



Abstract

# Probability Density Function of a Random Area and Its Application to Wildfires <sup>†</sup>

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**Abstract:** We show that the probability density function (PDF) of a burned area enclosed by a random fire perimeter is driven by the PDF of the bounding-box sides. In particular, the random value of the area emerges to be proportional to the random position of the bounding-box sides times an averaged coefficient dependent on the geometry of the burned area. Therefore, the two PDFs are functionally equal. This means that the PDF of the burned area is driven and functionally equal to the PDF of the position of the head of the fire. The displacement of the head of the fire is given by the rate of spread (ROS); thus, the PDF of the burned area is driven and equal to the PDF of the ROS. This result holds in general whenever a fire exhibits an advancement in a main direction. The main theoretical result has been tested by different families of stochastic processes and also by using the operational fire simulator PROPAGATOR, which is based on a cellular automata approach. By using PROPAGATOR, the criteria for the validity of the derived result in realistic cases has been established by analyzing different configurations of orography and wind. This study can be understood as a start for the development of a theory of stochastic dynamics of wildfire propagation with the aim, for example, to provide physically grounded initial perturbations of wildfire perimeters for ensemble forecasting.

**Keywords:** wildfire propagation; burned area probability; head-of-fire probability



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