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Studies on Improving Seals for Enhancing the Vibration and Environmental Safety of Rotary Machines

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Abstract: There is a constant demand for higher equipment parameters, such as the pressure of a sealing medium and shaft rotation speed. However, as the parameters increase, it becomes more difficult to ensure hermetization efficiency. The rotor of a multi-stage machine rotates in non-contact seals. Seals' parameters have a great influence on vibration characteristics. Non-contact seals are considered to be hydrostatodynamic supports that can effectively dampen rotor oscillations. The force coefficients of gap seals are determined by geometric and operational parameters. A purposeful choice of these parameters can influence the vibration state of the rotor. It is shown for the first time that the initially dynamically flexible rotor, in combination with properly designed seals, can become dynamically rigid. Analytical dependencies for the computation of the dynamic characteristics are obtained. The resulting equations make it possible to calculate the radial-angular vibrations of the rotor of a centrifugal machine in the seals and construct the amplitude–frequency characteristics. By purposefully changing the parameters of non-contact seals, an initially flexible rotor can be made rigid, and its vibration resistance increases. Due to this, the environmental safety of critical pumping equipment increases.

Keywords: gap seals; hydromechanical system; vibrations; frequency characteristics; stability

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1. Introduction

Centrifugal pumps as a type of rotary machine are applied for transporting fluids in air conditioning [1,2] and refrigeration [3,4] systems for building and district heat and cold supply [5,6]. They are widely used in thermal and power energetics [7,8] for circulating hot water in cogeneration circuits [9,10] to remove the heat from combustion engines for their air cooling such as gas engines (GEs) [11,12], gas turbines (GTs) [13,14] and internal combustion engines (ICEs) [15,16], as well as in waste heat recovery circuits [17,18] to convert the heat released from primary engines in waste heat recovery chillers [19,20]. Centrifugal pumps feed exhaust gas boilers [21,22] in heat utilization systems [23,24] and facilitate the circulation of coolant in engine cooling systems for cooling inlets [25,26] and charged air [27,28] in ICEs and intercooling in GTs [29,30]. High-pressure pumps are used for feeding thermopressors for cooling charged air [31,32] in ICEs and intercooling in GTs [33,34]. They are also used for circulating coolants such as chilled water [35,36] in water-cooled heat exchangers, which are subject to stress and deformation during operation [37,38], as well as in cooling towers and radiators [39,40] or liquid refrigerant in the refrigeration circuits of heat exchangers [41,42] and multicomponent liquid media [43–45].

Vibration and environmental safety factors are of considerable importance for rotary machines subjected to dynamic loads during hard-loaded operation of railways [46,47] and especially in ship power plants [48–50].

The demand for centrifugal machines with high parameters, such as compaction pressure and rotor speed, is constantly growing [51]. Therefore, the task of ensuring their tightness and vibration reliability has become very important [52]. The rotor seals of high-speed centrifugal machines are developing into complex systems that determine the reliability of the units [53,54].

When creating sealing systems for non-standard operating conditions, it is necessary to take into account their influence on the vibration characteristics of equipment [55,56].

Increased demands are placed on the sealing units of energy pumps [57]. To ensure the required service life, seals with guaranteed controlled tightness are created for lubrication and cooling [58,59].

Rotary machines, the main unit of which is the rotor, i.e., a rotating shaft with certain working parts attached to it, constitute a very wide class of machines. During operation, the rotor is affected by harmonic disturbances, causing forced oscillations of the rotor [60]. As a rule, the vibration state of the rotor determines the technical level of such machines. Therefore, the problems of rotor dynamics are of great practical importance for a large number of design types of rotary machines [61–67]. Seal parameters have a great influence on vibration characteristics [68,69].

Among rotary machines in all industries, multi-stage high-pressure centrifugal pumps and compressors are widely used. They are characterized by a steady tendency to increase operating parameters such as feed, pressure, and rotation speed, that is, to concentrate increasingly higher power in individual units. The pressure developed by centrifugal machines is proportional to the square of the rotor speed, so increasing the speed is the most rational way to achieve high pressure. As a result, high-pressure centrifugal machines tend to have high speed, and for such machines, the problems of rotor dynamics are especially relevant.

Energy conversion in non-contact seals must be considered to determine radial stiffness that effectively reduces rotor vibration. In this case, non-contact seals play the role of additional dynamic supports of the rotor [70,71].

The rotor of the multi-stage machine rotates in non-contact seals. Hydraulic resistance arises due to the friction of a viscous fluid against the walls of the channels. The sealed medium acts on the walls of the non-contact seal channels, one of which belongs to the vibrating rotating rotor. This effect is especially evident when there are large differences in sealed pressure. Accordingly, positional, dissipative, gyroscopic, and inertial radial pressure forces and their moments act on the walls of the channels belonging to the rotor. The dynamics of the rotor are determined by these forces and moments, which, in turn, depend on the nature of the rotor's movement. Thus, the rotor and non-contact seals represent a closed hydromechanical system. When solving the problem of calculating the rotor dynamics of a centrifugal machine, it is necessary to take into account the influence of non-contact seals, which can change the critical frequencies of the rotor and influence the amplitude of its forced oscillations [72–75].

Operation experience shows that a significant part of the failures of centrifugal pumps and compressors is associated with the fatigue failure of individual components and parts. With increasing parameters, the danger of fatigue failure increases, since the overall level of specific energy intensity increases and, accordingly, the intensity at which additional alternating stresses caused by vibrations are superimposed also increases [76–80].

Increased vibrations are accompanied by work near critical frequencies, the calculation of which, due to the lack of reliable data on the rigidity of supports and many random factors, cannot guarantee reliable detuning from resonant modes [81].

The need for thermal power engineering for feed, main circulation, and other pumps of increasingly higher parameters prompted a detailed study of the hydrodynamics of slotted seals and their influence on the vibration state of rotors of centrifugal machines [82].

Clearance seals act as hydrostatic dynamic bearings, the radial stiffness of which is proportional to the throttled pressure drop. Typically, the stiffness of seals is either comparable to or superior to that of plain bearings. Due to this, seals act as additional intermediate supports [83].

The geometric and design parameters of seals determine their power characteristics. The analysis of the influence of clearance seals on the dynamics of the rotor makes it possible to select their design in such a way that the level of rotor vibration does not exceed acceptable limits throughout the entire operating range.

Therefore, assessing the influence of the geometric and operating parameters of seals on critical frequencies, the amplitudes of forced vibrations, and the stability of rotor motion is very important for increasing the vibration reliability of centrifugal machines.

2. Materials and Methods

2.1. Model of the Gap Seal

Figure 1 shows a model of the gap seal [81] which is an annular throttle formed by an inner cylinder (shaft) with a small taper angle ϑ_A and an outer cylinder (sleeve) with a taper angle ϑ_B; the total taper angle of the channel is $\vartheta_0 = \vartheta_B - \vartheta_A$, and the taper parameter of the channel is $\theta_0 = \vartheta_0 l/2H$, $|\theta_0| \leq 1$.

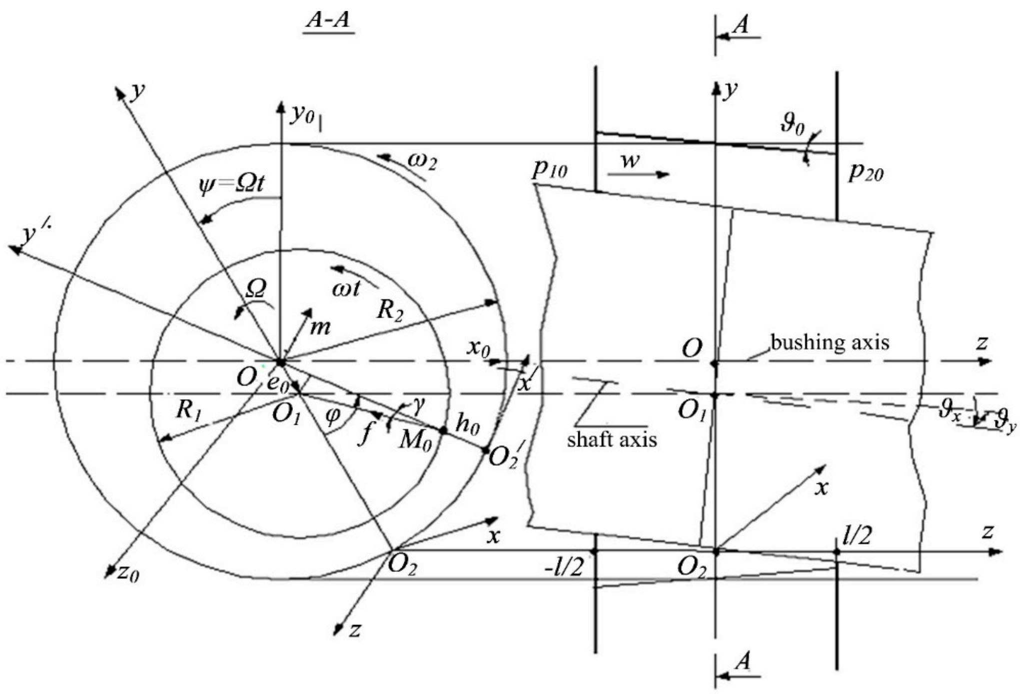


Figure 1. Model of the gap seal.

Shaft and bushing rotate around their own axes with the frequencies of their own rotation $\omega_1,\ \omega_2$. The axes themselves rotate around the fixed center O with precession frequencies $\Omega_1,\ \Omega_2$ and also perform radial and angular oscillations.

Thus, when developing gap seals, it is necessary to consider not only their direct purpose to reduce volumetric losses but also their equally important function, which has to provide the necessary vibration characteristics of the rotor.

The flow regime is characterized by the constants $C,\ n$ of the generalized Blasius formula for the friction drag coefficient $\lambda = C\mathrm{Re}^{-n}$. Local resistances are determined by the relative coefficients of hydraulic losses [82]. For laminar flows, the coefficient of local resistance was determined in [83] for slotted seals with annular grooves. The results showed that, in this case, the local resistances were close to zero.

Under the action of a sealable pressure drop, a liquid with a high (up to 70 m/s) axial velocity enters the annular gap formed by short ($l < 2R_0$) rotating cylinders. Due to the viscosity, the circumferential velocity of the particles adjacent to the rotating walls gradually spreads to the inner layers. The time during which the volume of liquid that enters the channel is in the gap $T = l/w_0$. By the end of this period, near the exit from the channel, the liquid acquires the maximum average circumferential velocity. At the entrance to the channel, the velocity is close to zero. Thus, the average circumferential speed and the swirl ratio vary along the length of the channel.

2.2. Radial Forces and Moments in Gap Seals

To further assess the influence of gap seals on the dynamics of the rotor, we determined the values of individual components of hydrodynamic forces and moments that arise in the gaps of gap seals. The derivation of these equations and the physical meaning of the quantities included in them are described in more detail in [84].

$$-F^*_{1x} = a_{11}\ddot{u}_x, \quad -F^*_{1y} = a_{11}\ddot{u}_y, \quad -M^*_{1x} = a_{11}j\ddot{\theta}_x, \quad -M^*_{1y} = a_{11}j\ddot{\theta}_y$$

$$-F^*_{2x} = a_{21}\dot{u}_x + a_{41}\dot{u}_y - \alpha_2\dot{\theta}_x + \alpha_4\dot{\theta}_y, \quad -F^*_{2y} - a_{41}\dot{u}_x + a_{21}\dot{u}_y - \alpha_4\dot{\theta}_x - \alpha_2\dot{\theta}_y,$$

$$-M^*_{2x} = j\left[15\alpha_2\dot{u}_x + 15\alpha_4\dot{u}_y + 2k_d\dot{\theta}_x + a_{41}\dot{\theta}_y\right],$$

$$-M^*_{2y} = j\left[-15\alpha_4\dot{u}_x + 15\alpha_2\dot{u}_y - a_{41}\dot{\theta}_x + 2k_d\dot{\theta}_y\right];$$

$$-F^*_{3x} = a_{31}u_x + a_{51}u_y - \alpha_3\theta_x + \alpha_5\theta_y, \quad -F^*_{3y} = -a_{51}u_x + a_{31}u_y - \alpha_5\theta_x - \alpha_3\theta_y,$$

$$-M^*_{3x} = j\left(-15\alpha_3 u_x + 5\alpha_5\frac{N\Delta\chi}{1+2\Delta\chi}u_y - 10a_{31}\frac{\chi_m}{\theta_0 + N\chi_m}\theta_x + a_{51}\theta_y\right)$$

$$-M^*_{3y} = j\left(-5\alpha_5\frac{N\Delta\chi}{1+2\Delta\chi}u_x - 15\alpha_3 u_y - a_{51}\theta_x - 10a_{31}\frac{\chi_m}{\theta_0 + N\chi_m}\theta_y\right).$$

Additional moments from elastic forces are as follows:

$$\Delta M_{3x} = -2l_c\Delta F_{3y} = -2a_{31}Hm\frac{l_c^2}{l}\theta_x, \quad \Delta M_{3y} = -2l_c\Delta F_{3x} = -2a_{31}Hm\frac{l_c^2}{l}\theta_y,$$

where the doubled force coefficients are determined as follows:

$$a_{11} = 2k_g, \quad a_{21} = 2\left(k_d + k_g K_i\theta_0\right), \quad a_{41} = k_g\kappa\omega, \quad \alpha_2 = \tfrac{2}{15}k_g\kappa\omega\theta_0, \quad \alpha_4 = \tfrac{4}{5}k_d\theta_0;$$
$$a_{31} = 2k_p(\theta_0 + N\chi_m), \quad a_{51} = k_d\kappa\omega, \quad \alpha_3 = \tfrac{2}{5}k_d\kappa\omega\theta_0, \quad \alpha_5 = 2k_p(1 + 2\Delta\chi).$$

The method for calculating additional moments and radial forces acting on the rotor in seals is given in [85].

In [86,87], the authors suggested considering non-contact seals as automatic control systems. Using this approach, an algorithm for constructing the dynamic characteristics of the rotor in non-contact seals was proposed.

3. Results

3.1. Model of the Hydromechanical System with "Rotor-Slotted Seals"

The model of the hydromechanical system with "rotor-slotted seals" is shown in Figure 2.

As can be seen from the presented model of the hydromechanical system, there are feedbacks between the parameters of the sealing channels and the parameters that affect the nature of the rotor oscillations.

The constructed model shows that slot seals not only reduce the loss of the sealed medium but also affect the vibration characteristics of the rotor of a centrifugal machine.

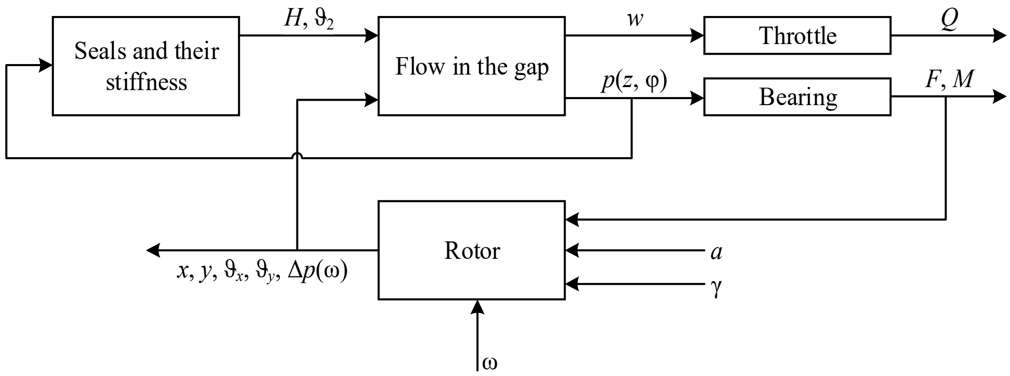


Figure 2. The model of the hydromechanical system with rotor-slotted seals.

3.2. Joint Radial–Angular Oscillations of the Rotor in Gap Seals

Radial forces and moments in gap seals, the formulas for which are proposed in Section 2, are included in the rotor oscillation equations as coefficients [88].

Typical rotor models are reviewed in [81]. In a typical design, the disk rotates in the plane of the curved axis of the shaft and moves in the radial direction. The gyroscopic moment of the disk arises due to the inertial resistance to rotation. The weightless elastic shaft rotates in rigid supports, and the entire mass of the rotor is concentrated at the disk center of mass.

The rotor is statically and dynamically unbalanced: The center of mass is displaced relative to the geometric center by an amount of eccentricity, which represents static unbalance. The main central axes of inertia of the disk, due to skewed landing or other technological errors, deviate from the main axes of the shaft section (the main axes of shaft rigidity) at angles that characterize the dynamic imbalance of the rotor. The imbalance parameters are considered to yield small values.

The rotor–seal system under consideration is an eighth-order oscillatory system with four generalized coordinates: u_x, u_y, θ_x, θ_y. If the system oscillates about a stable equilibrium position, then the roots of the characteristic equation are four pairs of complex conjugate numbers.

For isotropic systems, in which the force coefficients and external loads are identical in all directions in the plane perpendicular to the axis of rotation, all points of this axis move along circular trajectories. In this case, we can move on to complex variables and disequilibria. Let us first note that operations with an imaginary unit have a peculiarity due to the equivalence of $+i$ and $-i$, as can be seen from the relation $(\pm \mathrm{i})^2 = -1$. The rotor under the influence of gyroscopic forces and moments can perform both direct and reverse precession. In order not to lose the ability to detect additional movements when moving to complex variables, it is necessary to multiply the equations by equivalent ones $\pm$i. The system of equations describing the forced joint radial–angular oscillations of the rotor at a constant pressure drop across the seals takes the following form [88]:

$$\begin{aligned}
&a_1\ddot{u} + a_2\dot{u} + a_3 u \mp i\left(a'_4\dot{u} + a'_5 u\right)\omega - \left(\alpha'_2\dot{\theta} + \alpha'_3\theta\right)\omega \mp \\
&\mp i\left(\alpha_4\dot{\theta} + \alpha_5\theta - \alpha_0\theta\right) = \omega^2 a^* = \omega^2|a^*|e^{\pm i\omega t}, \\
&b_1\ddot{\theta} + b_2\dot{\theta} + b_3\theta \mp i\left(b'_4\dot{\theta} + b'_5\theta\right)\omega + \left(\beta'_2\dot{u} - \beta'_3 u\right)\omega \mp \\
&\mp i\left(\beta_4\dot{u} + \beta_5 u + \beta_0 u\right) = \left(1 - \mathrm{j}_0\right)\omega^2\gamma^* = \left(1 - \mathrm{j}_0\right)\omega^2|\gamma^*|e^{\pm i\omega t};
\end{aligned} \tag{1}$$

Using standard programs, you can immediately find a numerical solution to these equations. However, the traditional approach used here allows us to consider the analytical expressions of amplitudes and phases (the coefficients of the system's own operator and the operators of external influences) to determine how various forces and moments influence them.

By substituting the solution of Equation (1) in the form

$$u = u_a e^{i(\omega t+\phi_u)} = \widetilde{u}e^{i\omega t}, \ \theta = \theta_a e^{i(\omega t+\phi_\vartheta)} = \widetilde{\theta}e^{i\omega t},$$

we obtain a system of algebraic equations for the complex amplitudes A and Γ as follows:

$$\begin{aligned} &\left[-a_1\omega^2 + a_3 + a_4\omega^2 + i(a_2 - a_5)\omega\right]\widetilde{u} - \left[(\alpha_3 - \alpha_4)\omega + i\left(\alpha_2\omega^2 + \alpha_5 - \alpha_0\right)\right]\widetilde{\theta} = \mathrm{A}\omega^2 \\ &\left[-(\beta_3 - \beta_4)\omega + i\left(\beta_2\omega^2 - \beta_5 - \beta_0\right)\right]\widetilde{u} + \left[-b_1\omega^2 + b_3 + b_4\omega^2 + i(b_2 - b_5)\omega\right]\widetilde{\theta} = \Gamma\omega^2. \end{aligned} \quad (2)$$

After a series of transformations, Equation (2) takes the following form:

$$\begin{aligned} (U_{11} + iV_{11})\widetilde{u} + (U_{12} + iV_{12})\widetilde{\theta} = \mathrm{A}\overline{\omega}^2, \\ (U_{21} + iV_{21})\widetilde{u} + (U_{22} + iV_{22})\widetilde{\theta} = \Gamma\overline{\omega}^2. \end{aligned} \quad (3)$$

Here, $U_{11} + iV_{11}$, $U_{22} + iV_{22}$ are the own operators of the independent radial and angular oscillations correspondingly. Cross-sectional operators $U_{12} + iV_{12}$, $U_{21} + iV_{21}$ characterize the influence of angular oscillations on radial and the effect of radial on angular, i.e., the interconnection of these oscillations, with $\varpi = \omega/\Omega_{u0}$—dimensionless frequency.

3.3. Frequency Responses and Dynamic Stability

From the system of non-homogenous algebraic Equation (3), after a series of transformations, we obtain the amplitudes and phases expressed in terms of external disturbances as follows:

$$\begin{aligned} u_a &= \overline{\omega}^2\sqrt{\tfrac{(\mathrm{A}U_{22}-\Gamma U_{12})^2+(\mathrm{A}V_{22}-\Gamma V_{12}\ )^2}{U_0^2+V_0^2}}, \\ \theta_a &= \overline{\omega}^2\sqrt{\tfrac{(\Gamma U_{11}-\mathrm{A}U_{21})^2+(\Gamma V_{11}-\mathrm{A}V_{21}\ )^2}{U_0^2+V_0^2}}, \\ \phi_u &= -arctg\tfrac{(\mathrm{A}U_{22}-\Gamma U_{12})V_0-(\mathrm{A}V_{22}-\Gamma V_{12})U_0}{(\mathrm{A}U_{22}-\Gamma U_{12})U_0+(\mathrm{A}V_{22}-\Gamma V_{12})V_0}, \\ \phi_\vartheta &= -arctg\tfrac{(\Gamma U_{11}-\mathrm{A}U_{21})V_0-(\Gamma V_{11}-\mathrm{A}V_{21})U_0}{(\Gamma U_{11}-\mathrm{A}U_{21})U_0+(\Gamma V_{11}-\mathrm{A}V_{21})V_0}. \end{aligned} \quad (4)$$

Using Formula (4), the amplitude–frequency and phase characteristics are calculated.

The stability is determined using the Routh–Hurwitz criterion for a system of fourth order [88] as follows:

$$a_2(a_2a_3 + a_4a_5) - a_1a_5^2 > 0,$$

which reduces to the following form:

$$\omega_u^2 < \frac{a_{21}^2\Omega_{u0}^2}{a_1a_5^2 - a_{21}^2a_{31} - a_{21}a_4a_5} \quad (5)$$

From inequality (5), it is clear that the circulation force (coefficient a_5) destabilizes, and damping a_{21}, gyroscopic force a_4, and bending rigidity of the shaft Ω_{u0} stabilize the rotor in the seals. A detailed explanation of the physical processes occurring in gap seals is presented in [55].

4. Discussion

Frequency diagrams of the dependences of natural frequencies on rotational speed are shown in Figure 3 for constant, speed-independent pressure drops. Δp_o = (1.5; 3, 98; 13.3) MPa.

The pressure throttled at the slot seals of centrifugal machines is proportional to the square of the rotor rotation speed. This affects the type of frequency characteristics, since the compacted pressure ceases to be an independent external influence but is related to the rotational speed by an additional relationship $\Delta p_0 = B\omega^2$.

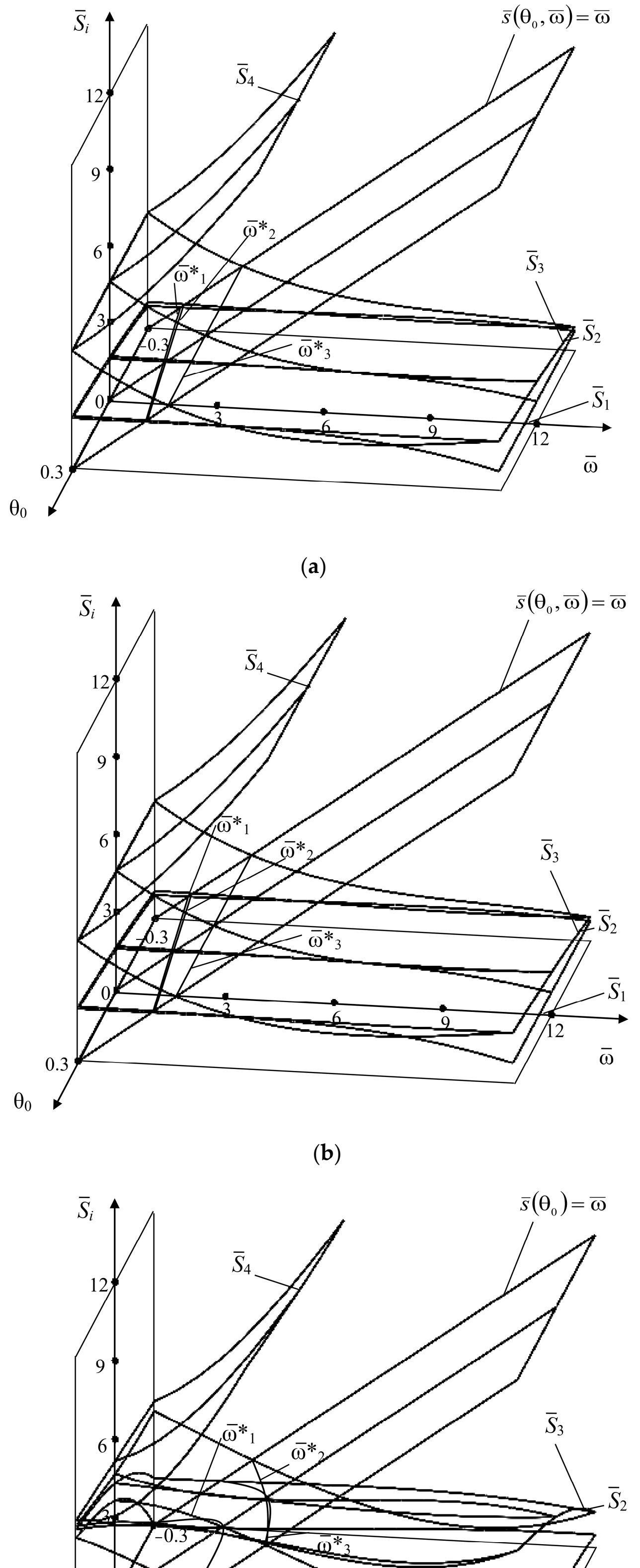
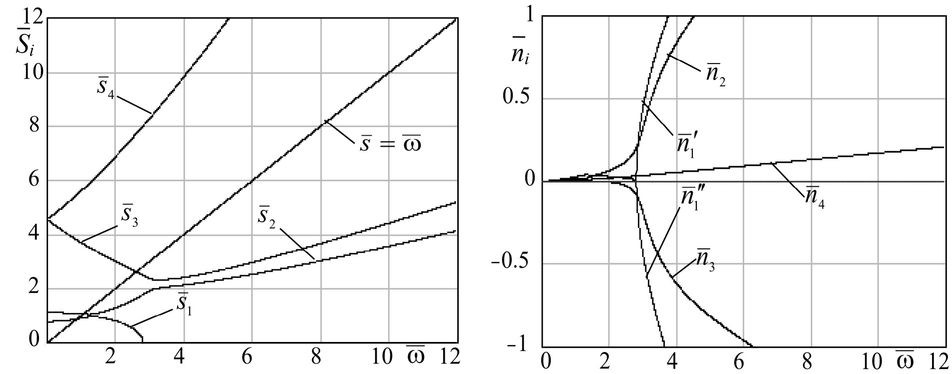


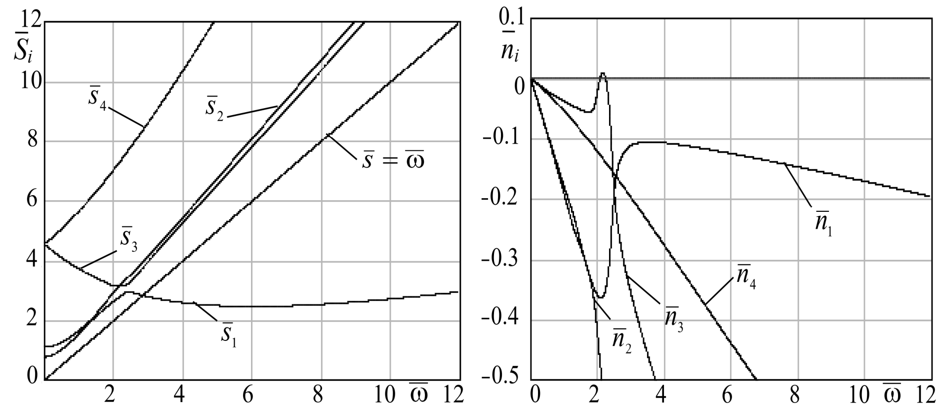
Figure 3. Frequency diagrams for constant differential pressure Δp_0: (**a**) 1.5 MPa; (**b**) 3.98 MPa; (**c**) 13.3 MPa.

Since many coefficients of Equation (1) depend on the pressure drop, they also depend on the rotation speed, and this is reflected in the shape of the frequency characteristics—the dependences of natural frequencies on the rotor speed.

Therefore, the external influence is only the rotation speed, and the rotor in the seals acquires greater dynamic rigidity (Figure 4). In turn, in each of the figures, the dependences of natural frequencies on the parameter of the annular gap taper of slotted seals in the range of $-0.3 \leq \theta_0 \leq 0.3$ and the rotational speed are plotted. The results of the analysis of frequency diagrams are presented below.



(**a**)



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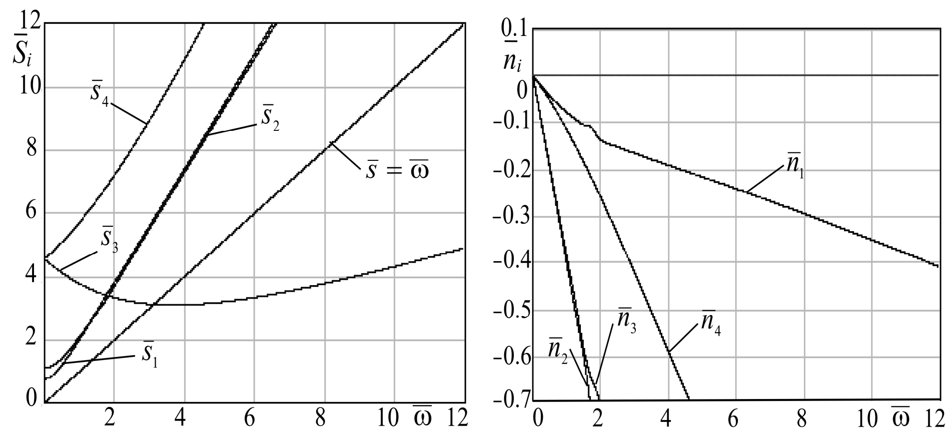


(**c**)

Figure 4. Frequency diagram and graphs of damping coefficients at $\Delta p_0 = B\omega^2$, $B =$ const: (**a**) $\theta_0 = -0.3$; (**b**) $\theta_0 = 0$; (**c**) $\theta_0 = 0.3$.

In the cylindrical $\theta_0 = 0$ and confusor $\theta_0 = 0.3$ gaps, there is an intensive increase in natural frequencies (except for the first) with increasing rotation frequency. Because of this, three critical frequencies, which are determined by the points of the intersection of frequency diagrams with straight lines $\bar{s} = \bar{\omega}$, exist only for the rotor in slotted seals with a diffuser shape of the annular gap. In seals with cylindrical and confusor gaps, only the first critical rotation speed occurs.

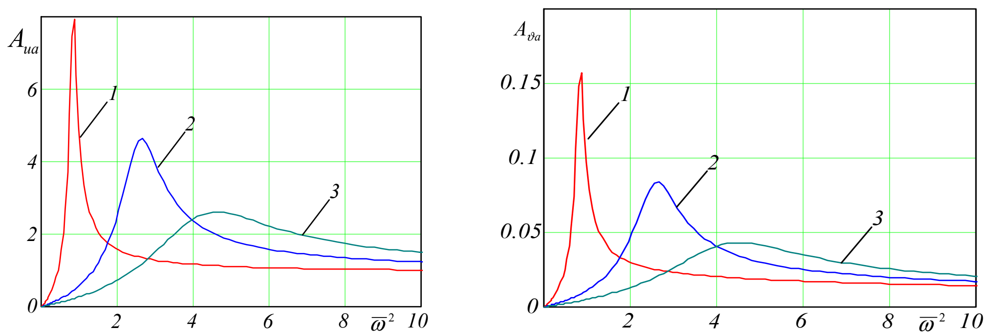
Figure 4 also shows graphs of the real parts of the roots of the characteristic equation for the generalized constant B. From the graphs of the real parts, one can assess the stability: If among the roots, there are roots with a positive real part, then the rotor is unstable at the corresponding rotation frequencies. In particular, the rotor in seals with a diffuser gap $\theta_0 = -0.3$ is unstable at all frequencies. The rotor in confusor seals remains stable throughout the entire range of rotation speeds under consideration. Thus, the destabilizing effect of slotted seals with a diffuser gap and the pronounced stabilizing effect of confusor channels are confirmed.

An increase in the confusor $(\theta_0 > 0)$ and pressure drop (independent of rotational speed) increases the first two natural frequencies $\bar{s}_1,\ \bar{s}_2$, which differ little from one another and are close to the partial frequencies $\bar{s}_{u1},\ \bar{s}_{u2}$ of independent radial oscillations. Only at high-pressure drops $\Delta p_0 > 5MPa$ and rotational frequencies $\overline{\omega} > 4$ is the difference between these natural frequencies noticeable. The two highest natural frequencies $\bar{s}_3,\ \bar{s}_4$ are practically independent of the pressure drop and taper and are close to the partial frequencies $\bar{s}_{\vartheta 1},\ \bar{s}_{\vartheta 2}$ of independent angular oscillations. With an increase in the number of revolutions, the second and third natural frequencies approach each other.

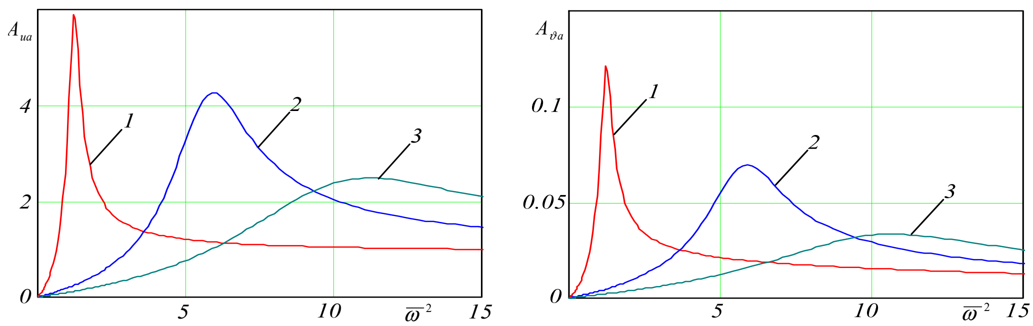
Critical frequencies are located on the lines of the intersection of the plane $\bar{s}(\overline{\omega}, \theta_0) = \overline{\omega}$ with surfaces $\bar{s}_{1-4}(\overline{\omega}, \theta_0)$.

There is no fourth critical speed for the examples under consideration: the gyroscopic moment causes the self-tightening of the rotor.

Numerical calculations were carried out for the rotor model with a disc between the seals. The gap seals with three taper parameters were considered (Figure 5).

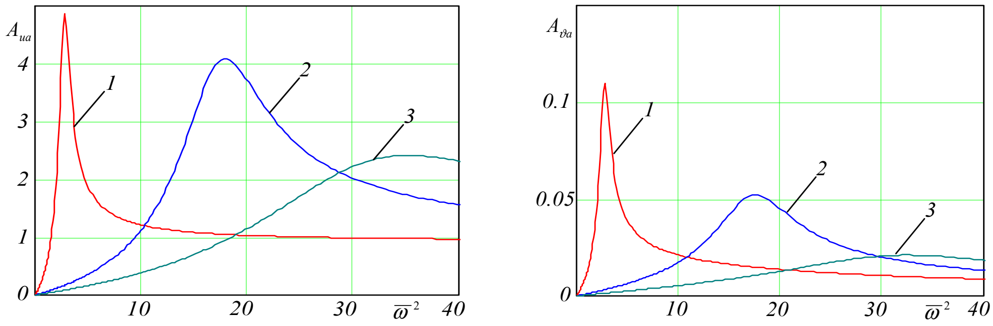


(**a**)



(**b**)

Figure 5. *Cont.*



(**c**)

Figure 5. Amplitude–frequency characteristics as a response to statistic unbalance: (**a**) Δp_0 = 1.5 MPa = const; (**b**) Δp_0 = 4 MPa = const; (**c**) Δp_0 = 13.3 MPa = const. 1—θ_0 = −0.3; 2—θ_0 = 0; 3—θ_0 = 0.3.

A comparison of the results of the calculations of frequency characteristics according to the obtained expressions with the data of experimental studies (Figure 6) shows that the calculation errors do not exceed 5%, which suggests the possibility of using the obtained formulas.

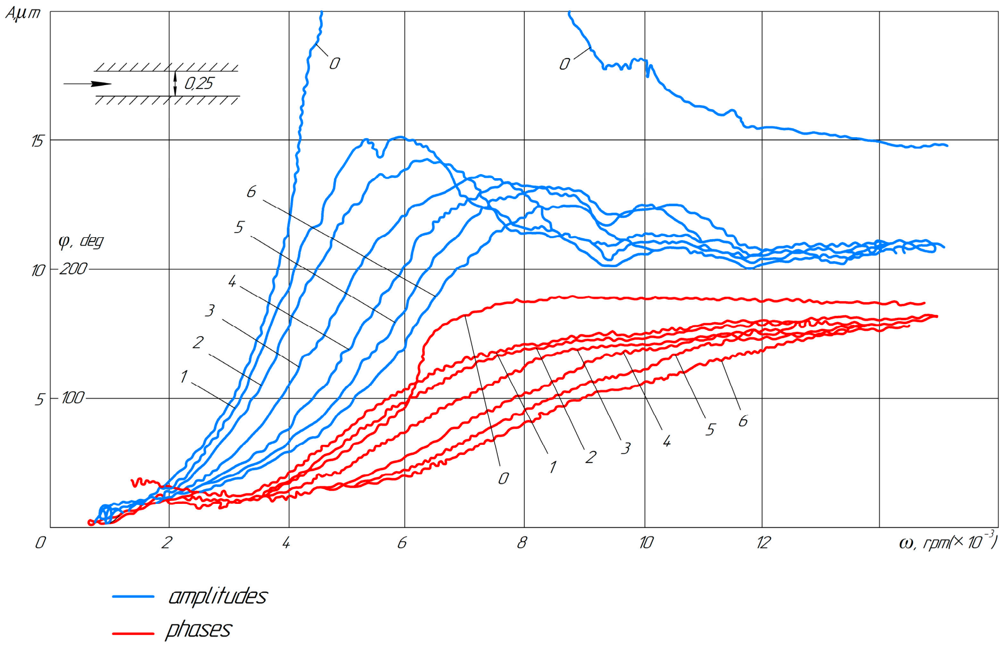


Figure 6. Amplitude (A) and phase (φ) frequency characteristics of the rotor in gap seals (experimental data); ω is a rotor rotation frequency; compaction pressure in MPa: 0—0; 1—0.18; 2—0.2; 3—0.4; 4—0.6; 5—0.8; 6—1.0.

5. Gap Seals for Energy Pumps

The turbo-feed pump (Figure 7) was designed to pump (1350 m^3/h) water at a temperature of 165 °C under a pressure of 35 MPa into steam boilers of steam turbine blocks of thermal power plants with a capacity of 800 MW [83].

The pump was driven by an 18 MW steam turbine with a rotor speed of 5500 rpm. End seals were composed of a 1, 6—slot type with a supply of cold locking condensate with 2—slotted stepped front seals of impellers, and 3—smooth rear seals. In the system of the automatic balancing of axial forces acting on the rotor (in the hydraulic heel), radial (4) and axial (5) slotted seals were used.

In the example of the considered pump design, it is clearly seen that the slotted seals are located quite tightly along the entire length of the rotor. Therefore, they significantly affect the vibration state of the rotor and the pump as a whole. Moreover, each stage of the pump develops a pressure of 4 to 6 MPa, and a pressure of more than 20 MPa is throttled on the hydraulic heel.

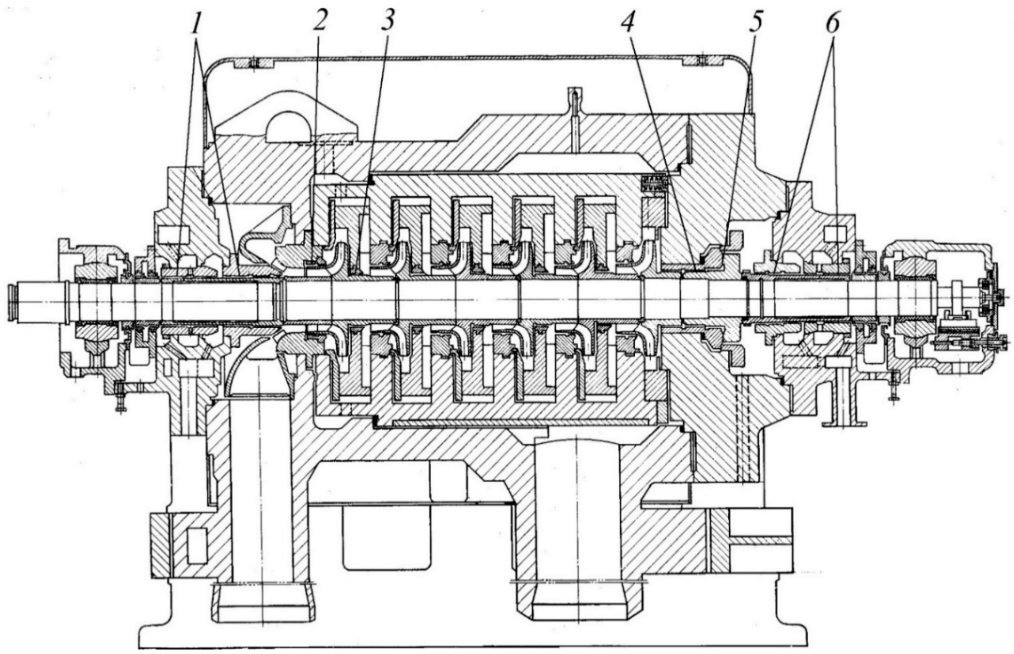


Figure 7. Turbo-feed pump for 300 MW power units.

6. Conclusions

To analyze the vibration state of rotary machines, we proposed to consider the rotor–non-contact seal system as an automatic control system. By changing the geometric parameters of the seals, it is possible to improve the dynamic characteristics of rotary machines and ensure their vibration resistance.

It is shown for the first time that the initially dynamically flexible rotor, in combination with properly designed seals, can become dynamically rigid.

Hydromechanical models of a slotted seal and a rotor in slotted seals were created. Based on the study of these models, analytical dependences were obtained that describe the radial–angular oscillations of the rotor in seals.

This is especially important for centrifugal machines with high pressure and as a result high shaft rotation speed.

Studies have shown that through the purposeful selection of seal parameters, it is possible to reduce the amplitude of forced oscillations of the rotor by 3–4 times. Thus, the results of the studies show directions for increasing the vibration resistance of centrifugal machines and their environmental safety.

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Abbreviations

Symbols and Units

a	the eccentricity of the mass center	m
a_{ij}	parameters of the gap seals	-
b_i	coefficients of total radial moments	
E	the isothermal volumetric module of the sealed medium elasticity	Pa, N/m^2
F	hydrodynamic forces arising in the sealing gap	N
H	mean radial clearance of gap seal	m
M	hydrodynamic moments arising in the sealing gap	Nm
$p(z, \varphi)$	gap pressure	Pa, N/m^2
p_n	nominal discharge pressure	Pa, N/m^2
u, $\widetilde{u}$	radial component of amplitude and complex amplitude of forced vibrations	
U_i, Vi	real and imaginary parts of differential operators	
w	gap fluid flow rate	m/s
x, y	radial vibrations of the rotor	m
α_i	coefficients of hydrodynamic forces depending on the angular oscillations of the rotor	
β_i	coefficients of hydrodynamic moments depending on the radial oscillations of the rotor	
θ, $\widetilde{\theta}$	angular component of the amplitude and complex amplitude of forced oscillations	
θ_0, θ_{0*}	taper parameter of the annular channel and its critical value	
ϑ_2	mean radial taper	rad
ϑ_x, ϑ_y	rotor angular oscillations	rad
ω	rotor speed	s^{-1}
Ω_{u0}	the bending stiffness of a shaft	N/m

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