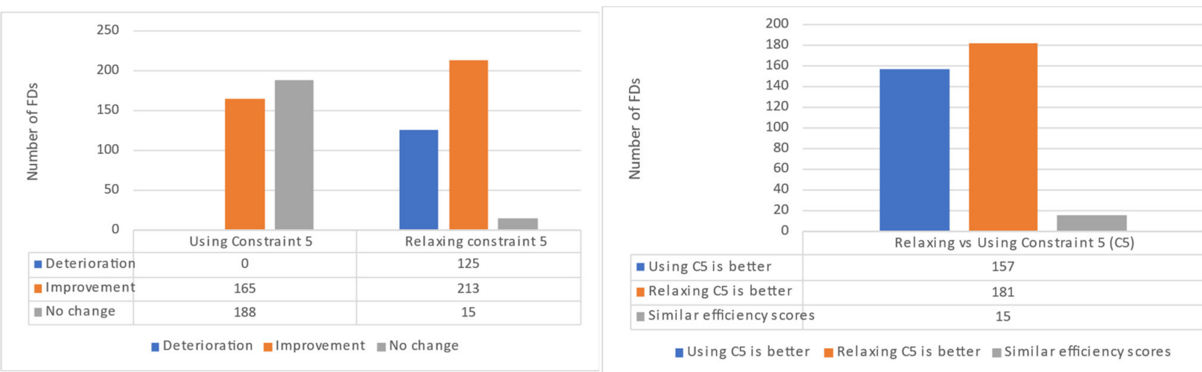


Figure 6. Efficiency impact of RAFD with and without C5: (**left**)—FDs' efficiency changes post-RAFD implementation; (**right**)—comparison of RAFD methods on FDs' efficiency improvement.

On the district level, as shown in Figure 7, the results of the PT-RAFD model under C5 demonstrate improvements across all PT districts; however, upon relaxing C5, while many districts experience significantly higher efficiency increases, there is an overall decrease in efficiency in five districts: Aveiro, Beja, Braga, Porto, and Santarém. This indicates that although removing C5 leads to considerable positive changes in PT-FDs' efficiency levels, certain districts see a negative impact on their FPS performance due to the new budget allocation. This underscores the need for attention from the ANEPC and FPS decision-makers in Portugal to establish an optimal constraint that allows for greater improvements in PT FDs' performance while also mitigating the negative effects of RAFD.

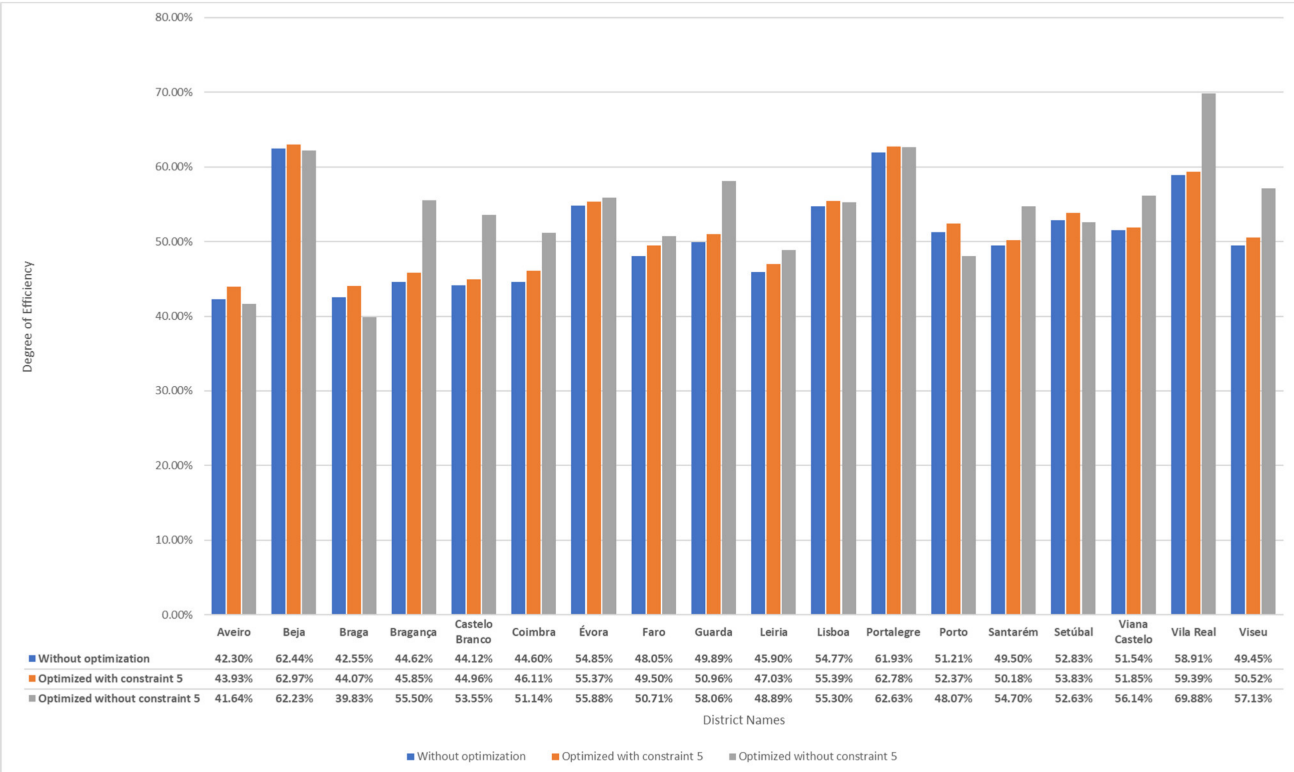


Figure 7. Efficiency changes at the district level with and without using the C5 in PT-RAFD.

The PT-RAFD model in both runs, as shown in Figure 8, led to an increase in the number of districts with over 50% efficiency. Before optimization, there were eight districts exceeding this threshold. After utilizing the PT-RAFD with C5 to optimize their budget, the number of districts with over 50% efficiency increased to 11. Furthermore, relaxing C5 resulted in 14 districts surpassing the 50% efficiency mark. However, after relaxing C5, two of the five districts with deteriorated efficiency experienced significant declines: Braga with a decrease of −2.72% and Porto with a decrease of −3.14%. The other three districts saw changes of less than −1%.

Further investigation revealed that the primary cause of deterioration in the PT-RAFD version with relaxation of C5 is that the optimization changes in 307 FDs, out of 353, exceeded the threshold set by C5. Additionally, this method reduced the budget of 203 FDs, predominantly those with higher budgets, while increasing the budget of 150 low-performing FDs. Conversely, in the PT-RAFD version using the change threshold of C5, the numbers were reversed: more FDs experienced an increased budget (189 FDs) and there were 164 instances of budget reduction. As one of the key inputs of the FDPA model for analyzing FD performance is their budget, and one of the outputs is their total cost, the relaxed method ultimately resulted in more efficient FDs, while the method using C5 led to a more balanced distribution of efficiency without deterioration in FD performance.

In comparison to the recent FDPA study in Portugal by Eslamzadeh et al. (2023) [8], the results of this study demonstrate a significant improvement in the percentage of FDs with less than 50% efficiency across all districts of Portugal. The comparison provided in Table 4 reveals that after optimizing budget allocation, the number of low-performing FDs decreases in all districts. Moreover, employing the RAFD optimization method with relaxed C5 yields the most favorable outcomes. It is important to note that this study marks the first implementation of RAFD in Portugal, making direct comparisons with previous studies limited; however, the improvements observed in FD efficiency underscore the potential of RAFD to enhance the performance of FPS.

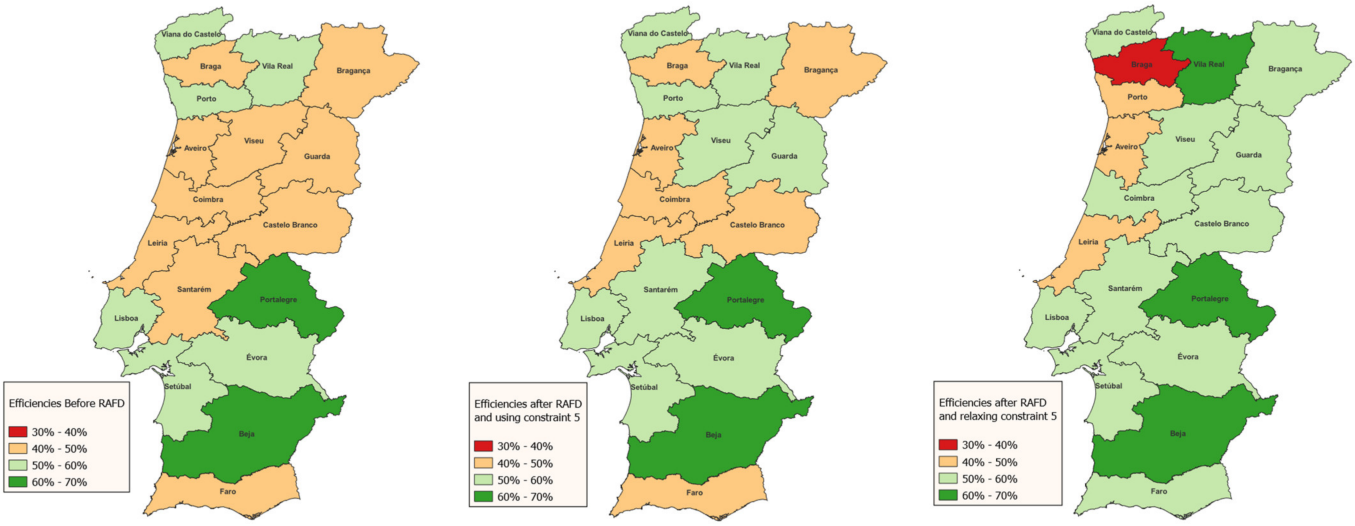


Figure 8. Portuguese districts' efficiency changes: (**left**): before RAFD; (**middle**): after RAFD by using C5; (**right**): after RAFD by relaxing C5.

Table 4. Comparison of the number and percentages of FDs with less than 50% efficiency before and after using the RAFD optimized budget with and without C5, relative to Eslamzadeh et al., 2023 [8].

Districts and No. of FDs	Recent Study's Results		Using RAFD with C5		Using RAFD Relaxing C5	
Aveiro (24)	23	95.83%	19	79.17%	18	75.00%
Beja (12)	5	41.67%	3	25.00%	4	33.33%
Braga (18)	16	88.89%	16	88.89%	13	72.22%
Bragança (14)	10	71.43%	9	64.29%	6	42.86%
Castelo Branco (12)	7	58.33%	7	58.33%	6	50.00%
Coimbra (17)	14	82.35%	13	76.47%	9	52.94%
Évora (9)	4	44.44%	3	33.33%	3	33.33%
Faro (11)	10	90.91%	8	72.73%	7	63.64%
Guarda (16)	11	68.75%	9	56.25%	4	25.00%
Leiria (23)	21	91.30%	14	60.87%	13	56.52%
Lisboa (46)	20	43.48%	18	39.13%	17	36.96%
Portalegre (7)	3	42.86%	3	42.86%	3	42.86%
Porto (42)	30	71.43%	26	61.90%	29	69.05%
Santarém (17)	12	70.59%	8	47.06%	4	23.53%
Setúbal (23)	16	69.57%	10	43.48%	12	52.17%
Viana Castelo (10)	5	50.00%	4	40.00%	2	20.00%
Vila Real (23)	10	43.48%	10	43.48%	5	21.74%
Viseu (29)	12	41.38%	17	58.62%	10	34.48%

The findings in this paper offer valuable insights for ANEPC decision-makers. Given the complexity of FD performance, influenced by factors such as socioeconomic conditions, spatiotemporal dynamics, and FD resources, this research provides a nuanced understanding of how different RAFD optimization strategies impact FD efficiency and how efficiency scores can guide the RAFD. The detailed analysis of allocated budgets and efficiency metrics equips decision-makers with actionable information for optimizing RA and enhancing FPS effectiveness at both national and district levels.

Recommendation 4: Embrace a broader scope for changes beyond the 95 to 110% threshold of FDs' last year's resources, as evidenced by the findings of this study. This expansion demonstrates the potential to elevate FD performance without adverse effects on other FDs. By adopting this adjustment, RAFD optimization can effectively elevate the overall FPS performance at both national and district levels, fostering a more robust and efficient firefighting system in Portugal.

The detailed allocated budget in both PT-RAFD methods, along with the slack results for inefficient FDs and their efficiency distance from the target FDs in the PT-FDPA, are additional outcomes of this research that offer valuable insights for corrective actions. This information is readily available upon readers' request, providing an opportunity to delve deeper into the specific budget allocations and performance metrics of individual FDs. By reviewing and analyzing these details, FPS stakeholders can gain a comprehensive understanding of the allocation process and identify areas for improvement, thereby facilitating informed decision-making and strategic planning within the firefighting sector.

The findings presented in this paper hold significant managerial implications for ANEPC's decision-makers and other stakeholders involved in FPS management. By clarifying the complex link between RA techniques and FD efficiency, this research provides actionable insights for optimizing RAFD and enhancing FPS effectiveness. Specifically, the nuanced analysis of allocated budgets and efficiency metrics equips decision-makers with the information needed to make informed decisions at both national and district levels.

The recommendation to record and update components of the total fire cost, as well as the emphasis on the importance of accurate and comprehensive data collection practices, addresses key deficiencies in current methodologies. Moreover, the integration of efficiency metrics into the RAFD process, as suggested, enables decision-makers to comprehensively assess different RAFD strategies and select the most suitable option based on its impact on FD efficiency. Additionally, the suggestion to embrace a broader scope for changes in resource allocation thresholds offers a pathway to elevate FD performance without compromising the efficiencies of other FDs. Finally, the detailed examination of allocated budgets and efficiency metrics, along with the results for inefficient FDs, provides valuable information for corrective actions and strategic planning within the FPS sector. By leveraging these insights, ANEPC decision-makers and FPS managers can refine their resource allocation strategies and enhance the overall efficiency and performance of FPS in Portugal.

6. Conclusions

In conclusion, this paper has significantly contributed to our understanding of the critical role played by RAFD in shaping the performance of Portuguese FDs within the FPS framework. Through the development of a comprehensive RAFD model tailored to the Portuguese context and an in-depth analysis of various optimization strategies, valuable insights have been provided regarding their impact on FD efficiency. The examination was conducted at both district and FD levels, revealing substantial performance improvements when FD efficiency scores were integrated into RAFD. Furthermore, this study underscores the importance of incorporating legal constraints, such as budget allocation limits, into the PT-RAFD model. While adherence to these constraints led to modest efficiency gains of nearly 4%, relaxing specific RAFD thresholds resulted in more significant improvements of nearly 8%. However, it is noteworthy that this approach led to performance deterioration in a minority of FDs. The detailed analysis of budget allocations and efficiency metrics provides decision-makers at ANEPC and similar organizations with actionable insights for optimizing RA and enhancing FPS effectiveness across both national and district levels.

This approach aligns with existing research highlighting the importance of performance-based resource allocation in improving FPS performance. By grounding RAFD decisions in FD performance metrics, our study offers a novel framework for optimizing RA strategies in FPS. Additionally, our findings highlight the need for a nuanced approach to RAFD optimization, considering both legal constraints and performance metrics to balance efficiency gains with potential trade-offs.

For decision-makers at the ANEPC and counterparts worldwide, the results and recommendations of this study offer practical guidance for optimizing RAFD and improving FPS effectiveness. By leveraging the insights gained from the analysis, decision-makers can make informed decisions to enhance FD performance and ensure community safety.

Despite these valuable insights, it is essential to acknowledge this study's limitations. The focus on urban areas in Portugal may limit the generalizability of findings to other regions or countries with different contexts. Urban areas typically have different risk profiles, resource availability, and operational challenges compared to rural areas. Therefore, the applicability of the findings to rural settings remains uncertain. Future research should address this by expanding studies to include rural areas and regions with varying risk profiles.

The reliance on available data and models introduced another limitation. While this study utilized the best available data, certain key data points, such as risk indices, operational costs, and property losses, were not included. These omissions can affect the comprehensiveness of the analysis. Future research should aim to collect and incorporate these missing data points to enhance the robustness of the findings.

Additionally, the study's model does not fully capture the dynamic nature of fire risks and resource allocation needs. Fire risks and resource demands can fluctuate due to various factors, including seasonal changes, changes in the characteristics of the residential areas, and local events. Future research should explore dynamic models that can adapt to these fluctuations, providing more responsive and effective resource allocation strategies.

Finally, the absence of certain socioeconomic and spatiotemporal indicators (e.g., income level, historical neighborhoods, critical buildings and infrastructure, and regional fire risk index) in the current model limits the ability to fully understand their influence on FD performance. These indicators can provide deeper insights into how different variables affect fire department operations and resource needs. Future research should investigate these indicators to offer a more comprehensive understanding of the factors influencing FD performance.

Looking ahead, future research should consider emerging factors like climate change and technological advancements in preventive and suppression activities. Examining how these factors affect FD performance and resource allocation could uncover new FPS strategies for enhancing efficiency and effectiveness. Collaborating with international counterparts to compare RAFD frameworks across different countries may also yield valuable lessons and best practices. By addressing these limitations and exploring new avenues, future research can continue to refine and improve resource allocation strategies for fire departments, ultimately contributing to safer and more resilient communities.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. List of abbreviations used in the paper.

Abbreviation	Full Phrase	Abbreviation	Full Phrase
ABM	Agent-Based Modeling	IP	Integer Programming
ANEPC	Autoridade Nacional de Emergência e Proteção Civil (National Authority for Emergency and Civil Protection)	MILP	Mixed-Integer Linear Programming
DEA	Data Envelopment Analysis	PT	Portuguese/Portugal
FD	Fire Department	RA	Resource Allocation
FDPA	Fire Departments' Performance Assessments	RAFD	Resource Allocation in Fire Departments
FPS	Fire Protection Services	SBM	Slack-Based Model
GIS	Geographic Information System	VSL	Value statistical life

Table A2. FDs' efficiency scores before optimization and after optimizing RAFD with/without using the constraint 5 (C5).

FD Code	FD Name	Curr Eff %	Relax C5%	Use C5%	FD Code	FD Name	Curr Eff%	Relax C5%	Use C5%	FD Code	FD Name	Curr Eff%	Relax C5%	Use C5%
101	Aveiro	37	39	35	903	Gouveia	48	48	57	1329	Leixões	100	100	100
102	Ílhavo	38	40	35	904	Pinhel	44	48	46	1330	Paços Ferreira	47	47	40
104	Ovar	43	45	40	905	Cast Rodrigo	70	70	66	1331	Avintes	63	63	55
105	OliVAzeméis	35	37	29	906	Meda	43	46	58	1332	Leça Balio	100	100	68
106	Aveiro	33	34	34	907	Trancoso	36	39	42	1333	Paço Sousa	46	49	49
107	Feira	50	50	48	908	Almeida	50	50	60	1334	Baião	41	43	45
108	Estarreja	41	43	39	909	VN Foz Côa	59	59	59	1335	Lordelo	54	54	48
109	Alberg Velha	32	33	32	910	Seia	38	41	45	1336	Trofa	39	41	35
110	Pampilhosa	54	54	56	912	Celorico Beira	37	39	48	1337	Vila Aves	53	53	52
111	Arrifana	61	61	57	914	Fornos Algodres	45	47	62	1338	Rebordosa	44	47	43
112	Mealhada	53	53	53	915	Aguiar Beira	57	57	62	1339	S Pedro Cova	49	49	46
114	S João Madeira	50	50	52	916	Manteigas	61	61	67	1340	Vila Meã	38	41	44
115	Vagos	46	48	40	917	São Romão	46	46	56	1341	Melres	100	100	100

Table A2. *Cont.*

FD Code	FD Name	Curr Eff %	Relax C5%	Use C5%	FD Code	FD Name	Curr Eff%	Relax C5%	Use C5%	FD Code	FD Name	Curr Eff%	Relax C5%	Use C5%
116	Esmoriz	42	45	46	919	Soito	61	61	77	1344	Pedrouços	100	100	100
117	Anadia	50	50	46	922	VFranca Naves	60	60	70	2203	Portuenses	72	72	60
118	Águeda	27	29	25	1001	Alcobaça	38	40	35	1404	Benavente	46	48	61
119	Sever Vouga	46	48	51	1003	Caldas Rainha	29	31	25	1405	Rio Maior	47	47	51
120	Vale Cambra	35	37	37	1004	Marinha Grande	44	47	37	1406	Ourém	27	29	29
121	Lourosa	39	41	34	1005	Martinh Porto	51	51	60	1408	Constância	40	42	56
122	OliVBairro	45	47	44	1006	Pombal	21	23	17	1409	VN Barquinha	53	53	62
123	Castelo Paiva	29	31	30	1007	Bombarral	55	55	61	1411	Torres Novas	33	35	32
124	Arouca	38	40	37	1008	Óbidos	37	39	44	1412	Salvat Magos	54	54	55
125	Murtosa	46	49	47	1009	Nazaré	53	53	51	1414	Mação	50	50	59
126	Fajões	49	49	51	1010	Peniche	36	37	36	1416	Golegã	75	75	87
201	Beja	28	30	27	1011	Figueiró Vinhos	37	40	49	1417	Ferreira Zêzere	53	53	60
202	Odemira	45	47	40	1012	Alvaiázere	47	47	53	1418	Entroncamento	62	62	62
203	Moura	39	41	38	1013	Vieira Leiria	50	50	61	1420	Almeirim	56	56	57
204	Aljustrel	70	70	67	1014	Cast Pêra	56	56	73	1421	Chamusca	62	62	62
206	Cuba	100	100	100	1015	Porto Mós	42	44	43	1425	Caxarias	58	58	62
207	F Alentejo	54	54	59	1016	Ansião	44	47	47	1426	Samora Correia	47	47	53
209	Almodôvar	72	72	73	1018	Batalha	39	42	38	1428	Fátima	42	45	46
210	Ourique	66	66	69	1019	Pataias	75	75	73	1429	Abrantes	35	37	35
211	Serpa	52	52	45	1020	Maceira	32	34	31	1502	Setúbal	50	50	41
213	Castro Verde	71	71	67	1021	Mira Aire	76	76	100	1503	Cacilhas	33	34	30
214	Vidigueira	65	65	69	1022	Leiria	27	29	24	1504	Sul e Sueste	39	41	41
215	VN Milfontes	87	87	93	1023	Juncal	65	65	69	1505	Sesimbra	38	40	34
303	Guimarães	39	42	29	1024	Benedita	47	47	51	1506	Montijo	40	43	37
304	Vizela	38	41	36	1025	Ortigosa	54	54	46	1507	Alcacér Sal	77	77	70
305	Barcelos	36	37	27	1101	Barcarena	49	49	56	1508	Almada	51	51	57
307	Fafe	33	35	29	1102	VFranca Xira	52	52	50	1509	Santiago Cacém	62	62	53
308	VN Famalicão	28	29	23	1103	Cascais	57	57	48	1510	Barreiro	45	48	44
309	Esposende	72	72	63	1104	Loures	36	38	35	1511	Trafaria	51	51	55
310	Póvoa Lanhoso	37	39	38	1105	Arruda Vinhos	54	54	54	1512	Moita	55	55	48
311	Amares	45	47	46	1106	Colares	53	53	65	1513	Palmela	43	45	42
312	Barcelinhos	33	35	27	1107	Sintra	34	35	43	1514	Sines	62	62	58
313	Vila Verde	39	41	37	1109	Bucelas	68	68	74	1515	Alcochete	50	50	51
314	Fão	69	69	54	1110	Oeiras	44	46	40	1516	Grândola	41	43	41
315	Celorico Basto	36	38	35	1111	Paço Arcos	77	77	66	1517	Pinhal Novo	46	49	46

Table A2. *Cont.*

FD Code	FD Name	Curr Eff %	Relax C5%	Use C5%	FD Code	FD Name	Curr Eff%	Relax C5%	Use C5%	FD Code	FD Name	Curr Eff%	Relax C5%	Use C5%
316	Famalicenses	26	28	21	1113	Odivelas	42	44	36	1519	Cercal Alentejo	61	61	77
317	Vieira Minho	48	48	51	1114	Sacavém	36	38	35	1520	Seixal	28	29	27
318	Cab Basto	43	45	49	1115	Alhandra	65	65	62	1521	Águas Moura	51	51	62
319	Riba Ave	49	49	48	1116	Algés	100	100	100	1522	Canha	46	49	57
321	Viatodos	50	50	51	1117	Torres Vedras	25	27	21	1524	Santo André	89	89	92
322	Terras Bouro	47	48	55	1118	Amadora	45	47	43	1525	Alvalade	100	100	100
401	Mirandela	32	34	31	1119	SPedro Sintra	46	47	51	1526	Amora	57	57	46
402	Bragança	25	27	25	1120	Carcavelos-S D R	50	50	49	1603	Ponte Lima	25	26	25
403	M Cavaleiros	35	36	39	1121	Dafundo	53	53	58	1604	Arcos Valdevez	46	48	49
404	F Espada Cinta	42	44	58	1122	Carnaxide	62	62	57	1605	Caminha	56	56	61
405	Carraz Ansiães	53	53	65	1123	S Monte Agraço	53	53	56	1606	Monção	48	48	52
406	Mogadouro	42	44	42	1124	Cadaval	53	53	57	1607	VPraia Âncora	73	73	79
407	Vimioso	57	57	61	1125	Queluz	56	56	55	1608	Valença	58	58	58
408	Torre Moncorvo	36	38	46	1127	Camarate	66	66	70	1609	PCoura	49	49	58
409	Alfândega Fé	50	50	65	1128	Belas	39	41	38	1610	Ponte Barca	56	56	64
410	Vinhais	37	40	42	1129	Parede	48	48	51	1611	VN Cerveira	54	54	58
411	Vila Flor	42	44	54	1130	Alverca	53	53	48	1612	Melgaço	52	52	57
412	Miranda Douro	59	59	64	1131	Alcabideche	39	41	39	1701	Peso Régua	43	45	51
413	Torre Chama	61	61	86	1132	Moscavide	100	100	100	1702	Flaviense	39	40	48
414	Sendim	54	54	100	1133	Mafra	38	40	41	1703	Verde-VReal	28	30	31
501	Covilhã	24	25	24	1134	Lourinhã	39	41	39	1704	Sanfins Douro	87	87	100
502	Sertã	20	21	29	1135	Fanhões	72	72	79	1705	Sabrosa	66	66	83
503	Fundão	20	22	21	1137	Ericeira	57	57	53	1706	Branca-VReal	23	25	29
504	Castelo Branco	21	21	21	1138	Agualva-Cacém	56	56	60	1707	Favaios	100	100	100
505	Penamacor	56	56	61	1139	Azambuja	58	58	56	1708	VPouca Aguiar	39	41	47
506	Oleiros	34	35	53	1140	Alcoentre	55	55	64	1709	Mondim Basto	45	45	63
507	Proença Nova	33	34	45	1141	Alenquer	33	36	31	1711	Murça	64	64	76
508	Idanha Nova	100	100	100	1142	Póvoa Sta Iria	67	67	60	1714	Montenegro	74	74	96
509	Velha Ródão	73	73	100	1143	Malveira	40	42	43	1715	Alijó	65	65	74
510	Belmonte	57	57	64	1144	Alg Mem-Martins	37	38	38	1716	Valpaços	46	48	50
511	Vila Rei	57	57	76	1145	Cast Ribatejo	84	84	78	1717	Chaves	64	64	62
512	Cern Bonjardim	35	37	49	1146	Vialonga	68	68	57	1718	Mesão Frio	56	56	67

Table A2. *Cont.*

FD Code	FD Name	Curr Eff %	Relax C5%	Use C5%	FD Code	FD Name	Curr Eff%	Relax C5%	Use C5%	FD Code	FD Name	Curr Eff%	Relax C5%	Use C5%
604	Coimbra	100	100	100	1147	Caneças	60	60	67	1719	Montalegre	47	47	50
605	Cantanhede	29	31	25	1148	Pontinha	87	87	90	1720	Fontes	60	60	80
607	Soure	30	32	32	1149	Merceana	59	59	65	1721	Vidago	38	39	66
608	OliVHospital	32	34	35	1150	Montelavar	57	57	66	1722	Boticas	47	47	60
609	Condeixa Nova	35	38	33	1201	Portalegre	34	37	35	1724	Ribeira Pena	60	60	74
610	Penacova	31	33	32	1203	Ponte Sôr	30	32	33	1725	de Cerva	100	100	100
611	Montemor Velho	38	40	38	1204	Elvas	34	36	33	1726	Sta M Penaguião	100	100	100
612	Arganil	58	58	54	1205	Nisa	62	62	59	1727	Salto	63	63	100
613	VN Oliveirinha	47	47	63	1209	Campo Maior	73	73	79	1802	Lamego	38	40	36
614	Tábua	40	43	48	1210	Avis	100	100	100	1803	Castro D'Aire	38	40	43
616	Lagares Beira	61	61	66	1213	Monforte	100	100	100	1804	Pedro Sul	53	53	61
617	Miranda Corvo	34	36	37	1302	Matosinhos-Leça	56	56	47	1805	Vouzela	37	39	45
618	VN Poiares	52	52	54	1303	Póvoa Varzim	42	44	34	1807	SJ Pesqueira	57	57	74
620	Coja	43	46	57	1304	Santo Tirso	50	50	49	1808	Santa Comba Dão	40	43	47
621	Pampilhosa Serra	43	45	100	1305	Penafiel	39	41	38	1809	Nelas	44	46	62
622	Penela	35	37	44	1306	Paredes	45	47	46	1810	Tondela	37	39	37
623	Mira	50	50	52	1307	Lixa	41	43	41	1811	Mortágua	42	45	45
701	Évora	34	36	30	1308	Valongo	43	45	43	1813	Moimenta Beira	34	36	40
702	Vendas Novas	56	56	58	1309	Felgueiras	38	40	35	1814	Mangualde	38	41	40
703	Montemor Novo	39	42	34	1310	Coimbrões	49	49	41	1815	Farejinhas	67	67	73
704	Estremoz	47	47	48	1311	Carvalhos	45	47	42	1816	Oliveira Frades	60	60	70
705	Arraiolos	62	62	59	1312	Vila Conde	34	36	26	1817	Canas Senhorim	51	51	63
706	Regueng Monsar	53	53	58	1313	Gondomar	53	53	45	1818	Armamar	59	59	67
707	Vila Viçosa	53	53	64	1314	Valadares	56	56	51	1819	Cabanas Viriato	53	53	71
710	Redondo	72	72	73	1315	Mamed Infesta	54	54	52	1820	Tabuaço	43	46	56
712	Portel	76	76	79	1316	Amarante	29	30	30	1821	Carregal Sal	47	47	54
802	Lagos	46	49	45	1317	Ermesinde	45	47	46	1822	Penalva Castelo	49	49	54
804	VR Sto António	37	39	38	1318	Areosa-Rio Tinto	66	66	56	1823	Resende	45	47	53
806	Silves	39	41	39	1319	Entre-os-Rios	49	49	52	1824	Ervedosa Douro	73	73	86

Table A2. *Cont.*

FD Code	FD Name	Curr Eff %	Relax C5%	Use C5%	FD Code	FD Name	Curr Eff%	Relax C5%	Use C5%	FD Code	FD Name	Curr Eff%	Relax C5%	Use C5%
807	Portimão	29	30	29	1320	Marco Canaveses	29	31	25	1825	Sernancelhe	62	62	69
809	S. Brás Alportel	57	57	61	1321	Aguda	55	55	46	1826	Cinfães	33	35	43
811	Monchique	100	100	100	1322	Côte	47	47	53	1827	Penedono	79	79	100
812	Aljezur	41	44	55	1323	Moreira Maia	33	35	27	1828	Nespereira	47	47	54
813	S Bart Messines	43	46	47	1324	Valbom	55	55	52	1829	Tarouca	52	52	52
814	Albufeira	29	30	36	1325	Baltar	45	48	42	1830	VNova Paiva	47	47	58
815	Lagoa	33	35	37	1326	Tirsenses	38	40	41	1831	Sátão	43	46	47
816	Vila Bispo	73	73	72	1327	Lousada	35	37	33	1832	Vale Besteiros	64	64	55
902	Sabugal	42	44	55	1328	Freamunde	35	37	40					

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