

# Centenary of Alexander Friedmann's Prediction of the Universe Expansion and the Quantum Vacuum

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**Abstract:** We review the main scientific pictures of the universe developed from ancient times to Albert Einstein and underline that all of them treated the universe as a stationary system with unchanged physical properties. In contrast to this, 100 years ago Alexander Friedmann predicted that the universe expands starting from the point of infinitely large energy density. We briefly discuss the physical meaning of this prediction and its experimental confirmation consisting of the discovery of redshift in the spectra of remote galaxies and relic radiation. After mentioning the horizon problem in the theory of the hot universe, the inflationary model is considered in connection with the concept of quantum vacuum as an alternative to the inflaton field. The accelerated expansion of the universe is discussed as powered by the cosmological constant originating from the quantum vacuum. The conclusion is made that since Alexander Friedmann's prediction of the universe expansion radically altered our picture of the world in comparison with the previous epochs, his name should be put on a par with the names of Ptolemy and Copernicus.

**Keywords:** quantum vacuum; Friedmann universe; general theory of relativity



**Citation:** Klimchitskaya, G.L.; Mostepanenko, V.M. Centenary of Alexander Friedmann's Prediction of the Universe Expansion and the Quantum Vacuum. *Physics* **2022**, *4*, 981–994. <https://doi.org/10.3390/physics4030065>

Received: 28 July 2022

Accepted: 16 August 2022

Published: 31 August 2022

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## 1. Introduction

According to Immanuel Kant [1], the starry heavens is one of two things which “fill the mind with ever new and increasing admiration and awe. . .” Questions about what our universe is, how it was created and how long it will exist have always aroused great interest. In the pre-scientific era, answers to these questions were usually given on the basis of various myths, religions, and philosophical systems. In ancient Greece, for the first time in the history of mankind, the foundations of a scientific approach to the study and attempts to answer these questions were laid.

In this review, we briefly list the main scientific pictures of the universe, developed during the period of time from ancient Greece to Albert Einstein, and emphasize one characteristic feature common to all of them. This common feature is that the universe has always been thought as a stationary system. The concept of a stationary universe was questioned only 100 years ago in 1922 when Alexander Friedmann, on the basis of the general theory of relativity, demonstrated that our universe expands with time. This prediction was confirmed experimentally very soon and became the basis of modern cosmology given every reason to include the name of Alexander Friedmann on a par with the names of greatest scientists who completely revised our understanding of the world around us.

Another important point is that the Friedmann universe exists for a finite time after its creation at a point “from nothing”. This initial state of the universe called the “cosmological singularity” makes a link between the Friedmann discovery of expanding universe and the concept of quantum vacuum. According to modern views, the first moments of the evolution of the universe were governed by quantum theory. In the framework of a

semiclassical model, where the gravitational field remains classical but the fields of matter are quantum, it is possible to consider the stress-energy tensor of the vacuum of quantized fields as the source of gravitational field. Under the influence of quantum vacuum the universe expands exponentially fast which is known as the cosmic inflation. At slightly later time, inflation gives way to the Friedmann expansion following the power law. At the present stage, an expansion of the Friedmann universe is accelerating under the impact of dark energy. One of the most popular explanations of this mysterious substance is again given by the quantum vacuum which leads to a nonzero cosmological constant.

The paper is organized as follows. After a discussion of various pictures of the universe from ancient times to Einstein in Section 2, we briefly consider the main facts of Alexander Friedmann's scientific biography in Section 3. Section 4 is devoted to Friedmann's prediction of the universe expansion made 100 years ago. The experimental facts confirming that the universe is really expanding are presented in Section 5. Section 6 is devoted to the cosmic inflation and creation of the universe from the quantum vacuum. In Section 7, we consider the accelerated expansion of the universe and its explanation in terms of dark energy originating from the vacuum of quantized fields. In Section 8, the reader will find the discussion, and the paper ends with conclusions in Section 9.

## 2. Pictures of the Universe—From Ancient Times to Albert Einstein

The first scientific picture of the universe based on observations was created by Claudius Ptolemy in the first century AD. The Ptolemy system was geocentric which means that the Earth was placed at the center of the world. All remaining celestial bodies, i.e., the Moon, the Sun and five planets known at that time (Mercury, Venus, Mars, Jupiter, and Saturn), rotated around the Earth in circular orbits. According to Ptolemy's system of the world, beyond Saturn there is a firmament to which the fixed stars are attached. It was assumed that the stars and the firmament do not obey the same physical laws as all bodies on the Earth. Despite the presence of some nonscientific elements, the Ptolemy system gave the possibility to perform calculation of both future and past positions of the Moon, Sun and all five planets with rather high accuracy. In fact, this system was successfully used until the 16th century. Needless to say, Ptolemy's picture of the universe was stationary. It did not vary with time.

Important change in our picture of the world has been made by investigations of Nicolaus Copernicus, Johannes Kepler, and Galileo Galilei, performed in the 16th and 17th centuries. They developed on a scientific basis and supported by observations the long-proposed hypothesis that our universe is in fact heliocentric. According to the heliocentric system, the Earth and all five planets orbit the Sun whereas the Moon orbits the Earth. Johannes Kepler made an important discovery that the orbits of planets are not circles but ellipses with the Sun at one of the ellipse's focuses. Both Copernicus and Kepler believed Ptolemy's idea that the stars are fixed points attached to the firmament. However, Galilei elaborated methods to determine the shape of stars and made estimations of their radii. The radically new picture of the world established by Copernicus, Kepler, and Galilei retains its validity in the scales of Solar system up to the present. However, they persisted in the belief that the universe is stationary in a sense that planets followed and will always follow the same predetermined orbits.

The next dramatic step in our understanding of the universe was made by Isaac Newton who developed the first physical theory, Newtonian mechanics, and laid foundations of the mechanical picture of the world. In his book [2] *Mathematical Principles of Natural Philosophy* published in 1687, Newton formulated the three laws of mechanics and the law of gravitation which must be obeyed by all material bodies on the Earth and in the sky. Newton arrived to the fundamental conclusion that the inertial mass,  $m_i$ , of each material body is equal to its gravitational mass,  $m_g$ , which is responsible for the gravitational attraction. This was the first formulation of the equivalence principle used by Einstein as the basis of general relativity theory 230 years later.

With the second law of mechanics and the Newton law of gravitation, one finds that the force acting between the test mass  $m_i = m_g$  and the Earth of the mass  $M$  and radius  $R$  can be expressed in two ways:

$$F = m_i a = \frac{G m_g M}{R^2}, \quad (1)$$

where  $G$  is the gravitational constant and  $a$  is the acceleration of the test mass. Then, using the equivalence principle, one obtains from Equation (1):

$$a = \frac{GM}{R^2}, \quad (2)$$

i.e., the conclusion is that all bodies in the vicinity of Earth surface fall down with the same acceleration independently of their mass. The law that light and heavy bodies falling to the ground from the same height reach the ground at the same time was experimentally discovered by Galileo Galilei. Newton derived it theoretically. This fundamental law of Nature, which defies common sense, was destined to play a huge role in elucidating the structure and evolution of our universe.

Newton's concept of the universe pushed its boundaries far beyond the solar system. According to Newton, the universe is infinitely large in volume and contains infinitely many stars. The space of the universe is homogeneous (i.e., all points are equivalent) and isotropic (i.e., all directions are also equivalent). However, keeping unchanged an important element of the previous pictures of the universe, Newton believed that our universe is of an infinitely large age and it will exist forever. In this sense he considered the universe to be stationary.

Newton's picture of the universe was universally accepted until the early 20th century despite some unresolved problems. For instance, according to Olbers paradox proposed in 1823, in the case of an infinitely large universe, in every direction one looks, one should see a star. As a result, the entire night sky would shine like a surface of a star, which is not true. One more difficulty is the problem of the heat death of the universe discussed by Bailly in 1777 and elaborated on by Lord Kelvin in 1851 on the basis of the laws of thermodynamics. Since the Newtonian universe exists for an infinitely large time, it should already have reached a state where all energy is evenly distributed and all dynamical processes are terminated. Thus, the observed temperature differences are in contradiction to an assumption that the universe is infinitely old.

A new era in the study of the universe began in 1915 when Albert Einstein created the general theory of relativity starting from the equivalence principle. According to this theory, there is no gravitational force which attracts material bodies to each other. All bodies move freely along the shortest (geodesic) lines in the Riemann curved space-time of the universe which becomes curved under the impact of energy and momentum of these bodies. Thus, the general theory of relativity describes the self-consistent system where the space-time curvature is determined by the material bodies whereas their motion is caused by the character of this curvature.

The description of the universe as a whole in the framework of general theory of relativity is based on Einstein field equations,

$$R_{ik} - \frac{1}{2} R g_{ik} - \Lambda g_{ik} = \frac{8\pi G}{c^4} T_{ik}, \quad (3)$$

where  $R_{ik}$  is the Ricci tensor characterizing the space-time curvature,  $R = g^{ik} R_{ik}$  is the scalar curvature,  $g_{ik}$  is the metrical tensor,  $\Lambda$  is the cosmological constant,  $c$  is the speed of light, and  $T_{ik}$  is the stress-energy tensor of matter. The indices  $i$  and  $k$  here take the values 0, 1, 2, 3 and there is a summation over the repeated indices.

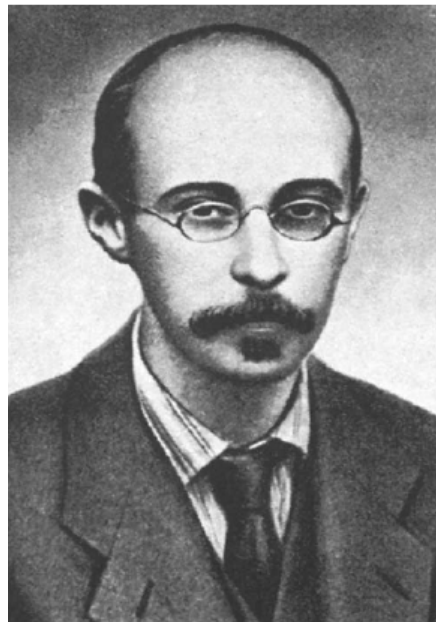
The term  $\Lambda g_{ik}$  was absent in the original Einstein's equations, published in 1915 [3] (see [4] for English translation). Einstein introduced it when applying his field equations to the universe as a whole in order to compensate the effect of gravity and make the

universe stationary [5] (see [6] for English translation). This means that he shared the opinion of Ptolemy, Copernicus, Galilei and Newton that the universe does not vary with time. Using the basic concepts of Newton's picture of the world, Einstein also assumed that the 3-dimensional space of the universe is homogeneous and isotropic. Based on this assumptions, he obtained the model of the stationary universe of finite spatial volume. This universe exists forever. It has never been created. There is a finite number of stars in the Einstein universe.

Thus, all the greatest scientists from Ptolemy to Einstein, who determined the views of mankind on the universe for two millennia believed that it is stationary. A new era in the understanding of the universe began with the paper by Alexander Friedmann [7], published 100 years ago, in 1922, in which he first proved that the universe expands. Before considering Friedmann's discovery, we briefly present the main facts of his scientific biography, making it clear how he came to such a radical conclusion.

### 3. Brief Scientific Biography of Alexander Friedmann

Alexander Friedmann (see his photo in Figure 1) was born on 6 June, 1888 in Saint Petersburg (the capital of Russian Empire) in the artistic family (a full description of his life can be found in [8,9]). Alexander Friedmann's father's name was also Alexander. He was an artist at the Court Ballet of the Imperial Theater and a ballet composer. Alexander Friedmann's mother's maiden name was Lyudmila Voyachek, she was a pianist. She graduated from the Saint Petersburg Conservatoire. Nothing indicated that a child born in such a family will become an eminent mathematician and physicist. After the divorce of Friedmann's parents in 1897, he lived with his father. In the same year, he started to study at the Second Saint Petersburg High School which was known for the highly qualified teachers in the field of mathematics and physics.



**Figure 1.** Alexander A. Friedmann (6 June 1888–16 September 1925), the founder of modern cosmology.

While still a schoolboy, Alexander Friedmann, in collaboration with his schoolmate Yakov Tamarkin (in the future, a famous mathematician), wrote his first paper, devoted to Bernoulli numbers. In 1906, this paper was published in *Mathematische Annalen* by the recommendation of David Hilbert [10].

Alexander Friedmann graduated from High School in 1906 with a gold medal and was admitted to the first course of the Department of Mathematics belonging to the Faculty of Physics and Mathematics at the Saint Petersburg University. During his student years at

the university, Alexander Friedmann obtained an intimate knowledge in different fields of mathematics and physics and his successes were always evaluated as “excellent”.

After a graduation from the Department of Mathematics in 1910, Alexander Friedmann stayed at the same department as a Postgraduate Researcher and to prepare to the Professor position. His supervisor was the famous mathematician academician Vladimir Steklov. From 1910 to 1913, Alexander Friedmann solved several complicated problems in mathematical physics, published many papers and delivered lectures in mathematics for students. Although he successfully passed examinations for a Master degree, he formally defended the Master thesis only in 1922. By that time, he was already Full Professor at the Perm University (1918–1920), at the Petrograd University, Petrograd Polytechnic Institute, and at the Institute of Railway Engineering (Petrograd, formerly Saint Petersburg, 1920–1925).

In 1913, Alexander Friedmann was employed by the Saint Petersburg Physical (later renamed in Geophysical) Observatory. During the work at this institution, he obtained several fundamental results in dynamical meteorology, hydrodynamics, and aerodynamics. His results retain their significance to the present day, and Friedmann’s name is well known to everyone working in these fields. Several important results were obtained by him during the visit to Leipzig University in the first part of 1914.

During the three years of the World War I, from 1914 to 1917, Friedmann served in the Air Force of the Russian Empire. During this period of his life, he personally piloted airplanes, organized the aerological service, and created the mathematical theory of bombing. His service during the war was marked with several military awards.

The period of Alexander Friedmann’s life from 1920 to 1925 was especially productive. During these years, he published several books and obtained outstanding results in the field of dynamical meteorology. As a recognition of his scientific merits, in 1925 Alexander Friedmann was appointed Director of the Geophysical Observatory of the Russian Academy of Sciences. Just during this period of time, he published two papers [7,11] containing an extraordinary prediction that our universe expands. Although on 16 September 1925 Alexander Friedmann tragically died of typhus at the age of 37, these papers made his name immortal. In the next section, we briefly discuss the essence of the obtained results and how the distinctive features of Friedmann’s scientific career helped him make a discovery that even the great Einstein himself missed.

#### 4. Friedmann’s Prediction of Expanding Universe

In his approach to the description of the universe in the framework of general relativity theory, Friedmann assumed that the three-dimensional space is homogeneous and isotropic. In this regard, he followed his predecessors Newton and Einstein. The assumption of homogeneity and isotropy alone gives the possibility to find the metric, i.e., the distance,  $ds$ , between two infinitesimally close space-time points,  $x^i$  and  $x^k$ , using the standard mathematical methods:

$$ds^2 = g_{ik}dx^i dx^k = c^2 dt^2 - a^2(t)[d\chi^2 + f^2(\chi)(d\theta^2 + \sin^2 \theta d\varphi^2)]. \quad (4)$$

Here,  $t$  is the time coordinate whereas  $\chi$ ,  $\theta$ , and  $\varphi$  are analogous to the spherical coordinates in the three-dimensional space. In doing so, the usual Cartesian coordinates and the radial coordinate are expressed as

$$\begin{aligned} x^1 &= r(t) \sin \theta \cos \varphi, & x^2 &= r(t) \sin \theta \sin \varphi, & x^3 &= r(t) \cos \theta, \\ r(t) &= a(t)f(\chi). \end{aligned} \quad (5)$$

The function  $f(\chi) = \sin \chi$ ,  $0$ , and  $\sinh \chi$  depending on whether the constant curvature of the three-space is equal to  $\kappa = 1$ ,  $0$ , and  $-1$ , respectively.

The function  $a(t)$  is called the “scale factor”. It has the dimension of length. In the case of  $f(\chi) = \sin \chi$  (the space of positive curvature),  $a(t)$  has the meaning of the radius of curvature. The space of positive curvature has the finite volume  $V = 2\pi^2 a^3(t)$ . The

spaces of zero and negative curvature have an infinitely large volume. In his first paper [7], Friedmann considered the space of positive curvature,  $\kappa = 1$ , whereas his second paper [11] is devoted to the space of negative curvature  $\kappa = -1$ .

In the Friedmann approach to the problem, it is important that he was a mathematician who used the rigorous analytic methods. He wished to see what is contained in the fundamental Einstein’s equations (3) in the case of a homogeneous isotropic metric (4) independently of our historical and methodological preferences. This approach, which proved to be very fruitful in all Friedmann’s diverse scientific activities, was based on the long-standing traditions of the Saint Petersburg mathematical school.

Substituting Equation (4) in Equation (3) and calculating  $R_{ik}$  and  $R$  by the standard expressions of Riemann geometry, Alexander Friedmann obtained two equations, which were later named after him:

$$\begin{aligned} \frac{d^2a}{dt^2} &= -\frac{4\pi G}{3c^2}a(\varepsilon + 3P) + \frac{1}{3}c^2a\Lambda, \\ \left(\frac{da}{dt}\right)^2 &= \frac{8\pi G}{3c^2}a^2\varepsilon - \kappa c^2 + \frac{1}{3}c^2a^2\Lambda. \end{aligned} \tag{6}$$

In these equations, it was taken into account that in the homogeneous isotropic space the stress-energy tensor,  $T_{ik}$ , is diagonal and that its component  $T_0^0 = \varepsilon$  has the meaning of the energy density of matter, whereas its components,  $T_1^1 = T_2^2 = T_3^3 = -P$ , describe the pressure  $P$  of matter. Note also that in his papers [7,11] Friedmann considered the so-called “dust matter” for which the relative velocities of its constituents are small as compared to the speed of light. This leads to the zero pressure,  $P = 0$ , but does not affect any of the fundamental conclusions following from Equation (6).

Friedmann found that for  $\kappa = 1$  Equation (6) admits the stationary solution in the special case when

$$\varepsilon + 3P = \frac{c^4\Lambda}{4\pi G}, \quad \frac{4\pi G a^2}{c^4}(\varepsilon + P) = \kappa. \tag{7}$$

In this case, from Equation (6) one has:

$$\frac{d^2a}{dt^2} = \frac{da}{dt} = 0, \quad a = a_0 = \text{const.} \tag{8}$$

This is the stationary universe obtained earlier by Einstein [5,6]. The stationary solution exists only in the case  $\kappa = 1, \Lambda \neq 0$ . If  $\Lambda = 0$ , the first equality in Equation (6) leads to  $d^2a/dt^2 < 0$  because for the usual matter it holds  $\varepsilon + 3P > 0$ . For  $\kappa = -1$  the stationary universe containing matter with  $\varepsilon > 0$  is impossible [11]. Thus, the cosmological solution found by Einstein is an exceptional case, whereas in all other cases (i.e., with  $\Lambda = 0$  or  $\Lambda \neq 0$  but with conditions (7) violated), the universe is nonstationary so that the scale factor  $a(t)$  depends on time.

Friedmann derived one more specific solution of Equation (6) for which the scale factor  $a$  depends on time but the scalar curvature  $R$  is constant. This is the solution previously found by de Sitter [12]. It is most simple to illustrate the de Sitter solution, which is determined by only the cosmological constant in the absence of usual matter,  $\varepsilon = P = 0$ , for the case  $\kappa = 0$ . Then Equation (6) is simplified to one equation:

$$\frac{da}{dt} = \sqrt{\frac{\Lambda}{3}}ca, \tag{9}$$

which has the solution

$$a(t) = a_0 \exp\left(c\sqrt{\frac{\Lambda}{3}}t\right). \tag{10}$$

The scalar curvature of the homogeneous isotropic spaces is given by

$$R = -\frac{6}{a^2} \left\{ \frac{1}{c^2} \left[ \left( \frac{da}{dt} \right)^2 + a \frac{d^2a}{dt^2} \right] + \kappa \right\}. \quad (11)$$

Substituting Equation (10) in Equation (11) for  $\kappa = 0$ , one obtains the constant scalar curvature of the de Sitter space–time  $R = -4\Lambda$ . We will return to the consideration of the de Sitter scale factor (10) in Section 6 in connection with the quantum vacuum.

As to the general solution of Friedmann equation (6), it is characterized by the zero initial value of the scale factor  $a(0) = 0$  and by the infinitely large values of the initial scalar curvature  $R(0)$  and energy density  $\varepsilon(0)$ . Thus, at the initial moment the space of the universe was compressed to a point and the time period from the creation of the world to the present moment is finite [7]. The initial state of the universe is called the cosmological singularity; see [13,14] for details about the solutions of Friedmann equation (6) for different types of matter and corresponding equations of state.

It should be noted that the first reaction of Einstein to Alexander Friedmann’s results was completely negative. Shortly after the publication of Friedmann’s paper [7], Einstein published a note [15] (see English translation in [16]) claiming that the cosmological solutions, found by Friedmann, do not satisfy the field equations of general relativity theory. In response to this criticism, Friedmann wrote the explanation letter [17] (for English translation, see [18]), which was put in Einstein’s hands by Yurii A. Krutkov during his visit to Germany. This letter contained exhaustive explanations which cannot be ignored. As a result, Einstein published one more note where he recognized that his criticism “was based on an error in calculations” [19] (see [20] for English translation). Even after Einstein had realized that his original criticism was incorrect, this did not make him a supporter of the concept of an expanding universe. For a long time Friedmann’s discovery went largely unnoticed. Its importance became obvious only after the publications by Georges Lemaître and others (see below).

In fact, Alexander Friedmann did not construct a new physical theory for the description of the universe. This was performed by Albert Einstein, who created the general theory of relativity. The greatness of Alexander Friedmann lies in the fact that he was the first to believe that the mathematical solution of Einstein equations, corresponding to an expanding universe, can indeed apply to the real world at the time when this idea was not considered plausible. In doing this, he changed our picture of the universe and gave start to grandiose cosmological investigations of the last century.

Although Friedmann himself believed that the observational data at our disposal are completely insufficient for choosing the solution of his equations that describes our universe [7], the first experimental confirmation of the universe expansion came very soon.

## 5. Experimental Confirmations of the Universe Expansion

For all nonstationary solutions of the Friedmann equation (6) distances between any two remote bodies in the observed universe increase with time. This is seen from Equation (4), where the spatial distance is proportional to the scale factor  $a(t)$ . As a result, if the universe expands, all observable galaxies should move away from the Earth. The galaxies are observed due to the light emitted by them. According to the Doppler law, the frequency of an electromagnetic wave emitted by a source moving away from the observer is decreased. This is the so-called “redshift” of the emitted light to the red end of the visible spectrum.

Actually, the first observation of the redshift of Andromeda Nebula was made by Slipher in 1913 [21], i.e., before the development of the general theory of relativity by Einstein. He interpreted this observation in the spirit of Doppler effect that the Andromeda Nebula moves away from the Earth. The universal law which connects the redshift in the spectra of remote galaxies with the universe expansion was experimentally discovered in 1927 by Georges Lemaître [22] and in 1929 by Edwin Hubble [23] who finally validated that

the nebulas are the galaxies outside the Milky Way. According to this law, the velocity of a remote galaxy is proportional to a distance to it,  $v = HD$ , where  $H$  is the Hubble constant which can be expressed via the scale factor as

$$H = H(t) = \frac{1}{a(t)} \frac{da(t)}{dt}, \quad (12)$$

i.e., it is in fact time-dependent.

The discovery of the redshifts in the spectra of remote galaxies was the first experimental confirmation of the universe expansion predicted by Alexander Friedmann. The next important step was made by George Gamow who elaborated the theory of a hot universe which provided a possibility to explain the creation of chemical elements and the formation of galaxies [24]. The basic point of Gamow's theory was an assumption that the early universe was dominated not by the dust matter but by radiation with the equation of state  $P(t) = \varepsilon(t)/3$ . In the framework of Gamow's theory of hot universe, Ralph Alpher and Robert Herman predicted the existence of the background relic radiation [25]. The discovery of this radiation served as the second most important experimental confirmation of the expansion of the universe.

The cosmic microwave background electromagnetic radiation, called also the "relic radiation", was discovered in 1965 by Arno Penzias and Robert Wilson [26]. It fills all space and was created in the epoch of formation of first atoms. The observation of relic radiation confirmed the origin of the universe from the cosmological singularity predicted by Alexander Friedmann as a result of the so-called "Big Bang". Based on the theory of the hot universe, it became possible to describe the various eras in the universe evolution starting from the Electroweak Era followed by the Particle Era, the Era of Nucleosynthesis, Eras of Nuclei, Atoms, and, finally, by the Era of Galaxies. This covers the period of the universe evolution from approximately  $10^{-33}$  s after the cosmological singularity to about 14 billion years which is the present age of the universe. As to the very early stage from 0 to  $10^{-33}$  s, it remained a mystery and could not be explained on the basis of the general theory of relativity.

## 6. Cosmic Inflation and Creation of the Universe from Quantum Vacuum

As was noted above, the basic assumption, used in Friedmann's cosmology and in the theory of hot universe, is that the space is homogeneous and isotropic. This assumption was confirmed by the approximately homogeneous and isotropic large-scale distribution of galaxies and, more importantly, by the properties of relic radiation. It was found that the relic radiation has a blackbody thermal spectrum at  $T = 2.726 \pm 0.001$  K average temperature and the variations of this temperature measured from different directions in the sky do not exceed  $\Delta T/T \sim 10^{-5}$ .

As discussed in Section 5, at the early stages of its evolution the universe was filled with radiation possessing the equation of state  $P = \varepsilon/3$ . In this case the solution of Friedmann equation (6) is given by  $a(t) \sim \sqrt{t}$  and the respective energy density behaves as  $\varepsilon(t) \sim 1/t^2$  when  $t$  goes to zero.

These behaviors, however, create a problem. The point is that if  $t$  is decreased down to the Planck time defined as

$$t_{\text{Pl}} = \sqrt{\frac{\hbar G}{c^5}} = 5.39 \times 10^{-44} \text{ s}, \quad (13)$$

the size of the universe turned out to be unexpectedly large  $a(t_{\text{Pl}}) \sim 10^{-3}$  cm as compared to the Planck length,

$$l_{\text{Pl}} = t_{\text{Pl}} c = \sqrt{\frac{\hbar G}{c^3}} = 1.62 \times 10^{-33} \text{ cm}, \quad (14)$$

traveled by light during the Planck time.



Thus, if the above scale factor were applicable down to  $t = 0$ , at Planck time the universe would consist of the  $10^{89}$  causally disconnected parts. This is in contradiction with the fact that the relic radiation in all places and all directions in the sky has the same temperature—the so-called “horizon problem”.

The horizon problem cannot be solved in the framework of the general theory of relativity. The point is that this is the classical theory and the space-time scales of the order of Planck length and Planck time are outside the region of its applicability. In the absence of quantum theory of gravitation, which is still unavailable in spite of repeated attempts to develop it undertaken during several decades, some semiclassical approaches are believed to lead to at least a partial solution of the problem.

In 1981, Alan Guth [27] found that the symmetry breaking caused by the scalar fields introduced in particle physics can cause the period of exponentially fast expansion of the universe. During this period, the scale factor varies as  $\exp(t)$  rather than  $\sqrt{t}$ . As a result, at the Planck time the universe has the Planck size which solves the horizon problem. It was Guth who introduced the term “inflation” for the exponentially fast expanding universe. The scalar field responsible for the inflation process was called the “inflaton field”. The theory of inflation was further developed by Andrei Linde [28]. Actually, the possibility of an early exponential expansion of the universe was predicted before Guth by Sergey Mamaev and one of the authors of this paper [29] and, independently, by Alexei Starobinsky [30] based on the semiclassical Einstein’s equations (see below). In doing so, the scale factor was expressed either via the proper synchronous time  $t$  [30] or, equivalently, via the conformal time  $\eta$  [29].

Due to a very fast expansion of the universe during the inflationary stage, the energy density of matter becomes very low. The conversion of the energy density of oscillating inflaton field into that of usual matter is called “reheating” after inflation. The theory of reheating was developed by Lev Kofman, Andrei Linde, and Alexei Starobinsky [31] using the effect of resonant particle production in the time-periodic external field revealed earlier by one of the authors and Valentin Frolov [32].

The weak point of the theory of inflation is that the physical nature of inflaton field remains unclear. In this situation, the question arises of whether there are other possibilities for obtaining the period of exponentially fast expansion in the evolution of the early universe. The possible answer to this question was given by the theory of quantum matter fields in curved space-time [33,34]. This theory is applicable under the condition that the gravitational field can be considered as the classical background, i.e., at  $t \gg t_{\text{Pl}}$ , which is already well satisfied at  $t \geq 10^{-40}$  s. One can assume that at  $t \sim 10^{-40}$  s all quantum matter fields are in the vacuum state  $|0\rangle$ , i.e., the number of particles of different kinds is zero. This does not mean, however, that the vacuum energy density and pressure are zero because vacuum is polarized by the external gravitational field.

The vacuum stress-energy tensor of quantum matter fields with different spins in the homogeneous isotropic space was calculated in the 1970s by several groups of authors. It is common knowledge that the quantities  $\langle 0|T_{ik}|0\rangle$  contain the ultraviolet divergences. These divergences can be interpreted in terms of the bare cosmological constant which is connected with the infinitely large energy density of the zero-point oscillations, the bare gravitational constant, and the bare constants in front of the invariant quadratic combinations of the components of Ricci tensor. The finite expressions for the renormalized values of  $\langle 0|T_{ik}|0\rangle_{\text{ren}}$ , obtained after the removal of divergencies, can be found in [33,34]. Based on these results, the so-called “self-consistent” Einstein equations were considered:

$$R_{ik} - \frac{1}{2}Rg_{ik} = \frac{8\pi G}{c^4}\langle 0|T_{ik}|0\rangle_{\text{ren}}. \quad (15)$$

In these equations, the vacuum of quantized matter fields  $|0\rangle$  is polarized by the gravitational field of the homogeneous isotropic space with metric (4) determining the left-hand side of Equation (15). In free Minkowski space-time, the physical energy density and pressure in the vacuum state obtained after discarding of infinities are equal to zero.

However, in an external field (regardless of whether it is electromagnetic or gravitational) after discarding of infinities the vacuum state is polarized, such as a dielectric in an electric field, i.e., it is characterized by some nonzero stress-energy tensor. On the other hand, the gravitational field described by the left-hand side of Equation (15) is created as a source by the vacuum energy density and pressure on the right-hand side. By solving the self-consistent equations (15), one can find the scale factor of the homogeneous isotropic space determined by the quantum vacuum of the matter fields.

It should be stressed that the theoretical approach based on Equation (15) is semi-classical. This means that the gravitational field and the corresponding metrical tensor in Equation (4) are still treated as the classical ones. It is assumed that only the fields of matter (scalar, spinor, vector, etc.) exhibit a quantum behavior. Recall that this approach is applicable at  $t \geq 10^{-40}$  s. At earlier moments down to the Planck time and to the domain of singularity in the solution of the classical general theory of relativity, one should take into account the effects of quantization of space-time, i.e., the effects of quantum gravity.

The solutions of Equation (15) for the massless matter fields were obtained in [29,30]. As an example, for a scalar field in the space of positive curvature the self-consistent scale factor is given by

$$a(t) = \sqrt{\frac{\hbar G}{360\pi c^3}} \cosh\left(t\sqrt{\frac{360\pi c^5}{\hbar G}}\right). \quad (16)$$

Using Equations (13) and (14), one can see that for  $t > t_{\text{Pl}}$  the scale factor (16) takes the form

$$a(t) = \frac{l_{\text{Pl}}}{\sqrt{360\pi}} \exp\left(\frac{\sqrt{360\pi} t}{t_{\text{Pl}}}\right), \quad (17)$$

i.e., it is the exponentially increasing with time scale factor of the de Sitter space (cf. with Equation (10)). This is the scale factor describing the cosmic inflation obtained on the fundamental grounds of quantum field theory without introducing the inflaton field.

Actually, in this approach the inflationary universe is spontaneously created from the quantum vacuum. It should be noted that the expressions for  $\langle 0|T_{ik}|0\rangle_{\text{ren}}$  contain the third and fourth derivatives of the scale factor, which lead to scalar and tensor instabilities. As a result, the de Sitter solution becomes unstable relative to the spatially homogeneous massive scalar modes (scalarons). Using this fact, Alexei Starobinsky [30] constructed the nonsingular cosmological model where the de Sitter universe describing inflation is spontaneously created from the quantum vacuum. Then, due to the generation of scalarons and the decay of scalarons into usual particles, the exponentially fast expansion of the inflationary stage is replaced by the power-type expansion law  $a(t) \sim \sqrt{t}$  of the theory of hot universe. According to current concepts, the inflationary stage of the universe expansion lasts from  $t \sim 10^{-36}$  s to  $\sim 10^{-33}$  s. More precise measurements of the spectrum of relic radiation, planned for the near future, should provide information concerning the gravitational radiation generated during the exponentially fast expansion. This will help to conclusively establish the main properties of the inflationary stage of the universe evolution.

## 7. Accelerated Expansion of the Universe, Dark Energy and the Quantum Vacuum

According to the commonly accepted views formed by the end of the 20th century, the present stage of the universe evolution is described by the Friedmann equations (6) with  $\Lambda = 0$  and a predominance of dust matter having the energy density  $\varepsilon(t) = \rho(t)c^2$ , where  $\rho(t)$  is the mass of matter per unit volume, and zero pressure  $P = 0$ . In doing so, the fraction of visible matter is by a factor of 5.4 smaller than that of invisible matter, which possesses the same properties as the visible one and is called the “dark matter”. We know about the existence of dark matter due to its gravitational action on visible bodies. In this situation, it was expected that due to the gravitational attraction of matter the expansion of the universe should be decelerating.

It was quite unexpected, however, when in 1998 two research teams (Supernova Cosmology Project and High-Z Supernova Search Team) observed clear evidence that, by

contrast, the expansion of the universe is accelerating (see the references and discussion below). This means that the velocities of remote galaxies moving away from the Earth increase with time. Actually, the inflationary stage of the universe evolution was also the period of accelerated expansion which, however, lasted for an infinitesimally short period of time. As to the accelerated expansion observed at present, it has already been going on for several billion years.

The matter which causes acceleration of the universe expansion was called “dark energy”. Unlike the dark matter, which is not seen but gravitates like ordinary matter, dark energy acts against the gravitational attraction, i.e., it is characterized by the negative pressure. The observed acceleration rate of the universe expansion requires that 68% of the universe should consist of dark energy. As to dark matter and usual visible matter, they constitute approximately 27% and 5% of the universe’s energy, respectively. There were many attempts in the literature to understand the physical nature of dark energy by introducing some new hypothetical particles with unusual properties [35]. However, the most popular explanation of the accelerated expansion returns us back to the concept of cosmological constant and quantum vacuum.

As discussed in Section 4, in the absence of background matter,  $\varepsilon = P = 0$ , the cosmological term  $\Lambda g_{ik}$  in the Einstein’s equations (3) determines the de Sitter space-time with a scale factor (10). In the homogeneous isotropic space, the presence of this term just corresponds to the energy density and pressure,

$$\varepsilon_{\Lambda} = \frac{c^4 \Lambda}{8\pi G} > 0, \quad P_{\Lambda} = -\frac{c^4 \Lambda}{8\pi G} = -\varepsilon_{\Lambda}, \quad (18)$$

i.e., results in some effective negative pressure. When the cosmological term is considered along with the stress-energy tensor of the ordinary matter  $T_{ik}$ , it just leads to the required acceleration of the universe expansion. Calculations show that an agreement with the observed rate of acceleration is reached for  $\Lambda \approx 2 \times 10^{-52} \text{ m}^{-2}$  and the corresponding energy density  $\varepsilon_{\Lambda} \approx 10^{-9} \text{ J/m}^3$  [36,37].

The question arises what is the nature of the cosmological constant. Actually, it can be considered as one more fundamental constant closely related to the concept of the quantum vacuum [38]. This statement is based on the geometric structure of the vacuum stress-energy tensor of quantized fields,

$$\langle 0|T_{ik}(x)|0\rangle = I g_{ik}, \quad (19)$$

where  $I$  is the infinitely large constant depending on the number, masses, and spins of quantum fields (see the pedagogical derivation of this equation by the method of dimensional regularization in [39]). The structure of Equation (19) is the same as the cosmological term in Einstein’s equation (3).

The only difficulty is that the constant  $I$  is diverging. By making the cutoff at the Planck momentum  $p_{\text{Pl}} = (\hbar c^3 / G)^{1/2}$ , one obtains the enormously large value of  $I \approx 2 \times 10^{68} \text{ m}^{-2}$  which exceeds the observed cosmological constant  $\Lambda$  by 120 orders of magnitude [36,40]. This discrepancy was called the “vacuum catastrophe” [37].

One can argue, however, that the large value of  $I$  is determined by the contribution of virtual particles and, thus, is of no immediate physical meaning. The energy density  $c^4 I / (8\pi G)$  determined by the constant  $I$  does not gravitate as the bare (non-renormalized) electric charge in quantum electrodynamics is not a source of the measurable electric field. This is also in some analogy to the Casimir effect [41] where only a difference between two infinite energy densities in the presence and in the absence of plates is the source of gravitational interaction [42,43] and gives rise to the measurable force. The quantity  $I$  takes the physical (renormalized) value of  $\Lambda \approx 2 \times 10^{-52} \text{ m}^{-2}$  only after the renormalization procedure. These considerations have already received some substantiation in the framework of quantum field theory in curved space-time, but could obtain the fully rigorous justification only after a construction of the quantum theory of gravitation.

There is a lot of different approaches to the problem of cosmological constants in connection with the origin and physical nature of dark energy. The number of articles devoted to this subject is large and we do not aim to review them here (see [44] for an introduction to the field).

## 8. Discussion

In previous Sections, we reviewed the main scientific concepts regarding the structure of the universe developed in the history of mankind from Ptolemy to Einstein. All of these concepts imply that the universe is stationary and its properties do not vary with time. Although Copernicus and Newton's pictures of the universe are significantly different from Ptolemy's picture, all of these pictures are similar in one basic point: each is time-invariant. This characteristic point was remained untouched by Einstein, who has had in his mind the predetermined aim to obtain the stationary cosmological model in the framework of the general theory of relativity developed by him. Actually, the mathematical formalism of this theory presumed a much more broad spectrum of cosmological models describing the expanding universe. However, the power of tradition was so strong that even a great innovator like Einstein chose to modify his equations by introducing the cosmological term for the sole purpose of keeping the universe stationary.

This gives us an insight into the fundamental importance of the scientific results obtained by Alexander Friedmann 100 years ago. By solving the same equations of the general relativity theory as Einstein, Alexander Friedmann demonstrated that their general cosmological solution describes the expanding universe whereas the stationary solution is only a particular case. This result was so unexpected that Einstein rejected it as a mathematical error, and only after a detailed written explanations passed to him by Friedmann, Einstein had to recognize that, in fact, he himself made an error.

We considered the main facts of Alexander Friedmann's scientific biography which provides an explanation why a mathematician, educated in the traditions of Saint Petersburg mathematical school, was able to make such a fundamental discovery in the field of theoretical physics. After the brief exposition of the properties of expanding universe based on the Friedmann equations, we discussed the main experimental confirmations of the universe expansion, i.e., the discoveries of redshifts in the spectra of remote galaxies and the relic radiation.

Although the theory of the hot universe raised the possibility to describe the main stages of its evolution, the problem of cosmological singularity and the problem of horizon remained unsolved. The solution of these problems suggested by the theory of cosmic inflation links them to the concept of the quantum vacuum. There are reasons to believe that the inflationary stage of the universe expansion is caused by the vacuum quantum effects of fields of matter rather than by some special inflaton field. This point of view may find confirmation in further developments of the quantum theory of gravitation, on the one hand, and by measurements of the spectrum of relic gravitational radiation, on the other hand.

The discovery of the acceleration in the universe expansion resumed an interest to the cosmological term in the Einstein equations originally introduced with a single aim to make the universe stationary. The point is that this term has the same geometrical form as the vacuum stress-energy tensor of quantized matter fields and provides a possible explanation of the observed acceleration of the universe expansion with some definite value of the cosmological constant. There is a great discrepancy between this value and theoretical predictions of quantum field theory which created a discussion in the literature. However, independently of the resolution of this issue, one can argue that the quantum vacuum bears a direct relation to both the earliest and modern stages of the evolution of our universe.

## 9. Conclusions

To conclude, Alexander Friedmann made a prediction of the universe expansion which radically altered our scientific picture of the world as compared to the previous epochs.

Later, this prediction was confirmed experimentally, and Friedmann equations became the basis of modern cosmology. Because of this, the Friedmann name should be put on a par with the names of Ptolemy and Copernicus, who created the previous, stationary, pictures of the world around us.

Friedmann's discovery was based on the classical general relativity theory and could not take into account the quantum effects. Nowadays, we know that the quantum vacuum plays an important role in the problem of the origin of Friedmann's universe from the initial singularity, governs the process of cosmic inflation, and can be considered as a possible explanation of the observed acceleration of the universe expansion. Future development of quantum gravity will make our current knowledge more complete but the concept of expanding universe created by Alexander Friedmann will forever remain the cornerstone of our picture of the world.

**Author Contributions:** Investigation, G.L.K. and V.M.M.; writing, G.L.K. and V.M.M. All authors have read and agreed to the published version of the manuscript.

**Funding:** This work was supported by the Peter the Great Saint Petersburg Polytechnic University in the framework of the Russian State Assignment for Basic Research (Project No. FSEG-2020-0024). The work of V.M.M. was supported by the Kazan Federal University Strategic Academic Leadership Program.

**Data Availability Statement:** Not applicable.

**Conflicts of Interest:** The authors declare no conflict of interest.

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