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Reinforcement of a Gas Transmission Network †

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Abstract: There have been many studies performed on the optimal design and expansion of a natural gas transmission network; however, very few works have addressed the problem of the reinforcement of an existing natural gas transmission network with limited application. This study is focused on the reinforcement of an existing natural gas transmission network with the aim to minimize investment cost. The compressor stations have been assumed operational in either direction. A mathematical model was developed for the problem, which is non-convex mixed integer nonlinear programming (MINLP) in nature; therefore, a convex relaxation was formulated to solve the problem easily in General Algebraic Modeling System (GAMS, GAMS Development Corp., Fairfax, VA, USA) using DICOPT and CONOPT solvers. The model was applied to a small transmission network for validation and the results proved its efficiency.

Keywords: natural gas; transmission network; reinforcement planning; optimal expansion; mixed integer nonlinear programming

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1. Introduction

The natural gas transmission network is the most critical part of a gas supply chain network. The different gas supply sources including the indigenous gas-producing sources, Liquefied Natural Gas (LNG) terminals, underground storage, and international pipeline entry points are connected to a natural gas transmission network at borders. Major industrial consumers such as gas-fired electric power plants, and fertilizer and cement plants are directly connected to a natural gas transmission network. Therefore, its efficient operation and optimal expansion are very critical to ensuring an uninterrupted supply of natural gas as well as minimal cost [1]. Natural gas transmission networks are high-pressure networks spanning entire countries involving large distances. The natural gas transmission networks expanded with time due to the addition of the new demand centers and the enhanced load at the existing ones. The flow dynamics of natural gas through a pipeline are very complex due to the nonlinear relationship between the natural gas pressure and flow [2]. Further, the combinatorial nature of pipeline diameters makes the problem mixed-integer in nature [3]. These characteristics make the problem an Mixed Integer Nonlinear Programming (MINLP), which further needs reformulations and relaxation for efficient results. In literature, the unidirectional operation of the compressor stations has been considered, which makes the compressor station nonoperational in the reverse direction, hence, it has no role in the capacity of the cyclic network in the reverse direction. In this study, it is assumed that they can operate in either direction to fully utilize their capacity, hence, the network. The objective of this study is to propose a mathematical formulation for the optimal expansion of a gas transmission network while considering the reinforcement of the existing network with the bidirectional flow of the compressor stations. The substantial costs can be saved by even minor improvements in design and operation conditions. The total cost can be minimized by optimizing system variables while ensuring that all network constraints are satisfied [4]. A typical natural gas transmission network is shown in Figure 1.

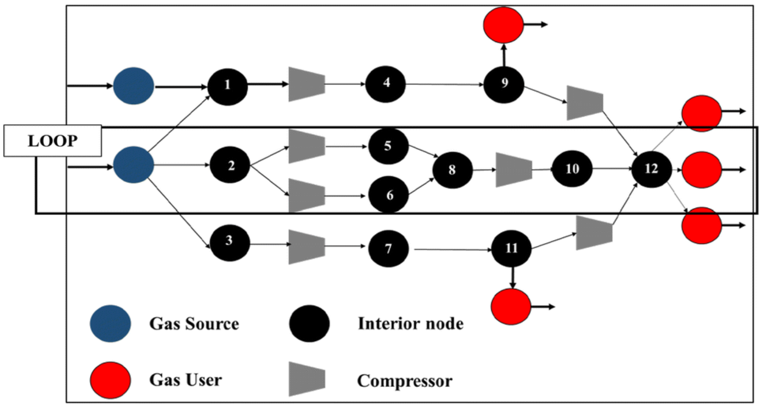

Figure 1. A Cyclic Gas Network.

2. Problem Formulation and Mathematical Modeling

We are dealing with the reinforcement of an existing gas transmission network with the following assumptions: The network under study is a cyclic gas network and is in a steady-state condition. The gas composition and temperature are assumed as constant over time and space. The variations in elevation of the network are neglected and it is considered horizontal. The demand at each delivery node must be fully satisfied, i.e., no shortages are allowed.

Let,

i, index of the nodes; a, index of the arcs;
N, the set of nodes;
N_s, the set of supply nodes;
N_t, the set of transit nodes;
N_d, the set of demand nodes;
A, the set of arcs;
A_{pipe}, the set of pipe arcs;
A_{comp}, the set of compressor arcs;
D, the set of commercially available diameters of pipe;
Q_a, the flow rate in a pipe (million standard cubic feet per day);
π_a^i, the inlet square pressure (or inlet head) of the pipe (psi);
π_a^j, the outlet square pressure (or outlet head) of the pipe (psi);
y_a, binary variable showing gas flow direction in a pipe;
k_a, auxiliary variable for a pipe arc;
L, the pipe length (miles);
D_a, the internal diameter of the pipe (inches);
M, nodes-arcs incidence matrix;
b, the vector of supply or demand;
P_{max}, the maximum pressure at a node (psi;
P_{min}, the minimum pressure at a node (psi);
${\pi_i}^{max}$, the maximum pressure head limit;
${\pi_i}^{min}$, the minimum pressure head limit;
C_a, the cost of pipe per mile per inch.

2.1. Objective Function

The objective function is to minimize the construction cost of new pipelines, which is given by:

$$\text{Min } Z = \sum_{a \in A_{pipe}} C_a \cdot D_a \cdot L_a \tag{1}$$

2.2. Constraints

There are some limitations posed by the technicalities of a gas transmission network, therefore, the model is subject to the following constraints:

$$\pi_a^i - \pi_a^j = \beta Q_a |Q_a|, \ \forall\ a\ \varepsilon\ A_{pipe} \tag{2}$$

$$\pi_i^{min} \le \pi_i \le \pi_i^{max}, \ \forall\ i\ \varepsilon\ N \tag{3}$$

$$\tau_a^{min} \le \tau_a \le \tau_a^{max}, \ \forall\ a\ \varepsilon\ A_{comp} \tag{4}$$

$$MQ = b \tag{5}$$

where β is a constant in the pressure loss equation; whereas, the modulus of Q_a shows the direction of gas flow in a pipe arc. Further, Equation (5) summarizes flow balance equations at each node.

3. Solution Methodology

The relation between the gas flow and pressure makes the problem non-convex in nature, which is difficult to solve; therefore, the convex relaxation is applied to the constraint (2) of the model, then, solved in GAMS.

3.1. Convex Relaxation

The model with the convex relaxation of (2) is as below:

$$\text{Min } Z = \sum_{a \in A_{pipe}} C_a \times D_a \cdot L_a \tag{6}$$

$$k_a = \beta Q_a^2 \tag{7}$$

$$k_a \ge \pi_a^j - \pi_a^i + 2y_a\left(\pi_i{}^{min} - \pi_j{}^{max}\right) \tag{8}$$

$$k_a \ge \pi_a^i - \pi_a^j + 2(y_a - 1)\left(\pi_i{}^{max} - \pi_j{}^{min}\right) \tag{9}$$

$$k_a \le \pi_a^j - \pi_a^i + 2y_a\left(\pi_i{}^{max} - \pi_j{}^{min}\right) \tag{10}$$

$$k_a \ge \pi_a^i - \pi_a^j + 2(y_a - 1)\left(\pi_i{}^{min} - \pi_j{}^{max}\right) \tag{11}$$

$$\pi_i^{min} \le \pi_i \le \pi_i^{max}, \forall\ i\ \varepsilon\ N \tag{12}$$

$$\tau_a^{min} \le \tau_a \le \tau_a^{max}, \ \forall\ a\ \varepsilon\ A_{comp} \tag{13}$$

$$MQ = b \tag{14}$$

The Equations (8)–(11) are called McCormick envelopes for the equation $k_a = 2(y_a - 1)(\pi_i - \pi_j)$ [5]. These envelopes result in exact reformulation because it is the product of a binary variable with a continuous variable.

3.2. GAMS as Modeling Language

The mathematical model developed in Section 3.1 is now modeled into a computer program in General Algebraic Modeling System (GAMS) [6], which is then solved through DICOPT and CONOPT solvers. GAMS is a high-level modeling language used for mathematical optimization. DICOPT is DIscrete Continuous OPTimizer that solves Nonlinear Programming (NLP) and Mixed Integer Programming (MIP) sub-problems alternatively. CONOPT is used due to a large number of nonlinear constraints [7].

4. Computational Experiments and Results

All the computational experiments were performed on a small gas transmission network with 16 nodes and 15 arcs. The details of some arcs are shown in Table 1. Two benchmarks were created from the base network; (i) the enhancement in diameter of the segment s1 which consists of arcs 1 to 3 as well as the higher pressure at the supply nodes (ii) the installation of a compressor station at the transient node to boost the pressure. The results show that the first benchmark is feasible, whereas, the second benchmark was infeasible despite increasing the pressure. The optimal solution for the first benchmark was achieved at a 12-inch diameter [8,9].

Table 1. Arcs details.

Arc	Length (Miles)	Arc Resistance
1.2	2.78	4.45×10^{-8}
2.3	13.51	2.16×10^{-7}
3.4	2.7	4.32×10^{-8}

5. Conclusions

In this study, we developed a mathematical model with convex relaxation for the reinforcement of a gas transmission network with a minimal investment cost objective. The computational experiments showed that the developed model is an efficient tool to solve the problem of the optimal expansion of a gas transmission network in a very short time.

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References

1. Bickel, T.; Himmelblau, D.; Edgar, T. Optimal Design of Gas Transmission Networks. *Soc. Pet. Eng. J.* **1978**, *18*, 96–104. [CrossRef]
2. Sundar, K.; Misra, S.; Zlotnik, A.; Bent, R. Robust Gas Pipeline Network Expansion Planning to Support Power System Reliability. In Proceedings of the 2021 American Control Conference (ACC), New Orleans, LA, USA, 25–28 May 2021; pp. 620–627.
3. Üster, H.; Dilaveroğlu, Ş. Optimization for Design and Operation of Natural Gas Transmission Networks. *Appl. Energy* **2014**, *133*, 56–69. [CrossRef]
4. Elshiekh, T.; Khalil, S.; Mawgoud, H. Optimal Design and Operation of Egyptian Gas-Transmission Pipelines. *Oil Gas Facil.* **2013**, *2*, 44–48. [CrossRef]
5. McCormick, G.P. Computability of Global Solutions to Factorable Nonconvex Programs: Part I—Convex Underestimating Problems. *Math. Program.* **1976**, *10*, 147–175. [CrossRef]
6. Li, T.; Liu, P.; Li, Z. Modelling and Optimization of a Natural Gas Supply System at a Transient Stage: A Case Study of China. *BMC Energy* **2019**, *1*, 5. [CrossRef]
7. Zheng, T.; Feng, H.; Wang, B.; Zheng, J.; Liang, Y.; Ma, Y.; Keene, T.; Li, J. Operational Optimization of a Cyclic Gas Pipeline Network with Consideration of Thermal Hydraulics. *Ind. Eng. Chem. Res.* **2021**, *60*, 2501–2522. [CrossRef]
8. Mak, T.W.K.; van Hentenryck, P.; Zlotnik, A.; Bent, R. Dynamic Compressor Optimization in Natural Gas Pipeline Systems. *INFORMS J. Comput.* **2019**, *31*, 40–65. [CrossRef]
9. dos Santos, S.P.; Eugenio Leal, J.; Oliveira, F. The Development of a Natural Gas Transportation Logistics Management System. *Energy Policy* **2011**, *39*, 4774–4784. [CrossRef]