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A Comprehensive Comparison of the Hargreaves Isothermal Model with the Schmidt Model for the Gamma Stirling Engine †

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Abstract: The Stirling engine, a type of external combustion engine utilizing a compressible fluid as its working medium, holds promise as a highly efficient device for converting heat into mechanical work at Carnot efficiency. This research conducts a detailed analysis, comparing the Hargreaves isothermal model and the Schmidt model specifically for the gamma-type Stirling engine. The study examines the impact of dead volume on the engine's performance, revealing that the engine network is solely influenced by these volumes. Furthermore, it highlights the effectiveness of the Hargreaves model for the performance analysis of gamma-type Stirling engines.

Keywords: Stirling engine; thermodynamics; power; efficiency

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1. Introduction

In 1816, a cleverly made Stirling engine was introduced. This special machine worked in a unique way that made it different from others. For almost 100 years, it proved itself as a dependable power source for many things. But as technology advanced, a different kind of engine, called the spark-ignition engine, became more popular and took its place [1].

Nowadays, things have changed. Stirling machines used to carry out a lot of work in industries, but now they have a new job. They are used as special heaters in businesses. They are really good at making things very cold and turning air into liquid in specific places where these things are needed. Even though the old-style Stirling engine is not used as much as before, it is not forgotten. Instead, it has become an exciting area where people learn and make new things. Scientists and inventors keep studying it to find out all it can achieve. Recently, some experiments were performed that showed how powerful and efficient the engine can still be. These new ideas prove that the Stirling engine is still interesting and has a lot of potential [2–4].

The analysis of a Stirling machine that includes a space, which is often referred to as the "dead volume", can be approached through a method developed by Schmidt, as mentioned in reference [5]. This method is applicable when the engine's operation is considered to involve consistent temperature changes (isothermal evolutions) and an optimal process of restoring energy (ideal regeneration). Second-order design approaches are initiated by first computing the theoretical maximum power that a system can generate. From this value, losses occurring independently due to factors like fluid friction and thermal inefficiencies are deducted. Subsequently, losses related to power transmission and conversion are subtracted, while also accounting for any additional losses during heat input or heat rejection processes. The model used to forecast this theoretical maximum power is typically more intricate compared to the simpler model employed in first-order analyses. There are many ways to estimate the performance of a Stirling engine, i.e., isothermal, adiabatic, finite time thermodynamics (FTT) [2], and finite physical dimensions thermodynamics (FPDT) [6], but the most common and popular basic approach is employing the isothermal method [7].

In this research work, the performance of a gamma-type Stirling engine is investigated using the Hargreaves isothermal model, and the results of this model are compared with other thermodynamic models. The work carried out and power obtained by the Hargreaves isothermal model is 13.6 J and 200.8 W, respectively. The percentage error in the work and power using this model is 81.6% and 80.03% respectively. The efficiency obtained by this model is equal to Carnot efficiency, which is 30.6%, and this efficiency is equal to the Schmidt isothermal model. Moreover, the performance of the Stirling engine is also affected by dead volume. If the dead volume of the Stirling engine is large, the performance of the Stirling engine will be low.

2. Engine Data

The engine under study is a gamma-type double-piston Stirling engine. In this engine, the exhaust gas of the diesel engine is used as the heat source. To heat the heater tubes of the Stirling engine, the cap is especially made for the Stirling engine. Further details of the engine data are given in Table 1 below [8].

Table 1. Engine data of the gamma-type double-piston Stirling engine [8].

Engine Data	Value	Engine Data	Value
Cooler temperature	294	Pressure (bar)	3.58
Heater temperature	424	Rotational speed (rpm)	882
Cooler void volume	223×10^{-6}	Phase angle (degree)	88°
Heater void volume	87.28×10^{-6}	Regenerator void volume (m^3)	308.93
Expansion swept volume (m^3)	221×10^{-6}	Compression swept volume (m^3)	194×10^{-6}
Expansion clearance volume (m^3)	24×10^{-6}	Compression clearance volume (m^3)	35×10^{-6}

3. Schmidt Model and Hargreaves Model

For the Schmidt model, the equation for the expansion and compression space volume calculation is given below, where V_{DE}, V_{DC}, V_{SE}, V_{SC}, $\varnothing$ are the expansion dead volume, compression dead volume, expansion swept volume, compression swept volume, and phase angle, respectively [8].

$$V_E = \frac{V_{SE}}{2}[1 - \cos(\theta)] + V_{DE} \tag{1}$$

$$V_C = \frac{V_{SE}}{2}[1 + \cos(\theta)] + \frac{V_{SC}}{2}[1 - \cos(\theta - \phi)] + V_{DC} \tag{2}$$

The Shmidt model was modified by Abdul Rab et al. [8] for this specific gamma-type Stirling engine. The mass of the Stirling engine is given by

$$M = m_C + m_{k1} + m_{k2} + m_r + m_h + m_E \tag{3}$$

$$M = \frac{p}{R}\left(\frac{V_C}{T_C} + \frac{V_{k1}}{T_C} + \frac{V_{k2}}{T_C} + \frac{V_r \log\left(\frac{T_E}{T_C}\right)}{T_E - T_C} + \frac{V_E}{T_E} + \frac{V_E}{T_E}\right) \tag{4}$$

The total work performed is defined by the following equation:

$$W = W_E + W_C = \oint p\left(\frac{dV_E}{d\theta} + \frac{dV_C}{d\theta}\right)d\theta \tag{5}$$

The Hargreave isothermal model for the gamma-type Stirling engine was modified for Wegnar et al. [9], in which the heater, regenerator, and cooler volume are considered as the dead volume. The mass in the Stirling engine is given by

$$M = \sum m_i = m_D + m_E + m_C \quad (6)$$

$$M = \frac{\rho}{R}.\left[\frac{V_E}{T_E} + \frac{V_D}{T_D} + \frac{V_C}{T_C}\right] \quad (7)$$

$$t = \frac{T_C}{T_E} \text{ (Temperature ratio)} \quad (8)$$

$$v = \frac{V_{SC}}{V_{SE}} \text{ (Swept volume ratio)} \quad (9)$$

$$s = \frac{V_D}{V_{SE}}.\frac{T_C}{T_D} \text{ (Dead volume ratio)} \quad (10)$$

By using the above equation, the mass in the Stirling engine is re-arranged as

$$M = \frac{P}{R.T_C}\left[t.V_E + V_C + V_D.\frac{T_C}{T_D}\right] \quad (11)$$

The pressure in the Stirling engine is given by

$$P = \frac{P_m.\sqrt{1-c^2}}{1 - c.\cos(\alpha - \delta)} \quad (12)$$

$$c = \frac{A}{B} \quad (13)$$

$$A = \sqrt{t^2 - 2.t + 1 + 2.(t-1).v.\cos\varphi + v^2} \quad (14)$$

$$B = t + 1 + v + 2.s \quad (15)$$

$$\delta = \arctan\left(\frac{v.\sin\varphi}{t - 1 + v.\cos\varphi}\right) \pm (0; \pi; 2\pi; \ldots\ldots\ldots) \quad (16)$$

The expansion, compression, and total work performed are given by

$$W_C = \int_0^{2\pi} P.dV_C \quad (17)$$

$$W_E = \int_0^{2\pi} P.dV_E \quad (18)$$

$$W = W_e + W_c \quad (19)$$

4. Results and Discussion

Model Validation

In this research work, the results of the Hargreave model for the gamma Stirling engine are compared with the literature isothermal model, adiabatic, and experimental models. The work obtained by the Hargreave model is 13.6 J and the Schmidt isothermal model 9.08 J [8]. It is important to note that in the mentioned isothermal model, the thermal or heat transfer losses are not included, whereas in experimental results, all losses are included, as referred to in the literature. The experimental value of the work obtained is 7.50 J [8]. When the Hargreave model is compared with the experimental results, it is found that there was a +81.6% error. The case in the power calculation is similar, whereby the power obtained by the Hargreave model is 200.6 W, which is very high compared to experimental values at 111.43 W [8]. The percentage error in the power calculation by this model is 80.03%. But the efficiency of this engine by this model is equal to Carnot efficiency, which is 30.6, and this efficiency is equal to other thermodynamics models. Table 2 shows the summary of the thermodynamic models and their comparison; Figure 1 shows the comparison of the p–V diagram curves with the other thermodynamics models. It can be seen from the p–V diagram curve that the area under the curve of the Hargreaves model is more than that of the other thermal models. Therefore, this indicates that the power using this model is more than other of the thermal models and experimental results.

Table 2. Summary of the thermodynamic models.

Parameter	Isothermal [8]	Experimental [8]	Hargreave Model
Work (J)	9.08	7.50	13.6
Power (W)	133.8	111.43	200.8
Efficiency (%)	30.70	24.70	30.6

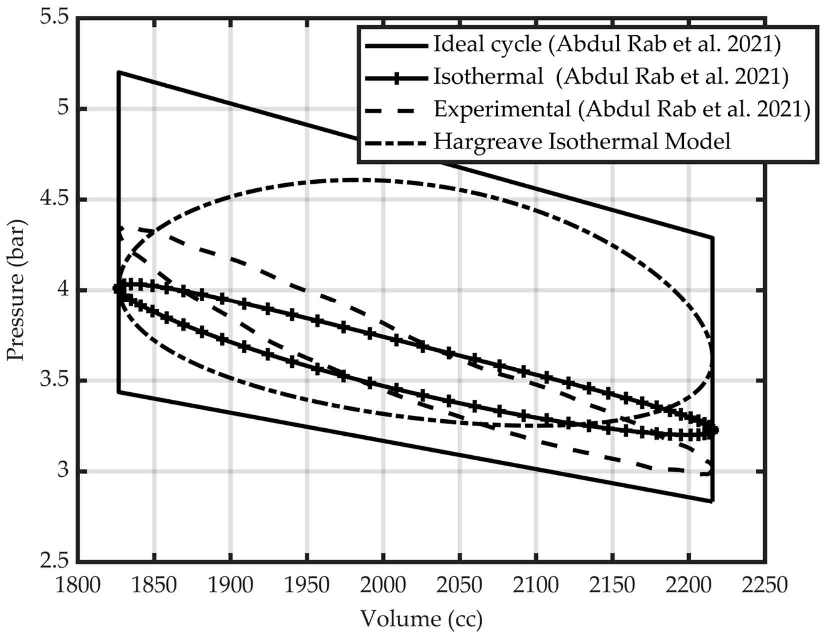

Figure 1. Comparison of the p–V diagram with other thermodynamic models [8].

Figure 2 represents the derivative work carried out in the expansion, compression and total work performed using the Hargreaves model. The peak positive value of the derivative curve of the total work performed is approximately 100 J and the negative peak value is −60 J. The total work performed is calculated by taking the average values of the work carried out at every crank angle. Moreover, the dead volume of the Stirling engine plays a very important role in the design of the Stirling engine. If the dead volume is large, the performance of the Stirling engine will be low.

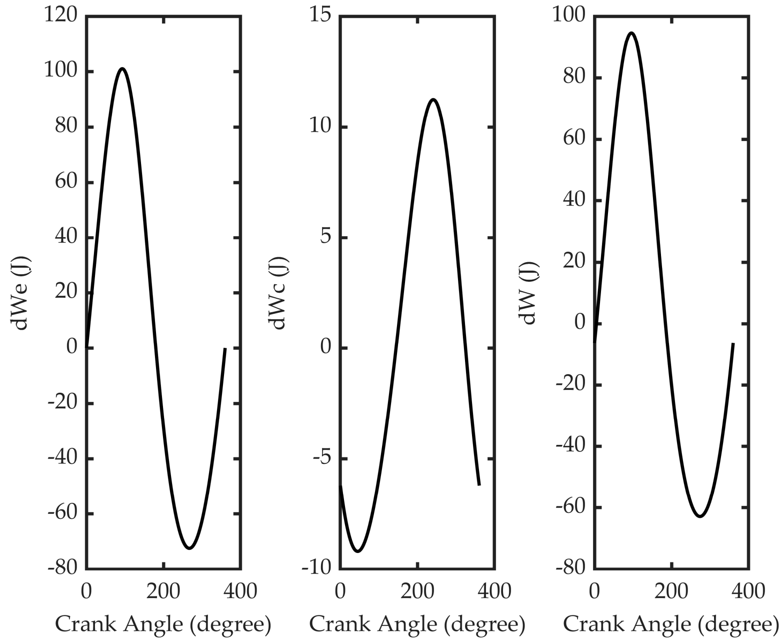

Figure 2. Derivative of the expansion, compression, and total work performed by the Hargreaves model.

5. Conclusions

This research focuses on evaluating the performance of a gamma-type Stirling engine through the utilization of the Hargreaves isothermal model. A comparison is made between the outcomes of this model and those of alternative thermodynamic models. According to the Hargreaves isothermal model, the achieved work and power stand at 13.6 J and 200.8 W, respectively. The corresponding percentage errors for work and power using this model are 81.6% and 80.03%, respectively. Notably, the efficiency attained from this model aligns with Carnot efficiency, registering at 30.6%, and coincides with the Schmidt isothermal model's efficiency. Additionally, the Stirling engine's effectiveness is influenced by its dead volume, whereby a larger dead volume results in diminished engine performance.

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