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Cost Optimization of Reinforced Concrete Section According to Flexural Cracking

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Abstract: A series of distributed flexural cracks develop in reinforced concrete flexural elements under the working load. The control of cracking in reinforced concrete is an important issue that must be considered in the design of reinforced concrete structures. Crack width and spacing are influenced by several factors, including the steel percentage, its distribution in the concrete cross-section, the concrete cover, and the concrete properties. In practice, however, a compromise must be made between cracking, durability, and ease of construction and cost. This study presents the optimal design of a reinforced concrete cross-section, using the optimization method of mixed-integer nonlinear programming (MINLP) and the Eurocode standard. The MINLP optimization model OPTCON was developed for this purpose. The model contains the objective function of the material cost considering the crack width requirements. The crack width requirements can be satisfied by direct calculation or by limiting the bar spacing. Due to the different crack width requirements, two different economic designs of reinforced concrete sections were proposed. The case study presented in this study demonstrates the value of the presented optimization approach. A direct comparison between different methods for modelling cracking in reinforced concrete cross-sections, which has not been done before, is also presented.

Keywords: crack width; cost optimization; optimal design; mixed-integer non-linear programming

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1. Introduction

The study of cracking, the development of cracks under load, and the limitation of cracking must be considered in the design of reinforced concrete structures. Cracks form in flexural elements under applied loads. Since concrete has negligible tensile strength compared to steel, reinforcement is placed in the tension zone to resist the tensile force generated by the load. According to theory, crack width and crack spacing depend on the steel percentage, the distribution of steel reinforcement in the cross-section, the concrete cover, and the concrete properties. Several models have been developed for estimating the width and spacing of cracks in reinforced concrete flexural elements [1–4]. Numerical prediction of crack width and also crack propagation in concrete structures is presented in [5]. The crack width specifications requirements may be met by direct calculation, or by simply limiting the bar spacing. When calculations are made to predict maximum crack widths, they are based on the quasi-permanent combination of loads, and an effective modulus of elasticity of the concrete should be used to account for creep effects. The maximum allowable value for the crack width is given by the specifications [6,7]. Smaller crack widths are acceptable for important structural elements or in special cases where the structure retains water. The relationship between crack width and normalized reinforcement ratio, at serviceability limit state and ultimate limit state, has been analyzed, and it was confirmed that crack width is higher in beams with a low reinforcement ratio [8].

For optimal use of available materials and other resources, minimizing cost is the primary objective. The vast majority of works on cost optimization of concrete structures show significant savings [9–13]. The results show that it is necessary to conduct research on cost optimization of realistic structures [14–16]. The life cycle costs, mass, and construction costs of a floor structure, consisting of steel girders and a reinforced concrete slab, have also been analyzed using an optimization approach, but no information was provided on the crack widths in the concrete slabs [17]. Recently, the cracking modes of reinforced concrete slabs under applied bending moments have also been included in the optimal design of reinforced concrete slabs [18]. Various optimization algorithms were developed to increase computational efficiency and avoid local minima, both for individual structural elements and for the entire building. The reinforced concrete foundations of the building were optimized using both meta-heuristic algorithms [19–21] and deterministic algorithms [22,23]. Other important structural elements such as beams [24,25], slabs [26,27], and columns [28,29] were also optimized using deterministic and stochastic algorithms. By integrating ETABS-OAPI files into MATLAB programming, the complete 3D reinforced concrete structure was also optimized [30]. Mei and Wang [31] have provided a comprehensive review of previous research in structural optimization, including various objective functions, constraints, and algorithms. Reinforced concrete elements should be checked for ultimate and serviceability limit states, the latter including the calculation of crack width. The main weakness of previous research is that optimization models rarely consider crack width checks. Moreover, powerful machine learning methods are capable of accurately determining the properties of reinforced concrete, as Ben Seghier, et al. [32] have shown.

In order to improve the economic and mechanical performance of reinforced concrete sections, this study presents the optimization of material costs. The optimization was performed using the mixed integer nonlinear programming (MINLP) approach. For this purpose, the MINLP optimization model was developed. The model included the objective function of the material cost subjected to the crack width requirements. The crack width requirements were satisfied in the first case by direct calculation, and in the second case by limiting the bar spacing. In both cases, the cross-section with minimum material cost was determined. Since the optimization model contained integer variables, the obtained results provide an actual solution applicable in engineering practice. A direct comparison between different methods for modeling cracking in reinforced concrete cross-sections, which has not yet been carried out, will make it possible to assess whether the different methods lead to different solutions once the cross-sections are optimized. The main objective of this study was to explain the importance of the choice of crack width calculation method in optimization models for reinforced concrete design. However, it should be noted that the ultimate limit state conditions and the serviceability limit state conditions must be considered simultaneously, together with the crack width requirements in the optimal design of reinforced concrete slabs.

2. Mechanics of Flexural Cracking

The mechanics of flexural cracking can be illustrated by the behavior of a structural member subjected to a bending moment. A concrete slab, as shown in Figure 1, initially exhibits elastic behavior throughout the cross-section as the quasi-permanent bending moment M_{QP} is increased. Due to the increased load, the cross-section behaves nonlinearly. In addition, the distribution of the cracks is directly related to the spacing of the steel reinforcement.

According to specifications, the crack width and the crack spacing requirements can be met by direct calculation or by limiting the bar spacing. Both methods are summarized below.

2.1. Direct Calculation of Crack Width

According to Figure 1, the strain ε_1 located below the neutral axis for the distance y is calculated as:

$$\varepsilon_1 = \frac{y}{(d-x)} \cdot \varepsilon_s \tag{1}$$

where ε_s is defined as σ_s/E_s, and means the average strain in the main reinforcement; σ_s is the steel stress in the cracked section; and E_s is the modulus of elasticity of the steel reinforcement. The total width of all cracks over a unit length is equal to the strain per unit length, and is defined as:

$$\varepsilon_1 = \frac{y}{(d-x)} \cdot \frac{\sigma_s}{E_s} = \sum w \tag{2}$$

where the sum of all crack widths at distance y is defined as $\sum w$. The total number of cracks in the unit length of the concrete slab defines the width of the individual cracks. Including the average spacing (s_{rm}), the average crack width can be calculated:

$$w_{av} = \frac{\sum w}{\text{av.number of cracks}} = \frac{\varepsilon_1}{(1/s_{rm})} = s_{rm} \cdot \varepsilon_1 \tag{3}$$

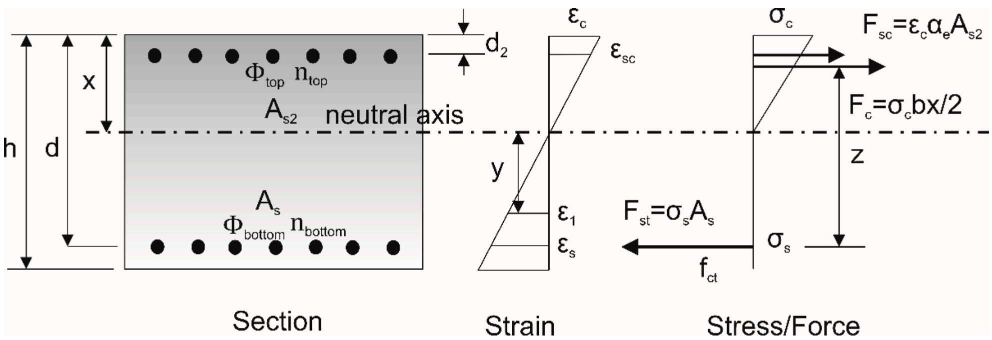


Figure 1. Cross-section of a reinforced slab, and corresponding strain diagram and stress block.

The maximum crack spacing, $s_{r,max}$, can be used to design the concrete slab, considering the maximum crack width, w_k. The design crack width on each plane defined by y in a structural member is given by:

$$w_k = s_{r,max} \cdot \varepsilon_1 = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm}) \tag{4}$$

where w_k is the design crack width; ε_{cm} is the mean strain of the concrete between cracks; and ε_{sm} is the mean strain of concrete, taking into account the effects of tensile stiffening of the concrete and shrinkage. The mean strain ε_{sm}, is less than the apparent value ε_1, and ($\varepsilon_{sm} - \varepsilon_{cm}$) is calculated by the equation:

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} \left(1 + \alpha_e \rho_{p,eff}\right)}{E_s} \geq 0.6 \frac{\sigma_s}{E_s} \tag{5}$$

where the tensile stress in the steel reinforcement σ_s is calculated in such a way that the cracked concrete section is taken into account. By including the factor k_t, the effect of load duration can also be taken into account (0.4 for long-term load, 0.6 for short-term load). Considering the most important factors affecting the crack width of a concrete section, the maximum crack spacing, $s_{r,max}$, is given in Equation (6) and is based on an empirical investigation:

$$s_{r,max} = 3.4 \cdot c + 0.425 \cdot k_1 \cdot k_2 \cdot \phi / \rho_{p,eff} \tag{6}$$

where ϕ is the size of the rebar (mm), or if different sizes of rebar are used, the average rebar size is used. The cover to the longitudinal reinforcement is defined by c, while k_1 takes into account the bond properties of the reinforcing bar (1.6 for plain bars, 0.8 for high bond), and k_2 the type of stress distribution, which can be assumed to be 0.5 for cracks due to bending. The effective reinforcement ratio $\rho_{p,eff}$ is:

$$\rho_{p,eff} = A_s / A_{c,eff} \tag{7}$$

where A_s is the area of reinforcement and $A_{c,eff}$ is an effective tension area of concrete.

When the maximum crack spacing exceeds 5·(c + ϕ/2), an upper bound on the crack width can be calculated by taking:

$$s_{r,max} = 1.3 \cdot (h - x) \tag{8}$$

2.2. Control of Cracking without Direct Calculation

By specifying a minimum area for steel reinforcement, and limiting the maximum clear spacing between reinforcing bars in the longitudinal direction, cracking due to the applied load is minimized. These limitations are shown in Table 1. This ensures that the maximum crack widths in the concrete do not exceed the limit value. The stress level (σ_s) calculation is required to determine the maximum spacing between bars. An acceptable approximation is to take σ_s as:

$$\sigma_s = \frac{f_{yk}}{1.15} \cdot \frac{G_k + 0.3Q_k}{(1.35G_k + 1.5Q_k)} \frac{1}{\delta} \tag{9}$$

where f_{yk} is the characteristic strength of the reinforcement and δ is the ratio of the moment distribution.

Table 1. Limitation of maximum bar spacing as a function of tensile stress in steel.

Steel Stress σ_s **(N/mm²)**	**Maximum Bar Spacing** s_{lim} **(mm)** $w_k = 0.3$
160	300
200	250
240	200
280	150
320	100
360	50

To make sure that the maximum crack widths in the concrete do not exceed 0.3 mm, the relationship between the maximum clear bar spacing and the stress level is given as follows:

$$s_{lim} = 500 - 1.25 \cdot \sigma_s \tag{10}$$

3. Discrete Optimization Model OPTCON

To obtain the most rational material cost for the concrete slab, cost optimization was proposed. In contrast to the engineering practice, in which the optimization of the design parameters is carried out by some iterative successive calculation tests, in this study the exact optimization of the parameters of the reinforced concrete cross-section is discussed on the basis of mathematical programming methods [33,34].

3.1. MINLP Problem Formulation

An optimization approach of mixed integer nonlinear programming (MINLP) was used for our method. This was done because the concrete slab problem contains integer variables and nonlinear relations in both the objective function and the constraint functions, which are defined by (in)equality equations. The mathematical representation of the MINLP optimization synthesis can be expressed in the following mathematical form:

$$\begin{gathered} Min\ z = \boldsymbol{c}^T \boldsymbol{y} + f(\boldsymbol{x}) \\ \text{subjected to}: \\ g(\boldsymbol{x}) \leq 0 \\ h(\boldsymbol{x}) = 0 \\ \boldsymbol{By} + C\boldsymbol{x} \leq \boldsymbol{b} \\ \boldsymbol{x} \in X = \left\{ \boldsymbol{x} \middle| \boldsymbol{x} \in \boldsymbol{R}^n, \boldsymbol{x}^{Lo} \leq \boldsymbol{x} \leq \boldsymbol{x}^{Up} \right\} \\ \boldsymbol{y} \in Y = \{0,\ 1\}^m \end{gathered}$$

In the above mathematical formulation of the MINLP problem, the vector of continuous variables is defined as $\boldsymbol{x}$ (within the compact set X) and the vector of discrete variables is defined as $\boldsymbol{y}$ (within the compact set Y). The objective function as well as the inequality and equality constraints are determined by the functions $f(\boldsymbol{x})$, $g(\boldsymbol{x})$, and $h(\boldsymbol{x})$.

The conditions that must be satisfied for discrete choices and structural configurations of all alternatives are formulated as $\boldsymbol{By} + C\boldsymbol{x} \leq \boldsymbol{b}$. The objective function z is divided into two parts. The first part represents the fixed costs ($\boldsymbol{c}^{\mathrm{T}}\boldsymbol{y}$), and the second part represents the costs that depend on certain variables contained in $f(\boldsymbol{x})$.

In the context of concrete slabs, variables include dimensions, cross-section characteristics, materials, stresses, etc.; binary variables are also used to select standard cross-sections. (In)equality constraints and the limits of variables formulate an exact system of load, resistance, and stress functions from mechanical analysis. In this study, an objective function was proposed, to minimize the material cost of the concrete slab.

3.2. MINLP Optimization Model

In accordance with the (MINLP) problem formulation, a MINLP optimization model OPTCON was developed. Since the model was developed in a form that allowed for the use of different input parameters, the optimization of the system could be performed for different material costs. The General Algebraic Modeling System (GAMS), which includes a language compiler and MINLP solver, was used to translate the optimization problems into computer code [35]. The proposed optimization model includes input data (constants), variables, and the cost objective function of the reinforced concrete section, subject to defined mechanical and design nonlinear and linear constraints.

3.2.1. Input Data

The input data were design and economic data (constants) for optimization. The design data (constants) included the compressive strength of the concrete f_{ck} (MPa), the tensile strength of the steel f_{yk} (MPa), the applied moment M (kNm), the maximum bar spacing s_{max} (mm), the minimum bar spacing s_{min} (mm), the concrete cover c_{cov} (mm), the modulus of elasticity of the concrete E_{cm} (GPa), the modulus of elasticity of steel E_s (GPa), the modular ratio α_{e} (-), the density of steel ρ (kg/m^3), the mean concrete strength at cracking f_{cm} (MPa), the mean concrete tensile strength f_{ct} (MPa), the design crack width w_k (mm), the cost of concrete C_{con} (€/m^3), and the cost of steel C_{steel} (€/kg).

3.2.2. Variables

The thickness of the concrete slab h (mm), the bottom bar diameter φ_{bottom} (mm), the top bar diameter φ_{top} (mm), the number of bottom bars n_{bottom} (-), and the number of top bars n_{top} (-) are given as variables in the optimization model OPTCON; see Figure 1. In order to obtain a reinforced concrete cross-section that is applicable in practice, discrete values of the variables were assigned; see Table 2.

Table 2. The values of discrete variables.

Variable	Allowable Discrete Values
h (mm)	100, 150, 200, 250, 300, 350, 400
ϕ_{bottom} (mm)	8, 10, 12, 14, 16, 18, 20
ϕ_{top} (mm)	8, 10, 12, 14, 16, 18, 20
n_{bottom} (-)	4, 5, 6, 7, 8, 9, 10
n_{top} (-)	4, 5, 6, 7, 8, 9, 10
h (mm)	100, 150, 200, 250, 300, 350, 400

3.2.3. Cost Objective Function

The objective function included the material cost of the reinforced concrete section COSTS (EUR); see Equation (11):

$$\mathrm{COSTS} = (b \cdot h \cdot 1)/1000000 \cdot C_{con} + \left(\pi \cdot {\phi_{bottom}}^2 \cdot n_{bottom}/4 + \pi \cdot {\phi_{top}}^2 \cdot n_{top}/4\right) \cdot 1/1000000 \cdot \rho_{steel} \cdot C_{steel} \tag{11}$$

The objective function COST included the thickness of the concrete slab, the diameter of the bottom bars, the diameter of the upper bars, the number of bottom bars, and the number of top bars, which depend on the (variable) cost. The material costs for concrete and steel depend on the material properties. The coefficients C_{con} and C_{steel} were therefore defined.

3.2.4. Mechanical Inequality Constraints

The study was concerned with the maximum crack width that can be expected to be exceeded with an acceptably low probability. For design purposes, the maximum crack width, w_k, can be related to the maximum spacing, $s_{r,max}$.

According to the Eurocode standards, three conditions in the form of three inequality constraints (Equations (12)–(14)) needed to be defined and entered into the OPTCON optimization model:

- Condition 1: The maximum crack width must be limited to an acceptable value.
- Condition 2: The compressive stress in the concrete must be limited to the design compressive concrete.
- Condition 3: The design tensile stress in the steel must be limited to the design tensile strength of the steel.

In this way, Condition 1 is verified by Equation (12), according to which the design crack width w_k (mm) must not exceed the maximum crack width w_{lim} (mm):

$$w_k \leq w_{lim} \tag{12}$$

where:

$$w_k = s_{\mathrm{r,max}} \cdot (\varepsilon_{\mathrm{sm}} - \varepsilon_{\mathrm{cm}}) \tag{12a}$$

Condition 2 is satisfied if the calculated design normal stress in concrete σ_{c} (MPa) is less than the design concrete compressive strength f_{cd} (MPa); see Equation (13):

$$\sigma_c \leq f_{cd} \tag{13}$$

where:

$$\sigma_{cd} = \frac{M}{\left(bx_c\left(d - \frac{x_c}{3}\right)/2 + (\alpha_e - 1)A_{s2}(d - d_2)(x_c - d_2)/x_c\right)} \tag{13a}$$

$$x_c = \frac{\left(-A_s\alpha_e - A_{s2}(\alpha_e - 1) + \left((A_s\alpha_e + A_{s2}(\alpha_e - 1))^2 - 2b(A_s\alpha_e d - A_{s2}d_2(\alpha_e - 1))\right)^{\frac{1}{2}}\right)}{b} \tag{13b}$$

$$f_{cd} = 0.6 \cdot f_{ck} \tag{13c}$$

When Condition 3 is considered, the calculated design stress in the tensile steel σ_s (MPa) is less than a design steel tensile strength f_{yd} (MPa); see Equation (14):

$$\sigma_s \leq f_{yd} \tag{14}$$

where:

$$\sigma_s = \frac{\sigma_{cd} \cdot \alpha_e \cdot (d - x)}{x} \tag{14a}$$

$$f_{yd} = 0.8 \cdot f_{yk} \tag{14b}$$

When controlling cracking without direct calculation, Equation (15) limits only the spacing of the bars:

$$\frac{b - n_{bottom} \cdot \phi_{bottom}}{n_{bottom}} \leq 500 - 1.25 \frac{f_{yk}}{1.15 \cdot 1.35} \cdot \frac{1}{\delta} \tag{15}$$

3.2.5. Design (in)Equality Constraints

The design (in)equality constraints determined the dimensions of the concrete slabs that needed to be calculated for the different discrete values of the variables. The reliability of the system depends on several geometric parameters, such as the thickness of the concrete slab h (mm), the bottom bar diameter φ_{bottom} (mm), the top bar diameter φ_{top} (mm), the number of bottom bars n_{bottom} (-), and the number of top bars n_{top} (-).

The thickness of the concrete slab h (mm) is limited; see Equation (16):

$$h^{LO} \leq h \leq h^{UP} \tag{16}$$

The bottom bar diameter φ_{bottom} (mm) varies between its lower and upper limits; see Equation (17):

$$\phi_{bottom}{}^{LO} \leq \phi_{bottom} \leq \phi_{bottom}{}^{UP} \tag{17}$$

The constraint (Equation (18)) defines the limits for the top bar diameter φ_{top} (mm):

$$\phi_{top}{}^{LO} \leq \phi_{top} \leq \phi_{top}{}^{UP} \tag{18}$$

The number of bottom bars n_{bottom} (-) is defined by Equation (19):

$$n_{bottom}{}^{LO} \leq n_{bottom} \leq n_{bottom}{}^{UP} \tag{19}$$

The number of top bars n_{top} (-) is defined by Equation (20):

$$n_{top}{}^{LO} \leq n_{top} \leq n_{top}{}^{UP} \tag{20}$$

4. Numerical Example

To interpret the proposed optimization approach, this study presents a numerical example of MINLP optimization of the material cost of a concrete slab. The optimization model presented above was developed in a general form so that an optimal design for a concrete slab can be determined for any project data (e.g., bending moment, concrete of strength class, steel strength, etc.). In order to obtain the optimal material cost and designs of the concrete slab, MINLP optimization was performed for the design parameters defined in Table 3. The concrete slab was subjected to a uniform moment of 100 kNm. Concrete of strength class C30/37 and steel S500 were used. The nominal cover of the concrete was 25 mm. The optimum design variables and the calculated values can be found in Table 4. The optimum reinforced concrete cross-section is shown in Figure 2. The material cost for the reinforced concrete cross-section is lower if the direct calculation of the crack width is used. In this numerical example, the material cost was reduced by 3%. It should be noted that both reinforced concrete cross-sections are optimal, but they are subject to different crack width and crack spacing requirements.

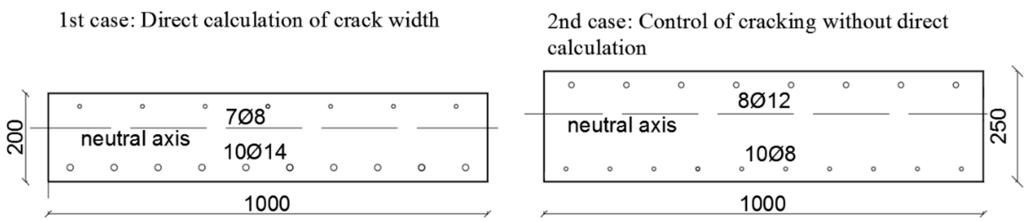


Figure 2. Optimum design cross-section of a concrete slab for different crack spacing requirements (direct calculation or by limiting bar spacing).

Table 3. Input data into the optimization model OPTCON.

Input Name	Symbol	Value
Compressive strength	f_{ck}	30 MPa
Mean concrete strength at cracking	$f_{cm,t}$	38 MPa
Mean concrete tensile strength	$f_{ct,eff}$	2.9 MPa
Modulus of elasticity of concrete	E_{cm}	32.8 GPa
Yield stress of steel	f_{yk}	500 GPa
Modulus of elasticity of steel	E_s	200 GPa
Density of steel	ρ_s	7850 kg/m^3
Width of concrete section	b	1000 mm
Applied moment	M	100 kNm
Cover of concrete	c	25 mm
Modular ratio	α_e	18.27
Final creep coefficient	φ	2.0
Minimum bar spacing	s_{min}	60 mm
Maximum bar spacing	s_{max}	200 mm
Moment redistribution	δ	1
Cost of concrete	C_{con}	95 €/m^3
Cost of steel	C_{steel}	1.0 €/kg
1st case: Direct calculation of crack width	w_{lim}	0.3 mm
2nd case: Control of cracking without direct calculation	s_{lim}	500–1.25·σ_s

Table 4. Optimal design variables and calculated crack width.

Optimal Design Variables	Symbol	1st Case: Direct Calculation of Crack Width	2nd Case: Control of Cracking without Direct Calculation
Thickness of concrete slab	h (mm)	200	250
Bottom bar diameter	φ_{bottom} (mm)	14	8
Top bar diameter	φ_{top} (mm)	8	12
Number of bottom bars	n_{bottom} (-)	10	10
Number of top bars	n_{top} (-)	7	8
Calculated values			
Fully cracked neutral axis depth	x_c (mm)	78.483	82.516
Concrete stress	σ_c (MPa)	17.897	12.608
Stress in tension steel	σ_s (MPa)	372.951	386.589
Effective tension area	$A_{c,eff}$ (mm^2)	38,966.237	55,325.476
Area of tension steel	A_s (mm^2)	1539.380	502.655
Area of compression steel	A_{s2} (mm^2)	351.858	904.779
Steel-to-concrete ratio ($A_s/A_{c,eff}$)	$\rho_{p,eff}$ (-)	0.040	0.009
Max. final crack spacing	$s_{r,max}$ (mm)	145.245	217.730
Average strain for crack width	$\varepsilon_{sm} - \varepsilon_{cm}$ (µstrain)	1611.973	1188.591
Calculated crack width	w_k (mm)	0.234	0.259
Material costs	$COSTS$ (€/m^2)	33.846	34.798

5. Parametric Analysis of an Optimally Designed Reinforced Concrete Slab

A series of optimizations of a reinforced concrete slab subjected to the crack width limit of 0.3 mm was performed for a combination of two different parameters, such as different values of the applied bending moment and strength classes of the concrete. Figure 3 shows the optimum thickness of the concrete slab h (mm) and the total area (compressive and tensile) of the steel reinforcement $A_{s,tot}$ (mm^2) for different bending moments M (kNm). The results of the optimization model, based on the direct calculation of the crack width, are shown in Figure 3a, while the results of the optimization model controlling the cracking without direct calculation are shown in Figure 3b:

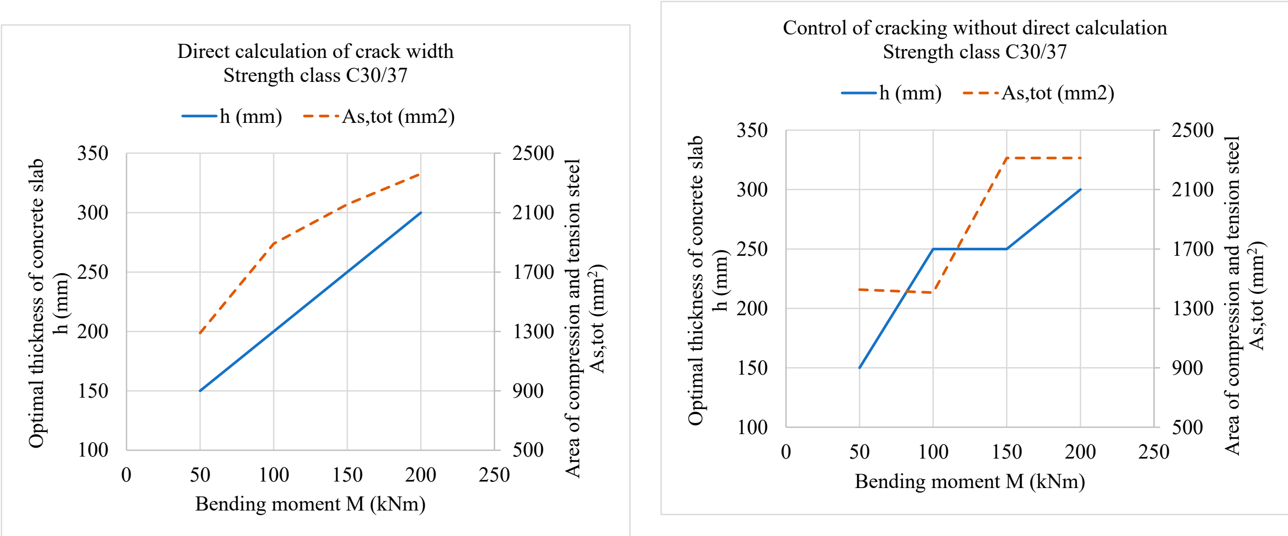


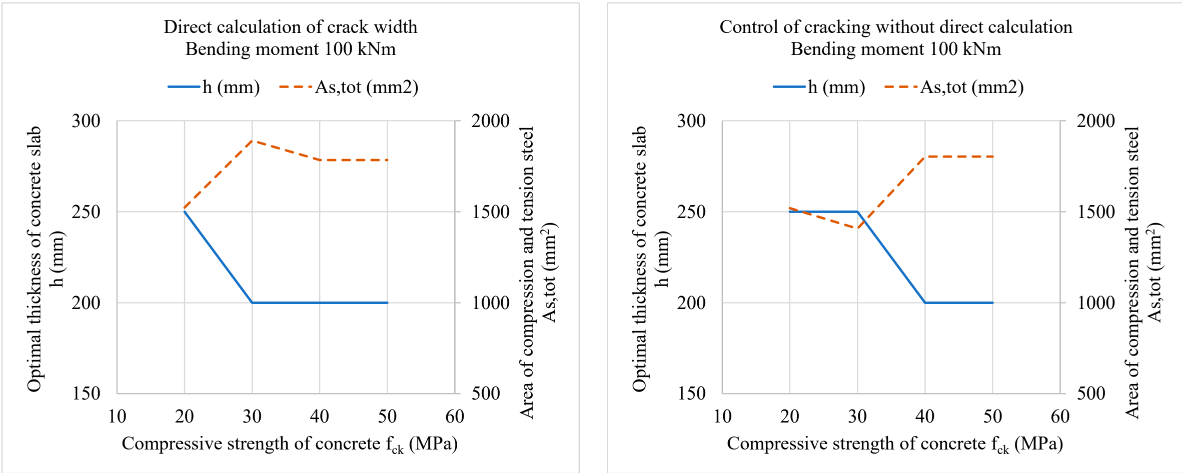
Figure 3. Optimum thickness of concrete slab (concrete strength class C30/37) and optimum area of steel reinforcement for different bending moments: (**a**) Direct calculation of crack width; (**b**) Control of cracking without direct calculation.

The input data for the parametric analysis of an optimally designed reinforced concrete slab consisting of different concrete strength classes are given in Table 5. Figure 4 shows the optimal thickness of the concrete slab h (mm) and the total area (compressive and tensile) of the steel reinforcement $A_{s,tot}$ (mm^2) for different concrete strength classes at a bending moment of M = 100 kNm. The results of the optimization model based on the direct calculation of the crack width are shown in Figure 4a, while the results of the optimization model controlling the cracking without direct calculation are shown in Figure 4b.

Table 5. Input data and optimum design of concrete slabs for different concrete strength classes at a bending moment load of 100 kNm.

Input Name	Design Data 1	Design Data 2	Design Data 3	Design Data 4
M (kNm)	100	100	100	100
Strength classes	C20/25	C30/37	C40/50	C50/60
f_{ck} (MPa)	20	30	40	50
f_{cm} (MPa)	28	38	48	58
f_{ct} (MPa)	2.2	2.9	3.5	4.1
E_{cm} (GPa)	30.0	32.8	35.2	37.3
C_{con} (€/m^3)	80	95	110	125
1st case: Direct calculation of crack width				
h (mm)	250	200	200	200
φ_{bottom} (mm)	12	8	10	10
φ_{top} (mm)	10	14	14	14
n_{bottom} (-)	5	10	7	7
n_{top} (-)	10	7	9	9
$COSTS$ (€/m^2)	31.961	33.846	36.008	39.008
2nd case: Control of cracking without direct calculation				
h (mm)	250	250	200	200
φ_{bottom} (mm)	12	12	12	12
φ_{top} (mm)	8	8	10	10
n_{bottom} (-)	10	10	10	10
n_{top} (-)	9	8	9	9
$COSTS$ (€/m^2)	31.936	34.798	36.156	39.156

The result of the parametric analysis also shows that expensive concrete with higher strength increases the cost of the concrete slab (see, Figure 5), even if a smaller slab thickness is required for the same bending moment. Figure 5 shows results based on a direct calculation of the crack width method, which was limited to w_{lim} = 0.3 mm.



(**a**) (**b**)

Figure 4. Optimal thickness of concrete slab built from different strengths of concrete, and optimal area of steel reinforcement at bending moment of 100 kNm: (**a**) Direct calculation of crack width; (**b**) Control of cracking without direct calculation.

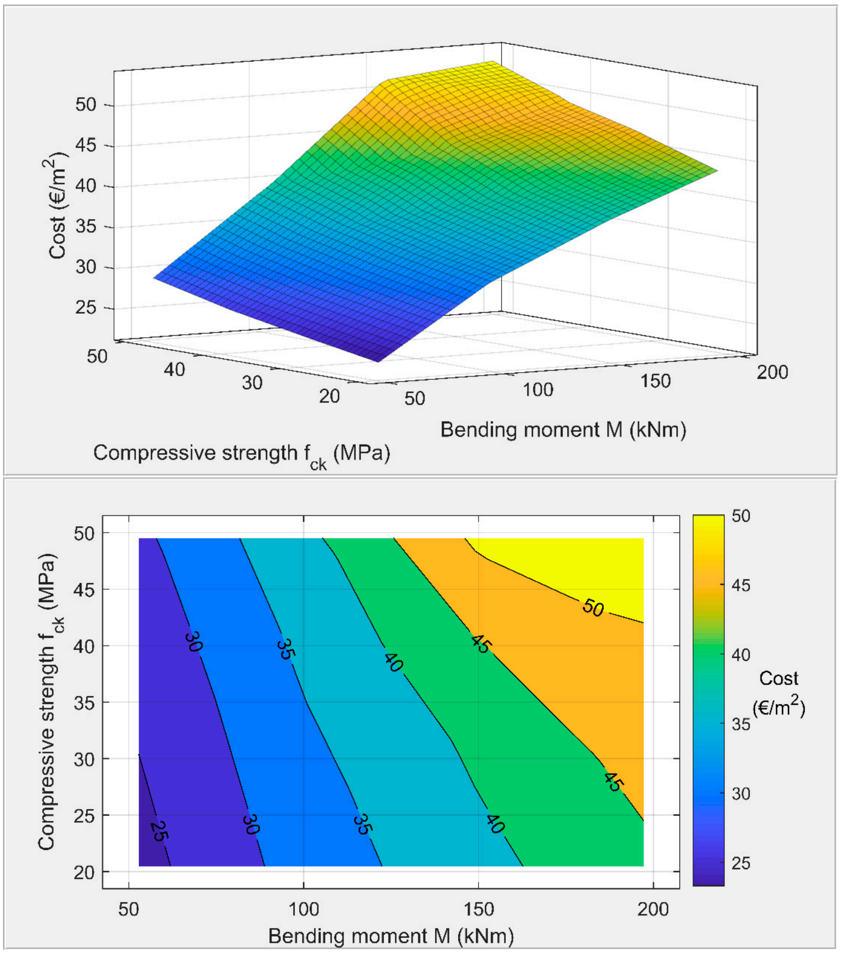


Figure 5. The relationship between the optimum cost of a concrete slab, the bending moment, and the compressive strength class of the concrete.

6. Conclusions

This study presents the material cost optimization of a reinforced concrete section. The optimization was performed using the mixed integer nonlinear programming (MINLP) approach. For this purpose, the MINLP optimization model OPTCON was presented, and the computer code was developed in the language compiler GAMS to solve the MINLP optimization problem. A deterministic optimization algorithm such as MINLP is faster and more efficient than other population-based algorithms in terms of convergence; population-based algorithms are more robust and can solve different types of optimization problems. The model includes the material cost objective function, which was subjected to mechanical and design constraints. Since the model was developed in a form that allowed for the use of different input parameters, the optimization of the system was performed for different material costs and different design parameters. In this way, the optimum design parameters for reinforced concrete slabs were determined, in which the conditions for crack width and crack spacing specified in the Eurocode standards were fully utilized without further resistance reserves. The generalized reduced gradient method was applied. The task of each optimization was to find the minimum material cost along with the design variables such as the thickness of the concrete slab, the bottom bar diameter, the top bar diameter, the number of bottom bars, and the number of top bars. The material cost for the concrete section was lower when the direct calculation of the crack width was used instead of limiting only the bar spacing. A direct comparison between different methods for modeling cracking in reinforced concrete cross-sections, which has not been done before, is also presented. In the case study presented, there was a small difference (by 3%) in the material cost of the reinforced concrete cross-section when the direct crack width calculation was used instead of just limiting the maximum bar spacing, but a much smaller reinforced concrete slab cross-section (by 25%) was optimal, which can be an important factor when constructing multi-story buildings. In the parametric analysis, the effects of the bending moment and the concrete strength class on the optimal design of the reinforced concrete slab were studied. In reviewing the literature on optimization models for reinforced concrete elements, it was found that crack width is rarely considered. Therefore, this study highlights this problem, and compares different methods for calculating crack width based on an optimization approach. While this study focuses only on crack width requirements, further research is needed to include all ultimate and serviceability limit states, to analyze the entire reinforced concrete structure and its optimal design. Moreover, not only the material costs but also the entire construction activity should be included in the optimization process, to demonstrate the applicability of the optimization models. The results obtained in this work are based on the Eurocode standard; the use of other specifications for crack calculation should also be investigated.

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