vibration

MDPI

Article

Methodology for Designing Vibration Devices with Asymmetric Oscillations and a Given Value of the Asymmetry of the Driving Force

Mihail D. Gerasimov [1], Nickolai S. Lubimyi [1,*], Andrey A. Polshin [1], Boris S. Chetverikov [1] and Anastasia Chetverikova [2]

[1] Department of Hoisting-and-Transport and Road Machines, Belgorod State Technological University named after V.G. Shukhova, st. Kostyukov 46, 308012 Belgorod, Russia; mail_mihail@mail.ru (M.D.G.); info@polshin.ru (A.A.P.); await_rescue@mail.ru (B.S.C.)

[2] Faculty of Foreign Languages, Institute of Pedagogy, The National Research University—Belgorod State University, st. Pobeda, 85, 308015 Belgorod, Russia; aanastasia.chetverikova@yandex.ru

* Correspondence: nslubim@bk.ru; Tel.: +7-951-136-40-17

Abstract: In mechanical engineering, the building industry, and many other branches of industry, vibration machines are widely used, in which circular and directed oscillations predominate in the form of movement of the working equipment. This article examines methods for generating asymmetric oscillations, which are estimated by a numerical parameter, namely by the coefficient of asymmetry of the magnitude of the driving force when changing the direction of action in a directed motion within each period of oscillations. It is shown that for generating asymmetric mechanical vibrations, vibration devices are used, consisting of vibrators of directed vibrations, called stages. These stages form the total asymmetric driving force. The behavior of the total driving force of asymmetric vibrations and the working equipment of the vibration machine are described by analytical equations, which represent certain laws of motion of the mechanical system. This article presents a numerical analysis of methods for obtaining laws of motion for a two-stage, three-stage, and four-stage vibration device with asymmetric oscillations. An analysis of the methodology for obtaining a generalized law of motion for a vibration device with asymmetric oscillations is performed based on the application of polyharmonic oscillation synthesis methods. It is shown that the method of forming the total driving force of a vibration device based on the coefficients of the terms of the Fourier series has limited capabilities. This article develops, substantiates, and presents a generalized method for calculating and designing a vibration device with asymmetric oscillations by the value of the total driving force and a given value of the asymmetry coefficient in a wide range of rational designs of vibration machines. The proposed method is accompanied by a numerical example for a vibration device with an asymmetry coefficient of the total driving force equal to 10.

Keywords: vibration machines; vibration device; asymmetric vibrations; driving force; summation of vibrations; asymmetry coefficient of the driving force

Academic Editors: Francesco Pellicano, Yuri Mikhlin, Konstantin V. Avramov and Antonio Zippo

Received: 12 November 2024
Revised: 8 January 2025
Accepted: 14 January 2025
Published: 15 January 2025

Citation: Gerasimov, M.D.; Lubimyi, N.S.; Polshin, A.A.; Chetverikov, B.S.; Chetverikova, A. Methodology for Designing Vibration Devices with Asymmetric Oscillations and a Given Value of the Asymmetry of the Driving Force. *Vibration* **2025**, *8*, 3. https://doi.org/10.3390/vibration8010003

1. Introduction

Vibrational processes are part of oscillatory processes, which are reflected in the fundamental works of scientists from different countries [1–6].

Theoretical developments that form the science of vibrational processes and technologies are based on the general laws of oscillation theory [7–12], which, in turn, are the theoretical basis for many areas of research and their practical use [13–16].

In accordance with the widespread use of vibration technologies and vibration machines in a number of industrial production of materials, works, and services, methods for their design are described to one degree or another in specialized technical literature [10,11,17]. A special place in the development of methods for general and special design and calculation of parameters of vibration machines is occupied by the works of Andronov A.A., Ashavskiy A.M., Bauman V.A., Blekhman I.I., Mandelstam L.I. Strelkov S.P.

In works [17–19], fundamental requirements and criteria for the development of [17] laws of change and control of the driving force of a vibration machine depending on the area of application and its functional tasks are formed in a sufficiently complete manner for the prospective design of vibration machines. In order to implement the requirements for the development of design methods and calculation of technological parameters of vibration machines, formulated in [17,18,20] and in a number of other specialized studies [21–24], when solving new technical problems, their formalization and additional analysis of the kinetics of processes occurring under the conditions of superimposed vibration effects are required each time [25,26].

Problems of this kind include, in particular, the development of a methodology for designing vibration devices with asymmetric oscillations or with an asymmetric driving force. Considering that this scientific direction is still in its infancy, some features of terminology and definitions should be noted. In this article, the terms 'asymmetric driving force' and 'asymmetric oscillations' are considered equivalent, as well as 'asymmetry coefficient of the driving force magnitude' and 'dynamic coefficient of the oscillatory system'.

The issues of parameter asymmetry in oscillatory processes are common topics in scientific publications. In the works [26–28], some classification features of asymmetry in oscillatory, and therefore, in vibrational processes are given, namely:

- geometric asymmetry,
- asymmetry of frictional properties,
- force asymmetry,
- temporal asymmetry of excitation,
- frequency asymmetry,
- functional asymmetry,
- kinematic asymmetry,
- structural (constructive) asymmetry,
- gradient asymmetry,
- wave asymmetry,
- initial asymmetry, i.e., associated with the initial conditions of motion.

Without going into a detailed description of each type of classification feature, it can be noted that the selected type of asymmetry can be closely related to another type of asymmetry or have conditions for the transition of one into another. The use of one or another type of asymmetry in the scientific or technical sphere is associated with specific conditions and priorities of the authors. In this work, force asymmetry is considered a complex parameter that can be assessed and characterized by a numerical value, namely the coefficient of asymmetry of the driving force (k_{as}) or the coefficient of dynamism of the oscillatory system (K_d or k_d) [29,30]. Asymmetric force is considered a force (F) of directed action along some straight line with a variable magnitude, whose magnitude in one direction significantly, or by some number of times, exceeds the magnitude of the component acting in the opposite direction while one of the components, for example, the larger one, can be taken as positive (+F), and in the opposite one, negative ($-F$). The magnitude of the asymmetry coefficient of the driving force (k_{as}) or the dynamic coefficient of the oscillatory system (K_d or k_d) can be calculated from the expressions:

$$k_{as} = K_d = k_d = -\frac{+F}{-F} = \frac{F}{|-F|} = \frac{F_{uw}}{F_{is}} \quad (1)$$

where F_{uw} is the magnitude of the driving force acting in the direction of performing 'useful work'; F_{is} is the magnitude of the driving force acting in the direction of performing 'idle stroke', in the opposite direction; and $(+F)$ and $(-F)$, respectively, are the components of the total value of the driving force, acting alternately in opposite directions.

The novelty of the study is that for the first time, an equation of the total driving force is given, ensuring the obtaining of any coefficient (for technical and technological tasks) of asymmetry of the driving force, for example, more than 10. This is achieved by the fact that on the basis of the analysis of a number of works in the field of development of methods for calculating and designing vibration machines, the authors introduce a new term in relation to the form of oscillations of the working equipment of vibration machines which is asymmetric oscillations. This type of oscillation has a numerical characteristic, namely, the asymmetry coefficient of the driving force. This work currently lacks a general methodology for designing and calculating vibration devices with asymmetric oscillations, and the available individual examples for vibration devices with low values of the asymmetry coefficient of the driving force, for example, in the range of 2–4, are of an exclusively local nature for specific conditions. The novelty is that the use of the method of decomposition of piecewise smooth or monotone functions into a Fourier series for creating a general law for designing and calculating vibration devices with asymmetric oscillations is not applicable. Since it does not allow for the obtaining of a coefficient of asymmetry of the driving force with a greater value, for example, more than four.

The purpose of this article is to develop an engineering methodology for designing vibration devices with asymmetric oscillations based on the initial parameters: the magnitude of the total driving force acting in the direction of performing useful work and the asymmetry coefficient of the magnitude of the total driving force.

2. Materials and Methods

This article uses methods of analytical, numerical, and comparative research, logically closely related to classical methods of vibration theory and real methods of calculation and design of technological vibration machines, with their parameters and conditions of the application area, such as vibration displacement, vibration sorting, vibration compaction, vibration driving of piles and others. To evaluate the existing patterns of designing vibration devices with asymmetric oscillations, numerical methods for estimating the coefficient of asymmetry of the magnitude of the driving force and the obtained values of its asymmetry coefficient are used. To evaluate the possibility of creating a general methodology for designing vibration devices with asymmetric oscillations and a generalized law for the formation of asymmetric oscillations, classical laws of analytical mathematics are used, i.e., expansion of piecewise smooth and continuous functions in a Fourier series. When creating a mathematical model of a vibration device with asymmetric oscillations, the d'Alembert principle was used to obtain the equilibrium equations of the systems under consideration. Theorems on the properties of optimal Maxwell–Fejer polynomials, which specify the optimal Maxwell–Fejer impulse, were used as a method for assessing the created general methodology for designing and calculating vibration devices with asymmetric oscillations. When deriving the general equation for the value of the total driving force, the search method by simplex (S^2-method) was used in problems of constructing a regular simplex for a given base point and scale factor.

3. Results

Vibratory machines occupy an important place in a number of technological processes, for example, in operations of sorting and separating various materials [31] into fractions, inter-operational transportation [32,33], compaction and molding [31,34,35], immersion of structural elements into the ground and their extraction from the ground [22,36,37], cutting [38,39], and destruction and digging [40,41]. In the building industry, vibration machines are used to produce building materials and perform construction and road construction work. In vibration machines, the driving force of the working equipment is the 'driving force'. The driving force serves to periodically move the structural elements of the machines with a certain amplitude A and frequency f and is characterized by the magnitude, modulus or amplitude value, direction or, line of action, point of application, and the nature of the behavior or position of the line of action in time [42–44].

Among the various types of vibration machines, a class of vibration inertial machines stands out, which the driving force of the vibration device is generated by one or several shafts rotating at an angular velocity ω (rad/s); with eccentric weight fixed to them, with a mass of m (kg), the center of gravity of which is shifted relative to the axis of rotation by a certain value, r (m), called eccentricity, and is described by the law of change [29,35].

$$F = m \cdot r \cdot \omega^2 \cdot cos\left(\omega t + \varphi\right) \tag{2}$$

where $F = m \cdot r \cdot \omega^2$ is the amplitude value of the driving force, H; t is the current time, s; φ is the initial phase of oscillations, deg, rad; m is imbalance mass, kg; r is imbalance eccentricity, m; and ω is the angular velocity of rotation of the unbalanced shaft, rad/s.

In the case where mechanical vibrations are generated by a rotating driving force (F), such vibrations are called circular or elliptical, depending on the ratio of resistance forces along the coordinate axes.

The magnitude of the driving force for a driving force of directional action, without asymmetry, i.e., when $k_{as} = K_d = 1.0$, acting in the direction of performing useful work (F_{uw}), is equal in magnitude to its action in the opposite direction, i.e., in the direction of performing an idle stroke (F_{is}).

However, the equality of the components of the total driving force in the direction of performing useful work and in the direction of performing idle stroke during vibration displacement and vibration transport of materials, during vibration immersion and vibration extraction of structural elements into and from the soil, as well as during vibration compaction is undesirable. In such a case, the following relationship must be fulfilled: $F_{uw} \gg F_{is}$, and accordingly, $k_{as} = K_d \gg 1.0$.

This problem can be solved by asymmetric oscillations, which allow obtaining a significant difference in the magnitude of the driving force acting in the direction of performing useful work, $F_{uw} = +F$, and in the direction of performing idle stroke, $|F_{is}| = \frac{F_{uw}}{k_{as}}$. In this case, the ratio of forces (F_{uw} and F_{is}) is estimated by the asymmetry coefficient of the driving force, k_{as}, or the dynamic coefficient of the oscillatory system (1).

Further improvement of vibratory machines is connected with the task of developing the methodology and principles of designing vibration devices with asymmetric oscillations. However, at present this direction is still in the state of development. In accordance with this, this article is aimed at developing and forming the methodology and principles of designing vibration devices with asymmetric oscillations. At the same time, the objective of this article is to analyze existing methods for designing asymmetric oscillations of vibration machines and comparative studies of the results of their use.

At present, there are some theoretical provisions [9,11,14,25,27,45] that allow to consider them as prerequisites for the formation of a methodology for designing vibration devices with asymmetric oscillations. The existing theoretical provisions do not yet have a generally accepted analytical completeness, but they have some general conclusions.

The first conclusion is based on the fact that asymmetric oscillations can be obtained by adding several directed oscillations. As a result of adding the driving forces of the directed oscillations, a total directed driving force is created that is asymmetric in magnitude in opposite directions. The second conclusion is that the rotation frequencies of the unbalanced shafts of each subsequent directed oscillation, starting with the second one, are multiples of the first, Table 1.

Table 1. Ratio of angular frequencies of vibrators with directional oscillations in a single vibration device with asymmetric oscillations.

Characteristics of Oscillations	First Stage	Second Stage	Third Stage		*n*th Stage
Frequency of single-directed oscillations	ω_1	ω_2	ω_3	...	ω_n
The ratio of the frequencies of single-directed oscillations to the frequency of the first stage	ω_1	$2\omega_1$	$3\omega_1$	...	$n\omega_1$

Attempts to use asymmetric vibrations in the building industry were made as early as the middle of the last century [31], although the term 'asymmetric vibrations' had not yet been introduced.

The vibration device [31] consists of four unbalanced shafts, installed two on top of each other, forming two pairs, each of which generates directed vibrations with equal driving forces (F). The vibration frequency of one pair of unbalanced shafts is 400 rpm, forming the first stage of the vibration device, and the rotation frequency of the second pair of unbalanced shafts is 800 rpm, forming the second stage of the vibration device.

Using the vibration parameters from the provisions of [31], it is possible to calculate the magnitude of the total driving force as the sum of two directed oscillations, forming a vibration device with two vibration stages.

$$F_{sum2} = F_1 + F_2 = m_1 r_1 \omega_1^2 cos\omega_1 t + 0.5 m_1 r_1 (2\omega_1)^2 cos(2\omega_1 t + \pi) \quad (3)$$

where F_1 and F_2 are, respectively, the driving force of the vibration device of the first and second stages of directed oscillations, kN; m_1 is the mass of the eccentric weight of the first stage of directed oscillation, kg; r_1 is the eccentricity of the eccentric weight of the first stage of directed oscillation, m; and t is the current time within one period of oscillations, s.

The following parameter values are accepted according to the recommendations [27]: m_1 = 10, m_2 = 5 (kg), r_1 = 0.02, r_2 = 0.01 (m), ω_1 = 41.9, and $\omega_2 = 2\omega_1$ = 83.8 (rad/s). The static moment of the imbalances of the two-stage vibration device (M_{st2}) of the first term in Equation (3) is $M_{1st2} = m_1 \cdot r_1 = 0.2$ (kg·m), and of the second term in (3) is $M_{2st2} = m_2 \cdot r_2 = 0.05$ (kg·m), and their ratio.

$$M_{1st2} : M_{2st2} = 100\% : 25\% \quad (4)$$

In this case, the oscillation frequency is f = 6.67 Hz, and the oscillation period is T = 0.15 s. Calculation of forces based on 20 values of the oscillation period shows that the value of the total driving force with the law of its change according to Equation (3) in the direction of performing useful work (F_{uw}) is $F_{sum2} = F_{uw} = -701.4$ kN, and in the direction of performing idle stroke is $F_{is} = 394.5$ kN. Hence, for a two-stage vibration device with asymmetric oscillations (3), the asymmetry coefficient will be: $k_{as2} = \frac{|-701.4|}{394.5} = 1.78$.

The graph of the change in the components and the total value of the driving force, according to the recommendations [27] based on the calculations, is shown in Figure 1. The direction of the total driving force (F_{sum2}) is toward the negative values of the ordinate axis.

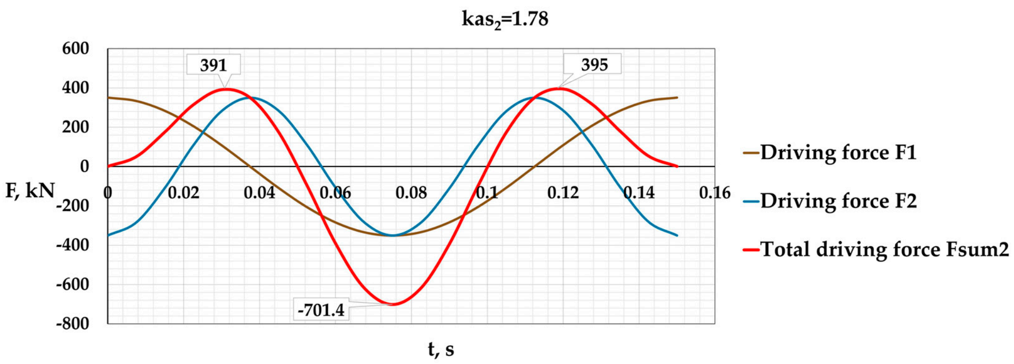


Figure 1. Adding two (F_1 and F_2) directed oscillations to obtain the total oscillation by initial conditions [31]. The asymmetry coefficient of the driving force is $k_{as2} = k_{d2} = 1.78$, (F_{sum2}).

In the graph, Figure 1, the direction of useful work performed by the driving force (F_{sum2}) is taken in the direction of negative values of the ordinate axis, for example, for driving piles.

On the one hand, the method of this technical solution [31] consists of doubling the angular frequency of rotation of the second stage eccentric weights in relation to the first. On the other hand, the mass and eccentricity of the second stage eccentric weights are halved, so that the amplitude values of the magnitude of the driving force of the first and second stages remain equal, $|F_1| = |F_2| = 350.7$ kN, and the sum is $|F_1| + |F_2| = 701.4$.

It should be noted that the largest number of publications and patents related to asymmetric oscillations of any parameter are associated with two-stage vibration devices, the asymmetry coefficient (k_{as2}) of which tends to two: $k_{as2} \to 2.0$. Where the index $(as2)$ characterizes the asymmetry of the parameter of a two-stage vibration device generating a total driving force by two coordinated directional harmonic oscillations with multiple frequencies.

Another example of the development of a methodology for designing a vibration device with asymmetric oscillations is the work [46]. In this work, the term asymmetric oscillations also does not appear. In [46], an equation is given for the total value of the driving force of forced oscillations, consisting of three components $y(t) = F_{sum3}$:

$$y(t) = 4.5sin\frac{\omega}{2}t + 3sin\left(\omega t - \frac{\pi}{2}\right) + 1.5sin(\frac{3}{2}\omega t - \pi) \tag{5}$$

where 4.5, 3, 1.5 are the amplitudes of harmonics in the spectrum of subharmonic oscillations [46]; $\varphi_i = 0, -\frac{\pi}{2}, -\pi$ constitute the initial phase angles; and ω is the angular velocity, rad/s.

To carry out a comparative calculation, it was assumed that ω = 104.7 rad/s, and the magnitude of the amplitude value of the function $y(t) = F_{sum3}$ = 9.0 [47].

The authors of [46] draw attention to the fact that Equation (5) is used to tune the resonant oscillations of a vibration machine with a piecewise linear characteristic of elastic connections and, under these specific conditions, is the law of change of the function $y(t)$. Using the parameters related to the vibration device [46], the calculation of the value of the total driving force has been performed, the graph of which is shown in Figure 2. The direction of action of the value of the total driving force (F_{sum3}) is chosen similarly to [46], upward, toward the positive values of the ordinate axis.

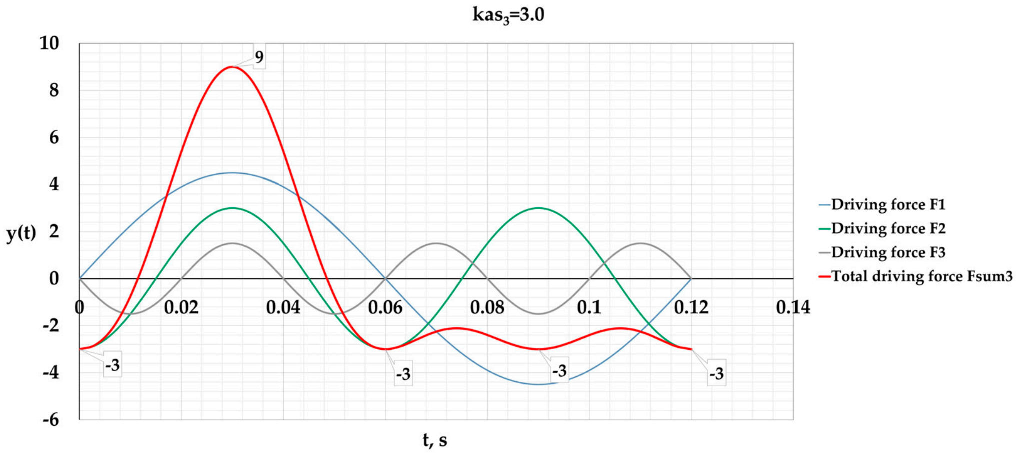


Figure 2. Addition of three directional oscillations (F_1 and F_2, F_3) to obtain a total oscillation (F_{sum3}) with asymmetry of the driving force $k_{as3} = k_{d3} = 3.0$, [46].

As can be seen from Figure 2, the total driving force (F_{sum3}) is the sum of three separate synchronized harmonic oscillations, that is, three stages of the vibration device with multiple frequencies, is asymmetric in magnitude, with amplitude values in the direction of performing useful work ($F_{sum3} = 9.0$) and in the direction of performing idle stroke ($F_{is3} = 3.0$) with an asymmetry coefficient: $k_{as3} = k_{d3} = \frac{9.0}{3.0} = 3.0$. It can be written that the amplitude value of the function.

$$y(t) = F_{sum3} = F_{13} + F_{23} + F_{33}, \tag{6}$$

where F_{13}, F_{23}, F_{33} are, respectively, the force of the first, second, and third components of the total driving force at three stages of the vibration device. Equation (6) can be written as follows:

$$y(t) = F_{sum3} = \lambda_1 \cdot F_{sum3} + \lambda_2 \cdot F_{sum3} + \lambda_3 \cdot F_{sum3} \tag{7}$$

where λ_1, λ_2, λ_3 are, respectively, the coefficients that determine the ratio of the component forces in fractions of the value of the total driving force. In the case of Equation (5), it can be written as follows:

$$F_{sum3} = 0.5 \cdot F_{sum3} + 0.33 \cdot F_{sum3} + 0.17 \cdot F_{sum3} \tag{8}$$

In this case, the value of the static moment of the unbalance of the three-stage vibration device (M_{st3}) is, for the first stage, $M_{1st3} = m_1 \cdot r_1 = 1.64$ (kg·m), for the second stage, $M_{2st3} = m_2 \cdot r_2 = 0.274$ (kg·m), and for the third stage, $M_{3st3} = m_3 \cdot r_3 = 0.061$ (kg·m). Their ratio is as follows:

$$M_{1st3} : M_{2st3} : M_{3st3} = 100\% : 16.67\% : 3.71\% \tag{9}$$

The next stage of development and improvement of the methodology for designing and calculating vibration devices with asymmetric oscillations can be attributed to the works [48,49].

In these works, the definition of 'asymmetric' oscillations also does not appear. At the same time, oscillations generated by vibration devices with similar parameters form oscillations with an asymmetric value of the total driving force. For a vibration device with four vibration stages, it is recommended to adopt the ratio of static moments of eccentric weight, expressed as a percentage:

$$M_1 : M_2 : M_3 : M_4 = 100 : 18.72 : 5.6 : 1.38 \tag{10}$$

The recommendations given in [49] for selecting the parameters of the vibration device make it possible to calculate the output parameters and plot a graph of the change in the components and the total driving force within the oscillation period, as shown in Figure 3. The magnitude of the total driving force (F_{sum4}) is calculated in the direction of the negative values of the ordinate axis, downwards.

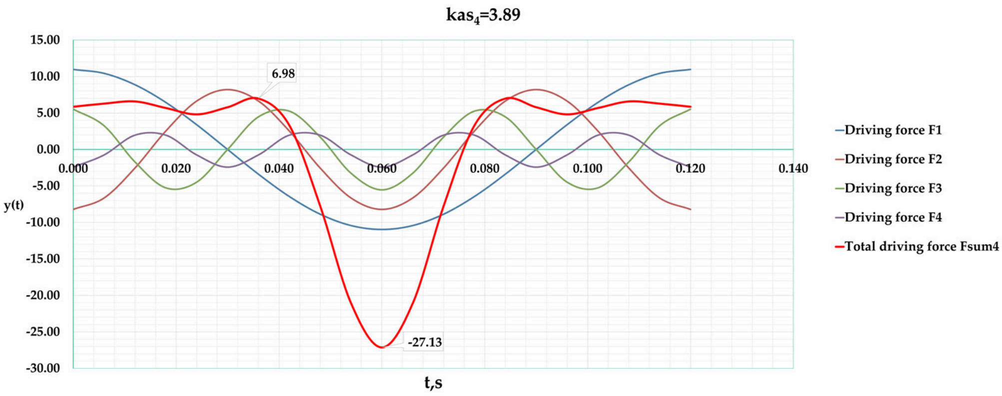


Figure 3. Addition of four directional oscillations (F_1 and F_2, F_3, F_4) to obtain a total oscillation with asymmetry of the driving force $k_d = 4.0$, ($k_{as4} = k_{d4} = 3.89$, (F_{sum4})).

Equation (10) can be considered as a method for designing a vibration device with an asymmetry coefficient of the driving force equal to four.

From the graph, Figure 3, it is evident that the magnitude of the total driving force is asymmetrical, since the force acting in the direction of performing useful work exceeds the force acting in the direction of performing idle stroke by the value of the asymmetry coefficient, which is equal to $k_{ac} = k_d = -\frac{-27.13}{6.98} = 3.89$.

The design methodology for a vibration device with asymmetric oscillations according to [49] for a given amplitude value of the total driving force is carried out in several steps, or stages, for example, for a four-stage vibration device.

At the first stage, the amplitude value of the total driving force and the asymmetry coefficient equal to $k_{as} = k_d = 4.0$. are determined.

At the second stage, the angular speed of rotation of the unbalanced shafts of the first and subsequent stages are assigned: ω_1; $\omega_2 = 2\omega_1$; $\omega_3 = 3\omega_1$; $\omega_4 = 4\omega_1$.

At the third stage, the parameters of the static moment, M_{1st}, of the unbalance of the first stage are selected based on its mass, m_1, and eccentricity, m_1: $M_{1st} = m_1 r_1$.

At the fourth stage, according to relation (10), the static moments of the unbalances of the second, third, and fourth stages, M_2, M_3, M_4, are determined.

At the fifth stage, according to the obtained values of the static moments, the masses of the unbalances of all stages, m_i, and their eccentricities, r_i, are determined.

At the sixth stage, the selected parameters are checked for the value of the total driving force, F_{sum4}.

Attempts to create vibration devices with an asymmetry coefficient greater than $k_{ac} \geq 4.0$ are not presented in [49].

The design methodology for a vibration device with asymmetric oscillations, which follows from [49], is artificially cumbersome for engineering calculations, since the static moments of the imbalances are intermediate parameters between the initial conditions and the value of the total driving force.

From the analysis of the methods of calculation and design of vibration devices with asymmetric oscillations, it is evident that in the considered sources of information related to mechanical oscillations, the term 'asymmetric' oscillations or vibration devices with 'asymmetric' oscillations are not actually used. The works considered above, which by the nature of the problems can be attributed to works connected with asymmetric oscillations of vibration machines, do not contain a unified methodology either from the point of view of the formulation of terminology or from the point of view of the formulation of the initial statement of problems and the sequence of their solution. Thus, there is a need to distinguish asymmetric oscillations in mechanical systems into an independent class and the need to obtain a general equation of the total driving force with a given asymmetry coefficient.

An important stage in the development of a methodology for designing vibration devices with asymmetric oscillations is associated with work, devoted to obtaining the parameters of real vibration devices using coefficients, obtained by expanding functions into a Fourier series [31].

The main theoretical basis is the works [31,40], in which the question of the formation of asymmetric oscillations has been determined using the coefficients of the Fourier series obtained by decomposing piecewise smooth or continuous monotone functions. It was assumed that, by successively accepting the coefficients of the terms under the summation sign as multipliers to the corresponding component forces, it is possible to establish a rational relationship of such components of elementary forces, which in sum makes it possible to obtain an asymmetric value of the total driving force.

The conducted studies on the expansion of piecewise smooth and monotone functions in a Fourier series in a wide range of possible variants established that the method of using the coefficients of the Fourier series has certain limitations [49,50]. These limitations are in the fact that the method allows obtaining the total driving force with the value of the asymmetry coefficient within the limits $k_{ac} = k_d = 1 \ldots 4$. The numerical analysis of the use of coefficients, terms of the Fourier series, is considered using the example of a function:

$$y = cos^{2m}(x/2), \tag{11}$$

and the corresponding series:

$$y(x) = \frac{(2m)!}{2^{2m}(m!)^2} + \frac{1}{2^{2m-1}} \sum_{k=0}^{m-1} \frac{(2m)!}{k! \cdot (2m-k)!} cos(m-k)x \tag{12}$$

Let us consider the problem of changing the value of the asymmetry coefficient of the total oscillation obtained by using, respectively, 1…6 terms of the Fourier series in the expansion of the function $y = cos^{12} \frac{x}{2}$.

There is the following series:

$$cos^{12} \frac{x}{2} = 0.226 + 0.387 \cdot cosx + 0.242 \cdot cos2x + 0.107 \cdot cos3x + 0.032 \cdot cos4x + 0.006 \cdot cos5x + 0.0005 \cdot cos6x \tag{13}$$

where the variable $x = \omega t$; ω is the angular velocity of the first unbalanced shaft; ω = 52.3 rad/s is taken; the oscillation frequency is f = 8.33 Hz; and the oscillation period is T = 0.12 s. As a result, a sequence of the value of the asymmetry coefficient of the total driving force (k_{asn}) is obtained, depending on the number (n) of terms of the series with the corresponding coefficients used, as shown in Figure 4.

It is obvious that for the first two or three components of the vibration device stages, the value of the asymmetry coefficient increases, practically, by one. At four stages, the growth of the asymmetry coefficient decreases, and at five and six stages, the growth of the asymmetry coefficient stops. This happens because the subsequent numerical values of the Fourier series coefficients decrease, tend to zero and become insignificant.

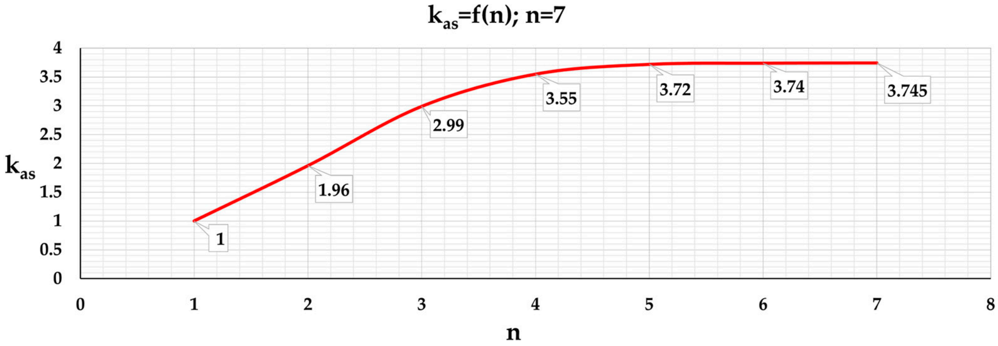


Figure 4. Graph of the change in the asymmetry coefficient (k_{asn}) of the total driving force of the vibration device, formed on the basis of the terms of the Fourier series of the function $= cos^{12}\frac{x}{2}$ for the first six terms (n = 1. . .6).

The main parameter determining the effect of asymmetric mechanical vibrations of a vibration device with asymmetric vibrations is the value of the total driving force obtained by adding two or more symmetric driving forces directed along a common straight line. Such components of driving forces are generated by separate vibrators called vibration stages. Thus, the value of the total asymmetric driving force, F_{sum}, is the sum of several elementary symmetric forces, F_i, directed along a common straight line, i.e.,:

$$F_{sum} = \sum_{i=1}^{n} F_i = F_1 + F_2 + \ldots + F_n \quad (14)$$

where i is the serial number of the driving force, the number of the stage of the vibration device with asymmetric oscillations; $i = 1, 2, \ldots, n$; n is the number of received components, the number of stages of the vibration device with asymmetric oscillations; n_i is the i-th stage of the vibration device with asymmetric oscillations, consisting of n stages; and F_i are forces that make up the total driving force.

A vibration device with asymmetric oscillations can be formed on the basis of a sequential connection into a single mechanism of paired vibrators with circular oscillations, each pair of which forms a stage of the vibration device, or on the basis of planetary vibrators with directional oscillations.

The following problems are of scientific interest:

- The procedure for determining the rational number of stages, and therefore the rational coefficient of asymmetry of the driving force of a vibration device with asymmetric oscillations, for the use of asymmetric oscillations in specific conditions [37];
- What ratio of driving forces of each stage in the value of the total driving force is rational;
- How to achieve the greatest coefficient of asymmetry of the total driving force and, therefore, the coefficient of dynamism of a mechanism with asymmetric oscillations;

A vibration mechanism is considered, consisting of n pairs of vibrators with circular oscillations, each equivalent pair, the stage of which generates a directed driving force corresponding to its parameters, as shown in Figure 5.

In Figure 5, φ_k is the angle of deviation of the k-th eccentric weight from the horizontal. The following designations are used for the k-th vibrator where m_k is the mass of the eccentric weight, R_k is the radius of the guide circle, φ_{k0} is the initial angle of deviation of the eccentric weight (initial phase), and ω_k is the angular velocity of rotation of the eccentric weight.

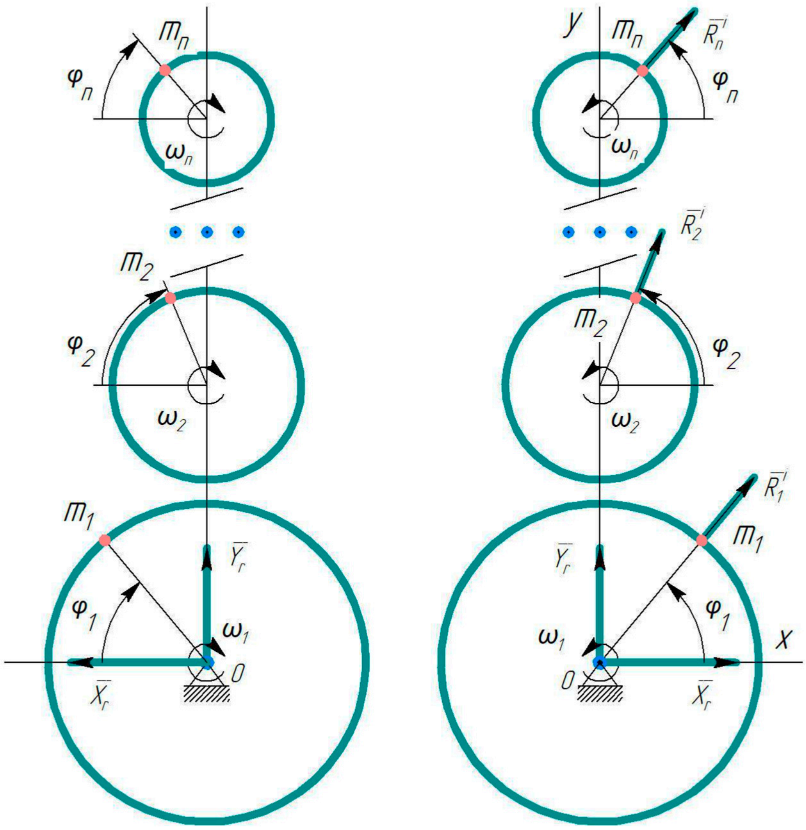


Figure 5. A diagram of a vibration mechanism consisting of *n* pairs of unbalanced vibrators, each pair of which forms a vibration block, a stage that generates directed oscillations.

Let us introduce the inertial forces $\overline{R}_k^i$, applied to the eccentric weight, and the components of the support reaction $\overline{X}_r$ and $\overline{Y}_r$. The total horizontal component of the support reaction of the mechanism, due to the mirror arrangement of the vibrators, is equal to zero. Neglecting the weight of the guide circles and eccentric weight due to their smallness in comparison with the inertial forces (at sufficiently large values of the angular velocities ω_k), based on d'Alembert's principle [22,37], the following equilibrium equation in projection onto the O_y axis is obtained:

$$Y_r + 2\sum\nolimits_{k=1}^{n} \overline{R}_k^i sin\varphi_k = 0, \tag{15}$$

where

$$\overline{R}_k^i = m_k R_k \omega_k^2,\ \varphi_k = \omega_k t + \varphi_{k0}. \tag{16}$$

Thus, the value of the vertical component of the support reaction can be calculated using the following equation:

$$Y_r = -2\sum_{k=1}^{n} \overline{R}_k^i sin\varphi_k, \tag{17}$$

or

$$Y_r = \sum_{k=1}^{n} \left(-2m_k R_k \omega_k^2\right) sin(\omega_k t + \varphi_{k0}), \tag{18}$$

Considering further only pairs of vibrators, the angular velocities of rotation of the eccentric weight that are related by the relation:

$$\omega_k = k\omega_1 \tag{19}$$

where ω_1 is the angular velocity of rotation of the pair of the first stage of eccentric weight, Equation (8) is written as shown below:

$$Y_r = \sum_{k=1}^{n} a_k sin(k\varphi + \varphi_{k0}), \omega_k = k\omega_1 \tag{20}$$

where $\varphi = \omega_1 t$, and parameters a_k can take any values (due to the choice of the value of the radii of the guide circles R_k and the mass of the eccentric weight m_k).

The minimum absolute value of Y_r, as a function of the angle φ, denoted by Y_{ri}, is the dynamic force acting in the direction of idle stroke. The maximum absolute value of Y_r, denoted by Y_{ra}, is the immersion force acting in the direction of performing useful work.

The ratio of these forces $\left(-\frac{Y_{ri}}{-Y_{ra}}\right)$ is less than one.

$$k_d = -\frac{Y_{ri}}{-Y_{ra}} \leq 1.0 \tag{21}$$

If the inverse ratio $\left(-\frac{-Y_{ra}}{Y_{ri}}\right)$ is used, then its value is greater than one.

$$k_d = -\frac{-Y_{ra}}{Y_{ri}} \geq 1.0 \tag{22}$$

It is called the coefficient of dynamism of the vibration system or the coefficient of asymmetry ($k_{as} = k_d$) of the driving force of the vibration mechanism (the minus sign in Equations (21) and (22) is introduced due to the fact that one of these quantities is negative, opposite in the direction of action to the other).

The task is to achieve the maximum value of the dynamic coefficient of the vibration mechanism k_d at a given immersion force Y_{ra}, which is further denoted as A, by combining several pairs of planetary vibrators and varying their parameters.

As follows from Equation (20), the minimum value of the function is achieved in the case when for all k:

$$sin(k\varphi + \varphi_{k0}) = -1 \tag{23}$$

The maximum immersion force A in this case is determined with the equation:

$$A = \sum_{k=1}^{n} a_k \tag{24}$$

Condition (23) is satisfied if:

$$k\varphi + \varphi_{k0} = \frac{3\pi}{2} + 2\pi m \tag{25}$$

where m can take any values.

From Equation (25) for the value of the initial phase φ_{k0}, the following equation is obtained:

$$\varphi_{k0} = \frac{\pi}{2}(k-1) \tag{26}$$

As an illustration, Figure 6 shows support reactions of each of the seven pairs of vibrators with coefficients a_k equal to one and initial phases φ_{k_0} equal to zero. The maximum immersion force of the first pair of vibrators is achieved at φ = 270°. At this angle, the immersion forces of the second and fourth pairs of vibrators are equal to zero, and the third pair of vibrators at this angle has the maximum value of the lifting force.

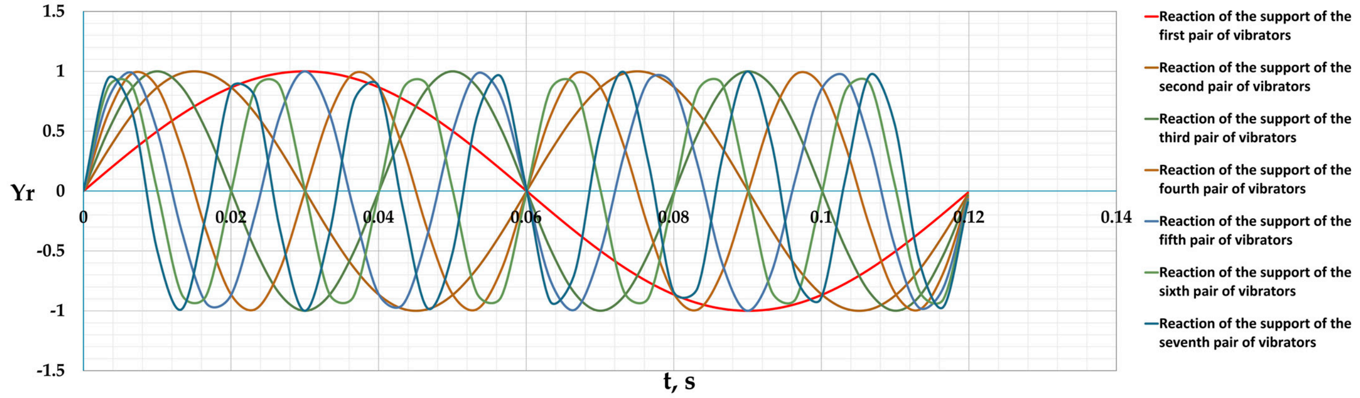


Figure 6. Support reactions of each of the seven pairs of vibrators with the value $a_k = 1.0$ and initial phases $\varphi_{k0} = 0$.

The sum of support reactions of all seven pairs of vibrators is shown in Figure 7. The amplitude value of the immersion force and the lifting force are equal to each other and are $Y_{ra} = Y_{ri}$ = 5.3, and the dynamic coefficient is 1, $k_d = 1.0$.

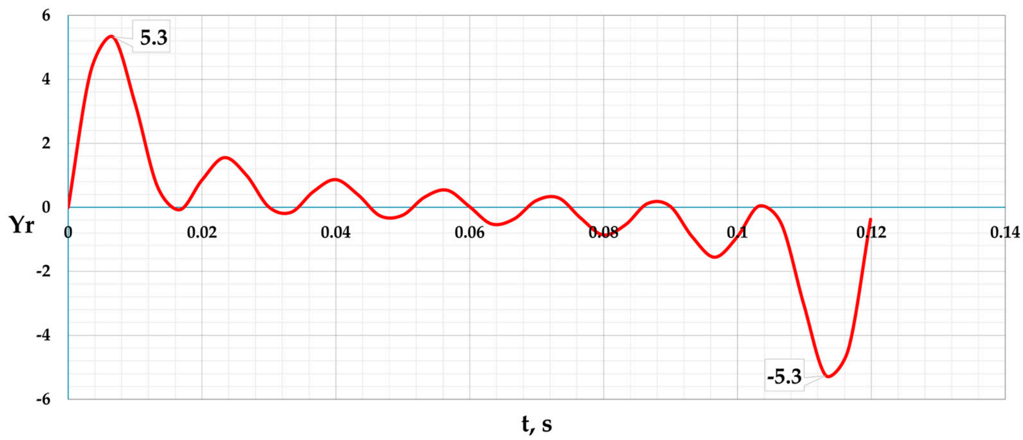


Figure 7. The sum of support reactions of all seven pairs of vibrators with the value $a_k = 1.0$ and initial phases $\varphi_{k0} = 0$.

Figure 8 shows the support reactions of each of the five pairs of vibrators with coefficients a_k equal to one and initial phases φ_{k0} calculated using Equation (26). The maximum values of the immersion forces of all pairs of vibrators are achieved at φ = 270°.

The sum of support reactions of all five pairs of vibrators is shown in Figure 9. The immersion force in this case is equal to 7.002, the lifting force is equal to 2.119, and the dynamic coefficient is equal to k_d = 3.3.

It is evident from the graph in Figure 9 that in the direction of useful work, the vibration device with asymmetric oscillations generates a total driving force of $F_{uw} = |6.99|$ kN. In the direction of idle stroke, a total value is equal to $F_{is} = 1.93$ kN. The asymmetry coefficient of the total driving force is $k_{as} = \frac{6.99}{1.93} = 3.62$. On the graph of the obtained function $Y_r = f(\varphi)$, there are two values that practically reduce the asymmetry coefficient from 7.0 to 3.62.

In the works [40], a problem is formulated related to the useful use of a directing or polyharmonic pulse. It is assumed that the mathematical model of a polyharmonic pulse is a trigonometric polynomial:

$$f_n(t, \lambda) = \sum_{k=0}^{n} \lambda_k cos(kt),\ t \in [0, \pi],$$
$$\lambda_k = (\lambda_0,\ \lambda_1,\ \ldots,\ \lambda_n) \tag{27}$$

and related functionality:

$$K_n(\lambda) = \frac{masf_n(t,\lambda)_t}{minf(t,\lambda)_t} \quad (28)$$

called the 'asymmetry coefficient'.

In this case, the problem of maximizing the functional $K_n(\lambda)$ over variations of λ is set:

$$K_n(\lambda) \to \sup_{\lambda}, \quad (29)$$

under the following conditions:

$$\int_0^{\pi} f_n(t,\lambda)dt = 0, \ \sum_{i=0}^{n} \lambda_i = c > 0 \quad (30)$$

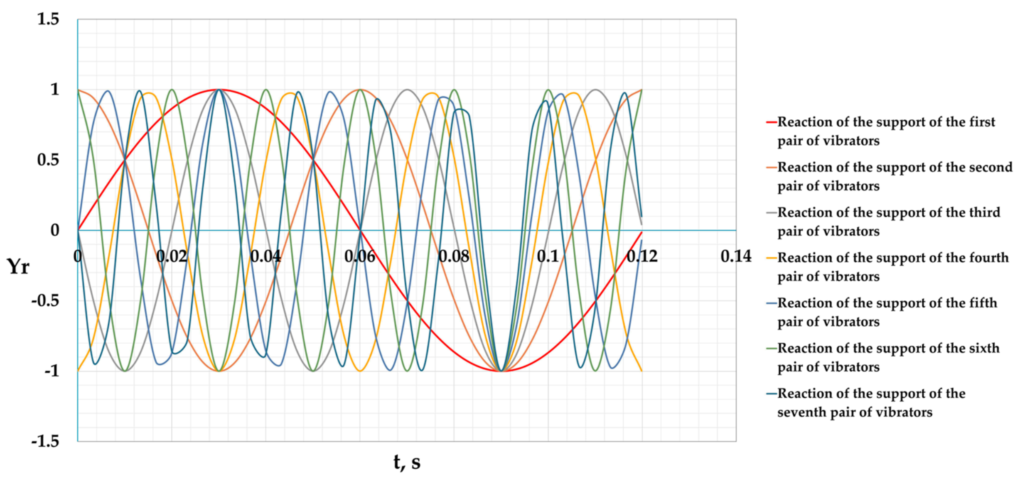


Figure 8. Support reactions of each of the seven pairs of vibrators with coefficients $a_{ki} = 1.0$ and initial phases $\varphi_{k0} = \frac{\pi}{2}(k-1)$, calculated using Equation (26).

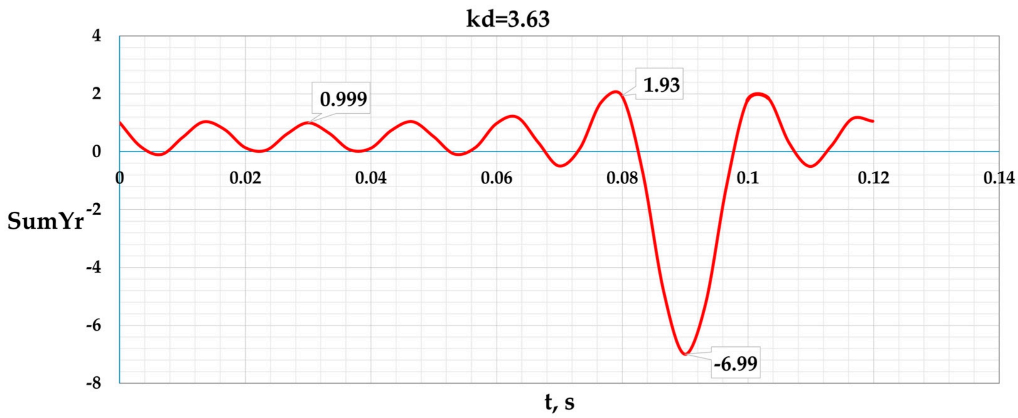


Figure 9. The sum of support reactions of all seven pairs of vibrators with coefficients $a_k = 1.0$ and initial phases $\varphi_{k0} = \frac{\pi}{2}(k-1)$, calculated using Equation (26).

As a result, in [40], the following is given:

Theorem 1. *A polynomial [40] is optimal if, up to a constant factor, it has the form of a Fejér sum*

$$f_n(t) = \sum_{k=1}^{n} (n+1-k)cos(kt) \quad (31)$$

In this case, there is equality:

$$\max K_n(\lambda)_\lambda = n \tag{32}$$

In addition, in [40], theorems on the properties of the optimal Maxwell–Fejer polynomials defining the optimal (in the sense of the asymmetry coefficient) impulse—the Maxwell–Fejer impulse have been formulated and proven. The graph of the Maxwell–Fejer impulse, which is important for engineering and technical problems, is shown in Figure 10. It is noted that a characteristic property of the Maxwell–Fejer impulse is the location of the minima at the same level.

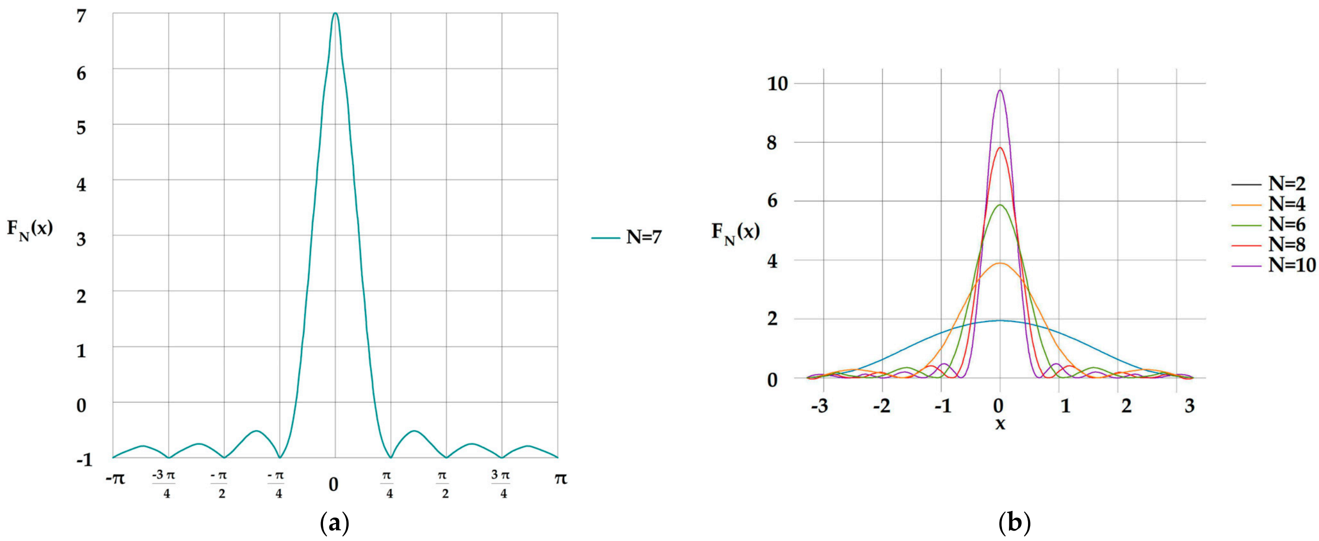


Figure 10. Fejer kernel, (**a**) is for N = 7, (**b**) is for several Fejer kernels [39–41].

It is obvious that the behavior of the graphs in Figures 9 and 10, reflecting the general trends of the calculation and design methods of vibration devices with asymmetric oscillations and the optimal value of the functional ($\max K_n(\lambda)_\lambda = \max k_d(\lambda)_\lambda = n$), do not ensure complete coincidence of the results.

The research must return to the previously calculated vertical support reaction Y_r (20) $Y_r = \sum_{k=1}^{n} a_k sin(k\varphi + \varphi_{k0})$, where $\varphi = \omega_1 t$. Thus, only pairs of vibrators are considered, angular velocities of rotation of the guide circles, which are related by the relation (26), $\varphi_{k0} = \frac{\pi}{2}(k-1)$. The conditions when the minimum value of Equation (30) is achieved, the maximum immersion force, A, which is determined with Equation (24), are considered.

For two pairs of vibrators in [31,37,40], it is theoretically proven that the maximum dynamic coefficient of the vibration mechanism $k_d = 2$ for a given immersion force A is achieved with coefficients:

$$\begin{cases} a_1 = 2m_1R_1\omega^2 = \frac{2}{3}A \\ a_2 = 8m_2R_2\omega^2 = \frac{1}{3}A \end{cases} \tag{33}$$

For a vibration mechanism consisting of more than two pairs of planetary vibrators, the determination of the optimal values of the parameters a_k, at which the maximum dynamic coefficient k_d is achieved (for a given immersion force A), was carried out numerically. The simplex search method (S^2-method) proposed in [47,50] was used. This method has no relation to the simplex method of linear programming, and the similarity of the names is accidental.

The implementation of the optimum search algorithm consists of two types of calculations: the construction of a regular simplex for a given base point and scale factor and the calculation of the coordinates of the reflected point.

The calculation of the coordinates of the vertices of the simplices for a given initial (base) point $a^{(0)}$ is performed using the following equations [51]:

$$a^{(i)} = \begin{cases} a_j^{(0)} + \delta_1, \; if\ j \neq i, \\ a_j^{(0)} + \delta_2, if\ j = i, \end{cases} \tag{34}$$

where the increments δ_1 and δ_2 depend on the dimension N of the problem and are determined with the equations:

$$\delta_1 = \left(\frac{\sqrt{N+1} + N - 1}{N\sqrt{2}} \right) \alpha \tag{35}$$

$$\delta_2 = \left(\frac{\sqrt{N+1} - 1}{N\sqrt{2}} \right) \alpha \tag{36}$$

where α is the scale factor (reduction coefficient).

Calculations of the second type are related to the reflection of a point relative to the center of gravity of the simplex. If $a^{(j)}$ is the point to be reflected, then the center of gravity of the remaining points of the simplex a_c is calculated using the following equation:

$$a_n^{(j)} = 2a_c - a^{(j)} \tag{37}$$

For calculations of a three-pair unbalanced vibrator, the immersion force $A = 6$ has been set, and the point with coordinates $a_1 = 1.0,\ a_2 = 1.0$ has been chosen as the base point. The coefficient a_3, in accordance with (24), has been calculated using the following equation:

$$a_3 = A - a_1 - a_2 \tag{38}$$

Since in the problem under consideration $N = 2$ (two variable parameters are a_1 and a_2), and the scale factor α has been chosen equal to one, the increments in this case are $\delta_1 = 0.9659$, $\delta_2 = 0.2588$. As the objective function, since the method used allows calculating the minimum of the function, the function $1/k_d$ has been used.

The calculation of the objective function values has been performed as follows. For the given values of the coefficients a_1 and a_2 and the coefficient a_3 calculated using Equation (38), the values of the function $Y_r(\varphi)$ have been calculated when the angle φ changed from 0° to 360° with a step of 1°, the minimum Y_{ri} and maximum Y_{ra} values of the function have been determined, and the dynamic coefficient (1) has been calculated

$$k_d = -\frac{Y_{ri}}{Y_{ra}}$$

and then the value $1/k_d$ has been used.

Figure 11 graphically presents the calculated simplices in the process of determining the optimal values of the parameters a_1 and a_2. Table 2 shows the coordinates of the vertices of the simplices. Figure 12 shows the values of the parameters at the vertices of the simplices and the corresponding values of the dynamic coefficient k_d.

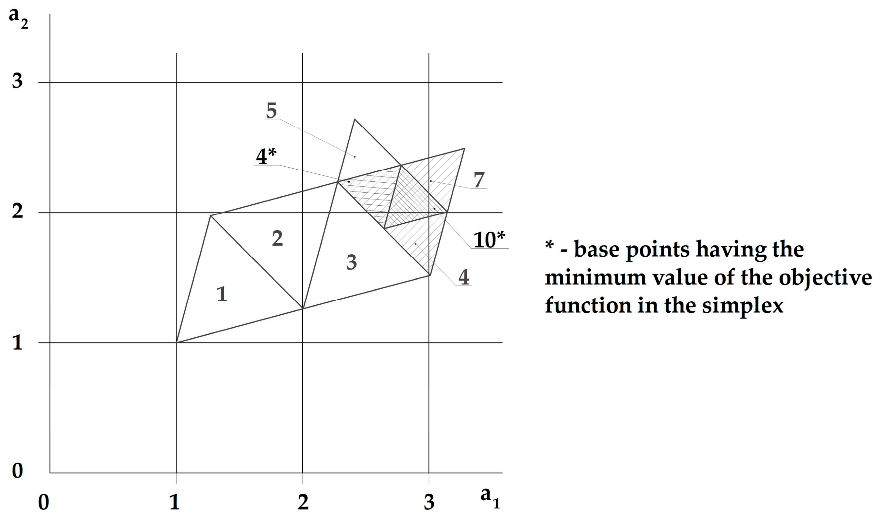


Figure 11. Calculation of simplexes in the process of determining the optimal values of parameters a_1 and a_2.

Table 2. Coordinates of the vertices of the simplices.

Vertices	1	2	3	4	5	6	7	8	9	10
a_1	1.00	1.26	1.97	2.22	2.93	3.19	2.35	2.71	2.58	3.06
a_2	1.00	1.97	1.26	2.22	1.52	2.48	2.71	2.35	1.87	2.00
$< a_3 >$	4.00	2.78	2.78	1.55	1.55	0.33	0.94	0.94	1.55	0.94
k_d	1.47	1.72	1.72	2.42	2.02	2.18	2.21	2.55	2.66	2.99

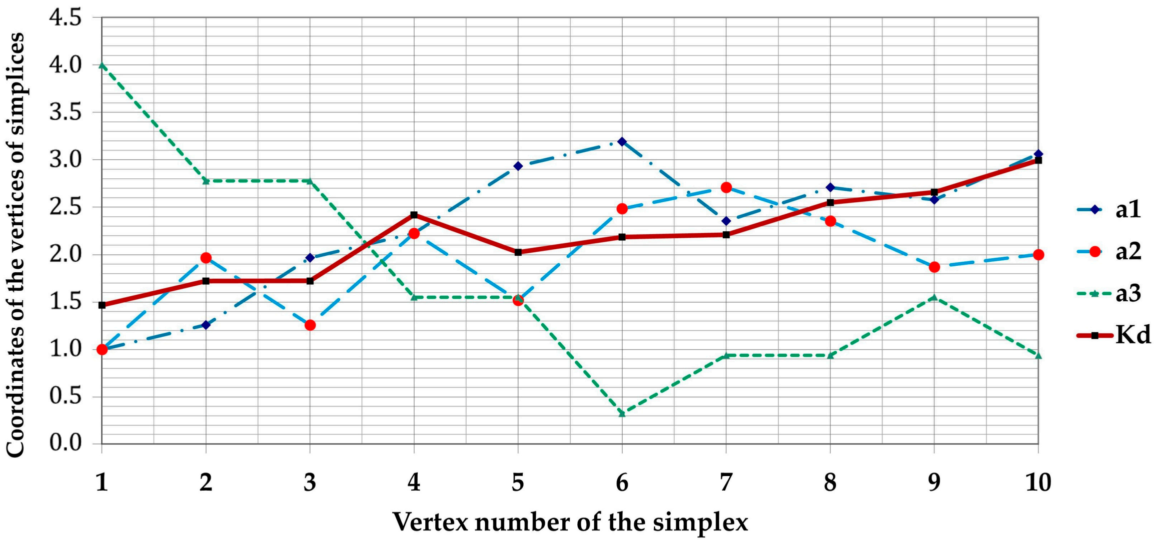


Figure 12. Values of parameters at the vertices of simplexes and the corresponding values of the dynamic coefficient k_d.

On the simplex '4', there was a coincidence in the minimum point, since the value of the objective function $1/k_d$ at the following iteration steps with a change of simplex vertices (they are not shown in the figures and in the table) turned out to be greater than the values at the vertices of the simplex of the previous iteration. The value of the reduction coefficient α was reduced to 0.5. Vertex '4' was taken as the base point, as having the minimum value of the objective function in the simplex. In Figure 11, it is marked with the symbol (*).

At the vertex '10' of the simplex '7', also marked with the symbol (*), $a_1 = 3.06$, $a_2 = 2.00$ (marked with a dot in Figure 11), the minimum of the objective function $1/k_d$ has been reached. The construction of further simplices, even with a decrease in the value of the reduction coefficient a, did not lead to a decrease in the objective function.

The results of the calculations led to the following conclusions: for a given immersion force A, the maximum value of the dynamic coefficient $1/k_d$ is achieved with the following values of the coefficients a_k:

$$\begin{cases} a_1 = \frac{1}{2}A = \frac{3}{6}A \\ a_2 = \frac{1}{3}A = \frac{2}{6}A \\ \quad a_3 = \frac{1}{6}A \end{cases} \tag{39}$$

Figure 12 and (39) show the results of calculating the support reaction $Y_r(\varphi)$ and the contribution of each of the three stages of unbalanced vibration blocks with directional vibrations for the above values of the input parameters.

Thus, for a given total immersion force A, the maximum possible dynamic coefficient for a two-stage unbalanced vibration mechanism with asymmetric oscillations is equal to two and is achieved with coefficients (33). For a three-stage unbalanced vibration mechanism with asymmetric oscillations, it is equal to three and is achieved with coefficients (39). For a four-stage unbalanced vibration mechanism with asymmetric oscillations, the dynamic coefficient can be equal to four and is achieved with coefficients (40)

$$\begin{cases} a_1 = \frac{4}{10}A, \\ a_2 = \frac{3}{10}A \\ a_3 = \frac{2}{10}A \\ a_4 = \frac{1}{10}A \end{cases} \tag{40}$$

Based on the results obtained and presented in this article, for a vibration mechanism consisting of n vibration stages, for example, n planetary vibrators with directional oscillations or n pairs of vibrators with circular oscillations, each pair generating directional driving forces, it follows that in order to obtain a total asymmetric oscillation, the following conditions must be met:

1. The maximum value of the total driving force acting in the direction of performing useful work is achieved when (26):

$$\varphi_{k0} = \frac{\pi}{2}(k-1)$$

in this case, the magnitude of the total driving force acting in the direction of performing useful work, for example, immersion, is determined with Equation (24):

$$A = \sum_{k=1}^{n} a_k$$

2. The maximum value of the asymmetry coefficient of the total driving force, or the dynamic coefficient of the oscillatory system ($k_{as},\ k_d$), is equal to n and is achieved at the following values of the coefficients of a number of vibration stages

$$a_i = \frac{n+1-i}{\sum_{k=1}^{n} k} \cdot A \tag{41}$$

where k_d is the dynamic coefficient of the oscillatory system (the asymmetry coefficient of the total driving force, k_{as}); k is the serial number of the vibration stage generating the directed oscillations (the directed driving force); n is the number of vibration stages in the vibration device with asymmetric oscillations; i is the serial number of the term of the total driving force included in the total driving force of the vibration device with asymmetric oscillations; A is the specified value of the total driving force acting in the direction of performing useful work; and a_i is the part of the value of the total driving force that falls on the i-th stage of the vibration device. When considering Equation (24), the sum of the values of a_i is always: $\sum_{i=1}^{n} a_i = 1.0$.

Using Equation (41), $a_k = (a_1, a_2, \ldots, a_n)$, the general equation of oscillations of a vibration device asymmetric with a total driving force can be written in the form:

$$F_{sum} = \sum_{i=1}^{n} a_i \cdot F_i = a_1 \cdot F_{sum} + a_2 \cdot F_{sum} + \cdots + a_n \cdot F_{sum} \quad (42)$$

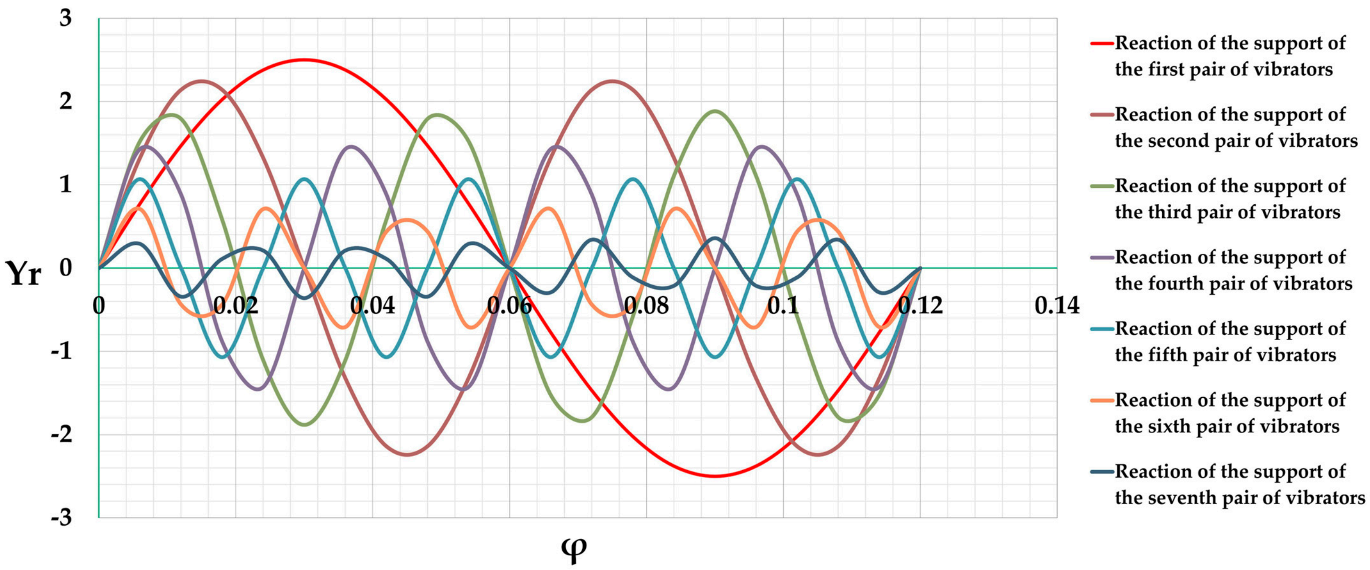
where a_i are the coefficients of the terms of the series of the total driving force of a vibration device with asymmetric oscillations when using the value A = 1.0 in Equation (42), and then $F_{sum} = 1.0$.

4. Discussion

The behavior of the total driving force $F_{sum} = A$ for an arbitrary value of its magnitude and a given asymmetry coefficient, using the equations obtained (24,26,41,42) as an example, have been considered. For comparison, the number of stages of the vibration device to be seven, n = 7.0, has been taken. The rotation frequency of the eccentric weight shafts, rpm, is, respectively, 500, 1000, 1500, 2000, 2500, 3000, and 3500. The total value of the driving force F_{sum} = 10 kN. The given asymmetry coefficient of the driving force is equal to $k_{as} = k_d = 7.0$. The graph of the change in $F_{sum} = f(\varphi)$ has the form shown in Figure 13.

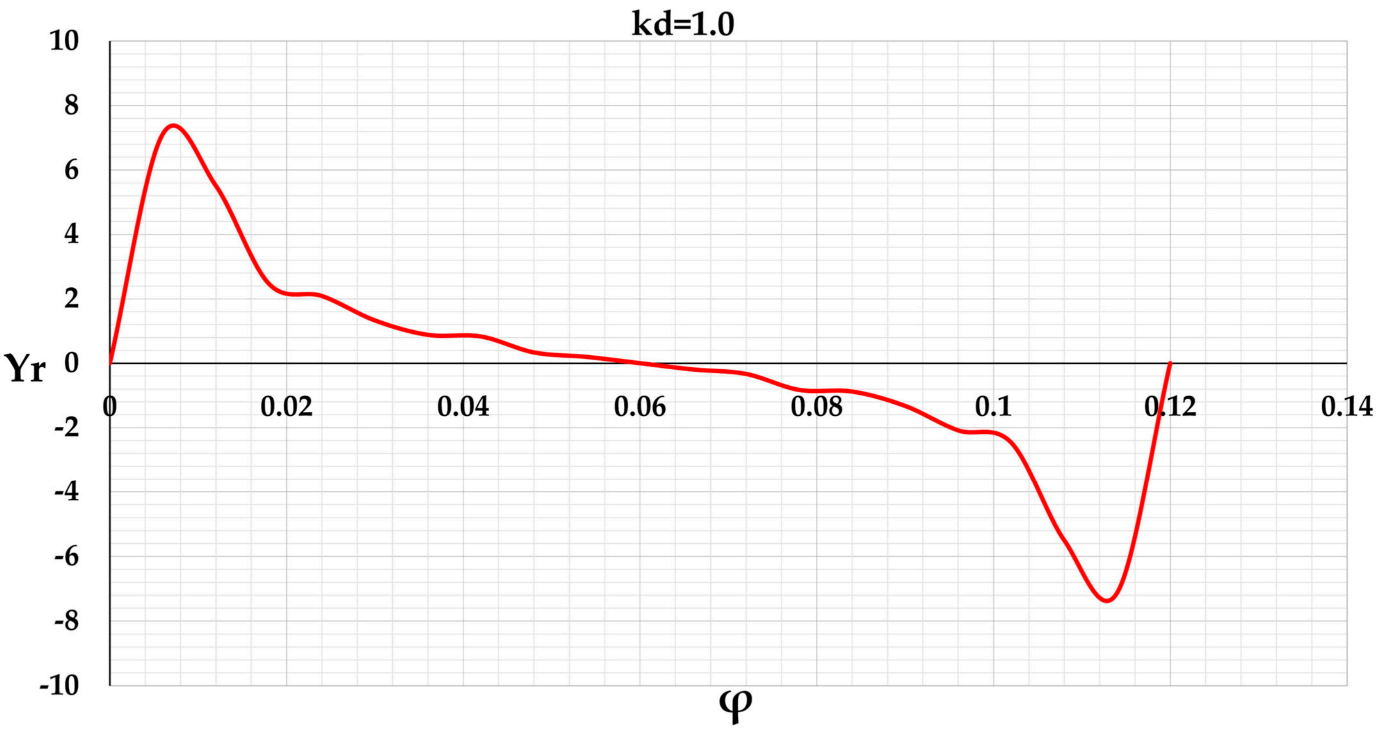
As the calculation result and the behavior of the graph in Figure 13d show, the obtained method for designing vibration devices with asymmetric oscillations fully corresponds to the optimality condition of the Maxwell–Feuillere polynomial, which assigns the optimal (in the sense of the asymmetry coefficient) impulse to the functional (28).

When using Equations (24), (26), (41) and (42) in designing a vibration device with asymmetric oscillations and a given coefficient of asymmetry of the total driving force, there is no need to calculate and plot graphs in Figure 13a–c, since the calculation provides for obtaining graph Figure 13d directly.

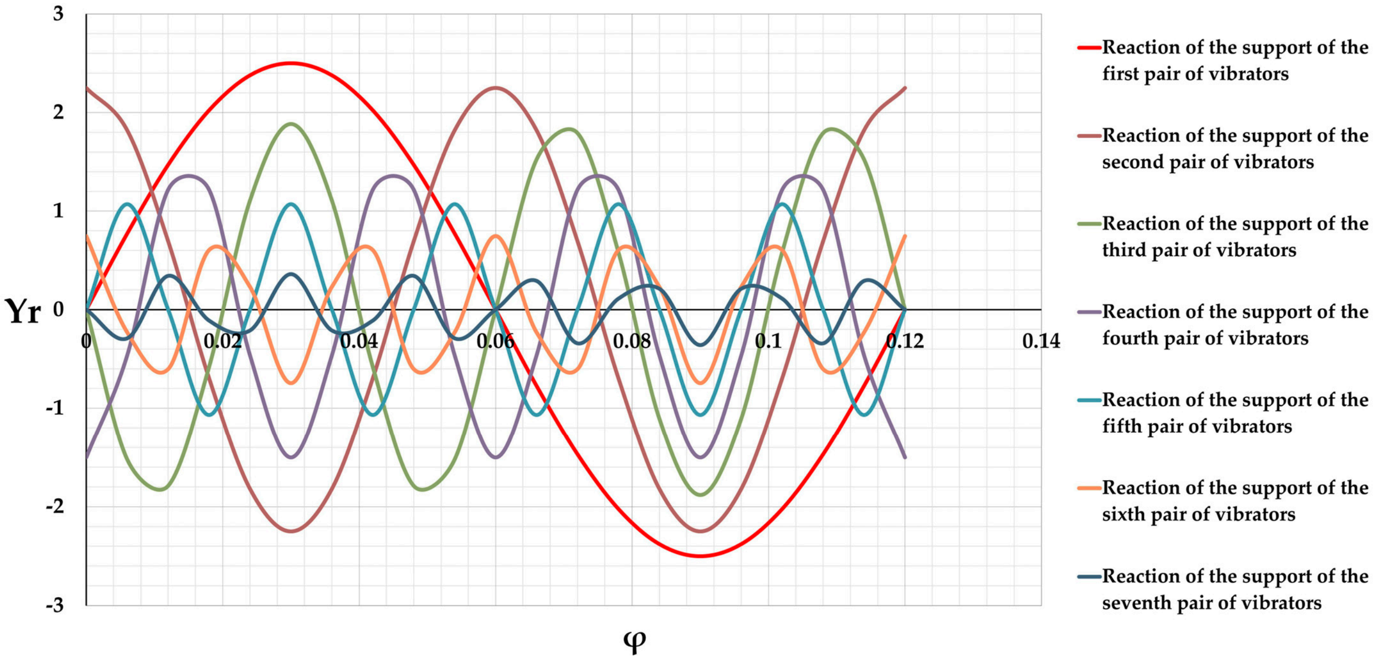


(**a**)

Figure 13. *Cont.*

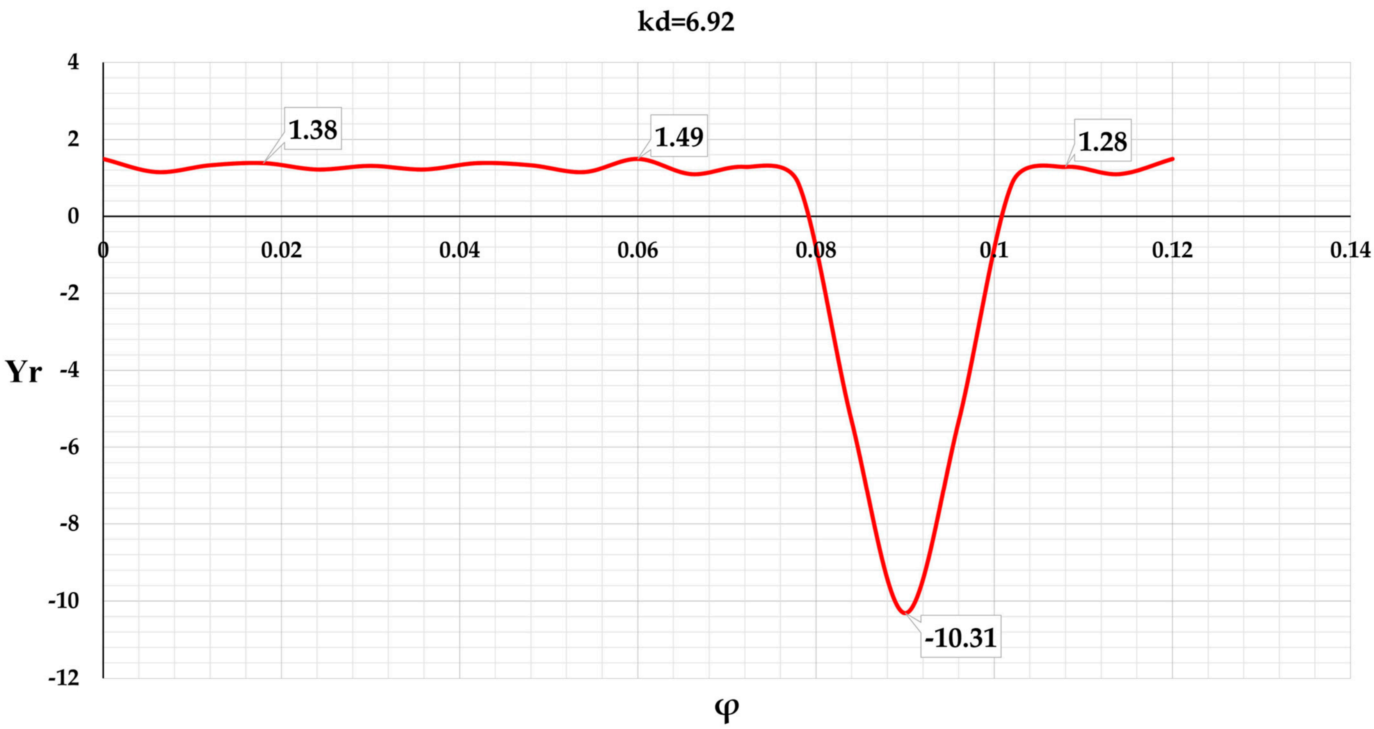


(**b**)



(**c**)

Figure 13. *Cont.*



(**d**)

Figure 13. Calculation graphs of the magnitude of the driving forces of a vibration device with asymmetric oscillations with the total driving force F_{sum} = 10 kN and the specified asymmetry coefficient k_{as} = 7.0. (**a**) is support reactions of each of the seven pairs of vibrators with the value F_{sum} = 1 kN and the initial phases $\varphi_{k0} = 0$; (**b**) is the sum of the support reactions of all seven pairs of vibrators with the initial phases $\varphi_{k0} = 0$; (**c**) is the support reactions of each of the seven pairs of vibrators with the initial phases $\varphi_{k0} = \frac{\pi}{2}(k-1)$, (**d**) is the sum of the support reactions of all seven pairs of vibrators with the initial phases $\varphi_{k0} = \frac{\pi}{2}(k-1)$.

5. Conclusions

The design methodology, in this case, comes down to the following actions.

1. Determination of the coefficients a_i for the terms of a series of seven members of the total driving force of a vibration device with asymmetric oscillations when used in Equation (24): A = 1.0;
2. Determination of the terms of forces of a series of seven members from the expression: $a_i \cdot F_{sum}$;
3. Checking the obtained value of the total driving force using Equation (42);
4. Assignment of the mass and eccentricity of the eccentric weight of each stage using known engineering methods based on the limitations associated with the dimensions of the specific area of application of the product.

$$m_i r_i \omega_i^2 = a_i \cdot F_{sum} \quad (43)$$

where m_i is the total mass of the eccentric weight of the i-th stage of the vibration device; r_i is the eccentricity of the eccentric weight of the i-th stage of the vibration device; and ω_i is the angular velocity of the unbalanced shaft of the i-th stage, related to the angular velocity of rotation of the unbalanced shaft of the first stage by a multiple ratio.

As a comparative example, a numerical example of the initial data is given: F_{sum} = 10 kN is the total value of the driving force with asymmetric oscillations and an asymmetry coefficient k_{as} = 10.

1. Calculation (41) of coefficients a_i for components of the total driving force $F_{sum} = 1$ kN is performed;
2. Rotation frequencies of unbalanced shafts of ten stages of the vibration device with asymmetric oscillations are assigned, $\omega_i = n{\cdot}\omega_1$;
3. Calculation of the value of components (F_i) of the total driving force $F_i = 10$ kN is calculated;
4. Based on design considerations and process kinetics features, masses (m_i) and eccentricity (r_i) of imbalances of the corresponding stages of the vibration device are assigned;
5. A general table of parameters is compiled, as shown in Table 3;
6. Calculation and control graphs of changes in the driving force value within the oscillation period are performed, as shown in Figure 14.

Table 3. The method of determining the parameters of a vibration device with asymmetric oscillations with the value of the total driving force $F_{sum} = 10$ kN and the asymmetry coefficient k_{as} =10.

Parameter Name	Unit of Measurement	Vibration Device Stage										Sum a_i
		1	2	3	4	5	6	7	8	9	10	
Coefficient a_i at F_{sum}= 1 kN	-	0.18	0.16	0.146	0.127	0.11	0.091	0.073	0.055	0.036	0.018	1.0
Magnitude of the component F_i at F_{sum} = 10 kN	kN	1.82	1.64	1.46	1.27	1.09	0.91	0.73	0.55	0.36	0.18	10
Angular velocity of imbalance of the *i*-th stage	s^{-1}	52.36	104.72	157.08	209.44	261.8	314.16	366.52	418.88	471.24	523.6	-
Moment of imbalance, $m_i{\cdot}r_i$	kg·m	3.14	4.19	3.14	2.09	2.62	3.14	3.67	2.09	1.41	1.05	-
Mass imbalance, m_i	kg	5.52	1.86	1.47	1.45	0.8	0.46	0.27	0.32	0.27	0.17	-
Eccentricity, r_i	m	0.06	0.04	0.02	0.01	0.01	0.01	.0.01	0.005	0.003	0.002	-

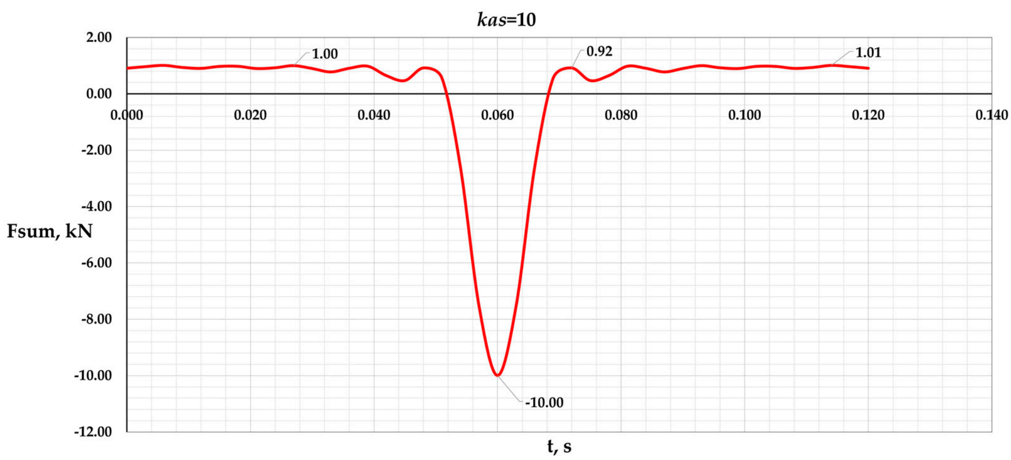


Figure 14. Graph of change in total driving force $F_{sum} = f(t)$.

Based on the calculation results, a graph of the total driving force $F_{sum} = f(t)$ is constructed within one oscillation period (T = 0.12 s).

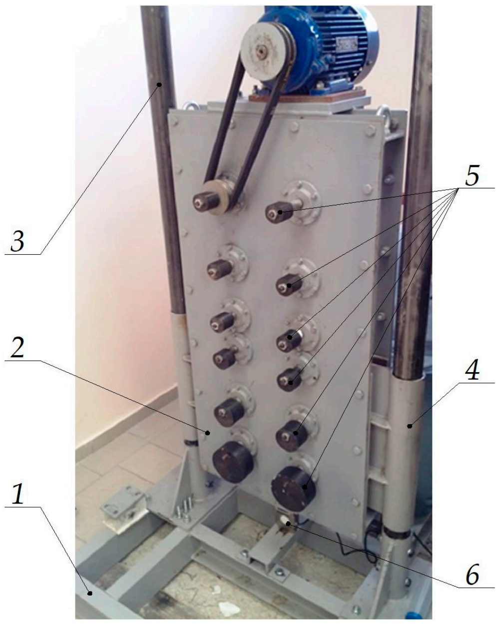
Thus, the magnitude of the driving force F_{sum} = 10 kN is obtained, acting in the direction of performing useful work with the asymmetry coefficient k_{as} = 10, which is generated by ten stages of the vibration device with directed oscillations with matched oscillation frequencies. The magnitude of the total driving force in the direction of the idle stroke is 1 kN.

In accordance with the stated objective of the research, a methodology and technique for calculating and designing the parameters of vibration devices with asymmetric oscillations for a given value of the total driving force and a given asymmetry coefficient have been developed.

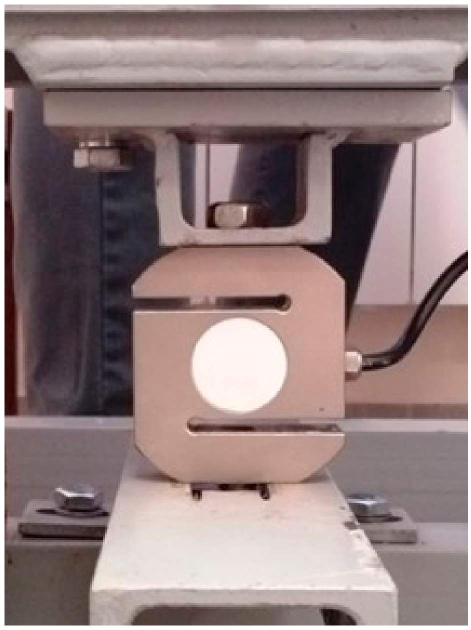
A general equation for the total driving force of a vibration device with asymmetric oscillations, ensuring any asymmetry coefficient specified from the point of view of technological feasibility, is obtained and proposed.

The proposed method of calculating and designing vibration devices with asymmetric oscillations using a generalized equation of the total driving force allows for the conversion of existing models of vibration machines from uniform oscillations to asymmetric ones, thereby increasing their operating efficiency with improved specific parameters.

The authors have developed and created a test bench, shown in Figure 15, consisting of 6 pairs of steps. The bench allows for the replacement of pairs of imbalances, changing and selecting the corresponding moments of inertia, masses of imbalances, eccentricity, and angular velocities. It allows for the obtaining of an asymmetric driving force with a given asymmetry coefficient according to the developed methodology. For the presented test bench, consisting of 6 steps, the asymmetry coefficient in the range of 2.0. . .6.0 is realized.



(**a**)



(**b**)

Figure 15. (**a**) The test bench with synchronized pairs of imbalances for multi-stage production of asymmetric oscillations: 1 is a base, 2 is a body, 3 is a guide rod, 4 is the sliders, 5 is the imbalances, 6 is a driving force sensor. (**b**) A driving force magnitude recording sensor.

Author Contributions: M.D.G.: Conceptualization, methodology; N.S.L.: data curation, writing—original draft preparation; A.A.P.: supervision, resources; B.S.C.: visualization, investigation, validation; A.C.: software. All authors have read and agreed to the published version of the manuscript.

Funding: The study was supported by the Russian Science Foundation Grant No. 23-79-10022, https://rscf.ru/project/23-79-10022/.

Data Availability Statement: The data presented in this study are available upon request from the respective author. The data are not publicly available as they are also part of ongoing research.

Acknowledgments: The authors are grateful to the team of the High Technology Center (HTC) of BSTU and V.G. Shukhov for assistance in providing design capacities.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Olukan, T.; Farshid, I. *Oscillations: Theory and Applications in AFM*; IOP Publishing: Bristol, UK, 2024; pp. 1–21. [CrossRef]
2. Yaïr, H.; Santos, S.; Elsherbiny, L.; Chiesa, M. Competition between transient oscillations and early stochasticity in exponentially growing populations. *Phys. Rev. Res.* **2024**, *6*, 033320. [CrossRef]
3. Chen, L.; Chen, J.; Tan, C.; Duan, P.; Liu, X.; Liu, M.; Fa, M.; Feng, J.; Zhang, X.; Zheng, B. The effect of channel size and wall erosion on the low-frequency oscillation in Hall thruster. *J. Phys. D Appl. Phys.* **2024**, *58*, 015203. [CrossRef]
4. Ginoux, J.-M. From Nonlinear Oscillations to Chaos Theory. In *The Foundations of Chaos Revisited: From Poincaré to Recent Advancements*; Springer: Berlin/Heidelberg, Germany, 2016; pp. 27–47. [CrossRef]
5. Gatti, P.L. *Applied Structural and Mechanical Vibrations: Theory and Methods*, 2nd ed.; CRC Press: Boca Raton, FL, USA, 2014; pp. 1–635. [CrossRef]
6. Ouali, M.; Magraoui, R. Contribution to Conditional Maintenance by Vibration Analysis of Rotating Machine Mechanical Failures and Proposed Solutions. *Rev. Gestão Soc.Ambient.* **2024**, *18*, e08541. [CrossRef]
7. Fatih, P.; İsmail, E.; Cihan, M. Vibration analysis of half rail vehicle model with 10 degrees of freedom based on mechanical-electrical analogy theory. *Int. J. Heavy Veh. Syst.* **2021**, *28*, 521–541. [CrossRef]
8. Polidor, B.; Nicusor, D.; Cornelia, D. Dynamic Performances of Technological Vibrating Machines. *Symmetry* **2022**, *14*, 539. [CrossRef]
9. Hordieiev, A.; Stariy, A.; Urbaniuk, E.; Hordieiev, A. Design of a vibrating machine module for pollution cleaning and a general method for calculating its elements. *Her. Khmelnytskyi Natl. Univ. Tech. Sci.* **2022**, *315*, 40–46. [CrossRef]
10. Lyan, I.; Panovko, G.; Shokhin, A. Creation and verification of spatial mathematical model of vibrating machine with two self-synchronizing unbalanced exciters. *J. Vibroeng.* **2021**, *23*, 1524–1534. [CrossRef]
11. Lyan, I.; Panovko, G.; Shokhin, A. Modeling of a vibrating machine with spatial oscillations of the working body. *IOP Conf. Ser. Mater. Sci. Eng.* **2021**, *1129*, 012038. [CrossRef]
12. Zhao, C.; Zhao, Q.; Gong, Z.; Wen, B. Synchronization of Two Self-Synchronous Vibrating Machines on an Isolation Frame. *Shock Vib.* **2011**, *18*, 73–90. [CrossRef]
13. Gurskyi, V.; Kuzio, I.; Krot, P.; Zimroz, R. Energy-Saving Inertial Drive for Dual-Frequency Excitation of Vibrating Machines. *Energies* **2021**, *14*, 71. [CrossRef]
14. Tian, Y.; Tu, C. Statistical Prediction and Analysis on Parameters of Vibrating Sinking Pipe Gravel Pile Machines. *J. Phys. Conf. Ser.* **2021**, *1744*, 022131. [CrossRef]
15. Jatsun, S.; Zaharov, I. Optimization of the Vibrating Technological Machines. In Proceedings of the 6th World Congresses of Structural and Multidisciplinary Optimization, Rio de Janeiro, Brazil, 30 May–3 June 2005; pp. 1–10.
16. Wu, F.; Zhang, Z. Abnormal Detection of Vibrating Sinking Pipe Gravel Pile Machines Based On Support Vector Machine Model. *J. Phys. Conf. Ser.* **2021**, *1744*, 022132. [CrossRef]
17. Astashev, V.; Krupenin, V.L. Quantitative evaluations of the effectiveness of vibrating and vibroimpact machines. *IOP Conf. Ser. Mater. Sci. Eng.* **2020**, *709*, 033041. [CrossRef]
18. Srikanth, V.; Kuznetsov, A. Symmetry-breaking of turbulent flow due to asymmetric vortex shedding in periodic porous media. *arXiv* **2024**, arXiv:2407.12955. [CrossRef]
19. Jiaguo, Y.; Ying, P. Research on Design of Vibration Platform of Molded Case Circuit Breaker and Reliability of Platform Frame. *Am. J. Eng. Technol. Manag.* **2018**, *3*, 35–41. [CrossRef]
20. Zhang, J.; Wang, X. Optimal Design and Experimental Study of Piezoelectric Vibrator With Multi-Order Bending Mode and Multi-Operating Frequency. *IEEE Access* **2023**, *99*, 45605–45617. [CrossRef]
21. Gurskyi, V.; Krot, P.; Korendiy, V.; Zimroz, R. Dynamic Analysis of an Enhanced Multi-Frequency Inertial Exciter for Industrial Vibrating Machines. *Machines* **2022**, *10*, 130. [CrossRef]
22. Xu, Y.; Luo, A. Paired asymmetric periodic oscillations in a pair of first-order asymmetric nonlinear circuit systems. *Mech. Syst. Signal Process.* **2022**, *171*, 108810. [CrossRef]
23. Fedyushkin, A. The Influence of Controlled Vibration Effects on Fluid Flow in Technological and Engineering Processes. In *Proceedings of the 10th International Conference on Civil Engineering*; Springer: Berlin/Heidelberg, Germany, 2024; pp. 656–664. [CrossRef]
24. Mousavi, S.; Fakhari, V. Design and development of closed-loop controllers for trajectory tracking of a planar vibration-driven robot. *J. Vib. Control* **2024**, 10775463241276026. [CrossRef]
25. Amiri, N.; Sohrabi, K.; Eftekharian, G.; Fakhari, V. Optimization and control of an energy-efficient vibration-driven robot. *J. Vib. Control* **2023**, *30*, 2184–2199. [CrossRef]
26. Hanns, W.T.; Roberta, L.K.; Bert, U.; Ralph, H. Good Vibrations: Asymmetric Vibrations for Directional Haptic Cues. In Proceedings of the World Haptics 2009—Third Joint EuroHaptics conference and Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems, Salt Lake City, UT, USA, 18–20 March 2009; pp. 285–589. [CrossRef]

27. He, W.; Tang, B.; Jia, S. Design, simulation and development of a broadband electromagnetic angular vibrator. *Int. J. Appl. Electromagn. Mech.* **2018**, *57*, 295–307. [CrossRef]
28. Gouskov, A.; Panovko, G.; Shokhin, A. To the Issue of Control Resonant Oscillations of a Vibrating Machine with Two Self-synchronizing Inertial Exciters. In *Proceedings of the 14th International Conference on Vibration Problems*; ICOVP 2019, Lecture Notes in Mechanical Engineering; Springer: Singapore, 2020. [CrossRef]
29. Du, Y.; Dang, P.; Liu, Z.; Qiao, C. Performance improvement of a low-frequency vibration generator by using iterative learning control. *Rev. Sci. Instrum.* **2022**, *93*, 105005. [CrossRef] [PubMed]
30. Fanghella, P.; Bruzzone, L.; Ellero, S. Dynamic Balancing of a Nutating Planetary Bevel Gear Train. *Mech. Mach. Sci.* **2015**, *25*, 23–33. [CrossRef]
31. Gerasimov, M.D.; Glagolev, S.N.; Mkrtychev, O.V. Determination of The Driving Force's Asymmetry Factor and The Vibrostand's Work's Analysis. *Int. J. Appl. Eng. Res.* **2015**, *10*, 45181–45185.
32. Kidokoro, J.; Mian, S.; Hoque, L.; Yu, P.; Okimura, K.; Nakanishi, T. Observation of transient aspects of self-sustained oscillations and the role of parallel capacitance in VO2-based planar devices. *J. Appl. Phys.* **2024**, *136*, 065303. [CrossRef]
33. Jazar, R.; Marzbani, H. Mechanical Vibrations Modeling. In *Vehicle Vibrations*; Springer: Cham, Switzerland, 2024; pp. 55–78. [CrossRef]
34. Noda, S.; Igari, R.; Endo, K.; Yamanobe, K.; Takayama, T. Switching backdrivability of a planetary gear by vibration: Design parameter setting and excited force estimation. *ROBOMECH J.* **2023**, *10*, 25. [CrossRef]
35. Yamamoto, T.; Ishida, Y.; Kazuo, A. On the Subharmonic Oscillations of Unsymmetrical Shafts. *Trans. Jpn. Soc. Mech. Eng.* **1978**, *44*, 1934–1942. [CrossRef]
36. Lubimyi, N.S.; Chepchurov, M.; Polshin, A.A.; Gerasimov, M.D.; Chetverikov, B.S.; Chetverikova, A.; Tikhonov, A.A.; Maltsev, A. Reducing the Cost of 3D Metal Printing Using Selective Laser Melting (SLM) Technology in the Manufacture of a Drill Body by Reinforcing Thin-Walled Shell Forms with Metal-Polymers. *J. Manuf. Mater. Process.* **2024**, *8*, 44. [CrossRef]
37. Gerasimov, M.; Lubimyi, N.; Yakovlev, E. Comparative analysis of the parameters of generators with circular and asymmetric oscillations. *IOP Conf. Ser. Mater. Sci. Eng.* **2020**, *945*, 012003. [CrossRef]
38. Petro, K.; Roman, S.; Oleksii, S.; Serhii, B. Vertical vibrating moveable device. *Vib. Eng. Technol.* **2022**, 29–35. [CrossRef]
39. Tyuremnov, I.; Shorokhov, D. Vibrating roller with compacted soil interaction modelling. *Russ. Automob. Highw. Ind. J.* **2024**, *21*, 202–216. [CrossRef]
40. Kostin, V.; Kostin, D.; Sapronov, Y. Maxwell-Fejer polynomials and optimization of polyharmonic impulse. *Dokl. Math.* **2012**, *86*, 512–514. [CrossRef]
41. Nessmann, A. Polyharmonic Functions in the Quarter Plane. *Electron. J. Comb.* **2024**, *31*, 1–63. [CrossRef] [PubMed]
42. Luan, T.; Khatir, S.; Thi, T.; Baets, B. Exponential-trigonometric optimization algorithm for solving complicated engineering problems. *Comput. Methods Appl. Mech. Eng.* **2024**, *432*, 117411. [CrossRef]
43. Andrukhiv, A.; Sokil, M.; Sokil, B.; Fedushko, S.; Syerov, Y.; Karovič, V.; Klynina, T. Influence of Impulse Disturbances on Oscillations of Nonlinearly Elastic Bodies. *Mathematics* **2021**, *9*, 819. [CrossRef]
44. Aoyama, Y.; Lehmamnn, P.; Theodorou, E. Second Order Stein-Variational Dynamic Optimization. *arXiv* **2024**, arXiv:2409.04644. [CrossRef]
45. Bai, J.; Zeng, Z. The Euler-Lagrange equations of nabla derivatives for variational approach to optimization problems on time scales. *Adv. Contin. Discret. Models* **2024**, *2024*, 33. [CrossRef]
46. Tsifansky, S.L.; Beresnevich, V.I. Method of Tuning a Vibration Machine with a Piecewise Linear Characteristic of Elastic Connections to Resonant Oscillations. 15 November 1987. (SU). Prior. SU 3910845/24-28. SU Patent 1,351,696, 17 June 1985. Available online: https://yandex.ru/patents/doc/SU1351696A1_19871115 (accessed on 12 September 2024).
47. Gerasimov, M.D.; Vorobyov, N.D.; Gerasimov, D.M. Mathematical model of a two-stage vibration mechanism. *Bull. BSTU Named After V.G. Shukhov* **2017**, *2*, 195–201. [CrossRef]
48. Albrecht, K.; Christian, H. Vibration Generator. 28 September 2010. (DE). Prior. US 2009/0243410. U.S. Patent 7,804,211, 1 October 2009.
49. Gerasimov, M.D. Addition of oscillations in vibration exciters. *Bull. BSTU Named After V.G. Shukhov* **2016**, *3*, 116–121.
50. Gerasimov, M.D. Method for Generating Directed Inertial Asymmetric Oscillations of Working Equipment of Vibration Machines. 12 January 2021. (RU). Prior. RU 2020121504. RU Patent 2,740,282, 5 August 2020.
51. Korn, G.; Korn, T. *Handbook of Mathematics for Scientists and Engineers*; The Main Editorial Board of Physical and Mathematical Literature; "Science": New York, NY, USA, 1973; pp. 1–832.