

Article

# Evaluation of AV Deadheading Strategies

Sruthi Mantri <sup>1,\*</sup>, David Bergman <sup>2</sup> and Nicholas Lownes <sup>1</sup> <sup>1</sup> Department of Civil and Environmental Engineering, University of Connecticut, Storrs, CT 06269, USA<sup>2</sup> School of Business, University of Connecticut, Storrs, CT 06269, USA

\* Correspondence: sruthi.mantri@uconn.edu

**Abstract:** The transition of the vehicle fleet to incorporate AV will be a long and complex process. AVs will gradually form a larger and larger share of the fleet mix, offering opportunities and challenges for improved efficiency and safety. At any given point during this transition a portion of the AV fleet will be consuming roadway capacity while deadheading, which means operating without passengers. Should these unoccupied vehicles simply utilize the shortest paths to their next destination, they will contribute to congestion for the rest of the roadway users without providing any benefit to human passengers. There is an opportunity to develop routing strategies for deadheading AVs that mitigate or eliminate their contribution to congestion while still serving the mobility needs of AV owners or passengers. Some of the AV fleet will be privately owned, while some will be part of a shared AV fleet. In the former, some AVs will be owned by households that are lower-income and benefit from the ability to have fewer vehicles to serve the mobility needs of the household. In these cases, it is especially important that deadheading AVs can meet household mobility needs while also limiting the contribution to roadway congestion. The aim of this study is to develop and evaluate routing strategies for deadheading autonomous vehicles (AVs) that balance the reduction of roadway congestion and the mobility needs of households. By proposing and testing a bi-objective program, this study seeks to identify effective methodologies for routing unoccupied AVs in a manner that mitigates their negative impact on traffic while still fulfilling essential transportation requirements of the household. Three strategies are proposed to deploy AV deadheading methodology to route deadheading vehicles on longer paths, reducing congestion for occupied vehicles, while still meeting the trip-making needs of households. Case studies on two transportation networks are presented alongside their practical implications and computational requirements.



**Citation:** Mantri, S.; Bergman, D.; Lownes, N. Evaluation of AV Deadheading Strategies. *Future Transp.* **2024**, *4*, 1059–1077. <https://doi.org/10.3390/futuretransp4030051>

Academic Editor: Lynnette Dray

Received: 11 June 2024

Revised: 10 August 2024

Accepted: 2 September 2024

Published: 12 September 2024



**Copyright:** © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

**Keywords:** AV routing; mixed traffic assignment; unoccupied AVs; AV policy

## 1. Introduction

Autonomous vehicles (AVs) have the potential to revolutionize transportation. They could make our roads safer, reduce traffic congestion, improve air quality and increase economic activity [1]. Research indicates that automated vehicles can substantially alleviate traffic congestion, with potential reductions of up to 75%. Additionally, average travel speeds can increase by as much as 300% compared to human-driven vehicles. These benefits are evident even when AVs represent a relatively small portion of the overall vehicle population [2].

The Federal Highway Administration (FHWA) has led communication and outreach efforts with highway stakeholders, including state departments of transportation (DOTs), public agencies and industry groups, to build understanding of the potential impacts of AVs on transportation, society and the economy [3]. The current research aligns with the Federal Highway Administration's (FHWA) research priorities for future planning of connected and automated vehicles [4]. Although the transition to a complete autonomous fleet is in the distant future, the integration of human-driven and AVs is a critical scenario and requires careful planning and coordination. As more and more vehicles become automated, there will be a growing number of vehicles deadheading (empty AVs enroute to next

passenger pickup) consuming capacity and contributing to congestion [5]. The routing strategy for these unoccupied AVs, whether they are shared or privately owned, should not be the same as an occupied vehicle. Deadheading AVs need not always be on the shortest route to their destination. In the case of a privately owned AV, the unoccupied vehicle need only serve the autonomous vehicle owner for the next household trip. Therefore, as long as the vehicle reaches the next departure point in time, the unoccupied vehicle does not necessarily need to travel on the quickest return path. The travel needs of occupied vehicles (AVs and human-driven vehicles) should be prioritized with unoccupied AVs rerouted to minimize their impact on occupied vehicles.

However, if deadheading AVs are assigned a route that is too circuitous the AV may not be able to meet a household's trip-making needs and additional vehicles for that household may be required, thus eliminating the household cost savings of owning an AV. This work proposes a differential route assignment for occupied vehicles and deadheading vehicles by prioritizing occupied vehicles. However, this methodology allows for flexibility for the deadheading AV to be on the shortest path in the case of the AV not meeting the household trip-making needs. This assumes that the availability of a personal AV in a household may allow for fewer vehicles at considerable cost savings, as the trips needed for multiple household members can be satisfied by a single AV.

## 2. Research Background

Research into the various impacts of AV implementation has grown significantly with the efforts analyzing the impacts from a wide array of viewpoints. Most literature has focused on population surveys regarding adoption rates, technology comfort and willingness to pay. A majority of the studies on AVs present them as a shared autonomous fleet, which replicates current rideshare systems. The likely impacts of shared AVs on household vehicle ownership were studied and it was found that shared AVs have the potential to significantly reduce household vehicle ownership [6]. A stated preference survey was conducted and the socio-economic characteristics that affect choosing a shared versus private AV were examined [7]. The study found that households with older household members and higher levels of income, education, household size and urbanization are more likely to choose a shared AV.

The factors that influence the public's acceptance of private autonomous vehicles (AVs) were investigated and perceived value of time, perceived risk, willingness to use time more efficiently and perceived AV motion sickness are concluded to be the most important factors in willingness to use private AVs [8]. The literature demonstrates that when AVs are on the market, a significant portion of the AV fleet will be owned by private households [9]. These studies suggest a persistent preference for private ownership, which the current study addresses by developing strategies that cater to the needs and behaviors of private AV owners, ensuring a balance of reducing traffic congestion.

Using data from the US 2009 National Household Travel Survey, a 43% reduction in the vehicle fleet and a reduction in personal vehicle use from 2.1 to 1.2 per household were observed through eliminating existing trip overlap [10]. A similar study examined the possible reduction in vehicle ownership from households switching to private AVs. This study concluded that there would be a 9.5% reduction of private vehicles in the study region due to the household efficiency gains realized by AVs' ability to serve multiple household trips. This study also noted that nearly 30 unoccupied vehicle miles traveled (VMT) will be induced per day per reduced vehicle [11]. The usage of private AVs was investigated for parcel delivery, which can significantly reduce the number of vehicles on the road to reduce traffic congestion and improve air quality [12]. The impact of private AVs was investigated on the miles traveled near activity destinations. The results show that private AV owners traveled an extra 0.11 to 1.51 miles compared to a conventional private vehicle owner. The increase in VMT implied that planners must develop policies to reduce the AV deadheading miles near activity locations by redirecting them to less congested roads to reduce their burden on traffic [13]. These findings are foundational to the current

research, as they highlight the significant impact that more efficient vehicle usage, enabled by private AVs, could have on reducing the number of vehicles per household.

Numerous studies have explored the impact of empty autonomous vehicles (AVs) on traffic congestion, emphasizing the rerouting of unoccupied vehicles as they navigate between passenger pickups and drop [14–17]. The repositioning (relocation) of these unoccupied AV trips resulted in the reduction of congestion on the downtown Austin network [15]. This study utilized a genetic algorithm for repositioning unoccupied AVs by modeling the choice of parking location when AVs move away from the travelers' destination and adjusting the parking costs based on zones to encourage AVs to park at cheaper locations. The impact of city-wide AV fleets on traffic congestion was studied using a multiagent simulation of Berlin, revealing that while AVs could reduce congestion by increasing road capacity, they may exacerbate congestion if road capacity does not improve, despite making travel more convenient and affordable [18]. The outcome of this study suggested that the revised parking fees resulted in a reduction in congestion caused by unoccupied AVs [19]. An optimization-based strategy was proposed for repositioning of shared autonomous vehicles (SAVs) to improve wait times and service levels [20]. SAVs are proposed to reshape travel patterns with potential benefits of reduced car ownership, increased accessibility, enhanced productivity and reduced transportation costs [21].

The impact of using fine-grained (smaller areas) vs. coarse-grained (larger areas) forecasts was examined for repositioning empty vehicles to meet future demand [22]. They found that the accuracy of demand forecasts decreases with increased spatial resolution, and using more granular forecasts improves the operational efficiency of the SAV fleet. Potential equity impacts of AVs and the policies that have been proposed to address these impacts were explored by identifying three main categories of AV-related policies with equity implications: access and inclusion, multimodal transportation and community well-being [23]. A system of SAVs combined with park-and-rides was proposed in residential areas to which the deadheading AV was assumed to return to its initial point until the next request was made [24]. The potential impact of AVs on parking choices was examined and the results suggest that there would be an increase in distance traveled by AVs and reduced travel anxiety. This study also discusses policy interventions to manage parking demand and encourage sustainable AV use like parking pricing strategies and land use planning [25].

The current study expands on these concepts by proposing routing strategies that not only optimize parking choices but also guide AVs to less congested routes during their unoccupied phases, thereby reducing their overall impact on traffic flow as well as the reduction of household ownership. The hypothesis is that a deadheading routing strategy can alleviate congestion caused by empty AVs by using more circuitous routes and retain the benefit of reduced household vehicle ownership provided the deadheading vehicle reaches its destination on time for the next pickup.

Research into competition and cooperative traffic assignment was pioneered in 1985 [26]. A mixed equilibrium model was proposed in a study where certain areas of the road network are dedicated to AVs [27]. A mixed behavior network equilibrium model was formulated as variational inequalities (VIs) that simultaneously describe the routing behaviors of user equilibrium (UE), system optimum (SO) and counter Nash (CN) players [28]. A UE-SO mixed equilibrium strategy was proposed where the connected vehicles act as SO users and the conventional vehicles act as UE users in the network. A mathematical formulation for the UE-SO mixed traffic assignment has also been proposed [29]. A multiclass traffic assignment model was proposed with mixed flow of human-driven vehicles and AVs where the route choice of each class of users used a cross-nested logit (CNL) model and user equilibrium (UE) model, respectively [30].

A mixed equilibrium assignment was conducted and concluded that optimal flow can be achieved with as little as 13% and as many as 54% of agents in compliance [31]. A bi-level programming model was proposed to compute the worst-case equilibrium flow and network performance in a mixed traffic network of human-driven vehicles and AVs [32].

A mixed fleet (including both human-driven and AVs) is used in a study to note that AVs are controllable and thus could be instructed to choose suboptimal routes, which would improve the travel time of human-driven vehicles [33]. They adopted the framework that human-driven vehicles will behave in a user equilibrium (UE) manner in which they choose the shortest path, and AVs could choose a system optimal (SO) strategy that minimizes the total travel time for all users [34,35].

This current research builds on this framework of studies to present a bi-level model with the upper-level problem assigning occupied vehicles using UE assignment and the lower level applying SO for deadheading vehicles using a mixed equilibrium framework. Travel time restrictions of deadheading vehicles are incorporated in the solution method so that they can meet other household needs by limiting the divergence of unoccupied vehicles from the shortest path and ensuring they can make the next household pickup.

**3. Problem Statement**

Consider the simple example in Figure 1 in which 15 occupied vehicles wish to travel from location A to location B on three possible paths. If they follow a user equilibrium (UE) assignment,  $x_1 = 10, x_2 = 5, x_3 = 0$  and  $t_1 = t_2 = t_3 = 20$ . If four vehicles are unoccupied, then the unoccupied vehicles can follow path 3 and decrease the travel times of the occupied vehicles. The result is  $x_1 = 8, x_2 = 3, x_3 = 4$  and  $t_1 = t_2 = 18$  and  $t_3 = 24$ . This would reduce the total system travel time ( $x_1 t_1 + x_2 t_2 + x_3 t_3$ ) from 300 to 294. However, the AV may need to return to pick up another household member for another trip. If the time window for two vehicles is 22 units and that for two vehicles is 30 units, then this assignment scheme would result in some household(s) absorbing the cost of a missed trip or utilizing other transportation services. This paper proposes a methodology to reroute unoccupied vehicles to a longer route while considering the time constraint of the household. If two unoccupied vehicles that are experiencing delay are rerouted to UE assignment and the other two unoccupied vehicles follow path 3, the result is  $x_1 = 9, x_2 = 4, x_3 = 2$  and  $t_1 = t_2 = 19$  and  $t_3 = 22$ . This would reduce the total system travel time from 300 to 291 units without causing delays to the AV owner.

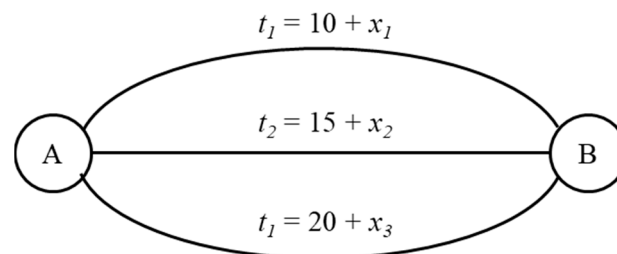


Figure 1. Example network.

**4. Contribution**

The contribution of this paper is a strategy of rerouting deadheading AVs to minimize their impact on occupied vehicles without increasing travel times to the extent that the deadheading AVs cannot meet the household trip-making needs. This study proposes differential route assignment for occupied versus unoccupied vehicles while considering the impacts of unoccupied AV route choice on AV owners.

The remainder of this paper is organized as follows. The proposed methodology is described and three solution strategies are presented in the first section. In the next section, numerical experiments with results are presented for the three strategies to study the system’s performance with the proposed methodologies. Finally, the last section concludes the paper with a summary of significant results, limitations and ideas for future research.

### 5. Methodology

Consider a transportation network  $G(N, A)$  with  $N$  nodes and  $A$  arcs. During the transition to a fully autonomous fleet, two classes of vehicles are using the network: occupied vehicles and unoccupied (deadheading) vehicles. The former is assigned to follow a user equilibrium (UE) traffic assignment whereas deadheading vehicles follow a system optimum (SO) traffic assignment. In a static UE assignment, each user chooses a route that minimizes their own travel time, which tends to better reflect human behavior. A static SO assignment is a model in which the total system travel time is minimized—since the deadheading vehicles are not carrying any passengers this goal places a priority on the persons utilizing the roadway over the AVs.

The objective of the deadheading vehicles is restricted by a return window ensuring the AV owner’s household travel needs are met. The time window is defined as the vehicle delay threshold ( $\theta$ ). The model in this paper was adopted from a previous study and was modified to accommodate delay in the deadheading vehicles [33]. Should delay in deadheading vehicles exceed a certain threshold, these vehicles are then treated as occupied vehicles. The rationale behind treating delayed deadheading vehicles as occupied vehicles is to balance the AV owner’s travel needs while optimizing system performance. This accommodation was performed using a heuristic approach. Three different strategies were proposed in this study in the application of the threshold  $\theta$ .

Strategy 1: Mixed equilibrium assignment with no vehicle delay threshold for the deadheading vehicles.

Strategy 2: Mixed equilibrium assignment with no vehicle delay of more than the 95th percentile for all vehicles.

Strategy 3: Mixed equilibrium assignment with fixed vehicle delay threshold.

### 6. Model Formulation

The adopted model is a recontextualization of the model found in a previous study for mixed equilibrium assignment by augmenting the model in rerouting the deadheading vehicles to UE that are delayed more than the allocated threshold. The model for UE and SO follows traditional formulations with modifications incorporating the segregation of occupied and deadheading vehicles [36]. In this study, we utilized the Bureau of Public Roads (BPR) travel time function to model traffic congestion and travel time variations within the transportation network [37]. BPR function is based on theoretical principles derived from traffic flow theory and offers quick estimations of urban setting parameters. Calibration of the parameters for specific geographical areas is recommended.

The upper-level objective function is:

$$\min_{x_0} z_1(x) = \sum_a \int_0^{x_a} [t_a(x_o + x_d)] dx \tag{1}$$

The lower-level objective function is:

$$\min_{x_d} z_2(x) = \sum_a [t_a(x_o + x_d)(x_o + x_d)] \tag{2}$$

These are constrained by path flow constraints requiring the flows across all paths  $k \in K$  between origin  $r \in R$  and destination  $s \in S$  satisfy demand between origin and destination. It is assumed that the demand between an OD pair remains constant and the demand is uniformly distributed between occupied and deadheading vehicles according to the value of  $e$  in the ten scenarios evaluated in this work.

$$\sum_k f_k^{r,s} = q_o^{r,s} + q_d^{r,s} \forall k, r, s \tag{3}$$

$$\sum_k f_k^{r,s} = q_k^{r,s} \forall k, r, s \tag{4}$$

$$x_a = \sum_r \sum_s \sum_k f_k^{r,s} \delta_{a,k}^{r,s} \tag{5}$$

The mapping (5) produces arc flows  $x_a$  using the path flows  $f_k$  and the binary indicator  $\delta_{a,k}^{r,s}$  which takes the value 1 if the link is  $a$  on path  $k$  between origin  $r$  and destination  $s$ . This indicator can also be used to compute path travel time on an arc.

$$\sum_k f_k^{(r',s')} = q^{r',s'} \forall (r',s') : t_k^d(r',s') - t_k^{UE}(r',s') > \theta \text{ where } r' \subseteq r, s' \subseteq s \tag{6}$$

Constraint (6) is used to ensure deadheading vehicle travel times are falling below the necessary time windows for the household owners of the AVs. If the deadheading vehicles experience delays beyond  $\theta$ , the model adapts to treat them as occupied to fulfill the requirements of the AV owners. For the OD pairs that are experiencing a delay which is indicated as  $r'$  and  $s'$ , the total demand is considered occupied and follows a user equilibrium assignment. This accommodation is performed using a heuristic approach to stratify vehicle assignment for the lower-level problem. BARON 22.2.12 solver is used in solving the nonlinear SO program in the lower-level problem [38]. The methodology is implemented using three different strategies, shown in the following flowchart given in Figure 2 and Algorithm 1.

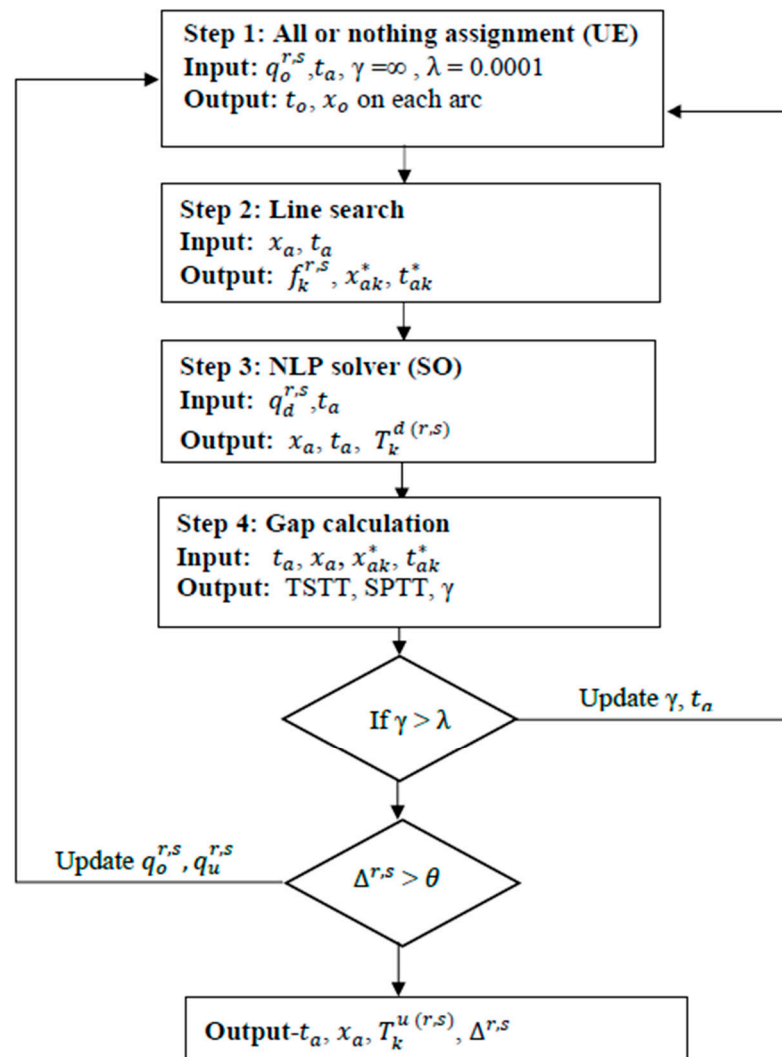


Figure 2. Flow of the Strategy.

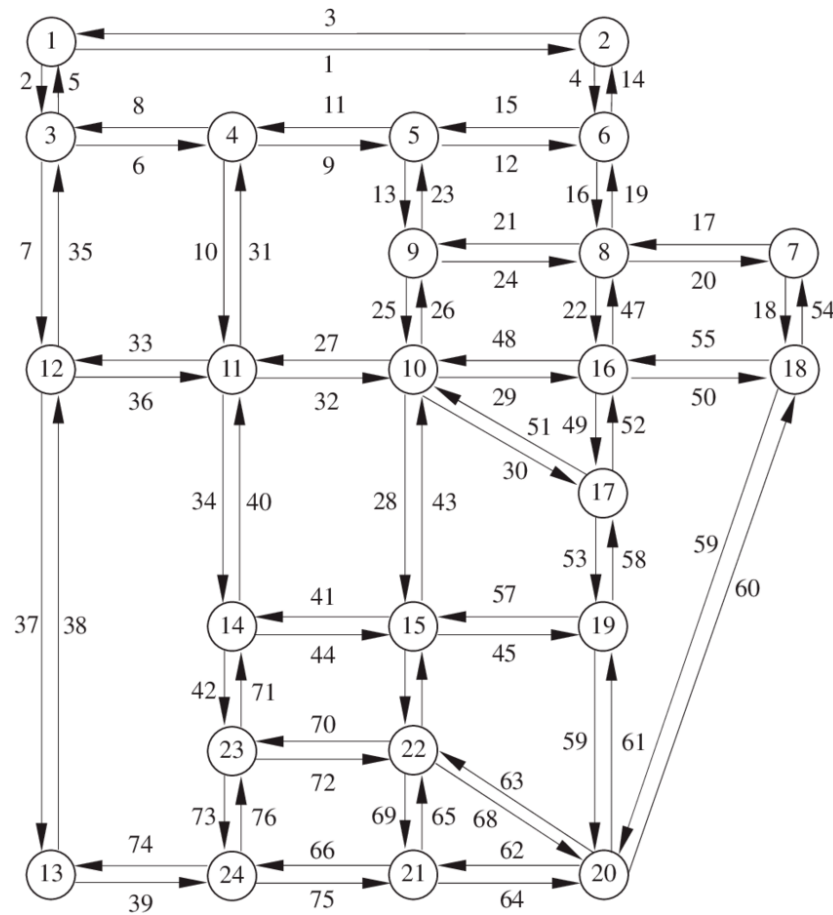
**Algorithm 1.** Strategy Algorithm

**Initialization:**  $q_u^{r,s} = e * q^{r,s}$ ,  $q_o^{r,s} = (1 - e) * q^{r,s}$ ,  $\gamma = \infty$  and  $\lambda = 0.0001$ ,  $it = 1$ ,  $\bar{x} = 0$ ,  $x_a = 0$ ;  $\Delta^{r,s} = \infty$ ,  $T_k^{UE(r,s)} = \sum_a t_a \delta_{a,k}^{r,s}$  from UE assignment,  $\Delta_e^{r,s} = \infty$ ,  
 $\theta = \infty$  in Strategy 1  
 $\theta = \Delta_e^{r,s}$  in Strategy 2  
 $\theta =$  fixed threshold in Strategy 3  
**Begin:**  
for  $(r, s): r \in R$  and  $s \in S$   
    While:  $\gamma > \lambda$  do # While gap is greater than the accuracy  
    for  $(r, s): r \in R$  and  $s \in S$   
         $f_{k^*}^{r,s} = q_o^{r,s}$  #All or nothing assignment  
        for  $a \in A$   
             $\bar{x} = \sum_r \sum_s f_{k^*}^{r,s} \delta_{a,k^*}^{r,s}$   
             $x_o = \alpha \bar{x} + (1 - \alpha)x_o$   
            if  $i = 1$   
                 $\alpha = 1$   
            else:  
                 $z_1(x(\alpha)) = z_1((1 - \alpha)x + \alpha \bar{x})$ ,  $\alpha \in (0,1)$  # Line search method  
            end  
             $t_a = t_o + \left[1 + 0.15 \left(\frac{x_o}{c_a}\right)^4\right]$  # Updating travel times  
            end  
        end  
         $\min_{x_u} z_2(x) = \sum_a [t_a(x_o + x_u)(x_o + x_u)]$  # System optimum assignment  
        for  $(r, s): r \in R$  and  $s \in S$   
             $\sum_{k \in p^{r,s}} f_k^{r,s} = q_d^{r,s}$   
            for  $a \in A$   
                 $x_d = \sum_r \sum_s \sum_k f_k^{r,s} \delta_{a,k}^{r,s}$   
                 $t_a = t_o + \left[1 + 0.15 \left(\frac{x_u + x_o}{c_a}\right)^4\right]$  # Updating travel times again  
                 $x_a = x_d + x_o$   
            end  
        end  
        TSTT =  $\sum_a x_a t_a$   
        SPTT =  $\sum_a x_{ak}^* t_{ak}^*$   
         $i = i + 1$   
    end  
     $t_k^{d(r,s)} = \sum_a t_a \delta_{a,k}^{r,s}$   
     $\Delta^{r,s} = t_k^{d(r,s)} - t_k^{UE(r,s)}$   
    If  $\Delta^{r,s} < \theta$ , # Check for delay in occupied vehicles  
        end  
    else,  
         $q_o^{r,s} = q^{r,s}$  and  $q_d^{r,s} = 0$  # Updating the demand for delayed vehicles to reroute

**7. Design of the Experiments**

The algorithm was scripted and implemented in Python 3.10 on a 24 GB RAM computer. Two transportation networks—Sioux Falls [39] (Figure 3) and Eastern Massachusetts [40]—were used to conduct the experiments. The Sioux Falls network has

24 nodes, 76 links, and 528 origin–destination pairs. Eastern Massachusetts has 74 nodes, 258 links and 5402 OD pairs.



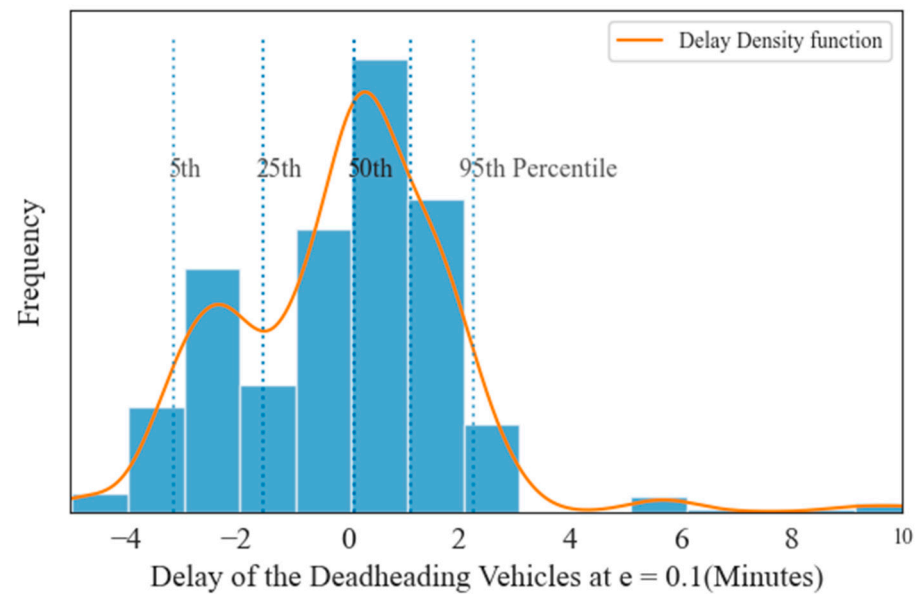
**Figure 3.** Sioux Falls Network, Sioux Falls, North Dakota, USA (with permission from Taylor and Francis).

In this paper,  $e$  varies from 0 to 100 in increments of ten. For example, if  $e = 10$ , 10% of all vehicles are deadheading. The market penetration of AVs is taken into account by using the % of unoccupied vehicles from  $e = 10$  to 90%. An assumption is made that the demand for an OD pair remains the same after the introduction of AVs. As long as deadheading vehicles reach their next destination on time, their value of time is considered zero since there is no human time to consider. Unoccupied vehicles that face considerable delays due to the SO routing will be rerouted in strategies 2 and 3 to avoid delays impacting their ability to serve the subsequent household trip. After the execution of the initial mixed equilibrium assignment, the delay of the unoccupied vehicles compared to occupied is examined.

Travel delays are restricted by a 95th percentile threshold in strategy 2 and a fixed threshold in strategy 3. This indicates that the unoccupied vehicle can take a longer route as long as it is not delayed by more than the threshold ( $\theta$ ) compared to the route that would be chosen in a user equilibrium traffic assignment. The fixed threshold is derived from the results of strategy 1 (Figure 4), indicating the distribution of the delay in this application of the deadheading vehicles has stabilized at 5 min. This value could change in different applications. The rationale behind these choices comes from the need to explore different scenarios and evaluate the impact of the parameter of the delay threshold on the overall system performance. The 95th percentile values give insights into the performance under conditions that exceed most instances. The fixed delay threshold, for instance, allows



for an examination of scenarios where extended delay is tolerated, capturing real-world conditions.



**Figure 4.** Distribution of the delay of the deadheading vehicles at  $e = 0.1$ .

## 8. Results and Discussion

The three strategies were implemented on two transportation networks (Sioux Falls and Eastern Massachusetts) and the results are analyzed for the total system travel time and travel time for the occupied vehicles. As expected, restricting a portion of the vehicle fleet to an SO assignment results in progressively lower total system travel time as  $e$  increases from 10 to 90. This suggests that increase in penetration of AVs in the market would lead to an increase in the number of deadheading vehicles traveling back to their original location, potentially saving time for the owners of such vehicles. This result is in alignment with the results of the previous study where the total system travel time decreases with an increase in the percentage of vehicles that are controlled by a central system [33]. The spatial distribution of the demand is an important factor to consider in traffic management strategies. However, this methodology involves the travel time calculations that consider network congestion, serving as an indirect indicator of the spatial distribution of demand.

Both Sioux Falls and Eastern Massachusetts networks indicate that adopting these strategies reduces total system travel time and mitigates delay impacts from the deadheading vehicles.

## 9. Sioux Falls Network

### 9.1. Strategy 1

Occupied vehicles are assigned according to the principles of UE assignment and deadheading vehicles are assigned using SO where  $\theta = \infty$ , which is, in effect, no threshold. With this mixed equilibrium assignment, 57 to 65% of the occupied vehicles have faster travel times compared to the UE routing assignment. The average delay of the occupied vehicles is negative, which indicates that occupied vehicles saw a reduction in travel time overall due to the rerouting of the unoccupied vehicles (Figure 5). Even for vehicles that experienced a delay, the maximum delay is 3 min. The 95th percentile of the delay is 2 min. Some deadheading vehicles did experience a delay; however, as long as the deadheading vehicle reached the destination in the time window, the value of time of the unoccupied vehicle was zero. The maximum delay with  $e = 0.9$  was 10.23 min.

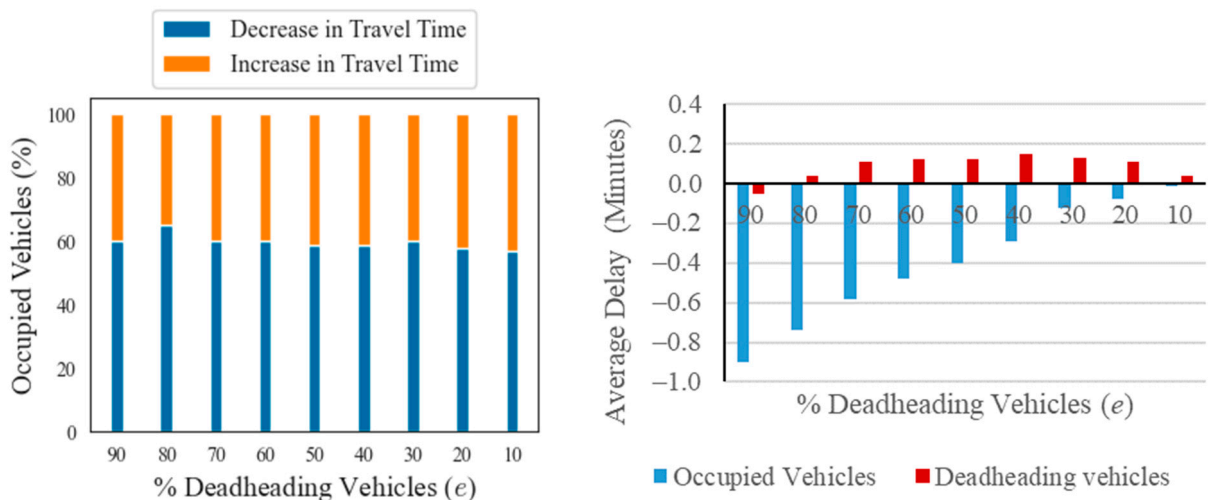


Figure 5. Delay ( $\mu^{r,s}$ ) of the OV and UVs—Strategy 1.

9.2. Strategy 2

Deadheading vehicles that have a delay of more than  $\Delta_e^{r,s}$  are removed from the SO assignment and rerouted according to UE with occupied vehicles. The values of  $\Delta_e^{r,s}$  ranged from 0.5 to 2 min. Additionally, 53 to 63% of the occupied vehicles have faster travel times compared to the user equilibrium routing assignment. The average delay of occupied vehicles is negative which indicates that most of the occupied vehicles have a faster travel time. The maximum delay of occupied vehicles is 2.8 min. The maximum delay for unoccupied vehicles is 2.1 min with  $e = 0.9$  (Figure 6).

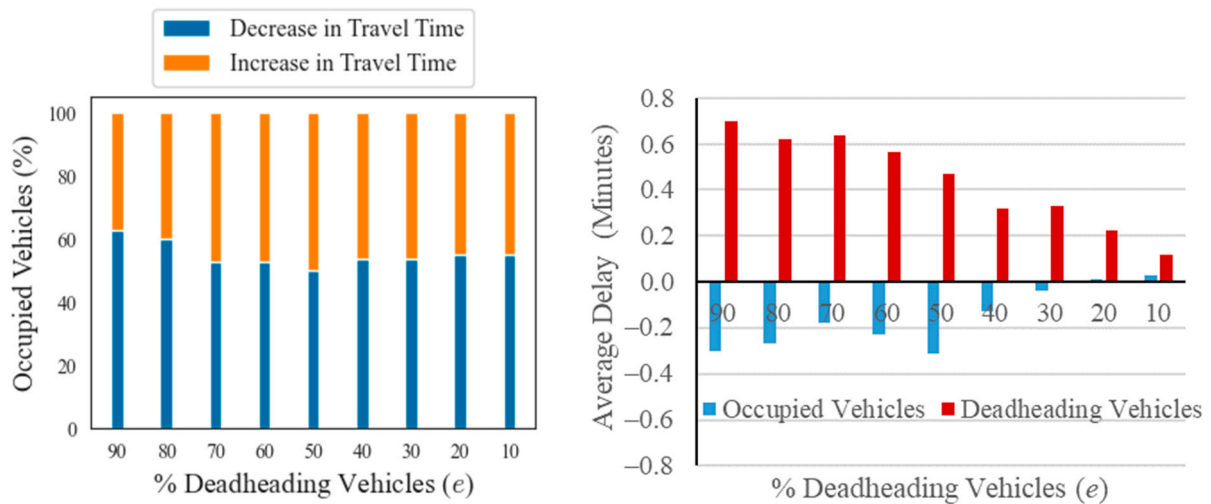


Figure 6. Delay ( $\mu^{r,s}$ ) of the OV and UVs—Strategy 2.

9.3. Strategy 3

Restricting the deadheading vehicles' delay to a fixed threshold of 5 min, the average delay of occupied vehicles is negative which indicates that most of the occupied vehicles have a faster travel time. In this case, 49 to 59% of the occupied vehicles have faster travel times compared to the user equilibrium routing assignment. The maximum delay of occupied vehicles is 3.3 min. The maximum delay for unoccupied vehicles at  $e = 0.7$  is 3.2 min (Figure 7).

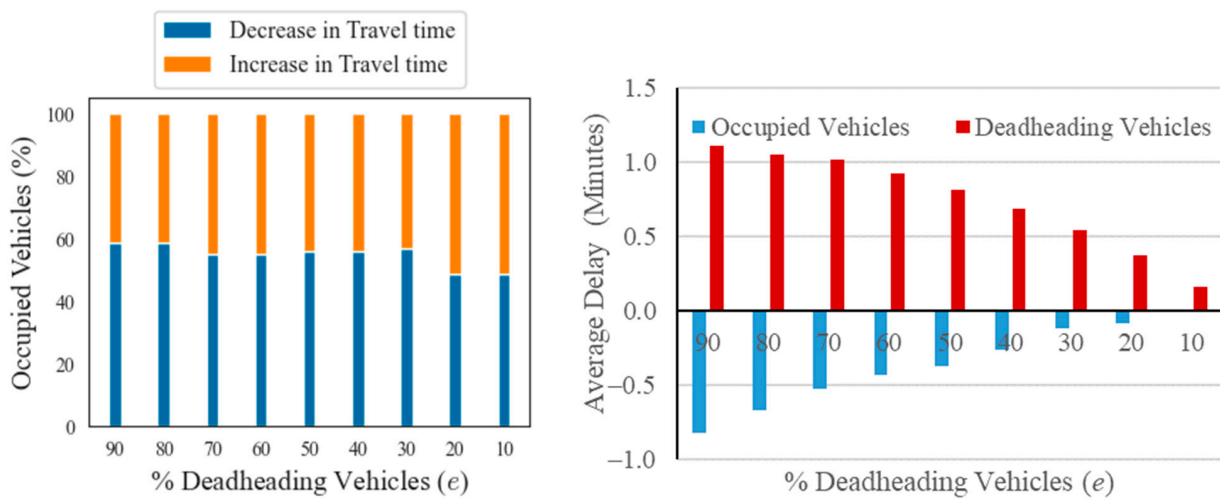


Figure 7. Delay ( $\mu^{r,s}$ ) of the OVs and UVs—Strategy 3.

9.4. Total System Travel Time Savings

In all three strategies, the TSTT improved compared to the UE traffic assignment. The total time savings is attributed to the rerouting of the deadheading vehicles. As the percentage of deadheading vehicles increases, the savings also increase. This indicates that a higher market penetration of AVs would result in an increased number of unoccupied vehicles returning to their point of origin, leading to potential time savings for owners of these vehicles.

Strategy 1 has the highest amount of savings as there is no delay threshold for the deadheading vehicles. However, to maintain the balance between system efficiency and AV owner trip satisfaction, strategies 2 and 3 are considered. Strategy 3 exhibits better performance compared to strategy 2. This improved efficiency is attributable to the fixed delay threshold employed in strategy 3, contrasting with the variable threshold ranging from 0.5 to 2 min in strategy 2. A direct correlation exists between increasing the delay threshold and maximizing savings (Figure 8).

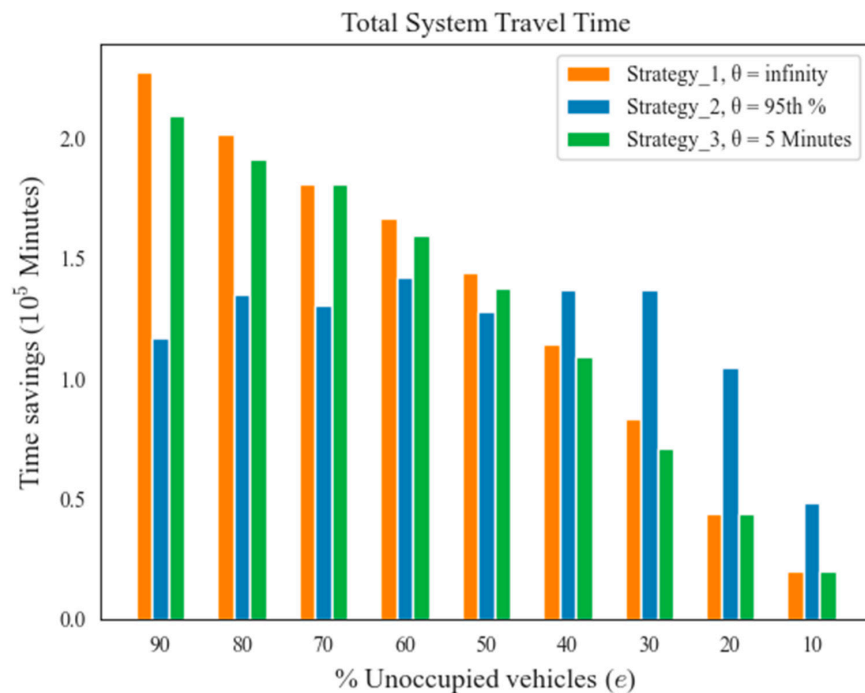


Figure 8. Total System Travel Time Savings.

### 10. Algorithm Convergence

The methodology is converging towards the objective of mixed equilibrium with a decrease in the observed gap over iterations. The total system travel time of occupied and deadheading vehicles increases to reach convergence over iterations (Figure 9).

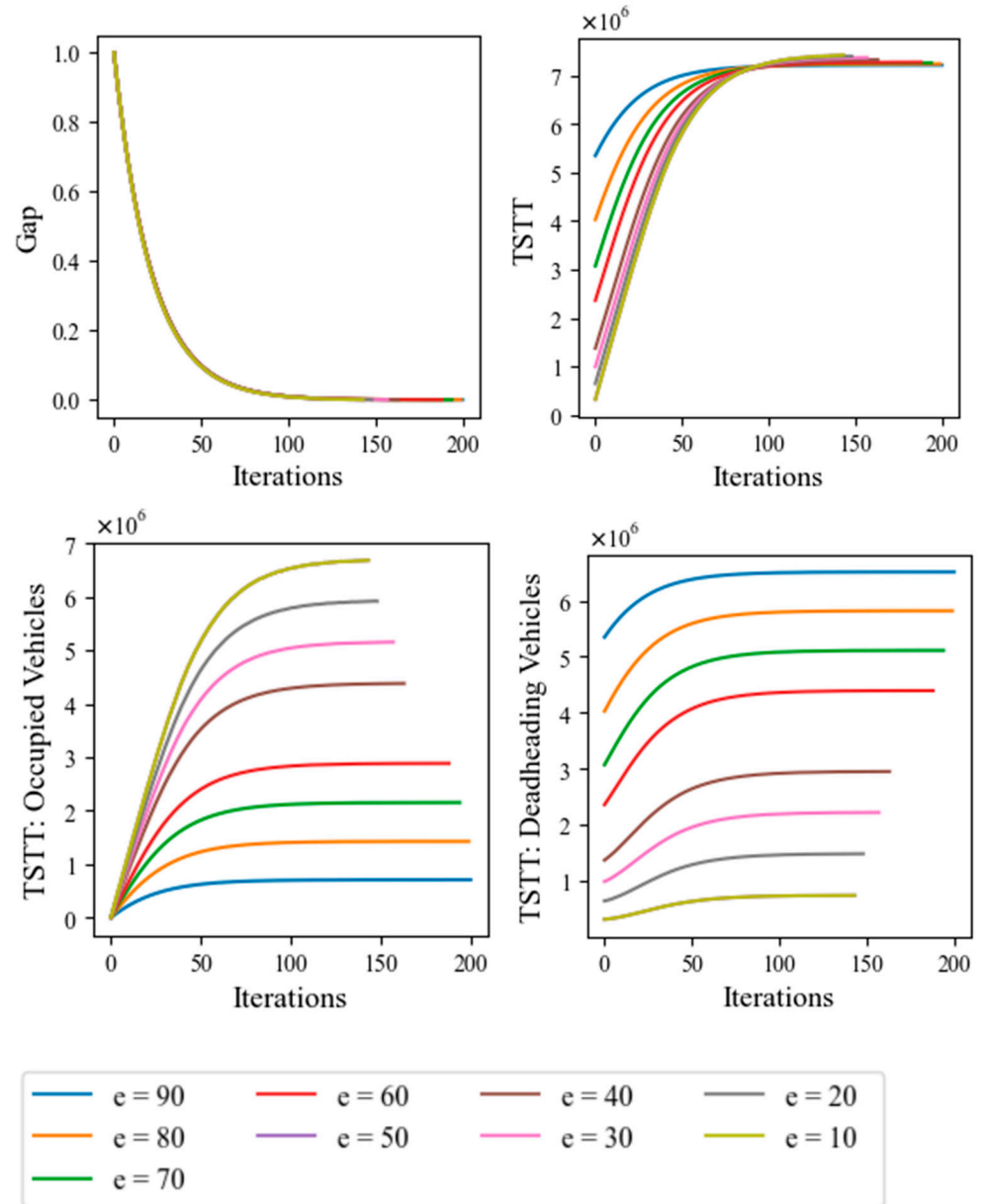


Figure 9. Algorithm Convergence for Strategy 1—Sioux Falls Network.

### 11. Eastern Massachusetts Network

#### 11.1. Strategy 1

Occupied vehicles are assigned according to the principles of UE and deadheading vehicles are assigned using SO. In this case, 57 to 73% of occupied vehicles saw a reduction in travel time due to the rerouting of deadheading vehicles (Figure 10). Some unoccupied vehicles experienced delayed, however, as long as the vehicle reached the destination in the time window, the value of time of the empty vehicle is considered to be zero. The maximum delay, in the case of  $e = 0.2$ , is 12 min.

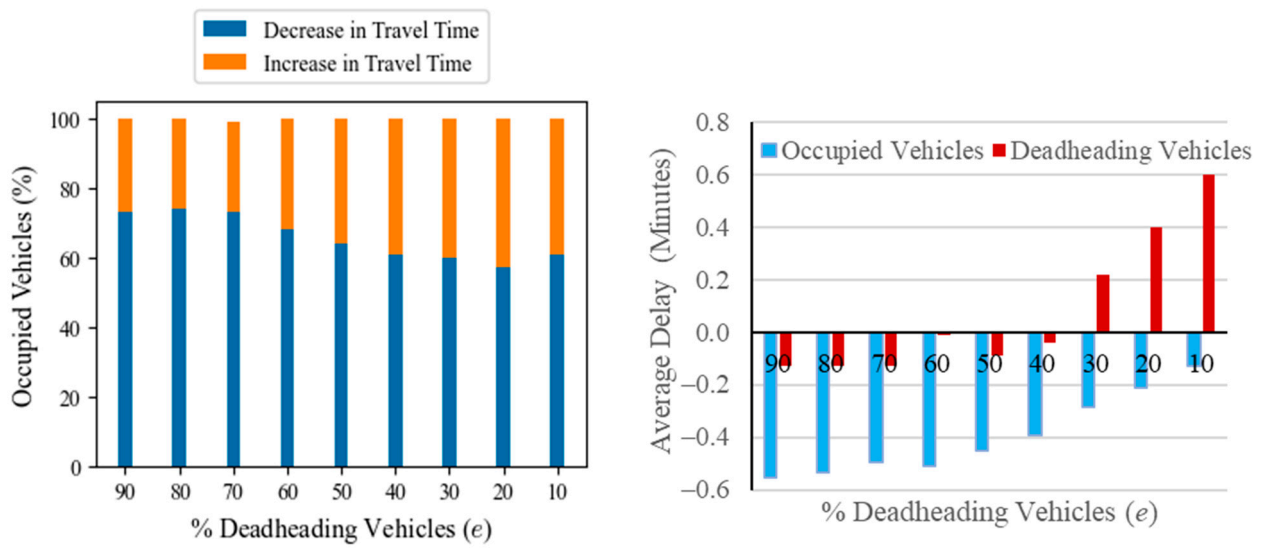


Figure 10. Delay ( $\mu^{r,s}$ ) of the OV and UVs—Strategy 1.

11.2. Strategy 2

Deadheading vehicles that have a delay of more than  $\Delta_e^{r,s}$  are removed from the deadheading assignment and rerouted according to UE with occupied vehicles. The average delay of occupied vehicles is negative which indicates that most of the occupied vehicles have a reduction in travel time. In this case, 40 to 58% of the occupied vehicles have a faster travel time compared to user equilibrium assignment. The values of  $\Delta_e^{r,s}$  ranged from 1.5 to 2.5 min. Hence, the maximum delay of occupied vehicles is 2.3 min at  $e = 0.9$ . The maximum delay for unoccupied vehicles is 2.4 min at  $e = 0.1$  (Figure 11).

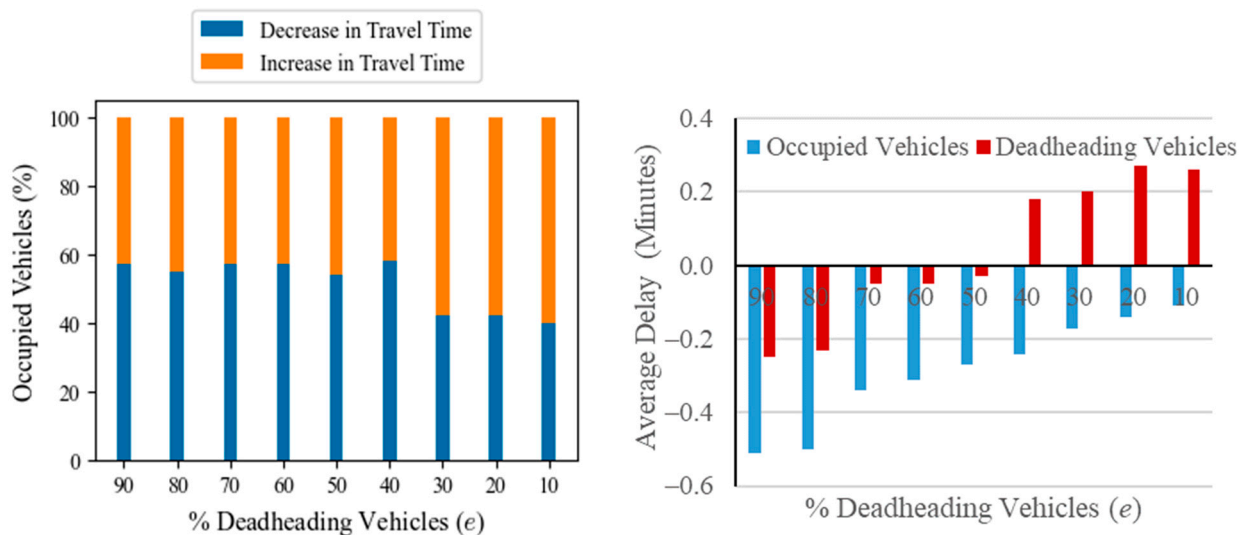


Figure 11. Delay ( $\mu^{r,s}$ ) of the OV and—Strategy 2.

11.3. Strategy 3

Restricting the deadheading vehicles’ delay threshold to 5 min, 57 to 72% of the vehicles have faster travel times. The maximum delay of occupied vehicles was still 2.6 min at  $e = 0.8$ . The average delay of occupied vehicles is negative which indicates that most of the occupied vehicles have a reduction in travel time (Figure 12).

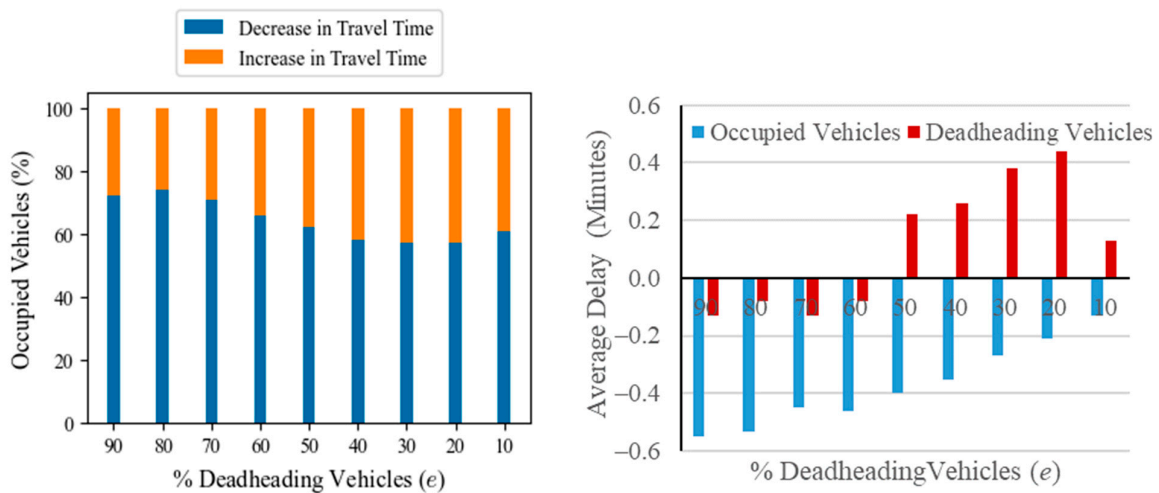


Figure 12. Delay ( $\mu^{r,s}$ ) of the OVs and—Strategy 3.

11.4. Total System Travel Time Savings

In all three strategies, the TSTT improved compared to the UE traffic assignment. The total time savings is attributed to the rerouting of the deadheading vehicles. As the percentage of deadheading vehicles increases, the savings also increase. This indicates that a higher market penetration of AVs would result in an increased number of unoccupied vehicles returning to their point of origin, leading to potential time savings for owners of these vehicles. Strategy 1 has the highest amount of savings as there is no delay threshold for the deadheading vehicles. However, to maintain the balance between system efficiency and AV owner trip satisfaction, strategies 2 and 3 are considered. Strategy 3 exhibits better performance compared to strategy 2. This improved efficiency is attributable to the fixed delay threshold employed in strategy 3, contrasting with the variable threshold ranging from 1.5 to 2.5 min in strategy 2. A direct correlation exists between increasing the delay threshold and maximizing savings (Figure 13).

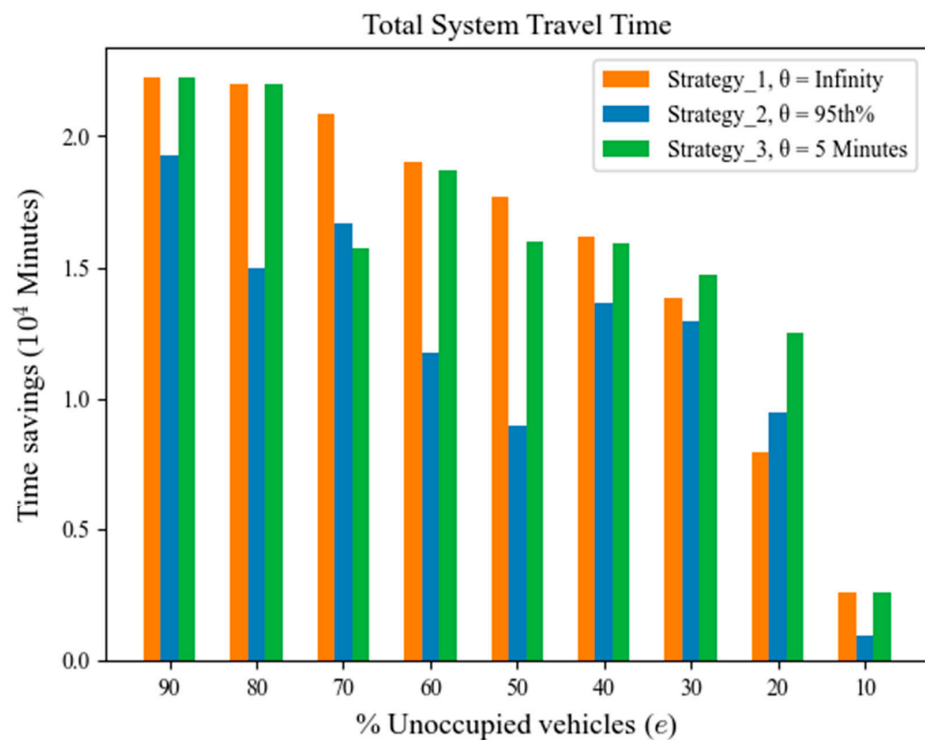


Figure 13. Total System Travel Time Plots.

Most occupied vehicles faced shorter travel times compared to the regular user equilibrium traffic assignment. None of the AV owners faced any significant delays when  $\theta$  was enforced. Another important aspect of this study is confirmation that AV owners would be able to accommodate multiple trips and thus potentially own fewer vehicles. These research findings align with the results of the existing research, supporting the idea that cooperative routing strategies can significantly improve overall traffic efficiency, in the context of mixed traffic conditions [41,42].

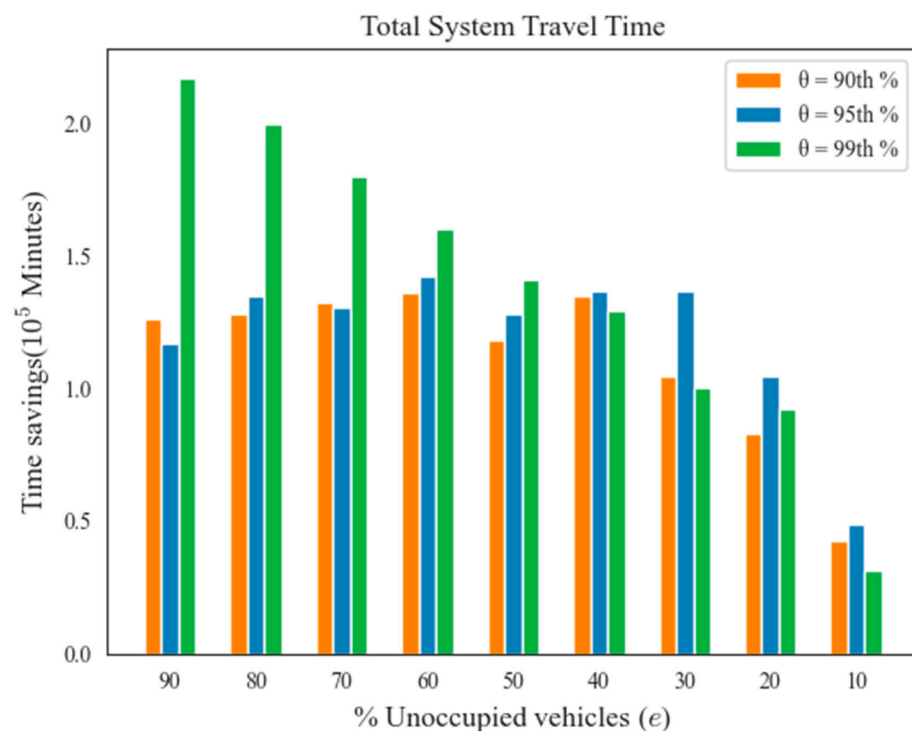
The computational time of the algorithm is higher for Eastern Massachusetts than Sioux Falls network as the complexity of the network is greater. A comparison of results for both the networks is presented in Table 1. The computational times provided were calculated using a computer equipped with 24 GB memory.

**Table 1.** Comparison of results for Sioux Falls and Eastern Massachusetts.

	Performance Metric	Sioux Falls	Eastern Massachusetts
Strategy 1	TSTT ( $10^6$ min)	7.19–7.43	1.66–1.68
	Delay $\Delta^{r,s}$ (min)	−0.1–0.15	0.01–0.6
	Computational time (h)	0.75–0.95	6.5–8.25
Strategy 2	TSTT ( $10^6$ min)	7.19–7.43	1.66–1.68
	Delay $\Delta^{r,s}$ (min)	0.11–0.63	−0.25–0.27
	Computational time (h)	0.75–1.25	12.5–19.25
Strategy 3	TSTT ( $10^6$ min)	7.19–7.43	1.66–1.68
	Delay $\Delta^{r,s}$ (min)	0.16–1.11	0.08–0.44
	Computational time (h)	5.0–7.5	6.5–12.5

### 12. Sensitivity Analysis

A sensitivity analysis is conducted on the parameter delay threshold ( $\mu^{r,s}$ ) using 90th, 95th, and 99th percentiles to analyze the TSTT savings in the Sioux Falls network. The findings suggest that the influence of the vehicle delay threshold parameter becomes more pronounced up to a threshold of  $e = 40\%$  (Figure 14).



**Figure 14.** Sensitivity Analysis of TSTT.

### 13. Conclusions

When AVs are a significant part of the vehicle fleet, unoccupied AVs will contribute significantly to traffic congestion and its cost in human time. To reduce the impact of the deadheading vehicles on occupied vehicles, a route assignment is implemented considering time constraints of privately owned deadheading vehicles. User equilibrium routing of occupied vehicles is followed by SO assignment of deadheading AVs in a mixed equilibrium assignment framework. Deadheading AVs that experience considerable delays due to rerouting are reconsidered as part of the UE fleet.

This study proposed a bi-objective program to evaluate and balance trade-offs between congestion reduction and the trip-making needs of households. This study examined three approaches for routing deadheading autonomous vehicles (AVs): in the first strategy, occupied vehicles are assigned to user equilibrium, while deadheading AVs are assigned to a system optimum with no consideration for the circuitousness of the rerouting. This resulted in significant savings in total system travel time, particularly with a higher number of deadheading AVs.

In the second strategy, 95th percentile values of delay were utilized to identify rerouting strategies that were too circuitous. The delay threshold values varied between 0.5 to 1.5 min in the Sioux Falls network and 1.5 to 2.5 min in the Eastern Massachusetts network. While the system has lower TSTT than user equilibrium assignment in both of the networks, the total travel time savings were lower compared to strategy 1 due to a larger number of deadheading vehicles opting for a selfish path. In the third strategy, with a fixed delay threshold of 5 min, the total system travel time savings were higher compared to strategy 2 but still less than strategy 1. This variation in strategies highlights the trade-offs between optimizing delay thresholds and overall system efficiency. A sensitivity analysis was also performed to observe the impact of the delay threshold parameter on the system efficiency. The major findings were:

- (a) Most occupied vehicles experienced shorter travel times compared to UE assignment of deadheading vehicles.
- (b) None of the AV owners faced significant delays when  $\theta$  was enforced.
- (c) The benefits of the method are more apparent when AV penetration is higher and there are a larger number of deadheading vehicles in the system.
- (d) Confirmation that AV owners can accommodate multiple trips with a single AV and thus potentially own fewer vehicles.

### 14. Limitations and Future Work

An underlying assumption in this work is that the AV immediately proceeds to its next pickup location after completing a household trip and any waiting time is accumulated at that pickup point. Intermediate points for vehicle storage while waiting for the next pickup are not considered in this analysis. It is also assumed that AVs operate similarly to human-driven vehicles regarding headway and platooning behavior.

Future research should consider intermediate points for vehicle storage and utilize pickup tolerance time windows from travel surveys paired with field traffic performance data. The heuristic used to apply the delay threshold  $\theta$  can be improved through the usage of exact methods, such as Lagrangian relaxation or Bender's decomposition. Even larger-scale implementations will be beneficial in future study for understanding the transferability of these results and highlighting new potential benefits and detriments of the method. Future research should prioritize the integration of traffic flow models. This approach will allow for a deeper understanding of how AV-specific behaviors, like platooning, interact with mixed traffic environments and inform more effective traffic management solutions. Considering the interconnected nature of the routing challenge for unoccupied autonomous vehicles (AVs), the dynamic traffic conditions on the road network and the spatial distribution of demand, future studies should explore the incorporation of dynamic user equilibrium (DUE) and dynamic system optimum (DSO) into their research endeavors.



### 15. Practical Applications

**Reduced Fleet:** This study confirms AV owners can accommodate multiple trips with a single AV and thus potentially own fewer vehicles.

**Traffic Optimization:** AV relocation can be used to redirect the empty AVs to less congested routes. This can reduce congestion for occupied vehicles and improve system efficiency.

**Fleet Management:** When there is a central system to monitor AV fleets, this study can be used to distribute the vehicles to manage areas with high demand.

**Author Contributions:** The authors confirm contribution to the paper as follows: study conception and design: S.M., N.L. and D.B.; data collection: S.M.; analysis and interpretation of results: S.M., N.L. and D.B.; draft manuscript preparation: S.M. and N.L. All authors have read and agreed to the published version of the manuscript.

**Funding:** The work is supported and funded by the Center for Advanced Multimodal Mobility Solutions and Education (CammSE), project ID: 2020 Project 09.

**Institutional Review Board Statement:** Not Applicable.

**Informed Consent Statement:** Not Applicable.

**Data Availability Statement:** Some or all data, models or code generated or used during the study is available from the corresponding author by request.

**Acknowledgments:** This work was supported by the CammSE (Center for Advanced Multimodal Mobility Solutions and Education). CammSE is a Tier 1 UTC (University Transportation Center) and a consortium of five universities led by The University of North Carolina, Charlotte (UNC Charlotte).

**Conflicts of Interest:** The authors declare no conflict of interest.

### Abbreviations

The following symbols are used in this paper:

$N$	Nodes
$A$	Arcs
$x_a$	Flow on arc $a$
$x_o$	Flow on arc of occupied vehicles
$x_d$	Flow on arc of deadheading vehicles
$t_o$	Arc travel time of occupied vehicles
$t_d$	Arc travel time of deadheading vehicles
$k^{r,s} \in K^{r,s}$	Path set $k$ from $r$ to $s$
$r \in R$	Origins
$s \in S$	Destinations
$(r', s')$	Origin–destination pair with delay
$f_k^{r,s}$	Flow on path $k$ between origin $r$ and destination $s$
$k^{*r,s}$	Shortest travel time path from $r$ to $s$ [43]
$q^{r,s}$	Demand between origin $r$ and destination $s$
$e$	Percentage demand of deadheading vehicles
$q_u^{r,s}$	Deadheading vehicle demand between origin $r$ and destination $s = e * q^{r,s}$
$q_o^{r,s}$	Occupied vehicle demand between origin $r$ and destination $s = (1 - e) * q^{r,s}$
$p^{r,s}$	Optimal number of paths in the system optimum assignment in the increasing order of travel time = $\{k_1, k_2, k_3, k_4 \dots k_p\}$
$\delta$	Binary variable indicating if arc $a$ is on path $k$ between origin $r$ and destination.
$\delta_{a,k}^{r,s}$	$\begin{cases} 1 & \text{if arc } a \text{ is on path } k \text{ between } r \text{ and } s \\ 0 & \text{if arc } a \text{ is not on path } k \text{ between } r \text{ and } s \end{cases}$
$t_0$	Free flow travel time on the arc $a$
$C_a$	Capacity of arc $a$
$t_a$	Travel time function on arc $a$ : $t_a = t_0 + \left[1 + 0.15 \left(\frac{x_a}{C_a}\right)^4\right]$ [37]
$x_{ak}^*$	Flow on arc $a$ on the shortest path $k^*$
$t_{ak}^*$	Travel time on arc $a$ on the shortest path $k^*$

$TSTT$	Total system travel time = $\sum_a x_a t_a$
$SPTT$	Shortest path travel time = $\sum_a x_a^* t_a^*$ on the shortest path $k^*$
$\gamma$	Relative gap; $\frac{TSTT}{SPTT} - 1$
$\lambda$	Accuracy
$z_1$	Lower-level objective function of occupied vehicles
$z_2$	Upper-level objective function of deadheading vehicles
$t_k^{UE(r,s)}$	Travel time on path $k$ between origin $r$ and destination $s$ in user equilibrium traffic assignment
$t_k^d(r,s)$	Travel time on path $k$ between origin $r$ and destination $s$ of deadheading vehicles
$\Delta^{r,s}$	Delay of the deadheading vehicles compared to user equilibrium traffic assignment; $t_k^d(r,s) - t_k^{UE(r,s)}$
$\mu^{r,s}$	Delay of the vehicle compared to user equilibrium traffic assignment for each OD pair
$\theta$	Delay threshold for the deadheading vehicle
$\Delta_e^{r,s}$	95th percentile of the $\Delta^{r,s}$ delay of the deadheading vehicles with $e$ as % of the demand of deadheading vehicles
$h$	Deadheading vehicle delay threshold in strategy 3

## References

- Fagnant, D.J.; Kockelman, K. Preparing a nation for autonomous vehicles: Opportunities, barriers and policy recommendations. *Transp. Res. Part A Policy Pract.* **2015**, *77*, 167–181. [CrossRef]
- Mavromatis, I.; Tassi, A.; Piechocki, R.J.; Sooriyabandara, M. On urban traffic flow benefits of connected and automated vehicles. In Proceedings of the 2020 IEEE 91st Vehicular Technology Conference (VTC2020-Spring), Antwerp, Belgium, 25–28 May 2020; IEEE: Piscataway, NJ, USA, 2020; pp. 1–7.
- Federal Highway Administration. Policy and Strategy Analysis Team. Available online: <https://www.fhwa.dot.gov/policy/otps/policyanalysis.cfm> (accessed on 17 May 2023).
- Gopalakrishna, D.; Carlson, P.J.; Sweatman, P.; Raghunathan, D.; Brown, L.; Serulle, N.U. *Impacts of Automated Vehicles on Highway Infrastructure*; Federal Highway Administration: Washington, DC, USA, 2021.
- Duarte, F.; Ratti, C. The impact of autonomous vehicles on cities: A review. *J. Urban Technol.* **2018**, *25*, 3–18. [CrossRef]
- Menon, N.; Barbour, N.; Zhang, Y.; Pinjari, A.R.; Mannering, F. Shared autonomous vehicles and their potential impacts on household vehicle ownership: An exploratory empirical assessment. *Int. J. Sustain. Transp.* **2019**, *13*, 111–122. [CrossRef]
- Nazari, F.; Noruzoliaee, M.; Mohammadian, A.K. Shared versus private mobility: Modeling public interest in autonomous vehicles accounting for latent attitudes. *Transp. Res. Part C Emerg. Technol.* **2018**, *97*, 456–477. [CrossRef]
- Zou, X.; Logan, D.B.; Vu, H.L. Modeling public acceptance of private autonomous vehicles: Value of time and motion sickness viewpoints. *Transp. Res. Part C Emerg. Technol.* **2022**, *137*, 103548. [CrossRef]
- Lavieri, P.S.; Garikapati, V.M.; Bhat, C.R.; Pendyala, R.M.; Astroza, S.; Dias, F.F. Modeling individual preferences for ownership and sharing of autonomous vehicle technologies. *Transp. Res. Rec.* **2017**, 2665, 1–10. [CrossRef]
- Schoettle, B.; Sivak, M. *Potential Impact of Self-Driving Vehicles on Household Vehicle Demand and Usage*; University of Michigan, Ann Arbor, Transportation Research Institute: Ann Arbor, MI, USA, 2015.
- Zhang, W.; Guhathakurta, S.; Khalil, E.B. The impact of private autonomous vehicles on vehicle ownership and unoccupied VMT generation. *Transp. Res. Part C Emerg. Technol.* **2018**, *90*, 156–165. [CrossRef]
- Schlenker, T.; Martins-Turner, K.; Bischoff, J.F.; Nagel, K. Potential of private autonomous vehicles for parcel delivery. *Transp. Res. Rec.* **2020**, 2674, 520–531. [CrossRef]
- Bahk, Y.; Hyland, M.F.; An, S. Private Autonomous Vehicles and Their Impacts on Near-Activity Location Travel Patterns: Integrated Mode Choice and Parking Assignment Model. *Transp. Res. Rec.* **2022**, 2676, 276–295. [CrossRef]
- Levin, M.W. Congestion-aware system optimal route choice for shared autonomous vehicles. *Transp. Res. Part C Emerg. Technol.* **2017**, *82*, 229–247. [CrossRef]
- Levin, M.W.; Smith, H.; Boyles, S.D. Dynamic four-step planning model of empty repositioning trips for personal autonomous vehicles. *J. Transp. Eng. Part A Syst.* **2019**, *145*, 04019015. [CrossRef]
- Metz, D. Developing policy for urban autonomous vehicles: Impact on congestion. *Urban Sci.* **2018**, *2*, 33. [CrossRef]
- Cohen, T.; Cavoli, C. Automated vehicles: Exploring possible consequences of government (non) intervention for congestion and accessibility. *Transp. Rev.* **2019**, *39*, 129–151. [CrossRef]
- Maciejewski, M.; Bischoff, J. Congestion effects of autonomous taxi fleets. *Transport* **2018**, *33*, 971–980. [CrossRef]
- Levin, M.W.; Wong, E.; Nault-Maurer, B.; Khani, A. Parking infrastructure design for repositioning autonomous vehicles. *Transp. Res. Part C Emerg. Technol.* **2020**, *120*, 102838. [CrossRef]
- De Souza, F.; Gurumurthy, K.M.; Auld, J.; Kockelman, K.M. An Optimization-based Strategy for Shared Autonomous Vehicle Fleet Repositioning. In *Vehits; 2020; Proceedings of the 6th International Conference on Vehicle Technology and Intelligent Transport Systems*; pp. 370–376. Available online: [https://www.cae.utexas.edu/prof/Kockelman/public\\_html/TRB21SAVrepositioning.pdf](https://www.cae.utexas.edu/prof/Kockelman/public_html/TRB21SAVrepositioning.pdf) (accessed on 10 June 2024).

21. Eluru, N.; Choudhury, C.F. Impact of shared and autonomous vehicles on travel behavior. *Transportation* **2019**, *46*, 1971–1974. [[CrossRef](#)]
22. Hyland, M.; Dandl, F.; Bogenberger, K.; Mahmassani, H. Integrating demand forecasts into the operational strategies of shared automated vehicle mobility services: Spatial resolution impacts. *Transp. Lett.* **2020**, *12*, 671–676. [[CrossRef](#)]
23. Emory, K.; Douma, F.; Cao, J. Autonomous vehicle policies with equity implications: Patterns and gaps. *Transp. Res. Interdiscip. Perspect.* **2022**, *13*, 100521. [[CrossRef](#)]
24. Zhou, Y.; Li, Y.; Hao, M.; Yamamoto, T. A system of shared autonomous vehicles combined with park-and-ride in residential areas. *Sustainability* **2019**, *11*, 3113. [[CrossRef](#)]
25. Zhang, W.; Guhathakurta, S. Parking spaces in the age of shared autonomous vehicles: How much parking will we need and where? *Transp. Res. Rec.* **2017**, *2651*, 80–91. [[CrossRef](#)]
26. Haurie, A.; Marcotte, P. On the relationship between Nash—Cournot and Wardrop equilibria. *Networks* **1985**, *15*, 295–308. [[CrossRef](#)]
27. Chen, Z.; He, F.; Yin, Y.; Du, Y. Optimal design of autonomous vehicle zones in transportation networks. *Transp. Res. Part B Methodol.* **2017**, *99*, 44–61. [[CrossRef](#)]
28. Yang, H.; Zhang, X.; Meng, Q. Stackelberg games and multiple equilibrium behaviors on networks. *Transp. Res. Part B Methodol.* **2007**, *41*, 841–861. [[CrossRef](#)]
29. Bagloee, S.A.; Sarvi, M.; Patriksson, M.; Rajabifard, A. A mixed user-equilibrium and system-optimal traffic flow for connected vehicles stated as a complementarity problem. *Comput.-Aided Civ. Infrastruct. Eng.* **2017**, *32*, 562–580. [[CrossRef](#)]
30. Wang, J.; Peeta, S.; He, X. Multiclass traffic assignment model for mixed traffic flow of human-driven vehicles and connected and autonomous vehicles. *Transp. Res. Part B Methodol.* **2019**, *126*, 139–168. [[CrossRef](#)]
31. Sharon, G.; Albert, M.; Rambha, T.; Boyles, S.; Stone, P. Traffic optimization for a mixture of self-interested and compliant agents. In Proceedings of the AAAI Conference on Artificial Intelligence, New Orleans, LA, USA, 2–7 February 2018; Volume 32.
32. Wang, J.; Wang, W.; Ren, G.; Yang, M. Worst-case traffic assignment model for mixed traffic flow of human-driven vehicles and connected and autonomous vehicles by factoring in the uncertain link capacity. *Transp. Res. Part C Emerg. Technol.* **2022**, *140*, 103703. [[CrossRef](#)]
33. Zhang, K.; Nie, Y.M. Mitigating the impact of selfish routing: An optimal-ratio control scheme (ORCS) inspired by autonomous driving. *Transp. Res. Part C Emerg. Technol.* **2018**, *87*, 75–90. [[CrossRef](#)]
34. Wardrop, J.G. Some theoretical aspects of road traffic research. *Proc. Inst. Civil Eng.* **1952**, *1*, 325–362. [[CrossRef](#)]
35. Beckmann, M.J.; McGuire, C.B.; Winsten, C.B. *Studies in the Economics of Transportation*; Yale University Press: New Haven, CT, USA, 1955.
36. Sheffi, Y. *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*; Prentice-Hall: Hoboken, NJ, USA, 1985.
37. Martin, W.A.; McGuckin, N.A. *Travel Estimation Techniques for Urban Planning*; National Academy Press: Washington, DC, USA, 1998; Volume 365.
38. Khajavirad, A.; Sahinidis, N.V. A hybrid LP/NLP paradigm for global optimization relaxations. *Math. Program. Comput.* **2018**, *10*, 383–421. [[CrossRef](#)]
39. Zhou, Z.; Chen, A.; Bekhor, S. C-logit stochastic user equilibrium model: Formulations and solution algorithm. *Transportmetrica* **2012**, *8*, 17–41. [[CrossRef](#)]
40. Zhang, J.; Pourazarm, S.; Cassandras, C.G.; Paschalidis, I.C. The price of anarchy in transportation networks: Data-driven evaluation and reduction strategies. *Proc. IEEE* **2018**, *106*, 538–553. [[CrossRef](#)]
41. Mansourianfar, M.H.; Gu, Z.; Waller, S.T.; Saberi, M. Joint routing and pricing control in congested mixed autonomy networks. *Transp. Res. Part C Emerg. Technol.* **2021**, *131*, 103338. [[CrossRef](#)]
42. Liang, Q.; Li, X.A.; Chen, Z.; Pan, T.; Zhong, R. Day-to-day traffic control for networks mixed with regular human-piloted and connected autonomous vehicles. *Transp. Res. Part B Methodol.* **2023**, *178*, 102847. [[CrossRef](#)]
43. Dijkstra, E.W. A note on two problems in connexion with graphs. *Numer. Math.* **1959**, *1*, 269–271. [[CrossRef](#)]

**Disclaimer/Publisher’s Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.