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Article

Wave Function and Information

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Abstract: Two distinct measures of information, connected respectively to the amplitude and phase of the wave function of a particle, are proposed. There are relations between the time derivatives of these two measures and their gradients on the configuration space, which are equivalent to the wave equation. The information related to the amplitude measures the strength of the *potential* coupling of the particle (which is itself aspatial) with each volume of its configuration space, i.e., its tendency to participate in an interaction localized in a region of ordinary physical space corresponding to that volume. The information connected to the phase is that required to obtain the time evolution of the particle as a persistent entity starting from a random succession of bits. It can be considered as the information provided by conservation principles. The meaning of the so-called "quantum potential" in this context is briefly discussed.

Keywords: information; nature of the wave function; Schrödinger equation; quantum particle; quantum potential; Bohmian trajectories

1. Introduction

In this short communication, we attempt to revisit the concept of the wave function of a quantum particle, proposing an entirely informational definition of it. We do this by introducing two distinct measures of information, functions of the point and the instant, on the configuration space C of the particle, enlarged with the addition of the time coordinate t. These two measures are connected respectively to the amplitude and phase of the wave function. The information related to the amplitude measures the strength of the *potential* coupling of the particle (defined in aspatial terms) with each volume of its configuration space, i.e., its tendency to participate in an interaction localized in a region of ordinary physical space corresponding to that volume. It substantially coincides with the quantum entropy *for a pure state* introduced in [1].

The information connected to the phase is that required to convert the time evolution of the particle from a random succession of bits to a persistent state in order to account for the existence of conserved quantities (spin, charge, momentum, etc.) associated with the free evolution of the particle.

These two measures of information become the components, imaginary and real, of complex information. The well-known wave equation for the non-relativistic scalar particle (Schrödinger equation) is then rewritten as a relation between the time derivative and the spatial gradient of this complex information, the latter defined as a row vector. In this new form, the wave equation appears as a complex (vectorial) generalization of the ordinary Hamilton–Jacobi equation, with the addition of a term of divergence of the spatial gradient. In particular, the divergence of the quantum entropy gradient is weighted by the reciprocal of this entropy; this weight constitutes a sort of "coupling constant" between each region of C and the particle (in itself aspatial), with respect to its possible localization in these regions. The term of the wave equation generated by this coupling is the so-called "quantum potential", which appears in the hydrodynamical representation of this equation [2–4].

This form of the dynamical equation, therefore, makes explicit the coupling, *in a potential sense*, of the quantum particle with each volume of its configuration space. This coupling is actualized when a quantum jump, induced by an interaction in a region of

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ordinary physical space corresponding to a given volume of C, localizes the particle in that volume [5–16].

We want to underline that although, for reasons of simplicity, we limit our considerations in this paperto the case of a single particle, they are immediately generalizable to systems with any number of particles and configuration spaces of any dimensionality.

The remainder of this paper is laid out as follows. In Section 2, we recall the essential notions of particle and interaction, underlining that the spatial environment of the particle (configuration space) is not the same as that of the interactions (ordinary physical space). The actual model is proposed in Section 3. Section 4 is divided into various subsections dedicated to the analysis of individual aspects of the model; the general sense is to show its continuity with the quantum formalism usually accepted and taught. In Section 5, we present the rewriting of the Schrödinger equation for a single non-relativistic scalar particle and discuss the meaning of the various terms that appear in it. Section 6 analyzes the physical basis of the model and discusses its relations with the typical aspects of the hydrodynamical representation of quantum mechanics, with particular reference to the quantum potential and the "Bohmian trajectories". In Section 7, we report some concluding remarks. To our best knowledge, the contents of Section 3, Section 5, and Section 6 should be considered as original.

2. Particles and Interactions

In this communication, we do not intend to delve into the problem of the ontology of quantum particles. For our purposes, we have adopted a minimalist definition, according to which a quantum particle is the maximal set of physical quantities encoded in its wave function that would be conserved if its motion wasfree. This means that these quantities are compatible and that in the absence of external forces, they would be constants of motion with values definite or not depending on the initial conditions actually chosen. Energy, momentum, spin, isospin, etc. are examples of such quantities, which, in systems with many interacting particles, combine according to specific rules to give the total conserved quantities. The spatial or spatiotemporal position cannot be included among these quantities because, even in the absence of external forces, it is not a constant of motion. As a consequence of this definition, the particle does not have a definite spatial position.

The aspatial packet of physical quantities that constitutes the particle is manifested as a whole in a single interaction event, in which the wave function of the particle is created or annihilated (and possibly replaced by a new wave function). This event, which constitutes a so-called "quantum jump" [5–16], occurs in a single volume of *ordinary* space at a definite instant. The simultaneous manifestation in different spatially separated volumes would be in fact incompatible with the local nature of the conservation principles. Let us consider, for example, the case of particle A with momentum $\boldsymbol{P}$, which manifests itself into two spatially separated and distinct regions V_1, V_2 by means of two releases of momentum $\boldsymbol{p}_1$, $\boldsymbol{p}_2$ such that $\boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2$. This would mean that at the moment of the interaction, in volume V_i, where $i = 1$ or 2, impulse $\boldsymbol{P}$ would enter and impulse $\boldsymbol{p}_i$ would exit. Within this volume, there would, therefore, be the disappearance of impulse $\boldsymbol{P} - \boldsymbol{p}_i$.

In other words, even though, according to our definition, the particle does not have a definite spatial position, it can nevertheless manifest itself integrally (as a whole) in a certain spatial region by means of a quantum jump induced by the interaction with other particles or systems. This means that ordinary space coordinates label possible interactions between particles, not some "position" actually occupied by the particle between two successive quantum jumps. Space is, in other words, the environment of interactions, *not* that of particles.

A special case is that of negative interactions [17–19], i.e., instances where an expected manifestation of the particle does not actually occur. Non-manifestation preserves the aspatial nature of the particle and, with it, the phase relation of its wave function. An example of negative interaction is offered by the particle crossing a double slit, made on an absorbing screen. The negative interaction with the screen annihilates the wave function

of the incident particle and creates a new one, different from zero only in the two regions occupied by the slits. The new function preserves the phase relation with the old one. The phase relation between the two components transmitted by the two slits produces the well-known self-interference effects downstream of the screen [20].

3. The Model

Let us consider a positive or zero real function $|\psi_0|$, defined on the configuration space C of a quantum system and enlarged with the addition of the time coordinate t. We indicate with α the square root of the average value of the square of this function on volume V of the configuration space:

$$\alpha(V) \quad = \quad \sqrt{\left\langle |\psi_0|^2 \right\rangle_V} \tag{1}$$

We introduce two complex-valued functions $h_f(V)$, $h_b(V)$, defined by two distinct conditions. The first condition is that their sum is real and corresponds to the information acquired by ascertaining the localization of the quantum system in V:

$$h_f(V) \; + \; h_b(V) \quad = \quad -\log\Big[(\alpha(V))^2 \; V\Big] \quad = \quad -\log|p(V)| \tag{2}$$

where $p(V)$ denotes the probability of the localization event $C \to V$. Equation (2) is satisfied if $|\psi_0|^2$ is identified as the probability density on the configuration space C and:

$$\int_C |\psi_0|^2 dV \quad = \quad 1 \tag{3}$$

Indeed, in this case:

$$-\log[p(V)] \quad = \quad \log\left[\frac{\int_C |\psi_0|^2 dV}{(\alpha(V))^2 V}\right] \quad = \quad -\log\Big[(\alpha(V))^2 V\Big] \tag{4}$$

is the information expected for the localization event $C \to V$.

If there were no conservation principles, the time evolution of the system starting from an initial condition would be an entirely random, transient process with decreasing probability. The conditional information associated with the confirmation of the existence of the system at a given time t would, therefore, grow with t. Denoting $p_V(t)$ as the integral of the localization probability density over the entire configuration space C at time t, under the condition that at time 0, the system is localized within V, the information associated with the existence of the system at time t is defined as $-\log(p_V(t))$. Subsequently, the second condition is that the differential $d(h_f(V) - h_b(V))/2i$, where i is the imaginary unit, represents, in this case, the increase undergone in the time interval dt by the information associated with the *existence* of the system, $-\log(p_V(t))$.

The effect of the conservation principles is to add, in the same interval, an exactly opposite decrease in information $-\lambda$, so that the total change in information in the interval is zero. The total information is then conserved in t; this ensures the conservation of the probability of existence, thus guaranteeing the persistence of the system at all times following the initial condition. We must, therefore, have:

$$d\left(\frac{h_f(V) \; - \; h_b(V)}{2i} \; - \; \lambda\right) \; = \; 0 \quad \Rightarrow \quad \frac{h_f(V) \; - \; h_b(V)}{2i} \quad = \quad \lambda(V) \quad + \quad constant \tag{5}$$

The quantity λ is, therefore, defined up to an additive constant, which we ignore in the following. From conditions Equations (2) and (5), it follows that, in the limit, $V \to 0$, and therefore, $\alpha(V) \to |\psi_0|$:

$$h_f = -\log\left(|\psi_0|\sqrt{V}\right) + i\lambda; \qquad h_b = -\log\left(|\psi_0|\sqrt{V}\right) - i\lambda \tag{6}$$

where λ is now a real valued function of the point x in which V is contracted and the instant t. Therefore:

$$h_f + h_b = -\log\left(|\psi_0|^2 V\right) \tag{7}$$

$$\frac{h_f - h_b}{2i} = \lambda = -\log(p_V(t)) \tag{8}$$

It is then possible to define two functions of the point and the instant:

$$\psi_f = \frac{e^{-h_f}}{\sqrt{V}} = |\psi_0| e^{-i\lambda}; \qquad \psi_b = \frac{e^{-h_b}}{\sqrt{V}} = |\psi_0| e^{+i\lambda} \tag{9}$$

which are identifiable with the "forward" and "backward" wave functions of the quantum system, the respective solutions of the retarded and advanced Schrödinger equation (respectively used in prediction and retrodiction problems). We note that, as a consequence of Equation (6), functions (9) do not depend on V; they only depend on the point and instant. Equation (9) provide an informational interpretation of the wave function. Since Equation (9) are complex conjugates, as are the wave equations of which they are solutions, in the following, we limit ourselves to considering only forward wave functions.

4. Remarks

4.1. Born Rule

We denote by ψ one of the two functions (9), defined on a basis of functions ψ_i ($i = 1, 2, \ldots$). Let us consider the transition $\psi \to a_1\psi_1$, with:

$$\psi = \sum_i a_i \psi_i; \qquad \langle \psi_i | \psi_j \rangle = \delta_{ij} \tag{10}$$

We assume that ψ is normalized to 1 on C. Before the transition (subscript "0"), we have:

$$\left(h_{f,0} + h_{b,0}\right) = -\log\left[(\alpha(V))^2 V\right]; \qquad \alpha(V) = \sqrt{\left\langle |\psi|^2 \right\rangle_V} \tag{11}$$

After the transition (subscript "1"), we have:

$$\left(h_{f,1} + h_{b,1}\right) = -\log\left[(\beta(V))^2 V\right]; \qquad \beta(V) = \sqrt{\left\langle |a_1\psi_1|^2 \right\rangle_V} \tag{12}$$

Therefore, the information acquired with the transition is:

$$\left(h_{f,1} + h_{b,1}\right) - \left(h_{f,0} + h_{b,0}\right) = \log\left[\frac{(\alpha(V))^2}{(\beta(V))^2}\right] \tag{13}$$

In the limit, $V \to C$ one has $\alpha^2 \to 1/V$, $\beta^2 \to |a_1|^2/V$, and then:

$$\left(h_{f,1} + h_{b,1}\right) - \left(h_{f,0} + h_{b,0}\right) \to -\log\left[|a_1|^2\right] \tag{14}$$

Equation (14) identifies $|a_1|^2$ as the conditional probability of the transition $\psi \to a_1\psi_1$. This identification constitutes the well-known Born rule [21].

4.2. Existential Information

In the absence of conservation principles, it should be:

$$d\left(\frac{h_f(V) - h_b(V)}{2i}\right) = -d\ \log\ p_V(t) \tag{15}$$

where $p_V(t)$ is the probability of existence of the system somewhere at time t, conditioned by the presence of the system within the volume V at time 0. From (5), (8) it follows that (15) equals $d\lambda$, and $p_V(t) = p_V(0)\exp[-\lambda(V)] = \exp[-\lambda(V)]$. In fact, the probability to find the system somewhere in C at time 0 is by construction 1, because at that instant, the system is in V; therefore, $p_V(0) = 1$. In the limit $V \to 0$, this relation becomes $p_{V \to 0}(t) = \exp[-\lambda(x, t)]$, where x is the point at which V is contracted. In the same limit:

$$\frac{d}{dt}\left(\frac{h_f(V) - h_b(V)}{2i}\right) = -\frac{d}{dt}\ \log\ p_{V\to 0}(t) = -\frac{\dot{p}_{V\to 0}}{p_{V\to 0}} = \dot{\lambda}\ (x,\ t) \tag{16}$$

If the system is stationary and admits a wave function:

$$\psi = |\ \psi(x)\ |\ e^{-i\omega t} \tag{17}$$

where x is the generic point on the configuration space and ω is a real number independent from that point and time; then, $\omega = \lambda/t$ from (9). Furthermore, from Planck's law, we have $\omega = E/\hbar$, where E is the total energy of the system. Therefore, $\lambda = Et/\hbar$ and $p_{V \to 0}(t) = \exp(-Et/\hbar)$. This last expression is the same one that would be obtained by applying the Wick rotation $t \to -it$ to the phase factor in (17) [22]. In addition, the expression (16) equates $\omega = E/\hbar$. Therefore, for a stationary state (17), the information added in the interval $(0, t)$ is nothing else than the de Broglie phase $\lambda = \omega t$, and the Planck constant h sizes the bit of this information, as evidenced by the relation $p_{V \to 0}(t) = \exp(-Et/\hbar)$, which identifies this bit as $\hbar\ln 2$. The quantization of the information of existence in time then takes the well-known form of the uncertainty principle $(Et)_{min} \approx h$.

4.3. Compatibility with Quantum Mechanical Rules

As we have seen, it is not generally possible to ascribe a position to the particle. It is the discontinuous transition of the particle from one wave function to another (quantum jump) that is localized. It is localized exactly in time, partially (depending on the diffusion of the outgoing wave packet) in space. The problem can be made quantitative, as follows. Let us first observe that from the relations already seen:

$$\psi = \sum_i a_i\ \psi_i \tag{18}$$

$$Probability(\ \psi\ \to\ \psi_i) = |\ a_i\ |^2 = \left|\ \int_C \psi\psi_i^*\ dV\ \right|^2 \tag{19}$$

with the further hypothesis of the existence of a linear Hermitian operator $\hat{O}$, such that (o_i is a real number):

$$\hat{O}\ \psi_i = o_i\ \psi_i\ ; \qquad \hat{O} = \hat{O}^+ \tag{20}$$

the following relation is obtained:

$$\langle\ \hat{O}\ \rangle_\psi \equiv \sum_i |\ a_i\ |^2\ o_i = \langle\ \psi\ |\ \ \hat{O}\ \ |\ \psi\rangle \tag{21}$$

In this sense, it is possible to associate the operator $\hat{O}$ with a physical quantity having a spectrum $\{o_i, p(o_i) \equiv Probability(\psi \to \psi_i) = |\ a_i\ |^2\}$. Let us consider the particular case where

this quantity is energy, that is, $\hat{O} = \hat{H}$, the Hamiltonian operator. Since the energy is the time rate of the information of existence, a stationary state:

$$\psi = |\psi_0| e^{-i\omega t} \tag{22}$$

must be an eigenfunction of $\hat{H}$. This is only possible if $\hat{H} = i\hbar\partial_t$, where ∂_t denotes the partial derivative with respect to t. In such a case, the eigenvalue is $\hbar\omega$, in accordance with Planck's law. For a material point, the relativistic transformation of the four-momentum $(E/c, 0) \to (E'/c, \boldsymbol{p})$ then implies the transformation between operators: $(i\hbar\partial_t/c, 0) \to (i\hbar\partial_{t'}/c, -i\hbar\nabla_x)$, from which the impulse operator $p = -i\hbar\nabla_x$ is obtained.

It follows that the energy E of a non-relativistic material point of mass m and impulse $\boldsymbol{p}$, subject to the potential V, takes on the operatorial form:

$$E = \frac{p^2}{2m} + V \to \hat{H} = i\hbar\partial_t = \frac{(-i\hbar\nabla_x)^2}{2m} + V \tag{23}$$

From (23), the equation of motion:

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla_x^2\psi + V\psi \tag{24}$$

follows, which we consider in the Section 5.

5. Schrödinger Equation

In this section, we use the notation of Ref. [1], with the additional position $\hbar = 1$. We start from the equation, which, in that article, defines quantum entropy (in the language of this article, the spatial localization information):

$$S_Q = -\frac{1}{2}\log\rho \tag{25}$$

where $\rho = |\psi|^2$ is the probability density on the configuration space. It is important to note that this entropy measure is relative to a pure state, and is, therefore, essentially different from the much better known quantum entropy measure introduced by Von Neumann [23] and the subsequent measures inspired by it (Rényi, Tsallis, etc.), which are instead null for a pure state [24,25].

We also indicate by S the *existential* information λ, defined in units $\hbar = 1$. It is then possible to introduce the complex energy as the opposite of the time derivative of the complex information $S + iS_Q$:

$$\Xi = \partial_t S + i\partial_t S_Q \tag{26}$$

We can further define the momentum as the row vector:

$$\Sigma = \left(\nabla S;\quad i\nabla S_Q\right) \tag{27}$$

The dynamical equation is the dispersion relation that connects quantities (26) and (27). Let us consider the simplest case: that of a scalar particle of mass m, subject to the potential V, in a non-relativistic regime. The corresponding quantum dynamical equation is the Schrödinger Equation (24). We can write it in the form:

$$-\Xi = \frac{1}{2m}\Sigma\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\Sigma^+ + V - \frac{1}{2m}D\,\Sigma^+ \tag{28}$$

where:

$$D = (\,i\nabla\cdot\,;\;\,i\,\nabla\cdot) \tag{29}$$

is a two-component divergence vector to which we will return later.

By substituting (26), (27), and (29) into (28) and expanding it, we obtain a complex scalar expression. By separating the real part of this expression from the imaginary one, we obtain two real equations:

$$-\partial_t S = \frac{(\nabla S)^2}{2m} - \frac{(\nabla S_Q)^2}{2m} + \frac{1}{2m}\frac{\Delta S_Q}{S_Q} + V = \frac{(\nabla S)^2}{2m} - \frac{1}{2m}\frac{\Delta R}{R} \quad (30)$$

where: $R = \exp(-S_Q)$.

$$\partial_t S_Q + \frac{\nabla S \cdot \nabla S_Q}{m} - \frac{\Delta S}{2m} = 0 \quad (31)$$

which are the fundamental equations of the hydrodynamical representation [2,26,27]. Equation (30) is the Hamilton–Jacobi equation with the quantum corrector (so-called "quantum potential"); Equation (31) is a rewriting of the continuity equation. Considered together, (30) and (31) are notoriously equivalent to the Schrödinger Equation (24), where $\psi = \exp(-S_Q + iS)$ [2,26,27].

We now come to the physical meaning of the terms that appear on the right side of (28). The first term, quadratic in Σ, is manifestly a complex generalization of the expression for kinetic energy. Its real part is the kinetic energy commonly understood, which, added to the potential V, provides the total *classical* energy distributed on the configuration space as a consequence of the "propagation of the particle". The imaginary part provides the second term of (31), which expresses the contribution of the coupling between the gradients of the two information S, S_Q to the temporal variation of S_Q.

The third term of (28) is a scalar product that measures the local inflow (or outflow) of momentum (27) into (from) the configuration space. The first addend of the scalar product is related to the S-momentum, the second addend is related to the S_Q-momentum. This latter is multiplied by the reciprocal of S_Q, which acts as a coupling constant, dependent on the point and the instant, to this contribution. It is, in fact, intuitive that the coupling will be greater where the particle localization density ρ is greater (and, therefore, S_Q is lower); it will instead be smaller where ρ is smaller (and, therefore, S_Q is larger).

The two addends of the scalar product give rise, respectively, to the third addend of (31), which measures the contribution of S to the temporal variation of S_Q, and to the quantum potential in (29). The latter constitutes the kinetic energy corrector derived from the inflow (outflow) of the momentum related to S_Q. This contribution represents an unknown effect in classical physics. In accordance with the aspatial nature of the particle, it is affected by variations in boundary conditions throughout the space, for example, the opening or closing of a slit.

In [28], the wave function is modeled as the exchange of a pair of charges $Q^{\pm}$ between individual points in space and an extra-spatial actuator. It is possible to connect the two narratives through the following transformations, relating to non-normalized wave functions:

$$\mathrm{Re}\left(\psi_f\right) = \mathrm{Re}(\psi_b) = \frac{Q^+ + Q^-}{2}; \qquad \mathrm{Im}\left(\psi_f\right) = -\mathrm{Im}(\psi_b) = \frac{Q^+ - Q^-}{2} \quad (32)$$

or:

$$|\,\psi_0\,| = \sqrt{\left(\frac{Q^+ + Q^-}{2}\right)^2 + \left(\frac{Q^+ - Q^-}{2}\right)^2} \quad (33)$$

$$tg(\lambda) = \frac{Q^+ - Q^-}{Q^+ + Q^-} \quad (34)$$

The derivation of the usual hydrodynamical representation (30), (31) of (24), starting from a notion of particle as an entirely delocalized entity, demonstrates that it is not necessary to consider the so-called drift velocity [2,29]:

$$\vec{v} = \frac{\nabla_x S}{m} = \frac{\hbar \nabla_x \lambda}{m} \tag{35}$$

as the speed of a corpuscle actually present on spacetime, in some way "guided" by ψ. Even putting aside the inevitable non-locality that this driving process would exhibit both in the case of multi-particle systems in the presence of entanglement and of single-particle systems (due to the non-local nature of the quantum potential), Equation (35) cannot be associated with the trajectory of a charge.

It is easy to realize this by observing that, if this were the case, for the electron of the ground 1s state of atomic neutral hydrogen (nascent hydrogen), one should have $\vec{v} = 0$; that is, it should be at rest with respect to the nucleus. The single hydrogen atom should, therefore, possess an electric dipole moment $\approx e \cdot \hbar^2/me^2$, easily observable but never actually observed. This moment would be zero only if averaged over the directions of space, as it would be if the averaging operation were performed on many atoms. The zero value of the electric dipole moment of each single hydrogen atom instead confirms the standard quantum mechanical predictions, which do not involve corpuscles.

One could think of maintaining the concept of trajectory of a charge by hypothesizing the existence of a stochastic motion superposed on (35), able to rebuild the 1s state as a temporal average [29]. In this case, the time average of the electric dipole moment of the single hydrogen atom would be zero. The problem with such a hypothesis, however, is that the velocity of the atomic electron would then be $\approx \alpha c \approx 10^8$ cm s^{-1}. The time it takes for the electron to cross the atomic radius ($\approx 10^{-8}$ cm) would then be $\approx 10^{-16}$ s. Current technology based on laser pulses with a width of the order of an attosecond (10^{-18} s) permits the monitoring of the electronic motion with a time resolution a hundred times bigger [30]. Nonetheless, to our best knowledge, no reports of significant deviation in the electronic motion from the behavior expected on the basis of quantum mechanical predictions can be found in the relevant literature. Of course, it is possible to argue that such deviations occur below the currently accessible time scale. Note, however, that the attosecond scale is only two orders of magnitude higher than the Compton scale, on which the valence electron ceases to exist and the electron–positron field coupled to the electromagnetic field appears [31]. The residual space for a stochastic solution, therefore, appears to be very limited.

6. Discussion

A particularly relevant point of the informational reading of the wave function seems to be constituted by non-classical relationship between particle and space. The quantum particle is not an object, unlike the classical material point. It is, instead, a set of quantities, which are conserved in the absence of external forces; among these quantities there is impulse. The spatial position is not part of this set because it is not a conserved quantity; in this sense, the particle is an aspatial entity. However, unlike what happens in classical mechanics, the position cannot be assigned independently on the impulse, as it is defined as the Fourier-conjugate quantity of the impulse. The absence of objects on the scale of the quantum of action h implies, in fact, the absence of rigid rods to measure distances and positions. Therefore, spatial coordinates can only be defined as conjugate quantities of conserved quantities (impulses), exchanged during micro-interactions. The Fourier transformations that define this conjugation relation are sized by h.

The impulse spectrum of a particle, therefore, encodes ordinary space through a non-local relation constituted by a Fourier transformation. Ordinary space, thus codified, becomes an aspect of the particle. Everything goes as if the particle maintained an *internal copy* of ordinary space; this copy is obviously labeled by the index i of the particle. This is the potential coupling between particle and space mentioned in the Introduction. Consequently, a system of N particles does not "live" in ordinary space, but in the space formed by the product of the N copies $i = 1, 2, \ldots, N$ of ordinary space, each constitute the internal space of the i-th particle. This product space is the configuration space of the system.

More particles can interact with each other, directly or through a field, in homologous regions of their internal spaces, by virtue of the *local nature of the interactions* (i.e., the diagonality of the interaction operators in the basis of the spatial coordinates). When an interaction changes the momentum spectrum of the interacting particles, inducing their spatial localization in correspondence with regions of their internal spaces homologous to a same region V of ordinary space, then each of those particles is "localized in V" by that interaction; this is called a *quantum jump*. The jump occurs both in the internal spaces of the particles (and, therefore, in the configuration space of the system) and in ordinary space, which is, in a certain sense, their trace. Ordinary space coordinates, therefore, label the position of the *jumps*, *not* that of the particles. This is the reason for the fact, already highlighted at the dawn of quantum theory, that only a wave function in the configuration space can account for the conservation principles [32].

The relationship between the internal space of each particle, an encoded copy of ordinary space, and ordinary space itself is, therefore, a potency–act relationship. The coordinates of the internal space of a particle label the *possible* positions of the jumps in which *that* particle can be involved. The quantum jump that localizes the particle in V is, in this sense, a transformation of the potency in act, i.e., an *actualization* of that particle in that specific interaction. In the description of the relationship between quantum particle and ordinary space, the actualization of a potentiality, and the *information* associated with it, therefore, becomes central. Hence, the possibility of summarizing the dynamics of the particle in terms of the dynamics of a wave function is realized. The latter, in fact, represents two measures of information, one of which is localization information. The equation of motion, in turn, exactly represents the interplay of this information with the other information, connected to the causal propagation.

It is also necessary to point out that in a quantum jump, the reshaping of the localization density involves the copy-space inside the particle. It does not involve any propagation of information or signals between *two events* in the external, ordinary space. Rather, the jump constitutes *a single event* in that space.

In our opinion, it is in this context that both the meaning of the streamlines of the vector field (35), the so-called "Bohemian trajectories" [26,27], and that of the quantum potential should be reconsidered. These two concepts concern S and S_Q respectively, i.e., causality and localization. The relation (35) was initially introduced by de Broglie in his famous doctoral thesis [33]. He noticed that, by imposing this relationship, the Maupertuis action of the particle coincided with the Fermat integral, giving the total propagation time of the particle. Consequently, the principle of least action coincided with Fermat's principle: the basic law of corpuscle mechanics coincided with the basic law of geometric optics. De Broglie, therefore, postulated that (35) was the group velocity of the wave packet associated with the particle. Let us indicate with $\boldsymbol{k} = \boldsymbol{k}(\boldsymbol{x},t)$, $\boldsymbol{x}$ being the current point of the particle configuration space, the wavenumber vector and with $\boldsymbol{\omega} = \boldsymbol{\omega}(\boldsymbol{x},t)$ the pulsation; if $\boldsymbol{v}$ is expressed by (35), and m is the particle mass, from the relations $\boldsymbol{p} = m\boldsymbol{v}$, $\boldsymbol{p} = \hbar\boldsymbol{k}$, one has $\boldsymbol{v} = \hbar\boldsymbol{k}/m$. Identifying, as indicated by de Broglie, the group velocity $grad_k(\omega)$ with $\boldsymbol{v}$, an elementary integration gives:

$$\hbar\omega = \frac{p^2}{2m} \tag{36}$$

where $p = |\boldsymbol{p}|$ and the constant of integration is eliminated by assuming $\omega = 0$ for $p = 0$. The relation (36) must be compared with (30); we then see that its left-hand side is the classical kinetic energy of the particle, with a correction given by the opposite of the quantum potential. So, the Bohmian trajectories are simply those along which the dispersion Relation (36) holds. As expected, based on their derivation from S, these trajectories are connected to *causality*. The inclusion of quantum effects on the left-hand side translates the interplay between S and S_Q.

This interplay can also be seen from another perspective, that of quantum potential. As can be seen from (30), in units $\hbar = 1$, this potential can be written as:

$$U_Q = -\frac{1}{2m}\frac{\Delta R}{R} = -\frac{1}{2m}\frac{\nabla \cdot (\nabla R)}{R} \tag{37}$$

Let us then consider, in the configuration space of the particle, an elementary volume ΔV enclosed in a surface Ω. We have:

$$\int_{\Delta V} dV\, U_Q R = -\frac{1}{2m}\int_{\Delta V} dV\, \nabla \cdot (\nabla R) \tag{38}$$

Under the normal hypotheses of applicability of the divergence theorem, we have:

$$\int_{\Delta V} dV\, U_Q R = -\frac{1}{2m}\int_{\Omega} d\vec{\Omega} \cdot (\nabla R) \equiv -\frac{\varphi_\Omega(\nabla R)}{2m} \tag{39}$$

where $d\vec{\Omega}$ is the surface element on Ω. In the limit $\Delta V \to 0$, this expression becomes:

$$U_Q = -\frac{1}{2mR}\frac{d\varphi_\Omega(\nabla R)}{dV} \tag{40}$$

From (40), we see that the quantum potential is different from zero where the "osmotic" velocity [2,29] $\nabla R/m = -(\nabla S_Q)R/m$ ceases to be solenoidal, presenting wells or springs. From (30), we see that this occurs when the difference between the energy $-\partial_t S$ and the kinetic energy constructed starting from (35) does not vanish, in other words, when the wave dispersion relation (36) diverges from the classical "corpuscular" dispersion relation. In the classical limit of the hydrodynamical representation, the velocity $\nabla R/m = -(\nabla S_Q)R/m$, connected in this case to an entirely classical positional probability distribution, is always solenoidal. As has already been highlighted in Section 5, it is the second term of the divergence operator (29) that makes the behavior of the osmotic velocity non-classical. The sink or source of osmotic velocity is, for the reasons already seen, the impulse spectrum of the particle.

Equation (40) can be applied to the case of a self-interference pattern in an experiment with photons or electrons. The dark fringes correspond to regions where the information S_Q is high, and it decreases moving away from these fringes. Therefore, the gradient of S_Q is negative in the points corresponding to the dark fringes: the osmotic velocity field enters the surfaces Ω surrounding these points. The flux is, therefore, negative, and the quantum potential is maximal. In the bright fringes, the information S_Q is low, and it increases moving away from these fringes. Therefore, the gradient of S_Q is positive in the points corresponding to the bright fringes: the osmotic velocity field comes out of the surfaces Ω surrounding these points. The flow is, therefore, positive, and the quantum potential is minimal. Finally, moving from a dark fringe towards a bright fringe, we will find points where the is flow canceled out by changing signs; at these points, the quantum potential is zero.

The alternation of the maxima and minima of the quantum potential at the fringes leads to the separation of the solution trajectories of (30) and (31) into distinct groups corresponding to the various bright fringes [34]. The boundaries of separation between the various groups are the dark fringes. Hence, the temptation arises to interpret these trajectories, i.e., the streamlines of (35), as actual trajectories of corpuscles; a problematic idea in any case [35]. Our hope is to have shown, with the present analysis, that the relationship between fringes and quantum potential does not imply any driving effect on the presumed corpuscles, being entirely informational in nature.

It is worth underlining that the topics covered in this section have, over recent years, been the subject of considerable theoretical and experimental research. In particular,

classical hydrodynamical analogs of (24) have been studied experimentally, measuring the equivalents of the Bohmian trajectories and the quantum potential [36,37]. Similar objectives have been achieved in the field of both classical and quantum optics, through experimental studies on both single photon systems [38] and systems with multiple entangled photons [39]. In a more strictly theoretical context, methods based on these concepts have been successfully used in computational applications, for example, to tunnel in solids [40].

7. Conclusions

In this paper, we propose a purely informational representation of both the wave function of a particle and the wave equation of which it is the solution. According to this proposal, two distinct measures of information appear, which can be considered respectively as the real and imaginary part of *complex* information, and as such, can be connected to the amplitude and phase of the wave function. The imaginary part represents the information gained in a quantum jump, which localizes the particle in a specific volume of its configuration space: the one corresponding to the region of ordinary space in which the interaction occurs. In this sense, it can be defined as the information linked to the spatial localization; it substantially coincides with the "quantum entropy" of Ref. [1].

The real part represents the information provided by the conservation principles to guarantee the persistence of the particle during its propagation between two subsequent quantum jumps. In other words, it is the existential information necessary to guarantee the causal nature of that propagation interval, or the information contained in causality. This information is linked to the quantum phase and the action variable. From a foundational point of view, it appears relevant that causality is connected to the existence of the phase, and that the introduction of a corpuscle carrying the interaction charges is not required. This introduction, moreover, would seem to be dissonant with some aspects of known physics, as argued in Section 5. On the other hand, the connection between causality and phase wave allows us to account for, in a natural way, the self-interference effects highlighted by experiments, such as that of the double slit.

The quantum phase is, therefore, a consequence of the aspatial nature of the particle, which "lives" in its own configuration space different from the ordinary physical space of our perception. The coordinates of the latter label not the possible positions of the particle but the positions of the interactions in which it can participate. Only when the particle is projected onto a portion of its configuration space by an interaction that occurs in a region of ordinary space corresponding to that portion, it becomes "present" in that region. And this presence is completely ephemeral, because it is limited to the sole moment of the interaction.

The two terms, real and imaginary, of the complex information are dynamically coupled by the evolution equations. The "quantum potential", the corrector of the kinetic energy in the hydrodynamical representation of the wave equation, expresses part of the action of the imaginary term on the real one. It is not associated with some force acting on some "corpuscle", and simply describes the dynamical relationship between the causality and the coupling of the particle with space. This relationship is clearly exhibited in experiments in which the particle crosses an absorbing screen, on which one or more slits have been made. The cancellation of the wave function on the screen, except in the areas corresponding to the slits, is the informational translation of the fact that no exchange of conserved quantities constituting the particle has occurred with the screen; consequently, the causal propagation of these quantities (i.e., the propagation of the phase wave) continues only at the slits. This implies that the localization information is reshaped on the configuration space, so as to provide a non-zero wave amplitude only at the slits. While the non-transfer of impulse, spin etc. to the screen represents an objective physical fact, the reshaping of the amplitude—i.e., the infamous "collapse of the wave function"—necessary to limit the causal propagation of quantities only to the spatial regions occupied by the slits, belongs to the informational sphere, as the generation of a new initial condition on the propagation of the particle [41]. The same mechanism works

in the case of an impact on the screen; since the exchange of conserved quantities occurs at the point of impact, those same quantities can no longer be exchanged elsewhere for the reasons seen in Section 2. It follows that the position information, as a function of the point, is reshaped in such a way as to guarantee the cancellation of the amplitude at all points of the configuration space other than the point of impact. It should be noted that the overall phenomenon thus defined involves a single experimental run and does not represent a purely statistical effect on the set of initial preparations of the micro-system. The informational reading of the wave function proposed here, therefore, seems to provide a balanced response to the difficulty raised by Einstein in the general discussion at the V Solvay Conference in 1927 [32].

In conclusion, we note that the real part of the complex information refers to a causal chain in the enlarged configuration space, which connects two actual interactions, while the imaginary part refers to the possible localizations in ordinary space. Actual interactions and localizations are synonymous with quantum jumps associated with micro-interactions, which represent objective physical facts in terms exemplified by the multiple slit screen experience. The information measures here introduced are, therefore, relative to physical facts, as it seems to us they should be in a physical theory, and do not constitute a primary layer of reality. The proposed approach, therefore, does not appear functional in circumventing the ontological problem through "it from bit" strategies [42].

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