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EC3-Compatible Methods for Analysis and Design of Steel Framed Structures

Sofia Antonodimitraki *, Pavlos Thanopoulos and Ioannis Vayas

Institute of Steel Structures, School of Civil Engineering, Zografou Campus, National Technical University of Athens, 9 Iroon Polytechneiou Str., 15780 Athens, Greece; pavlosth@central.ntua.gr (P.T.); vastahl@central.ntua.gr (I.V.)

* Correspondence: santonodimitraki@central.ntua.gr

Abstract: The behaviour of steel structures is affected by two nonlinearities—the geometric and material nonlinearity—and by the unavoidable presence of imperfections. To evaluate the ultimate capacity of a structure, these effects should be taken into consideration during the design process, either explicitly in the analysis or implicitly through the verification checks. In this context, Eurocode 3 provides several design approaches of different complexity and accuracy. The advantages and disadvantages of these approaches are discussed. Five different methods in conformity with the Eurocode provisions are applied for the design of four moment resisting steel frames of varying slenderness. The influence of nonlinearities and imperfections in respect to the slenderness of the structure is illustrated. The examined methods are compared in terms of the predicted ultimate capacity and their efficiency is assessed against the most accurate between them, i.e., an advanced geometrically and materially nonlinear analysis. It is shown that considerable differences arise between the methods. Nevertheless, except for the commonly used 2nd order analysis followed by cross-section verifications, the remaining methods are mostly on the safe side.

Keywords: design of steel frames; nonlinear analysis; imperfections; general method; buckling length; Eurocode 3

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1. Introduction

One of the major challenges when designing a structure is to strike a balance between safety and economy. Concerning steel structures, the European Standard dedicated to their design, Eurocode 3 [1], as well as the American Standard AISC 360 [2], do not impose a single design method but provide several alternatives instead. Hence, the selection of the most suitable design method for the problem at hand is an important issue, demanding guidance. The various design methods differ in the analysis theory employed and the design checks subsequently required. Each method results in a different level of safety and economy, since it approaches the behaviour of the structure with varying degree of accuracy.

1.1. Nonlinear Behaviour of Steel Structures

The behaviour (resistance, stiffness) of steel structures is affected mainly by two nonlinearities, i.e., the material and geometric nonlinearity. The collapse may be due to an exceedance of the material strength in one or more cross-sections (material nonlinearity) or due to instability of individual members or of the whole structure (geometric nonlinearity). The collapse of rigid structures is typically associated with the material nonlinearity, while in slender structures the geometric nonlinearity becomes critical. Structures of intermediate stiffness usually collapse due to an interaction of the two types of nonlinearity (inelastic buckling). Both nonlinearities have to be considered for the design, either directly during the analysis or implicitly through resistance and stability verification checks. Material nonlinearity should always be taken into account, while geometric nonlinearity can, under specific conditions, be neglected.

In framed structures, geometrically nonlinear or 2nd order effects involve P-Δ effects associated with the global instability of the frame and P-δ effects associated with the local instability of its members. Both of them result in additional moments or forces from the global horizontal displacements of the frame (P-Δ) and the deflection of its members from their initial straight geometry (P-δ) [3]. Eurocode 3 [1] specifies that P-Δ effects can be neglected if they increase 1st order internal forces and deformations less than 10%. Similarly, consideration of P-δ effects is not mandatory if the compression force in a member is lower than 25% of the critical Euler load assuming that both ends of the member are pinned.

1.2. Imperfections

Beside nonlinearities, imperfections, being inevitable in real structures, may affect intensely the behaviour of steel structures. Even though imperfections are not always unfavourable, their most adverse shape, which results in a structure with lower stiffness and resistance compared to the perfect one, has to be considered for the design. Main sources of imperfections are the deviation of the realised geometry from the perfect one (geometrical imperfections), the accidental eccentricity of the applied loads and the residual stresses due to manufacturing processes (structural imperfections). For design purposes, all types of imperfections can be merged into equivalent geometrical imperfections for the sake of simplicity. Two types of imperfections are relevant to framed structures: a global initial inclination of the frame (out-of-plumbness) and an initial curvature of its members (out-of-straightness). Global imperfections (associated with P-Δ effects) should always be considered in the analysis, regardless of its type, unless high horizontal loads coexist making their influence negligible. On the other hand, member imperfections (associated with P-δ effects) having already been incorporated in the European buckling curves, can be taken into account either via member design or by direct modelling, provided that a geometrically nonlinear analysis is performed.

1.3. Modelling–Analysis–Verification

As it has already been mentioned, during the design process, nonlinearities and imperfections should be considered either explicitly in the analysis or implicitly through verification checks. Any effect on the structure that cannot be captured in the analysis, either due to the model or the analysis theory employed, should be accounted for with subsequent design checks. Generally, cross-section resistance checks are required when the material nonlinearity is not taken into account in the analysis, while member stability checks are used when the geometric nonlinearity is not considered accordingly. However, even if a geometrically nonlinear analysis is carried out, buckling checks are necessary against the instability modes which may not be able to be captured by the model. This may be due to approximative simulation or not including all the necessary imperfections to trigger a critical instability mode.

Concerning hot-rolled steel framed structures, three types of instability may occur in members subjected to compression or combined compression and bending, i.e., flexural buckling (FB), flexural-torsional buckling (FTB) and local buckling (LB). Amongst these only FB can be captured by conventional 6 degrees-of-freedom (d.o.f.) beam elements. For FTB to be captured shell elements or 7 d.o.f. beam elements, which include the first derivative of twisting angle as an additional unknown, are required [4,5]. Shell elements can additionally capture LB, while recently the Continuous Strength Method [6] has been developed which, accompanied with appropriate strain limits, allows the consideration of LB by using beam elements [7]. LB is beyond the scope of the present work and will not be mentioned any further.

Apart from the appropriate model, the consideration of initial imperfections is also essential in order for the various instability phenomena to be detectable when a geometrically nonlinear analysis is carried out. An initial frame inclination is sufficient to give rise to global instability phenomena. Additionally, initial bow imperfections in both directions of the main cross-sectional axes are necessary for the flexural buckling of individual members to be captured, while imperfections combining an initial deflection towards the weak axis of the section with an initial twist of the member are associated with the FTB. It should be noted that both global and local imperfections should not be considered simultaneously in two directions. Alternatively, to discrete frame and member imperfections, a unique initial imperfection in the shape of the critical buckling mode of the frame may be used as firstly proposed by [8] and has been further developed in [9].

Consequently, if the model of the structure can describe any possible failure mode, appropriate initial imperfections are incorporated into it and a geometrically and materially nonlinear analysis is performed, no further verification is required. Otherwise, if the analysis is elastic, cross-section resistance verification (section design) should always follow, while if a 1st order analysis is performed, stability verifications (member design) against both flexural and flexural torsional buckling are required. Member design against FB and FTB is also required even if the analysis is geometrically nonlinear but only initial global imperfections are considered. In this case, the determination of the effective lengths needed for the FB checks should be made considering the frame as non-sway, since the P-Δ effects are already included in the internal forces. Finally, verification against FTB is required if the structural members, although susceptible to FTB, are modelled with 6 d.o.f. beam elements despite the execution of a geometrically nonlinear analysis including both global and local imperfections.

1.4. Advantages and Disadvantages of Different Methods

In principle, the optimum method is the simplest one that integrates all significant effects and leads to cost-effective structures. However, it generally holds that the more simplified the model and analysis, the more demanding the subsequently required verification checks. A geometrically and materially nonlinear analysis taking into account global and local imperfections is, obviously, the most complex method in terms of modelling and analysis. The foremost advantage of this method is that it approaches the real structural behaviour with the highest possible level of accuracy and therefore more economical solutions can be achieved. Additionally, Code predicted design checks can be bypassed rendering the design process more straightforward. On the other hand, GMNIA requires advanced software available as well as deeper understanding and significant experience of the Designer, not only to set up the model and adjust the algorithm parameters, but also for the post-processing and interpretation of the results. A serious concern associated with the advanced analysis is the correct consideration of imperfections. Beside the fact that their most adverse shape is not always obvious [10], their magnitude is also a point of debate [11–14]. A strategy for design by GMNI Analysis has been published in [15], however the complexity of the method itself and the high computational cost it entails are limiting factors preventing its wide application for common structures.

The common design practice follows a two-step procedure, involving a linear elastic analysis in the first step followed by cross-sectional and member verifications according to the Code design formulae in the second. The fast and relatively simple calculations for the determination of the internal forces of this type of analysis, as well as the applicability of the superposition principle are the main advantages of this method. Its major limitation, on the other hand, is that the structure is not designed as a whole but its members are treated individually. Since the evidence about the compatibility between the isolated member and the member as part of the structure is deficient [16], this method offers scant insight into the complete system behaviour. Additionally, in order to account for the various effects neglected in the analysis, the design formulae contain many auxiliary coefficients (effective length factor, interaction coefficients) the determination of which can be quite complex, making the execution of the verification checks an excessively laborious process. Moreover, even though extensive experimental and numerical research has been conducted for the development of the design formulae, their calibration is based on the behaviour of individual members with simple boundary and loading conditions [17,18]. This implies that the application of these formulae on members under different conditions or integrated into frames may lead to unsafe or overconservative design.

A compromise between the advanced numerical methods and those that rely exclusively on analytical formulae is the General Method of EN 1993-1-1 [1], which can be used in cases that are not covered by the interaction formulae of [1]. Although potentially promising, the mechanical background of this method is not very clear and the related literature is rather scarce. In the current version of Eurocode 3, General Method is limited only to in-plane loading and the determination of the proper buckling curve to be adopted is still under discussion. A numerical validation of the method is given in [19], a possible extension of its applicability for cases where both in-plane and out-of-plane loading coexist is presented in [20], while a consistent buckling curve for the load case of combined axial force and bending moment is derived in [21].

1.5. Literature Review

Comparisons between different methods of Eurocode 3 have been presented in several works [19,20,22–25]. Most of them, however, deal with individual members and they are not exhaustive rather than examine few of the possible methods each. Only [24,25] evaluate many alternative design methods applying them on planar frames. Bernuzzi et al. [24] compare four different methods based on elastic analyses. Their results indicate no significant differences concerning the resistance of the frames except for the method that involves section and member verifications against the internal forces of a 2nd order elastic analysis only with global imperfections, being asserted as unsafe. On the other hand, Fieber et al. [25] examine methods that involve both elastic and inelastic analyses. By comparing the results of the most accurate GMNI Analyses with those of the other methods of Eurocode 3 they conclude that all the methods of EC3 provide mostly safe results. Although quite comprehensive, these two works are limited only to the in-plane behaviour of the frames preventing any out-of-plane instability. Furthermore, in the opinion of the authors, there are some inaccuracies in the calculations of [24] leading to questionable results.

1.6. Objectives and Outline

In the present work five different analysis and design methods compatible with the Eurocode 3 provisions are applied on four frames of different slenderness in order to evaluate their performance. In extension to the already published works, this paper considers not only in-plane but also out-of-plane failure modes. Both elastic and plastic design approaches are examined covering almost any possible alternative design method. Guidance is provided on the application of the methods and the comparison between them demonstrates the suitability of each method in respect to the slenderness of the examined structure. The attention is given on slender frames sensitive to 2nd order effects. The examined frames are moment resisting, braced in the out-of-plane direction, having rigid joints and made of compact cross-sections. Only the columns of the frames are examined; any possible failure of the beams is prevented.

Firstly, in Section 2 the key features of the examined methods are presented. The formulae selected for the cross-sectional verification are justified by comparing the normative expressions [1] and the analytical ones proposed in [26] with numerical results. Afterwards, in Section 3 the examined structures and their loading as well as some modelling aspects are discussed. In Section 4, the main results of the investigations regarding the most unfavourable shape of the imperfections, the collapse mechanism of the structures and the comparison between the examined methods are reported. A discussion on the results and on the procedure employed for the determination of the effective lengths follows in Section 5. Finally, the conclusions drawn through this study are presented in Section 6.

2. Examined Analysis and Design Methods

The following five methods sorted in decreasing order regarding the complexity of the analysis theory employed but in increasing order regarding the complexity of the required verification checks are examined:

1. GMNIA: Geometrically and Materially Nonlinear Analysis with Imperfections
2. GNIA-SD: Geometrically Nonlinear Analysis with Imperfections—Section Design
3. GM: General Method
4. GNA-SMD: Geometrically Nonlinear Analysis—Section and Member Design
5. LA-SMD: Linear Analysis—Section and Member Design

The accuracy of the last four methods is evaluated with respect to the first one which is the most accurate among them. At the first step, the ultimate loads of each frame according to the GMNIA design method are calculated (i.e., the loads under which the ultimate load factor of GMNIA is $\lambda_{GMNIA} = 1$, see Section 3.2). By applying these loads in the remaining methods, their ultimate load factor (λ_{method}) is determined, i.e., the load factor under which the most critical verification check of each method is marginally satisfied. A result of $\lambda_{method} > 1$ indicates that the method is unsafe permitting higher loads than those the frame can actually bear. The application of the methods is outlined in the following paragraphs, while the key features of each method are summarized in Table 1.

Table 1. Key features of the examined methods.

Method	Modelling—d.o.f. of Beam Elements	Analysis	Modelled Imperfections	Section Design	Member Design	In Plane Buckling Length
GMNIA	7	GMNIA	Global & Local	No	No	-
GNIA-SD	7	GNIA	Global & Local	Yes	No	-
GM	6	GNIA/LBA	Global & in-plane local	Yes	General Method	-
GNA-SMD	6	GNA	Global	Yes	Yes	Non-sway
LA-SMD	6	LA	Global	Yes	Yes	Sway

2.1. GMNIA

To apply the GMNIA method the frames are modelled with 7 d.o.f. beam elements, both global (sway) and local (bow) imperfections are incorporated into the model and a geometrically and materially nonlinear analysis is performed. Since any possible collapse mode can be captured in the analysis, no verification checks are needed. The safety of the structure is verified if having applied the design loads the corresponding peak load factor is greater than one. The failure criterion of the method is the formation of a plastic mechanism or the instability of the structure. The collapse mechanism is identified by examining equilibrium paths accompanied by the stress distribution in the structure and its deformed shape.

2.2. GNIA-SD

The application of GNIA-SD method involves modelling the structure with 7 d.o.f. beam elements, performing an elastic geometrically nonlinear analysis taking into account both global and local imperfections and verification of cross-sectional resistance. Since any possible instability phenomenon can be detected in the analysis only cross-section checks are required. These are executed according to the formulae Equations (1)–(3) proposed in [26] considering the plastic resistance of the sections. The failure criterion of the method is the formation of the first plastic hinge in the frame. The formulae of [26] are selected over the equivalent ones of EN 1993-1-1 [1] due to their higher accuracy as demonstrated through a preliminary investigation where both formulae were compared with numerical results. The results concerning uniaxial and biaxial bending moment combined with axial force are presented in the form of interaction diagrams for a HEB 450 cross-section in Figures 1 and 2.

$$\left[\frac{m_y\cdot\left(1+\alpha_f\right)-\alpha_w+n^2/\alpha_w}{2\cdot\alpha_f}\right]^2+m_z+m_w\leq 1.0, \qquad if\ n\leq\alpha_w \tag{1}$$

$$\frac{\left[m_y\cdot\left(1+\alpha_f\right)\right]^2+\left[2\cdot(\alpha_w-n)\right]^2}{\left(2\cdot\alpha_f\right)^2}+m_z+m_w\leq 1.0, \qquad if\ n>\alpha_w \tag{2}$$

with the additional condition:

$$m_y\leq\frac{1-n}{1-0.5\cdot\alpha_w} \tag{3}$$

where n, m_y, m_z and m_w are the axial force, bending moments and bimoment, respectively, divided by the relevant plastic section resistances and α_f, α_w are the ratios between the cross-section areas of the flange and correspondingly the web to the total section area.

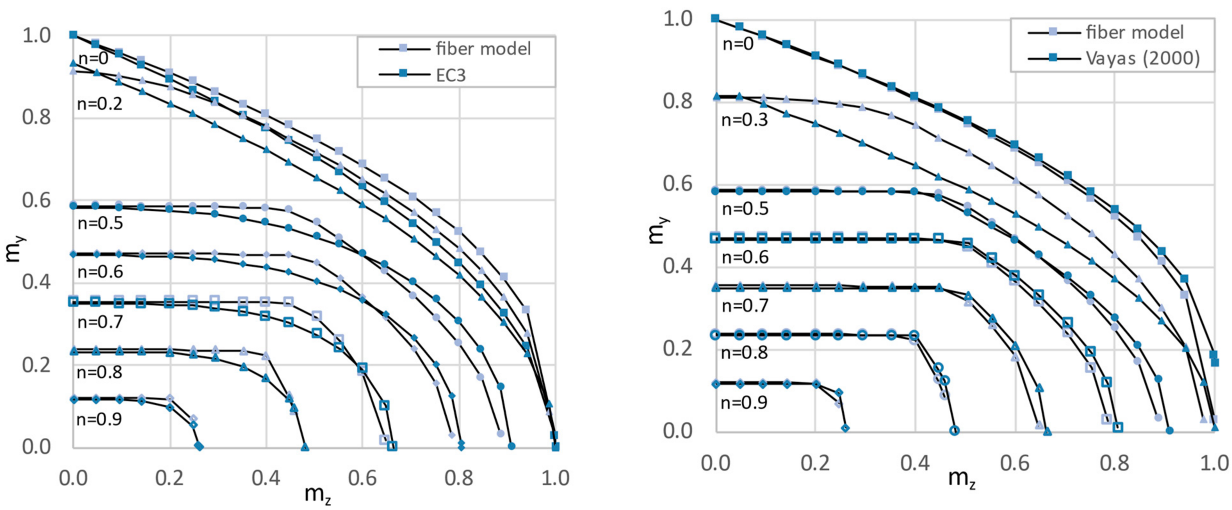


Figure 1. Comparison of interaction resistance diagrams n + m_y + m_z of numerical analysis (fiber model) and analytic formulae EC3 [1], Vayas (2000) [26] for a HEB 450 section.

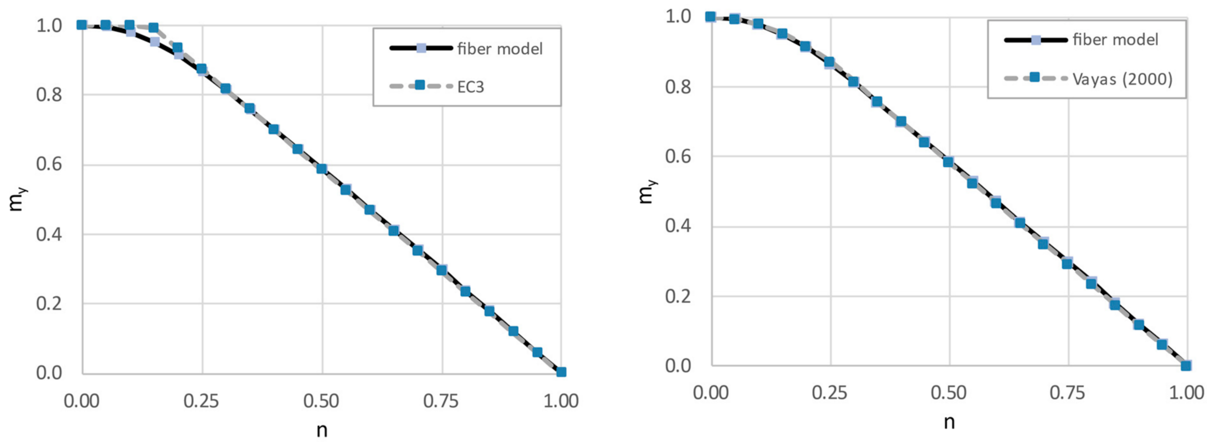


Figure 2. Comparison of interaction resistance diagrams n + m_y of numerical analysis (fiber model) and analytic formulae EC3 [1], Vayas (2000) [26] for a HEB 450 section.

2.3. General Method

For the application of the General Method two types of analysis are executed: an elastic geometrically nonlinear analysis with imperfections for the calculation of $\alpha_{\text{ult,k}}$, where global and in plane local imperfections are taken into account (GNIA) and a linearized buckling analysis (LBA) for the calculation of $\alpha_{\text{cr,op}}$. The members of the frames are modelled with 7 d.o.f. beam elements for both analyses, although for the purposes of GNIA 2D 3 d.o.f. beam elements would be sufficient. The $\alpha_{\text{ult,k}}$ coefficient is determined through an iterative procedure, as the minimum amplifier of the design loads, for the most critical cross-section of the frame to reach its characteristic plastic resistance. The plastic resistance of cross-sections is calculated with the interaction formulae Equations (4) and (5) proposed by [26] which are more accurate than the equivalent expressions of EN1993-1-1 [1] as can be seen in Figure 2. The $\alpha_{\text{cr,op}}$ coefficient, respectively, is taken equal to the eigenvalue of the first buckling mode involving the out-of-plane buckling of the most stressed column, from which the $\alpha_{\text{ult,k}}$ coefficient is determined.

$$m_y \leq \frac{1-\alpha_f{}^2-n^2}{1-\alpha_f{}^2}, \qquad if\; n \leq \alpha_w \tag{4}$$

$$m_y \leq \frac{1-n}{1-0.5\cdot\alpha_w}, \qquad if\; n \leq \alpha_w \tag{5}$$

where n and m_y are the axial force and bending moment about the strong axis, respectively, divided by the relevant plastic section resistances and α_f, α_w are the ratios between the cross-section areas of the flange and correspondingly the web to the total section area.

According to General Method, both strength and stability are verified by satisfying the condition:

$$\frac{\chi_{op}\cdot\alpha_{ult,k}}{\gamma_{M1}} \geq 1.0 \tag{6}$$

where

$$\chi_{op} = \min(\chi_z, \chi_{LT}) \tag{7}$$

is the buckling reduction factor taken as the minimum between the reduction factor for the out-of-plane flexural buckling (χ_z) and the reduction factor for the lateral torsional buckling (χ_{LT}).

2.4. GNA-SMD

For the GNA-SMD method the frames are modelled with 6 d.o.f. beam elements. An elastic geometrically nonlinear analysis is carried out taking into account only the global imperfections. Two alternatives are examined regarding the analysis: a rigorous geometrically nonlinear analysis based on the exact 2nd order theory (GNA) and an approximate one where the horizontal loads are multiplied by an amplification factor (Equatioin (8)) and the internal forces are calculated according to the 1st order theory (ASOA).

$$\alpha = \frac{1}{1-\frac{1}{\alpha_{\text{cr}}}} \tag{8}$$

Since the analysis is elastic and only global imperfections are incorporated into the model, both section and member verifications are required. The failure criterion of the method is the formation of the first plastic hinge or the stability loss of the most critical column, whichever occurs first. The cross-sectional verification is performed according to the expressions Equations (4) and (5), while the buckling verification according to the interaction formulae (6.61) and (6.62) of EN1993-1-1 [1], which for exclusively in-plane bending are written as:

$$\frac{N_{Ed}}{\chi_y\cdot N_{Rk}/\gamma_{M1}} + k_{yy}\cdot\frac{M_{y,Ed}}{\chi_{LT}\cdot M_{y,Rk}/\gamma_{M1}} \leq 1.0 \tag{9}$$

$$\frac{N_{Ed}}{\chi_z \cdot N_{Rk}/\gamma_{M1}} + k_{zy} \cdot \frac{M_{y,Ed}}{\chi_{LT} \cdot M_{y,Rk}/\gamma_{M1}} \leq 1.0 \tag{10}$$

where χ_y, χ_z, and χ_{LT} are the reduction factors for flexural buckling about the strong and weak axis of the cross-section and for lateral torsional buckling, respectively, N_{Rk} and $M_{y,Rk}$ are the characteristic resistances against axial force and bending moment about the strong cross-sectional axis and k_{yy}, k_{zy} are the interaction factors for combination of axial force and bending moment.

The interaction factors k are calculated according to Method 2 (German-Austrian approach), where C_{my} factor is taken equal to C_{my} = 0.9, as the frames buckle in a sway mode [18]. The effective lengths, however, are taken conservatively equal to the system lengths according to [1], as P-Δ effects have already been accounted for in the internal forces.

2.5. LA-SMD

For the LA-SMD method the frames are modelled with 6 d.o.f. beam elements, sway imperfections are included in the model and a linear elastic analysis is executed followed by cross-sectional and stability verifications. The failure criterion of the method is the formation of the first plastic hinge or the instability of the most critical member. The resistance of the cross-sections is checked according to the Equations (4) and (5) and the stability of the columns according to the interaction formulae Equations (9) and (10). Unless the condition $\alpha_{cr} > 10$ holds, the frames are considered sway and the effective lengths of their columns should be calculated accordingly. Here, for investigation reasons, the performance of the method is examined considering both the sway (LA-SMDs) and non-sway (LA-SMDns) in-plane buckling lengths of the columns, even if not permitted by the Code for all frames. The non-sway buckling lengths are taken equal to the system lengths, while the sway buckling lengths are calculated with the system buckling approach. Specifically, a linearized buckling analysis is performed to obtain the critical load factor (i.e., the eigenvalue of the critical sway buckling mode) which combined with the Euler's formula gives the effective length factor β (Equation (11)) [4] of each column. The out-of-plane buckling length is taken, conservatively, equal to the system length assuming that the frames are braced in that direction.

$$\beta = \frac{1}{L} \cdot \sqrt{\frac{\pi^2 \cdot E \cdot I}{\lambda_i \cdot N_{ref}}} \tag{11}$$

where λ_i is the eigenvalue of the critical (for each column) in-plane sway buckling mode, E the modulus of elasticity, I the moment of inertia in the examined plane and N_{ref} the design axial force of the column.

3. Examined Frames

3.1. Geometry and Material

Four frames of different slenderness ranging from fairly rigid to very slender are examined. In terms of Eurocode 3, the slenderness of a structure is expressed by the α_{cr} coefficient; the lower the α_{cr} the slenderer the structure. The value of α_{cr} is influenced by the dimensions of the structure, the cross-sections of its members, the modulus of elasticity of the material used, the boundary conditions and the magnitude of the vertical loads acting on the structure. In order for the examined frames to be comparable, all of the above-mentioned parameters, except for the applied loads, were kept constant for all frames. Different values of α_{cr} were achieved only by varying the ratio of vertical to horizontal loads.

The examined frames are moment resisting, with pinned supports and assumed braced in the out-of-plane direction. They have two storeys with three bays and all cross sections are compact (class 1) with a S355 steel grade. Their configuration, dimensions and cross-sections of their members are illustrated in Figure 3. The frames under consideration are in fact parts of a 2-, 3-, 5- and 7-storey building (named LS, MS, HS and VHS respectively). However, only the bottom 2 storeys are modelled, for simplicity of analysis.

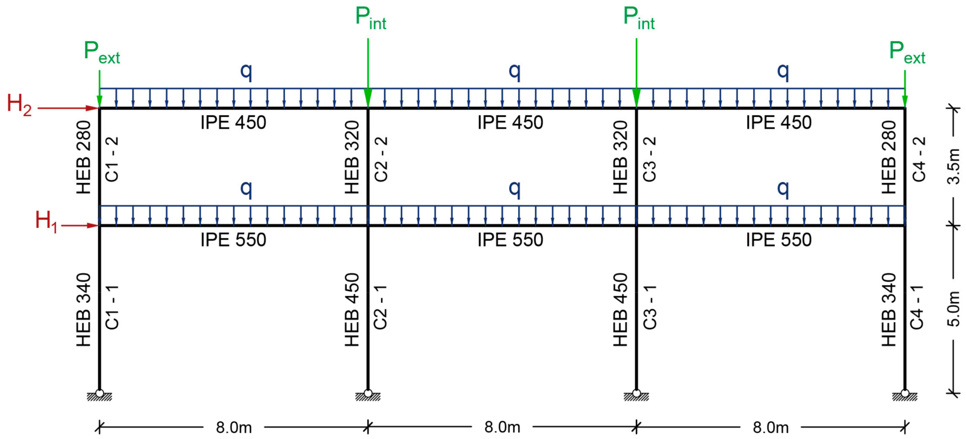
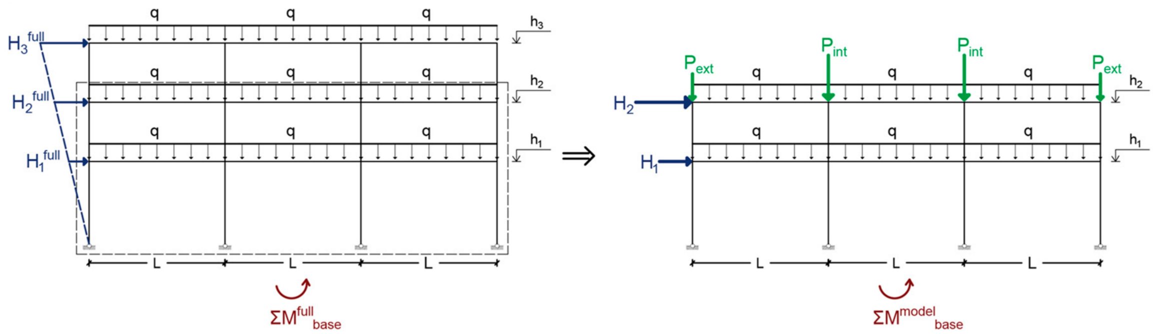


Figure 3. Configuration and load pattern of the examined frames and designation of their columns.

3.2. Loads

Both vertical and horizontal loads are imposed on the examined frames (Figure 3). The vertical loads consist of uniformly distributed loads on the beams and concentrated loads on the top of the columns of the 2nd floor. The horizontal loads are concentrated at the levels of the floors. The loading magnitude is such that the ultimate load coefficient as results from a GMNI Analysis is equal to 1 for all the frames and simultaneously the α_{cr} coefficient of each frame has a different value.

More specifically, the uniformly distributed vertical load has the same value in all frames corresponding to the value arising under the ultimate limit state load combination as specified by Eurocode 1 [27] for office buildings assuming that the beams have a 6 m influence width. The concentrated vertical loads are assumed to come from additional (not modelled) floors of the structure. For this reason, they are applied only on the top of the 2nd floor columns, while the force on the internal columns is twice as much the one on the external columns. The magnitude of these forces has been selected so that the α_{cr} coefficient of each frame has the desired value. As a result, LS, MS, HS and VHS frames are assumed to have 0, 1, 3 and 5 additional floors, respectively. Finally, the horizontal forces applied are such that the GMNIA ultimate load factor of each frame reaches unity. They are assumed to have a triangular distribution over the height of the full frame. The horizontal force at the level of the 1st floor is the same for both the full and the partial frame. However, at the level of the 2nd floor, for the simulation of the effects of the floors which are not modelled, the horizontal force applied on the partial frame is such that its base moment is the same with that of the full frame. This assumption results in different base shear between the two models; nevertheless, its influence on the structural behaviour and results was deemed insignificant, for the purposes of this study. The derivation of the loads applied on the examined frames is presented schematically in Figure 4, indicatively for the MS frame. Table 2 summarises the applied loads on each frame and the corresponding α_{cr} coefficient.



$$\Sigma M_{base}^{full} = H_1^{full} \cdot h_1 + H_2^{full} \cdot h_2 + H_3^{full} \cdot h_3$$

$$\Sigma M_{base}^{model} = H_1 \cdot h_1 + H_2 \cdot h_2$$

$$H_1 = H_1^{full}$$

$$\Sigma M_{base}^{model} = \Sigma M_{base}^{full} \Rightarrow$$

$$H_2 = \frac{H_2^{full} \cdot h_2 + H_3^{full} \cdot h_3}{h_2}$$

$$P_{int} = 2 \cdot P_{ext} = q \cdot L \cdot b_{eff}$$

$$\begin{Bmatrix} q \\ P_{ext}, P_{int} \\ H_1, H_2 \end{Bmatrix} \xrightarrow{\text{GMNIA}} \lambda_{collapse} = 1$$

Figure 4. Derivation of loads for the examined frames (MS).

Table 2. Applied loads and α_{cr} coefficients of the examined frames.

Frame	Slenderness	α_{cr}	Vertical Loads			Horizontal Loads	
			Distributed	Concentrated			
				Internal Column	External Column	1st Floor	2nd Floor
			q [kN/m]	P_{int} [kN]	P_{ext} [kN]	H_1 [kN]	H_2 [kN]
LS	Low	10.04	70	0	0	256.5	436
MS	Medium	6.66	70	576	288	89.5	456
HS	High	4.05	70	1727	863.5	15	289
VHS	Very High	2.86	70	2878	1439	0	0

3.3. Imperfections

Both global and member imperfections are considered in the form of equivalent notional loads. The global imperfection, taken into account in all analyses conducted, consists of an initial out-of-plumbness equal to 1/200 in the same direction with the horizontal forces. In addition to global imperfection, local bow imperfections are integrated into the model for the GMNI and GNI Analyses. Since their most unfavourable shape is not a priori known, four different shapes, as illustrated in Figure 5, are examined, while their magnitude is taken from the Table 5.1 of EN 1993-1-1 [1].

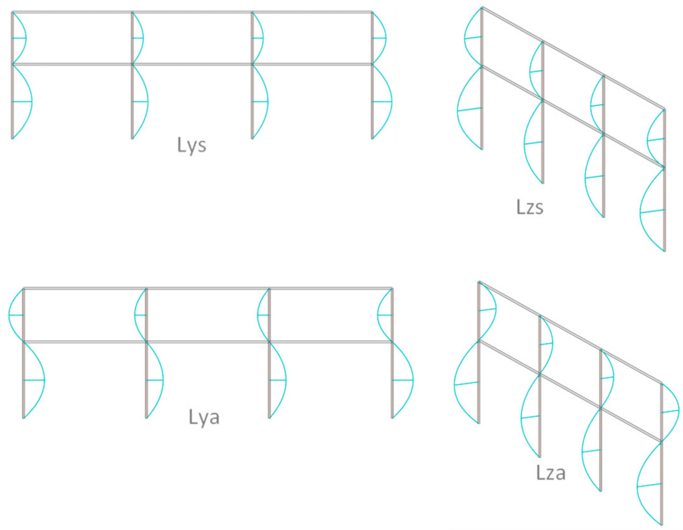


Figure 5. Examined shapes of local bow imperfections.

3.4. Modelling

Beam elements of 6 or 7 d.o.f. are used to model the members of the frames, as described in Section 2. Beam elements are sufficient for the purposes of this study, since all the members have Class 1 cross-sections, so local buckling is irrelevant. The elements' length, equal to 0.2 m, was selected after sensitivity analyses showing that by further halving the length the improvement in accuracy was insignificant (differences less than 1%).

Regarding connection properties, the columns are pinned at their bases, while beam to columns joints are considered fully rigid. Moreover, assuming that the frames are braced in the out of plane direction and that the lateral torsional buckling of the beams is prevented by the slabs and appropriate detailing, the out of plane displacements at the floor levels are restricted.

For the materially nonlinear analyses, steel is represented by a linear elastic-linear hardening plastic material model with input parameters: the modulus of elasticity E = 210 GPa, the yield stress f_y = 355 MPa, for the post-yield region a constant stiffness of $E_T = E \cdot 10^{-4}$ and the Poisson ratio v = 0.3. From a preliminary investigation, it was concluded that the differences in the overall capacity of the frames considering elastic or elastoplastic beams are very small, as shown in Figure 6. For this reason and because this study focuses on the influence of 2nd order effects on the behaviour of the frames, the material of the beams in the GMNI Analyses is considered elastic and the verification checks for all the other methods concern only the columns.

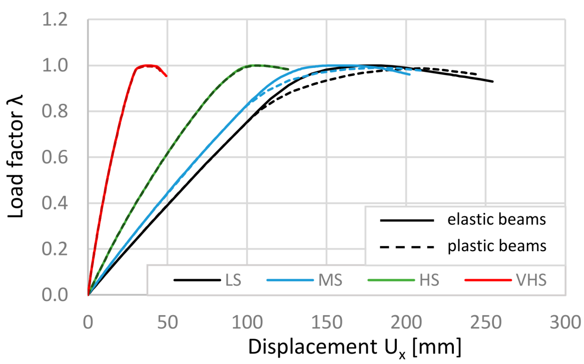


Figure 6. Comparison of the behaviour of the frames considering elastic and elastoplastic beams.

For the analyses, two different finite element programs, i.e., ADINA [28] and SOFiSTiK [29], were used, with the compatibility between them verified. More specifically, GMNI Analyses were executed with ADINA which, having incorporated a very robust algorithm using the arc-length method, is able not only to achieve easily a fast convergence but also to capture the unloading branch of the equilibrium path. All the other analyses were carried out with SOFiSTiK, which, being more practice-oriented, provides easier processing of the internal forces for the verification checks.

4. Results

4.1. Influence of Nonlinearities

It is known that the influence of geometric nonlinearity increases with the slenderness of the structure, whereas material nonlinearity affects more the more rigid structures. For the examined frames these influences are revealed in Figure 7, where the equilibrium paths of the examined frames as derived from different types of analysis are presented.

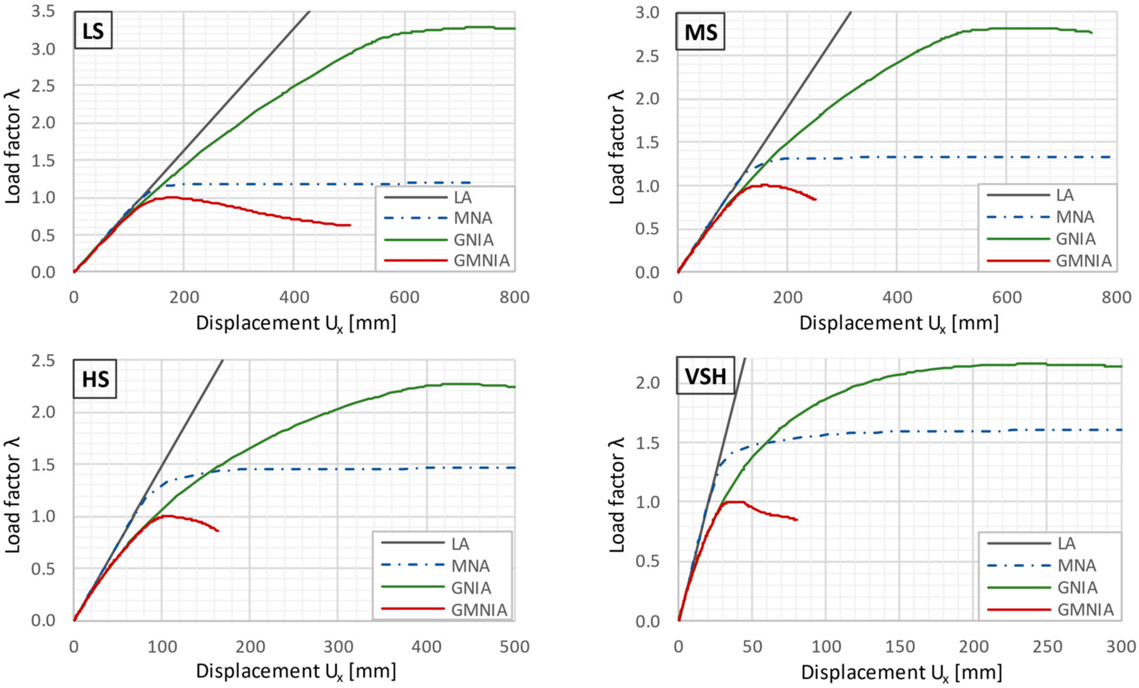


Figure 7. Equilibrium paths of the examined frames as derived with different methods of analysis.

4.2. Influence of the Imperfection Pattern

As mentioned in Section 3.3, four possible imperfection patterns (Figure 5) are examined to identify the most adverse one. The corresponding ultimate load factor, as derived from GMNIA and GNIA-SD methods is presented in Figure 8. More pronounced variations, increasing with the slenderness of the structure, are observed for GMNIA method, which for the slenderest frame exceed 20%. On the other hand, the variations for GNIA-SD method are moderate, less than 5% for all the frames. Consideration of out-of-plane imperfections leads to lower resistance for all frames according to GMNIA; the antisymmetric shape is the most adverse for the slenderest frame and the symmetric shape for the other ones. According to GNIA-SD, the symmetric out-of-plane imperfections are also the most adverse for the two more rigid frames, however for the two more slender frames, the in-plane ones turn to be more unfavourable.

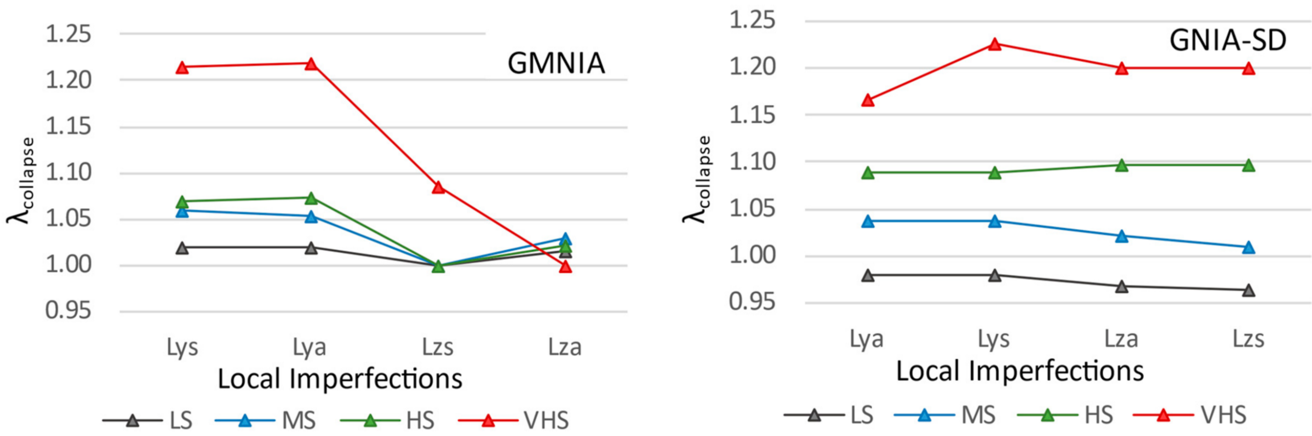


Figure 8. Variation of the ultimate load factor with the imperfection pattern for GMNIA and GNIA-SD methods.

4.3. Collapse Mechanism

GMNIA provides the most realistic estimation of the collapse mechanism. This is identified by examining appropriate equilibrium paths in combination with the distribution of the stresses and the deformed shape of the frame at the moment of collapse, presented for the frames LS and VHS in Figures 9a–d and 10a–d. The behaviour of the other two frames (MS and HS) is similar to the behaviour of LS. It may be observed that the collapse of the frames is due to instability and it occurs in the plastic range. More specifically, in the three more rigid frames the three right columns of the first floor buckle in a flexural torsional buckling mode, while in the slenderest frame the internal columns of both floors buckle also in the same mode. This can be verified by the equilibrium paths presented in Figures 9b,c and 10b,c which display the out-of-plane displacements and rotations about the members axis versus the applied loading for the most critical point of each column (marked with red in Figures 9a and 10a). Also, it can be noticed that no plastic mechanism has been formed in any frame before collapse (Figures 9d and 10d). Indeed, except for the most rigid frame where the cross-sections at the top of the three columns that buckle form a plastic hinge, all the others collapse without any of their cross-sections fully yielded. In fact, the slenderer the frame the smaller the part of its most critical section that has been plasticised at the moment of collapse.

Additionally, the utilisation level of the most rigid (LS) and most slender (VHS) frame when the failure criterion of the GNIA-SD, GNA-SMD and LA-SMDs methods is fulfilled is presented indicatively in Figures 9e–g and 10e–g. The utilization of the other two frames is similar to the LS frame. It may be observed that only the GNA-SMD method can capture the critical column in all frames. All the other methods correctly detect the critical column in the three most rigid frames, but fail in the slenderest one.

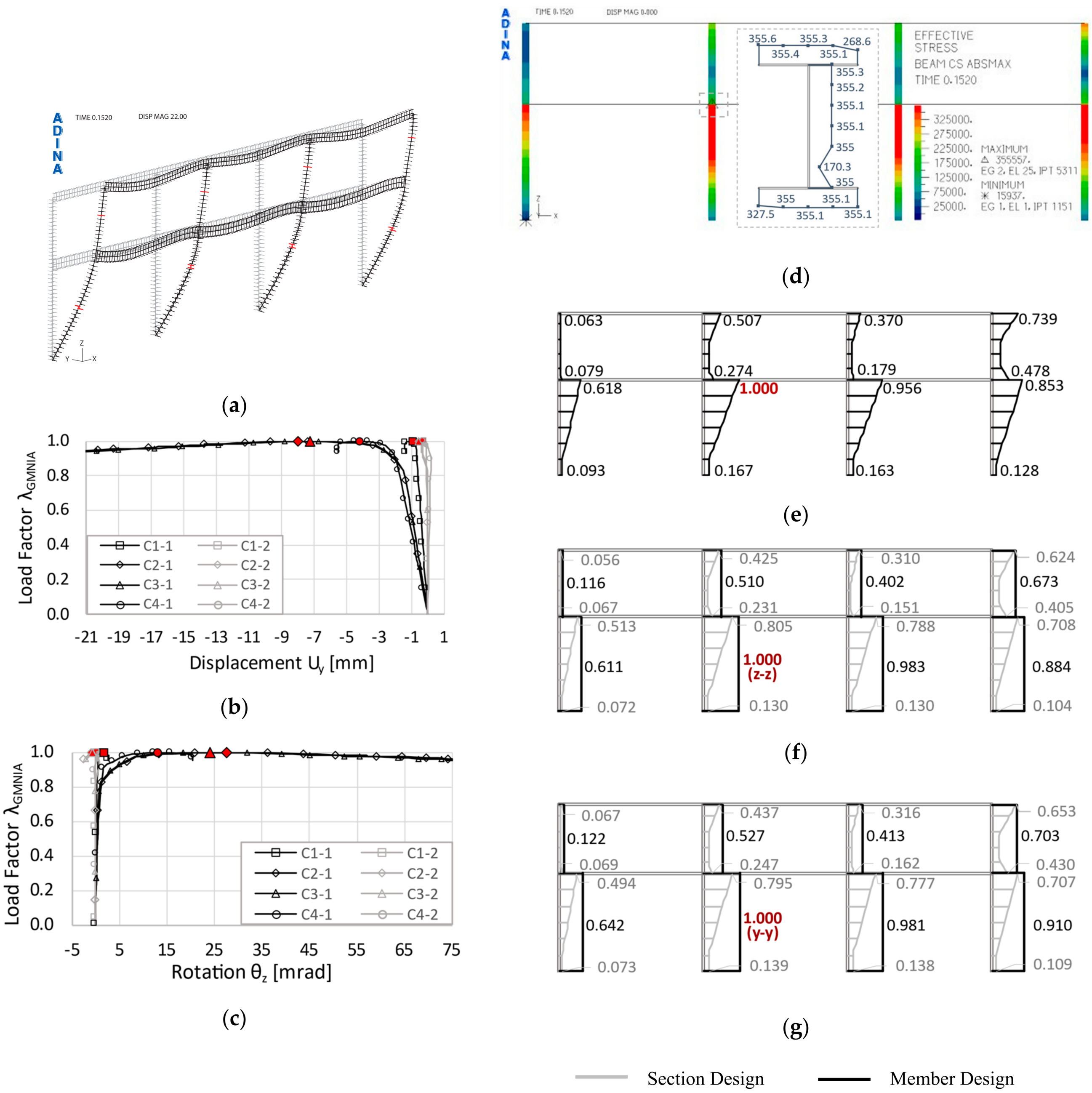


Figure 9. Collapse mechanism and utilization level of LS frame—(**a**) deformed shape (GMNIA); (**b**,**c**) equilibrium paths (GMNIA); (**d**) stress distribution (GMNIA); (**e**) utilization level (GNIA-SD); (**f**) utilization level (GNA-SMD); (**g**) utilization level (LA-SMDs).

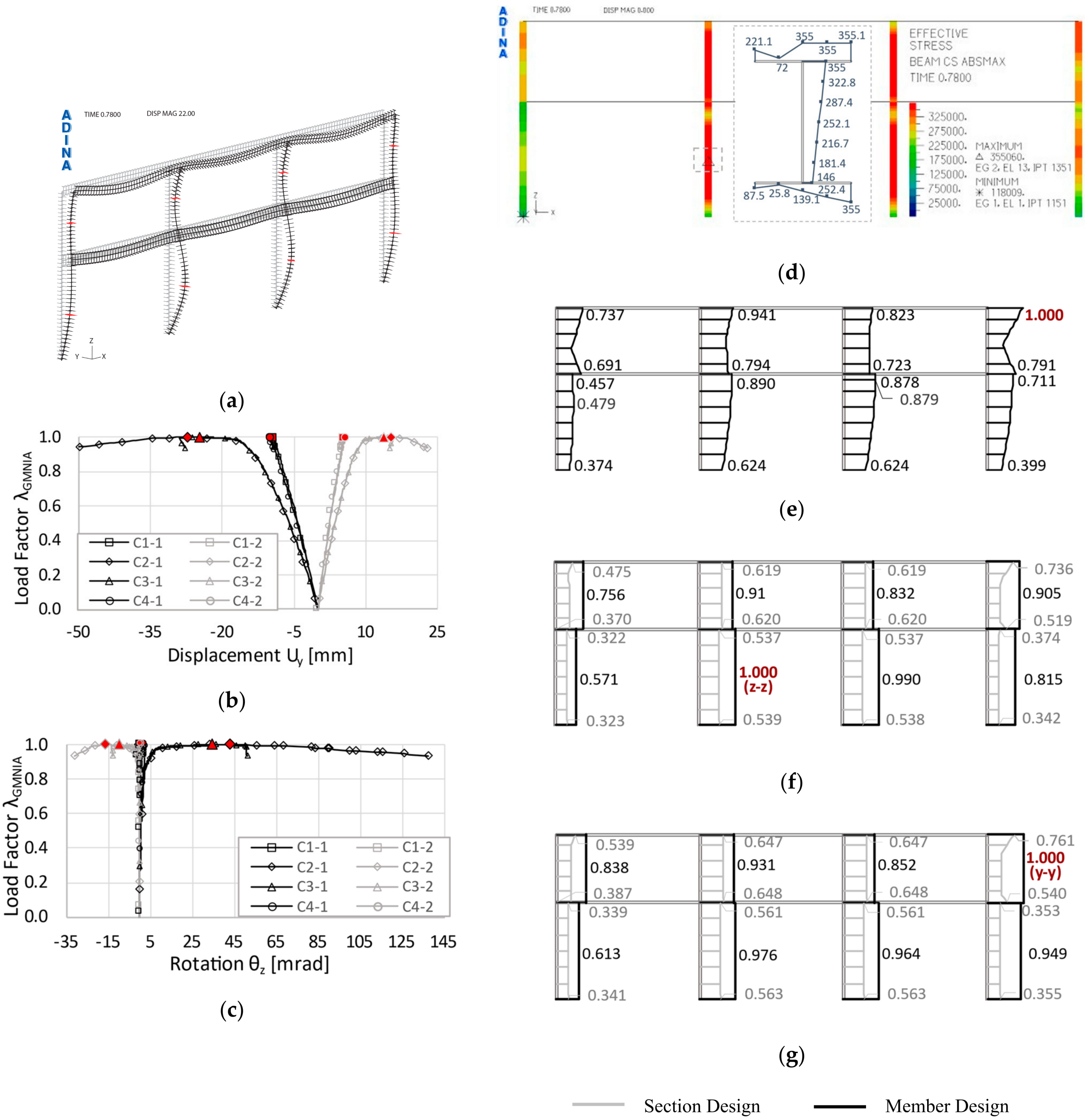


Figure 10. Collapse mechanism and utilization level of VHS frame—(**a**) deformed shape (GMNIA); (**b**,**c**) equilibrium paths (GMNIA); (**d**) stress distribution (GMNIA); (**e**) utilization level (GNIA-SD); (**f**) utilization level (GNA-SMD); (**g**) utilization level (LA-SMDs).

4.4. Ultimate Load Factors

The ultimate load factors of the frames as determined from each method are presented in Table 3 and graphically illustrated in Figure 11, where a green marker indicates that EC3 permits the use of the method for the specific frame and a red marker the opposite. Moreover, a comparison between the equilibrium paths of the examined methods marked at the point corresponds to the fulfilment of the failure criterion of each method is presented in Figure 12. The following observations may be made taking as reference the GMNIA method:

- All methods are on the safe side for the most rigid frame LS, where $\alpha_{cr} > 10$.

- All methods except for General Method are unsafe for the slenderest frame, where $\alpha_{cr} < 3$.
- Only the General Method is on the safe side for all frames.
- The commonly used in practice GNIA-SD method is safe only for the LS frame.
- The least conservative method is the GNIA-SD, while the most conservative is the ASOA-SMD, except for the VHS frame where the General Method is the most conservative.
- All methods become less conservative as the slenderness of the structure increases.
- Although for frames that $\alpha_{cr} < 10$, sway buckling lengths should be considered when a 1st order analysis is executed, results show that by taking them equal to the system lengths (method LA-SMDns) leads to safe results for the MS frame (α_{cr} = 6.66) and slightly (less than 5%) unsafe results for the HS frame (α_{cr} = 4.05).
- The differences between the exact GNA-SMD and the approximate ASOA-SMD method are very small. The approximate method is slightly more conservative for the three more rigid frames, while for the slenderest one, where its application is not permitted, the two methods lead to an identical ultimate load factor.

Table 3. Ultimate load factors.

Method	**Ultimate Load Factor $\lambda_{collapse}$**			
	LS (α_{cr} = 10.04)	**MS (α_{cr} = 6.66)**	**HS (α_{cr} = 4.05)**	**VHS (α_{cr} = 2.86)**
GMNIA	1.000	1.000	1.000	1.000
GNIA-SD	0.963	1.010	1.088	1.167
GNA-SMD	0.820	0.853	0.896	1.009
ASOA-SMD	0.804	0.834	0.878	1.009
LA-SMDs	0.874	0.904	0.957	1.054
LA-SMDns	0.877	0.939	1.011	1.082
General Method	0.869	0.905	0.922	0.960

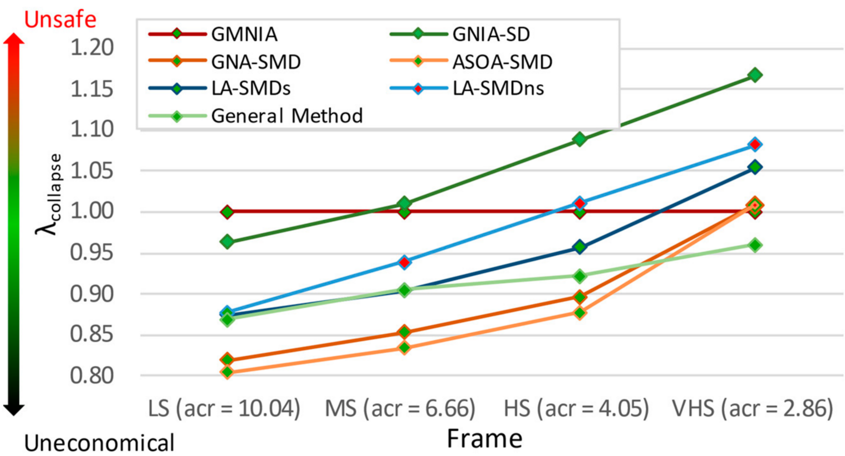


Figure 11. Ultimate load factors.

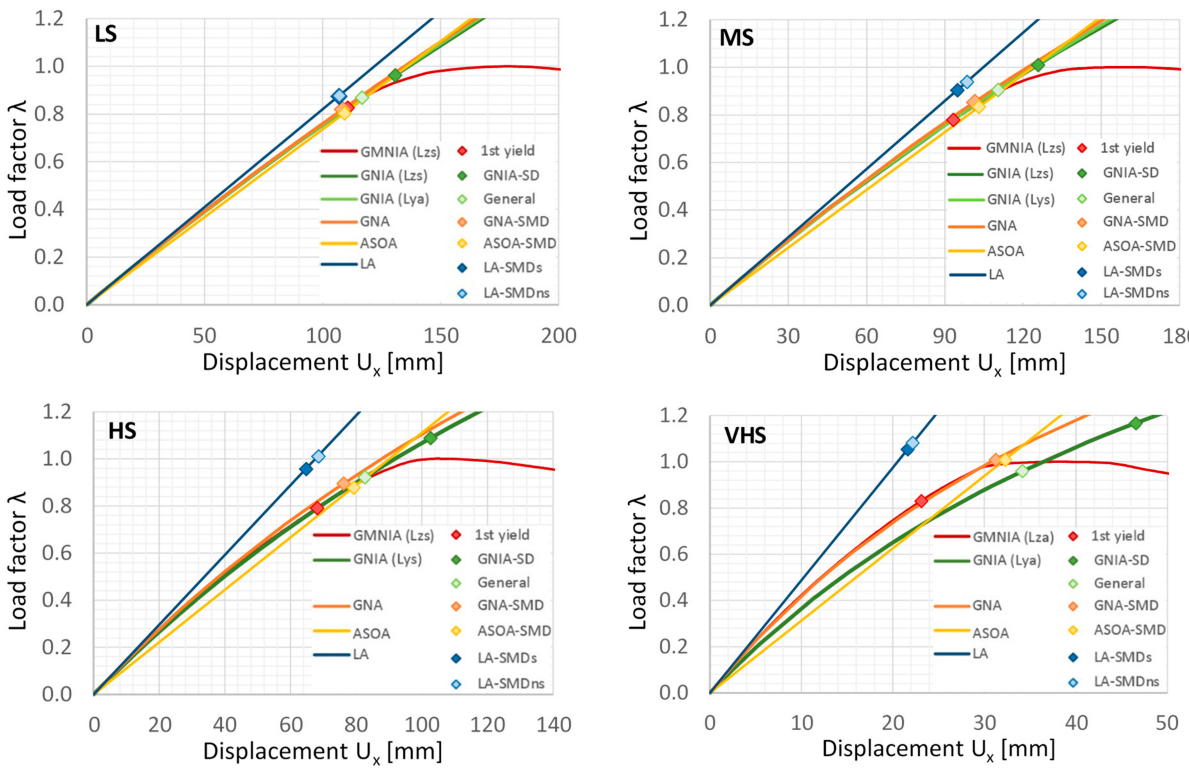


Figure 12. Comparison between the equilibrium paths of the examined frames marked with the point corresponds to collapse.

5. Discussion

5.1. Interaction of Nonlinearities

As identified in Section 4.3, the examined frames collapse due to inelastic buckling, thus due to the interaction of the geometric and material nonlinearity. GNIA-SD method neglects this interaction and for this reason is unsafe for the three slenderest frames. Indeed, these frames collapse before the full plastification of any of their cross-sections, which is the failure criterion of GNIA-SD method. Moreover, as ascertained in Section 4.3, the slenderer the frame, the smaller the plastic zone of its most unfavourable cross-section at collapse and for this reason the results of GNIA-SD become less safe as the slenderness of the frame increases. On the other hand, GNIA-SD is on the safe side for the rigid frame, because this frame collapses after its three more unfavourable cross-sections have been fully plasticised, thus at higher loading level than that required for the fulfilment of the failure criterion of the method.

5.2. Magnitude of Imperfections

The local imperfections for the analyses of the present study have been taken from the Table 5.1 of EN 1993-1-1 [1] as mentioned in Section 3.3. However, as already reported in [14] the values of that table are significantly higher than those extracted from the EC3 buckling curves through the Ayrton-Perry formulation. For example, the magnitude of the bow imperfection that Table 5.1 of EN 1993-1-1 proposes for a plastic analysis in the direction of the weak axis of the most critical column (C2-1) is e_0 = L/200. The corresponding value derived from the Equation (12) is e_0 = L/386. Therefore, the magnitude of the imperfection taken into account through the buckling check is 93% lower than that considered for the GMNIA analysis.

$$e_0 = \alpha \cdot \left(\overline{\lambda} - 0.2\right) \cdot W_{pl} / A \tag{12}$$

where α the imperfection factor, $\overline{\lambda}$ the non-dimensional slenderness, W_{pl} the plastic section modulus and A the cross-section area.

By repeating the GMNI Analyses taking into account local imperfections derived from the Equation (12) the ultimate load factors ($\lambda_{GMNIA(bc)}$) shown in Figure 13 are obtained. It may be observed that they are quite higher than those which correspond to the imperfections of Table 5.1 of EN 1993-1-1 and that the differences increase with the slenderness of the frame because the sensitivity to imperfections is also increasing. Moreover, Figure 13 reveals that if the methods which involve buckling verifications are compared to the GMNIA method where local imperfections from the Equation (12) are considered, then they are all on the safe side for all the frames.

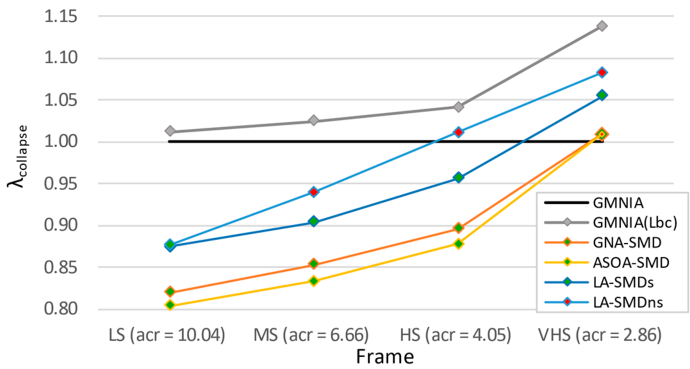


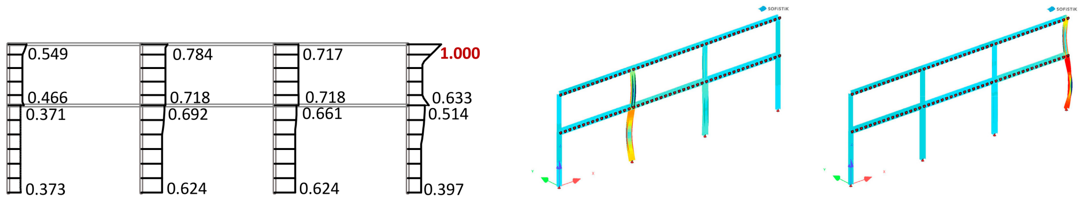
Figure 13. Comparison of the ultimate load factors of the methods that involve buckling verifications with those of GMNI Analyses taking into account local imperfections either from Table 5.1 of EN 1993-1-1 [1] or from the Equation (12).

5.3. GNA-SMD

In contrast to the results of [24], where it was concluded that the GNA-SMD method (or EC3-RAM according to the terminology therein) is the most economic among the other Eurocode 3 methods, in the present study it was found that this method together with the ASOA-SMD are the most conservative methods (except for the slenderest frame). This disagreement may be due to various reasons. First and foremost, in [24] the out-of-plane instability was not considered. In the present study, however, it was found to be the dominant instability mode causing the collapse of the frames. Another reason could be the value of C_{my} coefficient. More specifically, although in both studies the stability verification was performed with the same interaction formula (Equation (6.61) of EN 1993-1-1 [1]) and the interaction coefficient k_{yy} was evaluated according to the same method (Method 2), the C_{my} coefficient in [24] was not taken equal to 0.9 contrary to the calculations presented here. However, both EN 1993-1-1 [1] and [18] clearly state that C_{my} should be taken equal to 0.9 when the examined members buckle in a sway mode.

5.4. General Method

Special attention should be paid on the implementation of the General method when the first out-of-plane buckling mode does not involve the most stressed column from which the $\alpha_{ult,k}$ coefficient is determined. Being that the case for the VHS frame, it was demonstrated that the $\alpha_{cr,op}$ coefficient should be taken equal to the eigenvalue of the first out-of-plane buckling mode (see Figure 14c) that does involve the most stressed column (see Figure 14a). Otherwise, i.e., if the first out-of-plane mode of the frame is considered, but that mode does not engage the most stressed column (see Figure 14b), the General method leads to very conservative results (the ultimate load factor of the method for the VHS frame decreased from 0.959 to 0.882 (8%)).

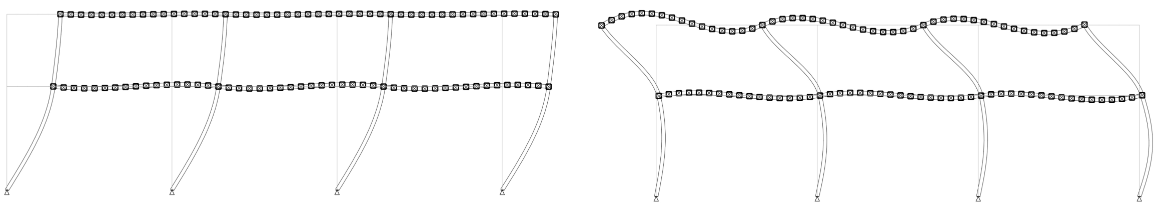


(**a**) (**b**) (**c**)

Figure 14. Implementation of the General method on the VHS frame: (**a**) utilization of the columns for the determination of $\alpha_{\mathrm{utl,k}}$; (**b**) first out-of-plane buckling mode of the frame; (**c**) first out-of-plane buckling mode that includes the most stressed column.

5.5. *Effective Buckling Length*

The most demanding task involved in the otherwise simple LA-SMD method is the determination of the buckling length. To this scope three methodologies have been developed: (a) the isolated subassembly approach, (b) the storey-based approach and (c) the system buckling approach [30,31]. As mentioned above, in this study the in-plane buckling length is calculated according to the third approach through the Equation (11), where λ_i is taken equal to the eigenvalue of the critical sway in plane buckling mode of each column. More specifically, it was identified that the critical buckling mode for the first-floor columns is the first sway mode of the frame (Figure 15a), while for the second-floor columns the second one (Figure 15b). The buckling length factors obtained with this assumption are depicted in Figure 16. As expected, all of them are greater than one, since the examined frames are unbraced in the in-plane direction.



(**a**) (**b**)

Figure 15. Two first in-plane sway buckling modes of the examined frames. (**a**)—1st sway buckling mode; (**b**)—2nd sway buckling mode.

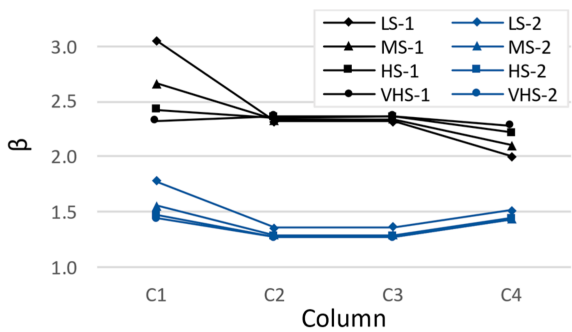


Figure 16. Buckling length factors determined from the 1st sway buckling mode for the 1st floor columns and the 2nd sway buckling mode for the 2nd floor columns.

It may be observed that the distribution of the buckling length factors between the columns of each storey correlates with that of the strut indexes of the columns (Figure 17). Strut index, given by Equation (13), is a parameter expressing the stress in a member with respect to its stiffness. A similar statement has been made in [32], where the authors affirmed that the buckling length factor is essentially a lateral stiffness coefficient.

$$\rho = L \cdot \sqrt{N_{Ed}/(E \cdot I)} \tag{13}$$

where L the member length, N_{Ed} the design axial force and $E\ I$ the flexural stiffness of the member in the buckling plane under consideration.

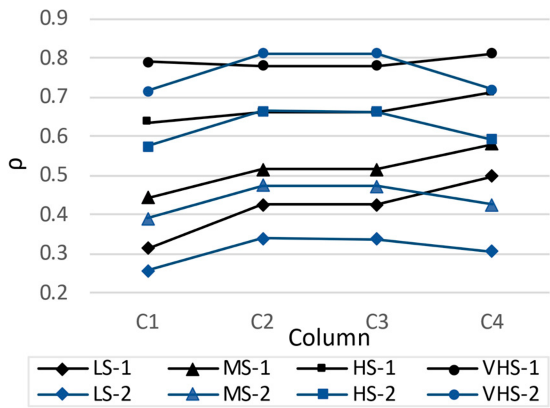


Figure 17. Strut indexes of the examined columns.

By comparing Figures 16 and 17, it is clear that the higher the strut index of a column in a storey the smaller its buckling length. Hence for both storeys of all frames, the internal columns of each storey have the same buckling length, always lower than that of the less stressed left external column, which has the highest buckling length among the columns of the storey. The right external column, being more stressed than the internal ones in the first storey has smaller buckling length than them, while the opposite applies for the second storey. This association can be explained by the interstorey interaction of the columns. In fact, sidesway buckling is a global phenomenon occurring not when the most stressed column reaches its sidesway buckling resistance, but when the total vertical loading on the frame becomes equal to the sum of the sidesway buckling resistances of the columns that provide lateral stiffness. To achieve this mechanism, the less stressed columns act as lateral restraints for the most stressed ones. So, even though they may have the same geometric properties and boundary conditions, the less stressed columns result in having higher buckling lengths than the most stressed ones in the storey. This mechanism, firstly explained by [33], has been reported in many works [4,30,31,34,35].

Furthermore, it may be observed that the differences between the buckling lengths within a storey decrease as the applied horizontal forces become smaller (from the LS to VHS frame). This is because the distribution of the axial forces between the columns of each storey becomes more uniform. The buckling length of the internal columns is almost identical among the frames, confirming that it does not depend on the magnitude of the axial force of a column, but on the distribution of the axial forces in the storey.

Finally, it has to be noticed that the buckllling lengths of the second storey columns are smaller than those in the first storey. This is mainly due to the different rotational boundary conditions. In fact, the first storey columns are pinned at their bases so their bottom ends are free to rotate, whereas the rotations at the top ends of the second storey columns are partially restrained by the adjoining beams.

The above observations give evidence that the buckling lengths calculated using the eigenvalue of the first sway buckling mode for the first floor columns and of the second one for the second floor columns are realistic. Usually, only the first buckling mode is considered critical for the examined system. However, this can lead to excessively large buckling lengths for the members that do not participate significantly in the buckling mechanism corresponding to the considered critical mode, as has already been reported by [30–32]. This is the case for the examined frames as can be realized by the shape of the two first sway buckling modes depicted in Figure 15. Actually, at the first mode only the first floor columns buckle sidesways, while the ones at the second floor practically just translate laterally. Conversely at the second mode, sidesway buckling takes place only in the second storey.

To confirm the validity of the employed methodology, the buckling lengths calculated according to it are compared to those obtained with the same approach but considering the first sway buckling mode as the critical one for the whole system and to those obtained with an analytical procedure [36] which is an improvement of the methodology provided by EN 1993-1-1 [1]. This procedure follows the isolated subassembly approach, where each column is examined individually and the contribution of the adjoining members to the rotational stiffness of its ends is taken into account approximately with equivalent springs.

Figure 18 presents the comparison of the buckling lengths resulted from the above-mentioned approaches. It is clear that the buckling lengths of the analytical procedure are close to those of the system buckling approach that correspond to the first sway buckling mode for the first storey and to the second one for the second storey. Actually, for the internal columns the agreement is very good, while for the external columns some discrepancies arise because the analytical procedure does not take into consideration the interstorey interaction of the columns. Finally, it has to be noticed that the calculation of the buckling lengths in both storeys based on the first sway buckling mode leads to two inconsistencies. More specifically, in contrast to what is theoretically expected, the buckling lengths in the second storey (a) are greater than those in the first one and (b) they seem to depend on the magnitude of the axial force, since they decrease as the difference between the axial forces in the two storeys decreases (from frame LS to VHS).

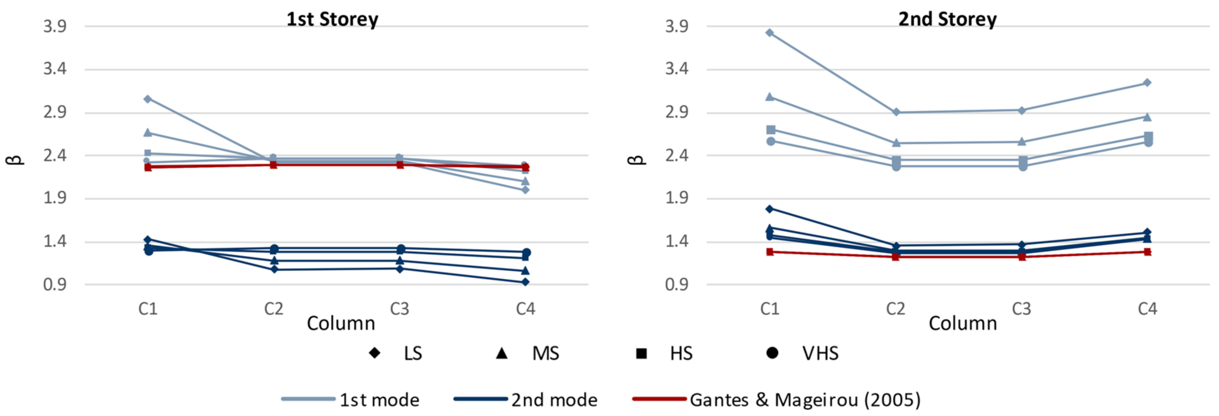


Figure 18. Comparison between effective lengths calculated with different approaches.

6. Conclusions

Five different methods of analysis and design which conform with EN 1993-1-1 provisions were applied to four plane frames of different slenderness ranging from relatively rigid ($\alpha_{cr} > 10$) to very slender ($\alpha_{cr} < 3$) structures. The accuracy of the methods was evaluated taking as reference the most accurate between them (GMNIA). The conclusions drawn from the investigations described in the previous paragraphs can be summarized as following:

1. The out-of-plane buckling dominates the behaviour of the examined frames. Their collapse mechanism involves inelastic flexural torsional buckling of their columns.
2. For the three more rigid frames ($\alpha_{cr} > 3$), all the EC3-permitted design methods, except for the GNIA-SD, are on the safe side.
3. The examined methods become less conservative as the slenderness of the structure increases. Hence, although they are all safe for the rigid frame, for the slenderest one ($\alpha_{cr} < 3$) all of them except for the General method are unsafe.

4. In slender structures ($\alpha_{cr} < 10$) the consideration of each type of nonlinearity separately but neglecting their interaction, i.e., the geometric nonlinearity in the analysis and the material nonlinearity through plastic cross-sectional verification, results in unsafe design.
5. The imperfections incorporated in the buckling curves are much lower than those proposed in Table 5.1 of EN 1993-1-1. If the imperfections extracted from the buckling curves are considered in GMNI Analyses, then all the methods that involve buckling verification are more conservative than GMNIA for all frames.
6. When the system buckling approach is adopted for the determination of the buckling lengths, the calculations should be based on the critical buckling mode for the storey that each column belongs to, namely the first buckling mode where this storey displays large drift.

The development of the next generation of Eurocode 3 [37] is in progress and incorporates some changes regarding both resistance and stability verifications as well as the magnitude of the imperfections to be considered for the design [38,39]. Thus, an extension of the present study according to the provisions of the next generation of Eurocode 3 would be an interesting update.

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Abbreviations

EC3	Eurocode 3
GMNIA	Geometrically and Materially Nonlinear Imperfection Analysis
GMNIA (Lbc)	Geometrically and Materially Nonlinear Imperfection Analysis (Local imperfections from buckling curves)
GNIA-SD	Geometrically Nonlinear Imperfection Analysis–Section Design
GNA-SMD	Geometrically Nonlinear Analysis–Section and Member Design
ASOA-SMD	Approximate Second Order Analysis–Section and Member Design
LA-SMDs	Linear Analysis-Section and Member Design (sway buckling length)
LA-SMDns	Linear Analysis-Section and Member Design (non-sway buckling length)
FB	Flexural Buckling
FTB	Flexural Torsional Buckling
LB	Local Buckling

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