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Estimating the Reliability and Sensitivity of a Public Address System Through the Markov Decision Process

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Abstract: This research aims to evaluate the reliability indices of a public address system (PAS) through the Markovian approach. Many organizations and businesses use this system to address or broadcast a message or pass on important information to the huge gatherings assembled at big premises. This system is the nerve of these organizations and businesses. The major components of a PAS are a microphone, mixer, amplifier, and speaker. These components should work in harmony with one another to execute the intended task. Any failure in these components leads to big issues for the public, and they may miss very important information. Therefore, the reliability assessment of this system is of utmost importance. The authors used the Markovian decision process to model the PAS by analyzing the various failure rates and repairs of the components. The explicit expressions for reliability, availability, and MTTF have been obtained for clear understanding about the PAS behavior with time as well as different failures. The sensitivity analysis of reliability is performed as well to determine the critical components of the system. The obtained results show that the reliability of the PAS at 2000 operated hours is 0.8. Also, the finding reflects that the PAS reliability is much sensitive with the failure rate of microphone, mixer, and amplifier.

Keywords: public address system; microphone; mixer; amplifier; reliability indices; Markov process

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1. Introduction

In the present scenario, a public address system (PAS) is an integral part of effective communication. It plays a crucial role in many places, especially where critical communication is needed for providing information to the mass gathering. It is used to amplify the voice of a human, pre-recorded/live audio, to effectively broadcast a message to a group of people at different places like railway stations, airports, schools, colleges, public events, military operations, and many more (some of the applications are listed in Figure 1).

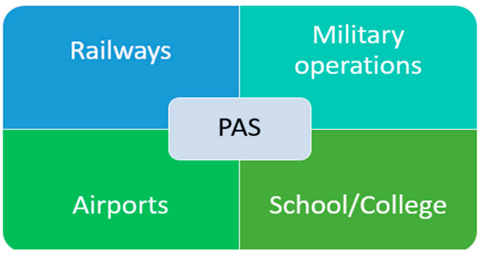

Figure 1. Some applications of a PAS.

A PAS consists of several components; the major components are a microphone, mixer, amplifier, and speakers. These components are interconnected to each other in a mixed configuration to perform the intended task. Also, the overall performance of a PAS is the result of the cumulative performance of its different components, and a major/minor failure in a component will degrade the performance of it. PAS converts acoustic energy to electric energy. This process starts with a human voice, either pre-recorded or live. Input is given to the system through a microphone. The microphone converts sound energy into electrical energy. The mixer is the central unit of the PAS. It allows for adjusting the volume levels, tone (treble and bass), and other audio settings for connected input devices, such as microphones, music instruments, or audio sources. The audio amplifier takes weak electrical signals from the mixer and boosts them up to a level that can power the speaker. It increases the audio signal's amplitude to deliver sufficient power to the speakers to produce audible sound. Speakers are the output devices of the PAS. They convert the amplified electrical signals back into sound waves that can be heard by the audience. The systematic flow diagram of a PAS is shown in Figure 2.

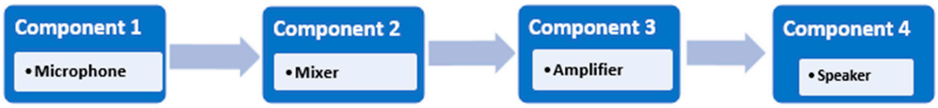

Figure 2. Flow diagram for a PAS.

In the literature, sound research has been carried out for the performance analysis of different real-time systems, including communication systems [1], IOT-based systems [2–5], as well as industrial systems [6,7]. When someone is talking about the performance analysis of a system, she/he means how much the system is reliable, for how much time the system will be available, how the system's MTTF behaves with time, how the system reacts with the failure and repair of its various components, and many more. Ryu and Kim [8] investigated a PAS under different failures through the redundancy operation method, which makes the PAS stable and cost-optimal during operation. Cohen et al. [9] conducted a survey in an international airport in the Philippines, which was oriented to evaluating the performance of a PAS in terms of audibility, intelligibility, and signal-to-noise ratios. The results reflect that only 42% of individuals are able to recall the latest announcement. Also, signal-to-noise ratios of 8.58 dBA as opposed to the recommended minimum of 10 dBA were found. The speech transmission index for the PAS was calculated for SNRA in the range of 5 db to 15 db [10]. Dziechciński [11] presented a study in which the effects of distortion of an amplifier on speech transition through PAS were discussed.

Kumar et al. [12] investigated a wireless sensor network (WSN) by considering its different components and their interconnection in the system. A mathematical model has been developed by using different failures and repairs of the WSN to find the various reliability measures for the same. Farahani et al. [13] explained the concept of system reliability in Markov and semi-Markov modeling. Patel et al. [14] presented an integrated AHP-RSM-PHM approach to calculate reliability indices of cutting tools used in a production plant. The author also optimized the number of cutting tools for a machine. Li et al. [15] presented a reliability modeling method for man-machine systems by using event trees and fault trees with probable common cause failure. The proposed approach is also validated.

The Markov decision process, which is a special case of stochastic process, is presented in the literature [16–18] as a great tool to model a real-time system to understand the behavior of different reliability measures of the system. The mathematical modeling of a system can be carried out by considering the different subcomponents, their interconnections, as well as the occurrence of different failures and repairs in the system.

1.1. Problem Description

The associated literature related to PAS reflected that it is utmost needed that the PAS must be highly reliable along with reasonably low MTTF, which has not been carried out in the past. So, in the present study, the author proposed a study that is focused on performance analysis of a public address system by incorporating the concept of the Markov process. The main components of the system that are considered for the study are the microphone, mixer, amplifier, and speaker. Any failure in the system's components leads to either the failure of the system or the degradation of the system, which may cause huge trouble in broadcasting the information. In this problem, an area of 400 m is covered. Two speakers are installed at 100 m and 300 m distances to convey the announcement. If the first speaker fails, then people standing in the first 200 m cannot properly hear the announcement, and if the second speaker fails, then people standing at 200–400 m cannot hear the announcement properly. Considering the different failures and repairs, that can occur during operation and interconnection of components, a mathematical model has been developed to find out the different associated performance measures, e.g., reliability, availability, MTTF, and profit function of the PAS.

1.2. Contribution and Novelty

The above introduction provides concrete information about a public address system's needs, its various uses, and the literature related to it. It can be observed from the literature that the performance analysis of a PAS, in terms of its reliability, availability, MTTF, and sensitivity analysis, has not been performed so far. So, here, the authors propose a mathematical modeling approach through Markov process by considering a PAS's major components and their interconnection to evaluate the various performance measures of the PAS, which makes this work novel. The presented work outlines the following contribution to enrich the literature on PAS reliability.

- Time-dependent availability;
- Time-dependent reliability;
- Mean time to failure (MTTF);
- Sensitivity Analysis;
- Expected Profit function.

The proposed approach used the concept of the Markov process along with the concept of conditional probability to evaluate the reliability measures of the PAS. In the literature, PASs were investigated using the redundancy operation method [8] and also discussed by using some surveys in terms of audibility, intelligibility, and signal-to-noise ratios [9]. The advantage of the proposed methodology over existing methodologies is that it provides a detailed analysis of different reliability measures of a PAS, which can be taken as a reference to improve the performance of a public address system.

2. Notations and State Description of the Public Address System

This section gives a brief description of the different states (Table 1) and notations (Table 2) used in the present study.

Table 1. States descriptions.

States	Description
S_0 :	The system is in a perfect working state as all the components are in good working condition.
S_1 :	The system has reached a failed state because the microphone has malfunctioned.
S_2 :	The system has failed due to a malfunction in the mixer.
S_3 :	The system has failed because the amplifier is malfunctioning.
S_4 :	The system's state has degraded as a result of a speaker malfunction.
S_5 :	The system is in a failed state due to the failure of the microphone after the failure of a speaker.
S_6 :	The system has entered a failed state following the malfunction of the mixer subsequent to a speaker failure.
S_7 :	The system is in a failed state due to the failure of the amplifier after the failure of a speaker.
S_8 :	The system is in a failed state due to the failure of another speaker after the failure of the first speaker.

Table 2. Notations.

Notation	Description
t :	Time variable
s :	Laplace transformation variable
$P_i(t)$:	Probability of the system being in the state S_i, $i = 0, 1, \ldots, 8$
$\overline{P}_i(s)$:	Laplace transformation of $P_i(t)$
λ_{mic} :	Represents the failure rate of the microphone
λ_{mix} :	Represents the failure rate of the mixer
λ_{amp} :	Represents the failure rate of the amplifier
λ_{sp} :	Represents the failure rate of a speaker
μ_{mic} :	Represents the repair rate of the microphone
μ_{mix} :	Represents the repair rate of the mixer
μ_{amp} :	Represents the repair rate of the amplifier
μ_{sp} :	Represents the repair rate of a speaker
$Av(t)$	Time-depedent availability of PAS
$R(t)$	Time-depedent reliability of PAS
$E_P(t)$	Expected profit function

3. State Transition Diagram of the Public Address System

During operation, a PAS can be in different states based on the working/failure of its components. A transition state diagram (Figure 3) depicts the different states and their interconnection in a systematic order. Initially, when the PAS is working with all of its components in good working order, it is represented in state S_0. It represents the probability that the PAS is working with full efficiency at a given time "t". Now, from state S_0, the PAS will move to state S_1, S_2, S_3 and S_4 when the failure of the microphone, mixer, amplifier, or speaker occurs, respectively. Since S_1, S_2, S_3 are filed states, no further state change is allowed before the maintenance team fixes the issue. On the other hand, S_4 is a degraded state, in this state, the voice intensity level is low and the announcement cannot be properly heard (only those who are near the speaker's broadcasting range can hear the announcement properly) it is obtained after the failure of a speaker and can further move to states S_5, S_6, S_7, S_8 when a microphone, mixer, amplifier, or speaker failure occurs.

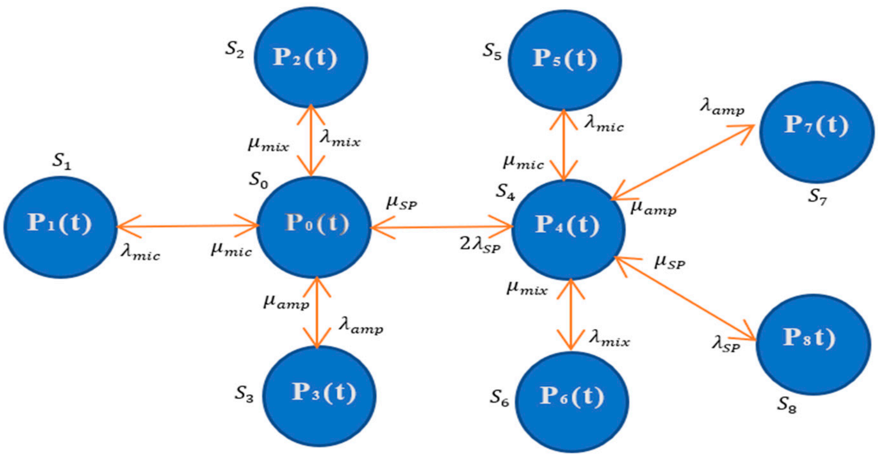

Figure 3. State transition diagram of a public address system.

4. Material and Methods

4.1. Mathematical Modeling and Methods

On the foundation of the above state transition diagram, the Kolmogorov-Chapman differential Equations (1)–(10) are developed and solved using Laplace transformation in the next section of the paper.

4.2. Components of the Public Address System

Microphone: The microphone model used in this analysis is a Shure BETA 58A. This microphone converts acoustic energy to electrical energy.

Mixer: The mixer is the central unit of the PAS. It allows you to adjust the volume levels, tone (treble and bass), and other audio settings for connected input devices, such as the microphone, musical instruments, or audio sources. The mixer model used in this analysis is an EPM8.

Amplifier: The audio amplifier takes weak electrical signals from the mixer and boosts them to a level that can power the speaker. It increases the audio signal's amplitude to deliver sufficient power to the speakers to produce audible sound. The amplifier model used in this analysis is the PARASOUND Zphono-MM/MC preamplifier with USB.

Speakers: Speakers are the output device of the PAS. They convert the amplified electrical signals back into sound waves that can be heard by the audience. The amplifier model used in this analysis is a two-way coaxial ceiling loudspeaker for EN54-24 applications.

4.3. Mathematical Formulations and Solution of the Problem

Based on the state transition diagram, the following set of Chapman-Kolmogorov differential equations, Equations (1)–(10), are generated, which govern the present mathematical model. Letting the system make the transition at time $t + \Delta t$ and letting $\Delta t \rightarrow 0$, the following set of differential equations is obtained:

$$P_0(t+\Delta\, t) = (1-\lambda_{mic}\Delta t)(1-\lambda_{mix}\Delta t)\left(1-\lambda_{amp}\Delta t\right)\left(1-2\lambda_{sp}\Delta t\right) \\ +\mu_{mic}P_1(t)\Delta t + +\mu_{mix}P_2(t)\Delta t + +\mu_{amp}P_3(t)\Delta t + \mu_{sp}P_4(t)\Delta t$$

$$\tfrac{P_0(t+\Delta\, t)-P_0(t)}{\Delta\, t} + \left(\lambda_{mix}+\lambda_{mic}+\lambda_{amp}+2\lambda_{sp}\right) = \mu_{mic}P_1(t) + \mu_{mix}P_2(t) \\ +\mu_{amp}P_3(t) + \mu_{sp}P_4(t)$$

$$lim_{\Delta t\rightarrow 0}\tfrac{P_0(t+\Delta\, t)-P_0(t)}{\Delta\, t} + \left(\lambda_{mix}+\lambda_{mic}+\lambda_{amp}+2\lambda_{sp}\right) = \mu_{mic}P_1(t) + \mu_{mix}P_2(t) \\ +\mu_{amp}P_3(t) + \mu_{sp}P_4(t)$$

$$\left[\tfrac{d}{dt} + \lambda_{mic} + \lambda_{mix} + \lambda_{amp} + 2\lambda_{sp}\right]P_0(t) = \mu_{mic}P_1(t) \\ +\mu_{mix}P_2(t) + \mu_{amp}P_3(t) + \mu_{sp}P_4(t) \tag{1}$$

$$\left[\frac{d}{dt} + \mu_{mic}\right]P_1(t) = \lambda_{mic}P_0(t) \tag{2}$$

$$\left[\frac{d}{dt} + \mu_{mix}\right]P_2(t) = \lambda_{mix}P_0(t) \tag{3}$$

$$\left[\frac{d}{dt} + \mu_{amp}\right]P_3(t) = \lambda_{amp}P_0(t) \tag{4}$$

$$\left[\tfrac{d}{dt} + \lambda_{mic} + \lambda_{mix} + \lambda_{amp} + \lambda_{sp} + \mu_{sp}\right]P_4(t) \\ = \mu_{mic}P_5(t) + \mu_{mix}P_6(t) + \mu_{amp}P_7(t) + \mu_{sp}P_8(t) + 2\lambda_{sp}P_0(t) \tag{5}$$

$$\left[\frac{d}{dt} + \mu_{mic}\right]P_5(t) = \lambda_{mic}P_4(t) \tag{6}$$

$$\left[\frac{d}{dt} + \mu_{mix}\right]P_6(t) = \lambda_{mix}P_4(t) \tag{7}$$

$$\left[\frac{d}{dt} + \mu_{amp}\right]P_7(t) = \lambda_{amp}P_4(t) \tag{8}$$

$$\left[\frac{d}{dt} + \mu_{sp}\right]P_8(t) = \lambda_{sp}P_4(t) \tag{9}$$

With the initial condition

$$P_0(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases} \tag{10}$$

Upon performing Laplace transformations of Equations (1)–(9), one can obtain the following set of equations:

$$\begin{gathered} \left[s + \lambda_{mic} + \lambda_{mix} + \lambda_{amp} + 2\lambda_{sp}\right]\overline{P}_0(s) = 1+ \\ \mu_{mic}\overline{P}_1(s) + \mu_{mix}\overline{P}_2(s) + \mu_{amp}\overline{P}_3(s) + \mu_{sp}\overline{P}_4(s) \end{gathered} \tag{11}$$

$$[s + \mu_{mic}]\overline{P}_1(s) = \lambda_{mic}\overline{P}_0(s) \tag{12}$$

$$[s + \mu_{mix}]\overline{P}_2(s) = \lambda_{mix}\overline{P}_0(s) \tag{13}$$

$$\left[s + \mu_{amp}\right]\overline{P}_3(s) = \lambda_{amp}\overline{P}_0(s) \tag{14}$$

$$\begin{gathered} \left[s + \lambda_{mic} + \lambda_{mix} + \lambda_{amp} + \lambda_{sp} + \mu_{sp}\right]\overline{P}_4(s) = \\ \mu_{mic}\overline{P}_5(s) + \mu_{mix}\overline{P}_6(s) + \mu_{amp}\overline{P}_7(s) + 2\lambda_{sp}\overline{P}_0(s) + +\mu_{sp}\overline{P}_8(s) \end{gathered} \tag{15}$$

$$[s + \mu_{mic}]\overline{P}_5(s) = \lambda_{mic}\overline{P}_4(s) \tag{16}$$

$$[s + \mu_{mix}]\overline{P}_6(s) = \lambda_{mix}\overline{P}_4(s) \tag{17}$$

$$\left[s + \mu_{amp}\right]\overline{P}_7(s) = \lambda_{amp}\overline{P}_4(s) \tag{18}$$

$$\left[s + \mu_{sp}\right]\overline{P}_8(s) = \lambda_{sp}\overline{P}_4(s) \tag{19}$$

Now, solving the above set of Equations (11)–(19), the following state transition probabilities are obtained:

$$\overline{P}_0(s) = \frac{1}{T_7} \tag{20}$$

$$\overline{P}_1(s) = \frac{\lambda_{mic}}{T_1 T_7} \tag{21}$$

$$\overline{P}_2(s) = \frac{\lambda_{mix}}{T_2 T_7} \tag{22}$$

$$\overline{P}_3(s) = \frac{\lambda_{amp}}{T_3 T_7} \tag{23}$$

$$\overline{P}_4(s) = \frac{2\lambda_{sp}}{T_4 T_6 T_7} \tag{24}$$

$$\overline{P}_5(s) = \frac{2\lambda_{mic}\lambda_{sp}}{T_1 T_4 T_6 T_7} \tag{25}$$

$$\overline{P}_6(s) = \frac{2\lambda_{mix}\lambda_{sp}}{T_2 T_4 T_6 T_7} \tag{26}$$

$$\overline{P}_7(s) = \frac{2\lambda_{amp}\lambda_{sp}}{T_3 T_4 T_6 T_7} \tag{27}$$

$$\overline{P}_8(s) = \frac{2\lambda_{sp}^2}{T_4 T_5 T_6 T_7} \tag{28}$$

where

$$T_0 = s + \lambda_{mic} + \lambda_{mix} + \lambda_{amp} + 2\lambda_{sp}$$
$$T_1 = s + \mu_{mic}$$
$$T_2 = s + \mu_{mix}$$
$$T_3 = s + \mu_{amp}$$
$$T_4 = s + \lambda_{mic} + \lambda_{mix} + \lambda_{amp} + \lambda_{sp} + \mu_{sp}$$
$$T_5 = s + \mu_{sp}$$
$$T_6 = 1 - \frac{\mu_{mic}\lambda_{mic}}{T_1 T_4} - \frac{\mu_{mix}\lambda_{mix}}{T_2 T_4} - \frac{\mu_{amp}\lambda_{amp}}{T_3 T_4} - \frac{\mu_{sp}\lambda_{sp}}{T_5 T_4}$$
$$T_7 = T_0 - \frac{\mu_{mic}\lambda_{mic}}{T_1} - \frac{\mu_{mix}\lambda_{mix}}{T_2} - \frac{\mu_{amp}\lambda_{amp}}{T_3} - \frac{2\mu_{sp}\lambda_{sp}}{T_6 T_4}$$

After obtaining the probability of each state, one can easily write the upstate and the downstate probabilities of the system. The upstate probability of the system is defined as the sum of the probabilities of those states in which the state is in a good working state or working in a degraded state. The downstate probability of the system is the sum of the probabilities of those states where the system is in completely failed states.

$$\overline{P}_{upstate}(s) + \overline{P}_{downstate}(s) = \frac{1}{s} \tag{29}$$

$$\overline{P}_{upstate}(s) = \overline{P}_0(s) + \overline{P}_4(s) \tag{30}$$

$$\overline{P}_{downstate}(s) = \sum_{i=1,2,3,5,6,7,8} \overline{P}_i(s) \tag{31}$$

5. Performance Measures of PAS

For the evaluation of various measures of reliability, data given in the following Table 3 will be used. These data have been taken from the organization's logbook. The failure and repair rate data have been converted as failure and repair rate in per hour.

Table 3. Failure and repair rates of different components of PAS.

S.No.	Component	Failure Rate (per h)	Repair Rate (per h)
1	Microphone	0.00002	1.0
2	Mixer	0.00005	1.5
3	Amplifier	0.00006	1.2
4	Speaker	0.00007	1.25

5.1. Availability of PAS

The availability of the system is a very important maintenance metric. It computes the probability that the system will not experience downtime when workers are required to use it. The upper bound for this metric is 1. In order to calculate the availability of the system, take the Laplace inverse of Equation (30) and set the repair and failure rate for the system components as given in Table 3. One can easily obtain the following availability function given in Equation (32).

$$\begin{aligned} Av(t) = 0.9998966711 - 1.2103572 \times 10^{-9} e^{-1.500300804\, t} + 0.00003333558699 \\ e^{-1.500050012\, t} - 4.0670741 \times 10^{-7} e^{-1.260028216\, t} + 4.2722323 \times 10^{-7} \\ e^{-1.241533050\, t} + 0.00004999275835\, e^{-1.200059996\, t} + 1.2284304 \times 10^{-8} \\ e^{-1.198558070\, t} + 0.00001999053893 e^{-1.000019992\, t} + 4.22255839 \times 10^{-9} \\ e^{-0.9999198591\, t} \end{aligned} \tag{32}$$

Now, varying the time unit t in Equation (32), one can obtain the following Figure 4 of the availability of the public address system.

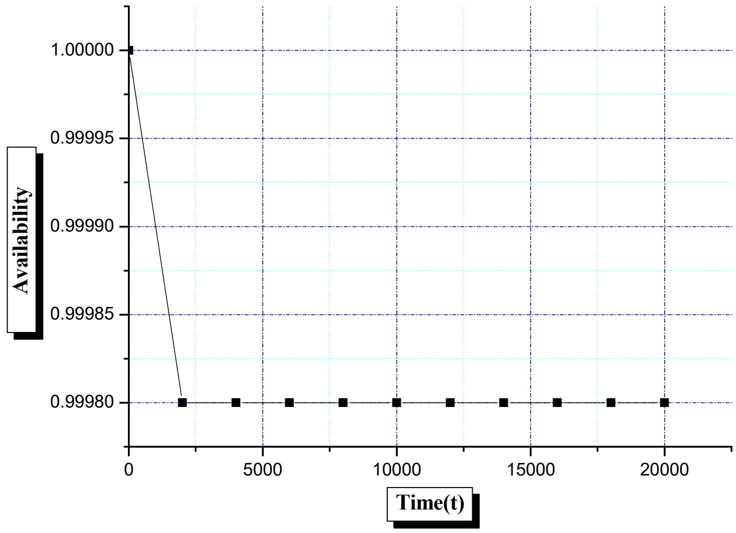

Figure 4. Availability of the PAS vs. time unit.

5.2. Reliability of PAS

Reliability is a very important metric of the system's performance. It is the probability that the device will not fail over the prescribed period of time when operated under the specified conditions. Mathematically, it is the probability that the system cannot fail before time t. The mathematical equation for reliability is given below.

$$R(t) = P(T > t) \tag{33}$$

In order to obtain the explicit expression for the reliability, set the failure rate as $\lambda_{mic} = 0.00002$, $\lambda_{mix} = 0.00005$, $\lambda_{amp} = 0.00006$, $\lambda_{sp} = 0.00007$ in Equation (30), set all the repair rates equal to zero, and take the inverse Laplace transform of the obtained expression. One can obtain the time-dependent reliability expression of the PAS as given below in Equation (34).

$$R(t) = e^{-0.00027\,t} + 4 \times e^{-0.000235\,t} \sin\mathrm{h}(0.0000035\,t) \tag{34}$$

Now, varying the time unit t in Equation (34), one can obtain the following Figure 5 of the reliability of the public address system.

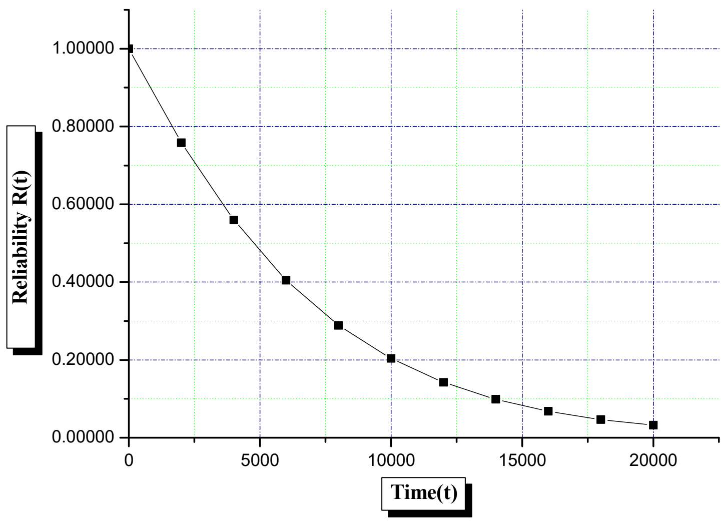

Figure 5. Reliability of PAS vs. time unit.

5.3. MTTF of the PAS

The mean time to failure (MTTF) is also a very important metric of the reliability of the system. It measures the average amount of time a non-repairable system operates before it fails. From an organizational point of view, a large MTTF value indicates the system's greater durability and ability to perform work, and a small MTTF value indicates a lower durability and ability to perform work. If organizations know the MTTF of a system beforehand, then they may plan the timely repair of the system. In order to calculate the MTTF of the system using Equation (30), setting all repair rates equal to zero and the taking the limit s $\rightarrow$ 0, one can easily obtain the MTTF of the public address system.

$$MTTF = \frac{1}{\lambda_{mic} + \lambda_{mix} + \lambda_{amp} + 2\lambda_{sp}} + \frac{2\lambda_4}{(\lambda_{mic} + \lambda_{mix} + \lambda_{amp} + \lambda_{sp})(\lambda_{mic} + \lambda_{mix} + \lambda_{amp} + 2\lambda_{sp})} \quad (35)$$

Now, in Equation (35), set the failure rates to $\lambda_{mic} = 0.00002$, $\lambda_{mix} = 0.00005$, $\lambda_{amp} = 0.00006$, and $\lambda_{sp} = 0.00007$.

Now, varying each failure rate one by one from 0.00001 to 0.00009 while fixing the other failure rates, one can easily obtain different values of the MTTF for the possible combinations of the failure rates. The MTTF of the system is given below in Table 4 and Figure 6.

Table 4. MTTF with respect to variation in the failure rates.

Variation in the Failure Rate	Variation in the MTTF with Respect to Failure Rate λ_{mic}	Variation in the MTTF with Respect to Failure Rate λ_{mix}	Variation in the MTTF with Respect to Failure Rate λ_{amp}	Variation in the MTTF with Respect to Failure Rate λ_{sp}
0.00001	6680.1619	8152.1739	8787.8787	7619.0476
0.00002	6296.2963	7598.0392	8152.1739	7450.9804
0.00003	5952.3809	7111.1111	7598.0392	7236.8421
0.00004	5642.6332	6680.1619	7111.1111	7002.8011
0.00005	5362.31884	6296.2963	6680.1619	6763.2850
0.00006	5107.5269	5952.3809	6296.2962	6526.3158
0.00007	4875.0000	5642.6332	5952.3809	6296.2963
0.00008	4662.0047	5362.3188	5642.6332	6075.5337
0.00009	4466.2309	5107.5268	5362.3188	5865.1026

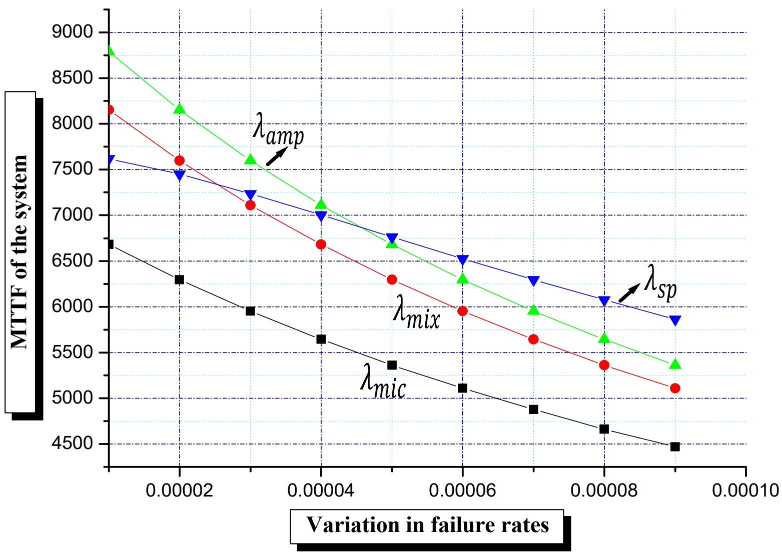

Figure 6. MTTF with respect to variation in the failure rates.

5.4. Sensitivity Analysis of PAS Reliability

In order to identify the critical components of a PAS, the authors performed a sensitivity analysis for the system's reliability. Sensitivity analysis is a method that indicates the effects of different failures on the reliability of a system. The following Table 5 and corresponding Figure 7 indicate the impact of different failure rates on the PAS's reliability.

Table 5. Sensitivity of the PAS with respect to reliability.

Time (h)	$\frac{\partial R(t)}{\partial \lambda_{mic}}$	$\frac{\partial R(t)}{\partial \lambda_{mix}}$	$\frac{\partial R(t)}{\partial \lambda_{amp}}$	$\frac{\partial R(t)}{\partial \lambda_{sp}}$
0	0	0	0	0
2000	−1515.7837	−1515.7837	−1515.7837	−350.2872
4000	−2236.2496	−2236.2496	−2236.2496	−877.8675
6000	−2426.9383	−2426.9383	−2426.9383	−1239.5462
8000	−2307.7433	−2307.7433	−2307.7433	−1385.1423
10000	−2034.6505	−2034.6505	−2034.6505	−1362.5954
12000	−1707.2641	−1707.2641	−1707.2641	−1237.2927
14000	−1383.1641	−1383.1641	−1383.1641	−1063.6464
16000	−1091.5924	−1091.5924	−1091.5924	−878.7942
18000	−844.1453	−844.1453	−844.1453	−704.6366
20000	−642.2939	−642.2939	−642.2939	−551.9623

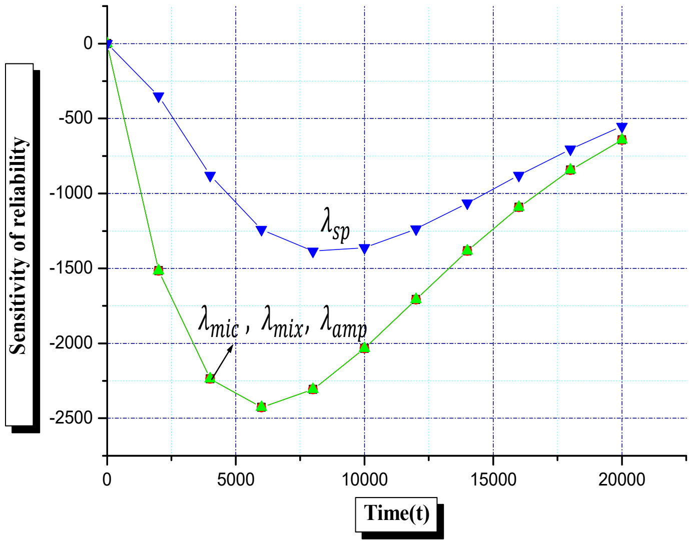

Figure 7. Sensitivity of the PAS with respect to reliability.

5.5. Expected Profit Analysis

It is always necessary to calculate the expected profit that a system will generate in the times to come. Therefore, the author calculated the expected profit from the public address system. This system is basically used to attract people so that the organizations may sell more and more products on the market by making attractive announcements to customers. The function below can be used to estimate the profit of the system in time interval [0, t).

$$E_P(t) = K_1 \int_0^t Av(t)dt - K_2 t \tag{36}$$

Here, the expression $\int_0^t Av(t)dt$ gives the total uptime of the system, and t gives the total time. The parameter K_1 represents the revenue generated by the system per unit of time and K_2 represents the expenditure of the system per unit of time. Now, setting $K_1 = 10$ and varying the value of K_2 and t, one can easily obtain the expected profit that the public address system can generate, indicated in Table 6 and Figure 8.

Table 6. Expected profit from the PAS.

Time (h)	$E_P(t)$ K_2=1	$E_P(t)$ K_2=2	$E_P(t)$ K_2=3	$E_P(t)$ K_2=4
0	0	0	0	0
2000	17997.93	15997.93	13997.93	11997.93
4000	35995.86	31995.86	27995.86	23995.87
6000	53993.80	47993.80	41993.80	35993.80
8000	71991.73	63991.73	55991.73	47991.73
10000	89989.66	79989.66	69989.66	59989.66
12000	107987.60	95987.60	83987.60	71987.60
14000	125985.53	111985.53	97985.53	83985.53
16000	143983.46	127983.46	111983.46	95983.46
18000	161981.40	143981.40	125981.40	107981.40
20000	179979.33	159979.33	139979.33	119979.33

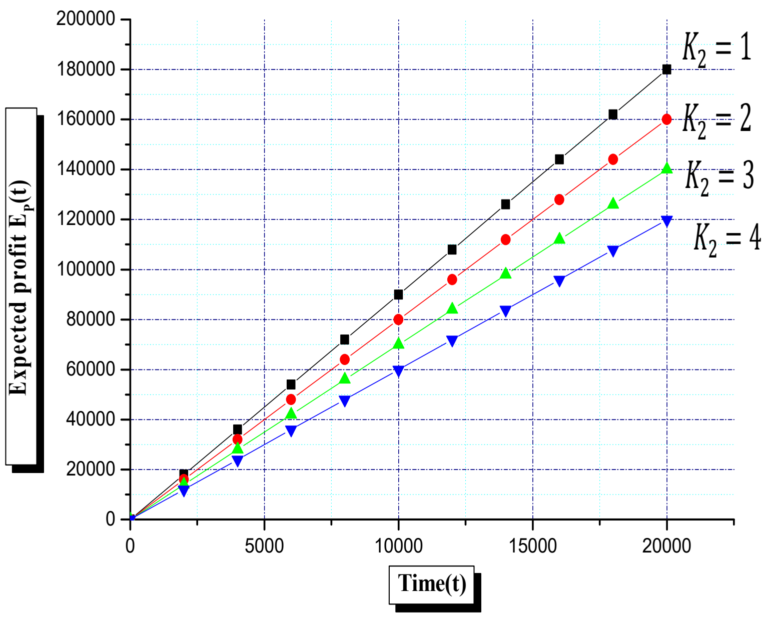

Figure 8. Expected profit from the PAS.

6. Results and Discussion

Here, in the present paper, the authors have discussed the performance of a public address system through a reliability approach. A mathematical model was developed by considering the different components of the PAS. The following are the pointwise results summary for the above.

- The PAS availability behavior is shown Figure 4. One can observe that the availability of the PAS is constant with a value of 0.9998 during the observed time.
- Figure 5 reflects the PAS reliability behavior with a time unit. The decrease in reliability is quite smooth and later on becomes almost constant. At 20,000 h, the reliability of the PAS becomes just 0.0321.

- Figure 6 shows the variation in the mean time to failure of the PAS concerning variation in different failure rates. It is quite clear from the graph that the system's MTTF is lower with respect to the microphone compared to other failures.
- Figure 7 shows the partial derivatives of the system reliability *R(t)* with respect to four different failure rates λ_{mic}, λ_{mix}, λ_{amp}, λ_{sp} at various time points (in hours). A negative partial derivative implies that an increase in the corresponding failure rate parameter leads to a reduction in the system's reliability, while a positive partial derivative would indicate the opposite. It is quite clear from the graph of the sensitivity of the reliability that the system's reliability is very sensitive with respect to the failure rate of the microphone, mixer, and amplifier.
- Figure 8 shows the expected profit of the public address system at different time points (in hours) for different values of the parameter K_2. It is quite clear that the profit decreases with the increase in the service cost K_2. Such information can be crucial for decision-making, financial planning, and resource allocation in optimizing the performance of the system.

7. Conclusions

In this paper, the authors analyzed the performance of a public address system. This system is crucial for many organizations to run their businesses well. The main components of this system are a microphone, mixer, amplifier, and speaker. It is important that all the components of the system work properly so that important information may be conveyed to large numbers of people. Now, based on the above results and discussion, we conclude that the MTTF of the system is very low in regard to variation in the failure rates of the microphone. Therefore, among all components, the microphone is the weakest component of the system. Hence, in order to improve the system's MTTF plan, the timely maintenance of the microphone is needed. The reliability of the system is affected by variation in the failure rate of the microphone, mixer, and amplifier. Therefore, to improve the overall performance of a PAS, the maintenance team should pay more attention to the failure/repair of the microphone, or an extra microphone can be used. It is expected that this research will be of great help for organizations to improve overall system performance.

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