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Are Unitary Accounts of Quantum Measurements in Relativistic Wigner's Friend Setups Compatible in Different Reference Frames?

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Abstract: Wigner's friend scenarios—in which external agents describe a closed laboratory containing a friend making a measurement—highlight the difficulties of quantum theory when accounting for measurements. The problem is to accommodate for unitary evolution from the point of view of the external agent with the measurements or other operations carried out by the friend. Here, we show in the context of a relativistic thought experiment that an operation that may be accounted for unitarily in a given reference frame cannot be described unitarily in a different reference frame. This result, based on the frame dependence of the state update in relativistic contexts, could point to some fundamental inadequacy when attempting to model actions taken by a complex agent as unitary operations.

Keywords: quantum measurement theory; Wigner's friend scenarios; relativistic quantum mechanics

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1. Introduction

Wigner's friend scenarios bring to the forefront the tension existing between the unitary evolution that, according to standard quantum theory, applies to closed systems, as well as the projection postulate that applies when an agent performs a measurement and observes a result. In his seminal paper [1], Wigner had already noted the ambiguity of an external observer's description of a perfectly isolated laboratory containing an agent making a measurement. Indeed, the friend having measured the spin of a particle cannot remain in a state of "*suspended animation*" until the external observer asks her what outcome she obtained. Wigner suggested in ref. [1] that the Schrödinger equation needs to be supplemented with a non-linear term when a measurement takes place in order for the state update to be the same for the friend and the external observer.

However, standard quantum mechanics has no provision for an additional non-linear term and remains ambiguous on how the state should be updated after a measurement took place. Should a closed laboratory be described as being no different from any isolated quantum system, even if it contains an agent (the friend)? In this case, for an external observer labeled W, the quantum state of the laboratory evolves unitarily, despite the fact that the friend obtained a definite outcome. Even assuming that W endorses a unitary description, there is an additional ambiguity, as the friend's outcome may be assumed to be unique and objectively defined for anyone [2], or instead, it may constitute a fact for the friend only. This fact may not be defined for external observers such as W, who do not apply the projection postulate [3,4]. An additional question that has not received significant attention until now is whether any arbitrary operation carried out by the friend can be described unitarily from the point of view of W. We examine this question in this paper by introducing relativistic considerations in Wigner's friend scenarios.

Indeed, a well-known relativistic constraint arises by considering different time orderings of space-like separated events. For instance, if event E_1 precedes a space-like separated event, E_2, in a given reference frame, there are inertial reference frames in which E_2 precedes E_1 (see Figure 1). This leads to well-known consequences concerning the state update [5,6]. For example, if Alice and Bob share two spin $1/2$ particles in an entangled state, say:

$$\alpha|+u\rangle|-v\rangle + \beta|-u\rangle|+v\rangle + \gamma|-u\rangle|-v\rangle, \tag{1}$$

where u (resp. v) is the direction chosen by Alice (resp. Bob) to measure the spin component. Then, in a reference frame in which Alice measures first, the state after her measurement is updated to $|-v\rangle$ (if Alice obtained $+1$) or to $\beta|+v\rangle + \gamma|-v\rangle$ (if she obtained -1). If Alice and Bob's measurements are space-like separated events, there is an inertial frame in which Bob's measurement happens first; hence, the state is updated either to $|-u\rangle$ or to $\alpha|+u\rangle + \gamma|-u\rangle$. The consensus [7] is that the frame dependence of the quantum states at intermediate times is not a problem, given that the outcomes and probabilities are identical in both reference frames (or more generally, they are related by a Lorentz transform). But, in situations in which an outcome might not be associated with a state update (in our case, the friend's measurement), it becomes necessary to analyze the implications of dealing with quantum states that might describe different physics in distinct inertial reference frames.

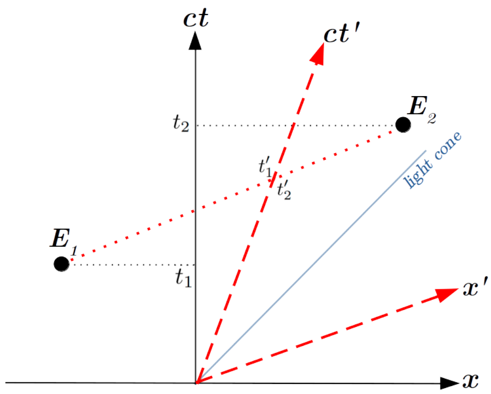

Figure 1. Space-like separated measurements of a bipartite entangled state in two reference frames. The measurement event E_1 takes place before E_2 in the frame (x, ct), while they are simultaneous in the frame (x', ct') (dashed red lines). During intermediate times ($t_1 < t < t_2$), the description of the quantum state is different in frame (x, ct) in which a measurement took place based on the description in the reference (x', ct') in which the state remains entangled.

In the original thought experiment devised by Wigner [1], it is straightforward to compute different probabilities for the external agent's outcome depending on whether this agent applies a state update after the friend's measurement (on the grounds that the projection postulate should apply), or a unitary evolution (on the grounds that the Schrödinger equation should be used since the laboratory is an isolated system). In all cases, the friend updates her state after completing her measurement. This gives rise, if unitary evolution is assumed, to a contradiction when considering two friends (each friend sitting in an isolated laboratory) sharing an entangled state (see the review [2] and refs. therein, as well as [8–15]). Technically, the contradiction is at the level of joint probabilities for measurement outcomes, given that a unitary account involves interferences, implying that such joint probabilities cannot be obtained as a marginal distribution—a well-known situation in quantum mechanics (e.g., non-contextual inequalities or Bell-type inequalities).

In a Wigner's friend scenario in a relativistic setting, an additional ingredient comes into play: if a state of the type given by Equation (1) describes a Wigner's friend setup, one might question whether the fact that the intermediate state after a measurement on an entangled pair is different in two inertial reference frames might not lead to a new type of contradiction if unitary evolution is assumed. Drawing on a recent work [16], we wish to examine in this paper the implications of what appears to be a generic property of relativistic Wigner's friend scenarios: a frame-dependence of the outcomes observed by an external observer. We will base our discussion on a definite example to be described in Section 3 after having briefly recalled the salient features characterizing the Wigner's friend scenarios and the role of relativistic constraints (Section 2). We will then analyze and discuss in Section 4 the main implications of relativistic models; we will argue in particular that describing a decision-making agent (the friend) by a quantum state evolving unitarily could be the underlying issue.

2. Wigner's Friend Scenarios

In the original Wigner's friend scenario (WFS), Wigner [1] introduces a sealed laboratory L in which a friend F performs a Stern–Gerlach experiment on an atomic spin, while an agent W is outside the isolated laboratory and ultimately measures the quantum state of the laboratory (on the same basis) by asking F what she obtained. The spin is initially in the following state:

$$|\psi(t_0)\rangle = \alpha|+z\rangle + \beta|-z\rangle, \tag{2}$$

and the isolated lab is assumed to be described by the quantum state:

$$|L(t_0)\rangle = |\psi(t_0)\rangle|m_0\rangle|\varepsilon_0\rangle, \tag{3}$$

where $|m_0\rangle$ and $|\varepsilon_0\rangle$ denote the initial states of the pointer and environment, respectively.

The issue is whether a sealed laboratory with an observer inside should be described by an external agent as evolving unitarily (because the laboratory is an isolated closed quantum system) or as a statistical mixture (assuming that any measurement implies a state update for all observers). Note that in ref. [1], Wigner supports the idea that the Schrödinger equation must be supplemented with nonlinear terms for conscious agents, though he will later change his mind [17,18]. In principle (assuming that the laboratory is still a quantum object after F's measurement), W can measure the laboratory L on a basis that is different from F's measurement. Indeed, if $|\psi(t_0)\rangle$ is measured on the $|\pm z\rangle$ basis, unitary evolution leads to the following:

$$|L(t)\rangle = \alpha|L_{+z}(t)\rangle + \beta|L_{-z}(t)\rangle, \tag{4}$$

where

$$|L_{\pm z}\rangle \equiv |\pm z\rangle|m_{\pm z}\rangle|\varepsilon_{\pm z}\rangle \tag{5}$$

are the states of the laboratory having inherited the $\pm z$ spin outcome; and W can choose to measure L in any basis spanned by $\{|L_{+z}\rangle, |L_{-z}\rangle\}$. However, such a measurement will then destroy F's measurement records (and any memory of the record) [19] so that no conflicting statements between the internal and external observers can be obtained (Of course, the computed probabilities for W are different according to whether unitary evolution or the projection postulate are applied).

Extended scenarios introduced in ref. [20] build on the WFS by combining more observers inside or outside isolated laboratories in order to formulate stronger assumptions, leading to a consistent description of the agents' observations. For example, F can open a communication channel through which she informs W that she obtained a definite outcome (without revealing this outcome) [20], somewhat circumventing the destruction of F's records by W's measurement. Indeed, the joint existence of F's and W's observations remains problematic, even when the postulates for which a WFS makes sense are endorsed. Additional assumptions concerning the validity of inferences made by the agents using the theory need to be made (see the review [2,21] for an early view with a more general discussion in the context of the measurement problem). Alternatively, the existence of joint facts can be denied in favor of observer-dependent facts [3]. Note that it can also be argued that claiming F made an observation while keeping interference terms between the branches corresponding to the friend's possible outcomes is contradictory, as it violates the uncertainty principle [22], but then this implies that unitary evolution should be discarded when a measurement takes place.

3. State Update in Relativistic Wigner's Friend Scenarios

3.1. Relativistic Constraints

An additional ingredient appearing in a relativistic setting is that for space-like separated events, the time ordering valid in one reference frame can be reversed in another inertial reference frame (see Figure 1 for an illustration). We will further assume that the state update of a separated multipartite quantum system takes place instantaneously in any inertial reference frame—an assumption that is more or less standard, if not consensual [6,7,23]. This implies that the description of the quantum state at intermediate times between preparation and final measurements will not be the same in different reference frames. These intermediate states are actually unrelated by any Lorentz transform, such as in the example given below in Equation (1). This is a generic situation in relativistic settings and is well-known—at least when the states describe unambiguous quantum systems—to have no observational consequence, given that the final measurement outcomes and their probabilities remain the same (or are related by a Lorentz transformation) in all reference frames [6,24]. However, in a WFS, the friend is an agent who is described by a quantum state. We now put forward a model in which the friend's actions depend on this intermediate quantum state, leading to an inconsistent description in different reference frames.

3.2. State Update and Agent's Actions

Let us start with two particles created in the entangled state:

$$|\psi_{FA}\rangle = 1/\sqrt{2}(|+z\rangle_F|+z\rangle_A + |-z\rangle_F|-z\rangle_A) \tag{6}$$

These are sent to the agents F (friend) and A (Alice). A and F will measure their respective particle's spin at space-like separated events. F is situated inside an isolated laboratory; next to this lab there is an external observer W equipped with several spin-measuring devices (see Figure 2 for a schematic representation). The total initial state is therefore as follows:

$$|\Psi(t_0)\rangle = |\psi_{FA}\rangle|m_0\rangle|\varepsilon_0\rangle, \tag{7}$$

where $|m_0\rangle$ and $|\varepsilon_0\rangle$ are the initial states of F's measuring apparatus and the lab's environment, respectively. The lab is described by the compound state $|L_{\pm z}\rangle = |\pm z\rangle_F|m_{\pm z}\rangle|\varepsilon_{\pm z}\rangle$, where $|\pm z\rangle_F$ represents F's spin state, while $|m_{\pm z}\rangle$ and $|\varepsilon_{\pm z}\rangle$ represent F's measuring device and the environment states inside the lab correlated with the measurement outcomes $\pm z$, respectively.

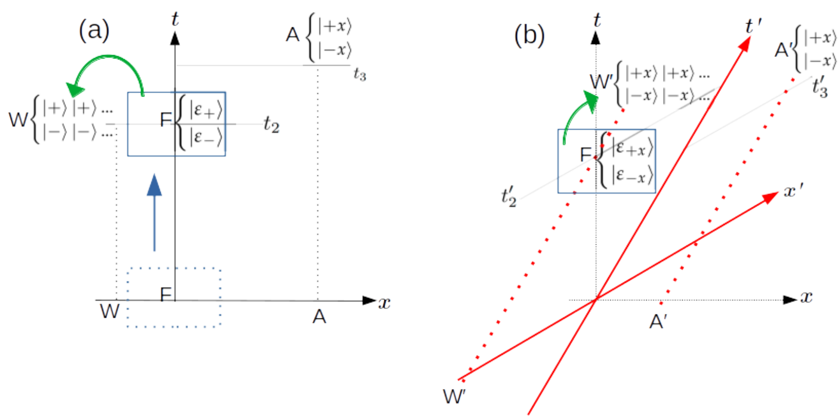

Figure 2. Schematic representation of the protocol detailed in Section 3.2 in the reference frames $\mathcal{R}$ (**a**) and $\mathcal{R}'$ (**b**). The sealed laboratory with the friend F inside is represented by the box at the time (t_2 in $\mathcal{R}$ and t'_2 in $\mathcal{R}'$). F sends qubits prepared in a state according to her observation, as determined by the environment states ($|\varepsilon_\pm\rangle$ in $\mathcal{R}$ and $|\varepsilon_{\pm x}\rangle$ in $\mathcal{R}'$).

Let us first describe the protocol in a reference frame $\mathcal{R}$ in which the laboratory is at rest.

- At time t_1, F measures her spin in the $|\pm z\rangle$ basis, and assuming unitarity, the spin superposition is inherited by the state of the laboratory, i.e., the apparatus and environment states of Equation (7) are transformed to mutually orthogonal states $|m_{\pm z}\rangle|\varepsilon_{\pm z}\rangle$ after coupling with the spins, so that according to an external observer:

$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}\big(|L_{+z}\rangle|+z\rangle_A + |L_{-z}\rangle|-z\rangle_A\big), \tag{8}$$

where $|L_{\pm z}\rangle = |\pm z\rangle_F|m_{\pm z}\rangle|\varepsilon_{\pm z}\rangle$ is the state of the laboratory correlated with a ± 1 outcome for F's measurement.

- Following her measurement, F immediately resets the states of the spin and the measuring device to a pre-assigned state $|s_0\rangle_F|m_0\rangle$. By defining $|\tilde{L}_{\pm z}\rangle = |s_0\rangle_F|m_0\rangle|\varepsilon_{\pm z}\rangle$, the state can now be written at $t = t_1^* > t_1$ as follows:

$$|\Psi(t_1^*)\rangle = \frac{1}{\sqrt{2}}\big(|\tilde{L}_{+z}\rangle|+z\rangle_A + |\tilde{L}_{-z}\rangle|-z\rangle_A\big). \tag{9}$$

The protocol for the reset mechanism and its unitary description by W is detailed in Appendix A.

- At time t_2, F declares the outcome she observes, which depends on the environment states $|\varepsilon_{\pm z}\rangle$ (as any record of the outcome must be part of the environment). She does so by creating many particles in the state $|+z\rangle$ (if she observes $+z$) or $|-z\rangle$ (if she observes $-z$). She sends the particles she has just generated outside the lab to W via a quantum communication channel so that $|\Psi(t_2)\rangle$ becomes the following after the particles' creation:

$$\frac{1}{\sqrt{2}}\big(|\tilde{L}_{+z}\rangle|+z\rangle_A|{+z}\rangle_W|{+z}\rangle_W \ldots + |\tilde{L}_{-z}\rangle|-z\rangle_A|{-z}\rangle_W|{-z}\rangle_W \ldots\big), \tag{10}$$

where the index W labels the particles sent to W. W must measure at least one particle in the z basis and one particle in the x basis; at this point, the state given by Equation (10) is updated to the $+z$ or the $-z$ branches, and W can conclude that F has observed $+z$ or $-z$, i.e., an outcome of a spin-z measurement. A protocol for F's particle creation procedure and its unitary description by W is given in Appendix B.

- At time t_3, Alice performs a spin measurement on her particle in the $|\pm x\rangle$ basis, which does not affect the final result obtained by W.

In another reference frame $\mathcal{R}'$ in motion relative to $\mathcal{R}$, the instant t_3' at which Alice measures in the $|\pm x\rangle$ basis takes place before t_2'; the time at which F creates particles according to the state she observes. In $\mathcal{R}'$, we can think of A′ and W′ as moving observers relative to the particle and the laboratory, who are synchronized to pass by the particle and laboratory at the times of measurements, t_3' and t_2', respectively. Hence, A′ measures the system in a state given by Equation (9), and after A′'s measurement, the state update leads to the following:

$$\begin{cases} |\Psi'_{+x}(t_3')\rangle = \frac{1}{\sqrt{2}}\big(|\tilde{L}_{+z}\rangle + |\tilde{L}_{-z}\rangle\big)|{+x}\rangle_A \equiv |\tilde{L}_{+x}\rangle|{+x}\rangle_A \\ |\Psi'_{-x}(t_3')\rangle = \frac{1}{\sqrt{2}}\big(|\tilde{L}_{+z}\rangle - |\tilde{L}_{-z}\rangle\big)|{-x}\rangle_A \equiv |\tilde{L}_{-x}\rangle|{-x}\rangle_A \end{cases}, \tag{11}$$

depending on whether A's outcome is $+x$ or $-x$. We have used $|\tilde{L}_{\pm x}\rangle = |s_0\rangle_F|m_0\rangle|\varepsilon_{\pm x}\rangle$ and defined the superposition of the environment states as $\big(|\varepsilon_+\rangle \pm |\varepsilon_-\rangle\big)/\sqrt{2} = |\varepsilon_{\pm x}\rangle$. Although it can be argued [16] that the states $|\varepsilon_{\pm x}\rangle$ correspond to environmental states after an outcome $\pm x$ has been observed, it is enough for the sake of the argument presented here to notice that the states $|\varepsilon_{\pm x}\rangle$ represent states that are distinct from those described by either $|\varepsilon_{+z}\rangle$ or $|\varepsilon_{-z}\rangle$. Since F's observation relies on records that are part of the environment states, F now observes either $+x$ or $-x$. She follows the protocol and sends qubits to W$'$ in the state $|+x\rangle$ or $|-x\rangle$ depending on whether she has observed $+x$ or $-x$. Therefore, at t'_2, after F creates the qubits, the quantum state in $\mathcal{R}'$ is either of the two following states:

$$\begin{cases} |\tilde{L}_{+x}\rangle|+x\rangle_A|+x\rangle_W|+x\rangle_W \dots \\ |\tilde{L}_{-x}\rangle|-x\rangle_A|-x\rangle_W|-x\rangle_W \dots \end{cases} \tag{12}$$

Again, W$'$ makes at least one measurement in the z basis and one in the x basis, and we assume he has received enough qubits in order to characterize F's outcome, namely $+x$ or $-x$.

Therefore, in $\mathcal{R}'$, the external observer would, following the assumptions of unitary evolution and instantaneous state update, receive qubits in different states than those received in the reference frame $\mathcal{R}$. Note that in $\mathcal{R}'$, the description of the friend's qubit creation procedure *cannot* be described unitarily by the same procedure used in frame $\mathcal{R}$—now a different unitary is needed. This inconsistency between accounts in different reference frames is a consequence of (i) the intermediate state being different in distinct reference frames (a generic property of a relativistic setting, as we have mentioned above) and (ii) the fact that F is an agent whose action depends indirectly on this intermediate quantum state (through the state of the environment inside the laboratory).

4. Discussion

4.1. Inconsistencies in Non-Relativistic Scenarios

In the usual non-relativistic Wigner's friend scenarios, the tension is between the application of the projection postulate after a measurement and a unitary description for a closed system. Mixing both types of evolution (assuming measurement outcomes exist objectively for any observer in conjunction with a unitary description by the external observers) leads to inconsistencies. At the formal level, this is strictly equivalent [22] to a double-slit experiment in which one would know which slit the particle took (this would correspond to a friend's measurement) while still maintaining an interference pattern at the screen (this is the description employed by the external observers measuring a friend's laboratory). Of course, at the interpretational level, such an analogy is unwarranted, given that the issue at stake is to characterize a measurement taking place inside a system that would be described unambiguously as following unitary evolution if an agent was not placed inside. Requesting the existence of all the agents' measurement records is mathematically equivalent to assuming the existence of a joint probability distribution for the outcomes of each friend and its corresponding Wigner [25,26]. Quantum theory has no provision for the existence of joint probability distributions for incompatible observables [8]—there is indeed no common eigenbasis—and by measuring the lab, Wigner's measurement destroys his corresponding friend's record [19].

The alternative to unitary evolution is to apply the state update after any measurement, irrespective of whether the measurement takes place in a closed system. Note that we are not interested here in giving a mechanism accounting for the state update (e.g., what would be proposed by objective collapse theories, or through effective collapse in some versions of the de Broglie–Bohm interpretation that discard for all practical purposes the action of the empty waves after a measurement). After all, this is nothing but the projection postulate as given in textbook quantum mechanics. Note also that conceptually, whether the state update should apply to all agents should not be conflated with the hypothesis concerning the application of quantum mechanics to isolated macroscopic laboratories containing an agent—if the latter hypothesis is not fulfilled, there are no Wigner's friend scenarios, but one can still endorse this hypothesis and reject a unitary description. Doing so leads to the conundrum encapsulated in Bell's question:"*What exactly qualifies some physical systems to play the role of the 'measurer'?*" [27].

4.2. Role of State Updates in Relativistic Contexts

In addition to the inconsistencies mentioned above, relativistic Wigner's friend scenarios have to deal with the consequences of frame-dependent intermediate states. Such states are not Lorentz transforms of one another. If the quantum state refers to particles, as in Equation (1), it is known that having different intermediate states in distinct inertial frames does not lead to any observational consequences [6]. Actually, it turns out that it is impossible to characterize an intermediate state by itself [28]. However, in Wigner's friend scenarios, the quantum state describes an agent and its environment. By definition, an agent acts, and the operations that the agent undertakes can only depend on the quantum state of the laboratory (as long as the laboratory is isolated). This is also the case in non-relativistic versions, where laboratories are prone to superposition, interference, and state updates.

Inconsistencies arise in a relativistic setting because the intermediate updated state is different in each reference frame. Then, the friend's operations become frame-dependent, and this can be inferred by an external observer making interventions on the laboratory. In the model of Section 3.2, in the frame $\mathcal{R}$, the environment is in one of the states $|\varepsilon_{\pm z}\rangle$, and the friend prepares qubits accordingly in the states $|\pm z\rangle$, whereas in $\mathcal{R}'$, the state of the laboratories is updated after A$'$'s measurement. This leaves the environment in states $|\varepsilon_{\pm x}\rangle$, and the friend now prepares and sends to W qubits prepared in the x basis in either of the states $|\pm x\rangle$. Hence, the inconsistency between the outcomes predicted in different reference frames is due to state updates in contexts in which the ordering of space-like events depends on the observers' reference frame. Note that if the external observer describes the friend's measurement by applying the projection rule rather than unitary evolution, then no contradiction between outcomes in different frames is obtained.

4.3. Unitary Agents

The inconsistent descriptions of Wigner's friend scenarios in two different reference frames are of course highly problematic. The problem arises when combining the asumptions generally endorsed in Wigner's friend scenarios with the instantaneous state update rule of standard quantum mechanics in a relativistic context. The validity of the state update rule in accounting for quantum correlations has never been questioned. However, the instantaneous character of the state update conflicts with the quantum state representation of an agent; this can potentially lead to signaling.

Indeed, consider the entangled state given by Equation (6) in which the friend measures her qubit in a sealed laboratory. Then, after the friend's measurement (there is no state update), A measures the θ component of her spin, so that we can write the following:

$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2}|L_{+z}\rangle + \sin\frac{\theta}{2}|L_{-z}\rangle\right)|+\theta\rangle_A \tag{13}$$

$$+ \frac{1}{\sqrt{2}}\left(-\sin\frac{\theta}{2}|L_{+z}\rangle + \cos\frac{\theta}{2}|L_{-z}\rangle\right)|-\theta\rangle_A. \tag{14}$$

After A's measurement, the state of the laboratory updates instantaneously to one of the expressions between $(\ldots)$ in Equation (13) or (14), which are here mutually orthogonal. Now, any operation through which the agent knows about the state of the laboratory gives rise to signaling; for instance, the friend could instantaneously determine the measurement direction θ chosen by A (a statement that does not make sense from a relativistic standpoint). We stress here that we allow the friend to perform operations that might not necessarily be represented by unitary transformations, on the grounds that since the friend is an agent, she can undertake any operations that an agent outside the sealed lab could perform. For instance, copying a known qubit in an arbitrary state is impossible to implement unitarily; doing so would break the linearity of quantum mechanics and lead to signaling [29]. Note further that, as emphasized by Peres [6], an instantaneous update of the quantum state should be tied to a mathematical computation rather than to a physical process to avoid conflicting with relativistic constraints.

This situation leads to fundamental questions concerning the validity of modeling an agent with simple quantum states obeying standard (linear) quantum mechanics. This question has not received much attention up to now in the context of Wigner's friend scenarios, but the operations available to an agent described by unitary quantum mechanics are restricted when compared to what one would expect from a classical agent. Put differently, describing a complex decision-making agent using simple wavefunctions evolving unitarily might turn out to be an oversimplification, leading to inconsistencies. This brings us back to the well-known difficulties in coping with the measurement problem—a theory whose dynamics are based on wavefunctions evolving unitarily cannot account for single outcomes [30], although this is precisely an assumption that is frequently made when studying Wigner's friend scenarios based on the "closed system" properties of the sealed laboratory.

5. Conclusions

We have analyzed the relativistic constraints that appear in Wigner's friend scenarios. While in non-relativistic Wigner's friend scenarios, inconsistencies arise when attempting to describe probabilities for joint outcomes of incompatible observables, an additional property that is generic to relativistic scenarios is that an instantaneous state update on entangled states leads to an intermediate quantum state that is specific to a given reference frame. We have seen in the example given in Section 3.2 that this leads to inconsistent outcomes in different reference frames depending on whether the update took place before or after the external agent's measurement. We have further argued that relativistic constraints bring to the foreground the possible inadequacy of accounting for decision-making agents by describing them using simple quantum states evolving unitarily.

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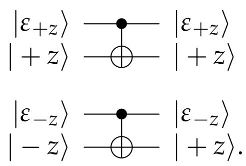
Appendix A

We aim to model the agent F resetting her qubit from the point of view of Wigner. This should be simple if W describes the process as an operation of a unitary CNOT gate with the environment state $|\varepsilon\rangle$ as the control qubit and the friend's qubit $|s\rangle$ as the target. We suppose that the predetermined reset state is $|s_0\rangle = |+z\rangle$. This operation is described as follows:

$$\mathrm{CNOT}_{\varepsilon s}|\varepsilon_{\pm z}\rangle|\pm z\rangle = |\varepsilon_{\pm z}\rangle|+z\rangle = |\varepsilon_{\pm z}\rangle|s_0\rangle; \tag{A1}$$

or, in circuit notation:



Note that the same description applies for resetting the measuring device. The state at t_1 can then be said to have undergone the following *unitary* process:

$$
\begin{aligned}
|\Psi(t_1^*)\rangle &= \mathrm{CNOT}_{\varepsilon m}\mathrm{CNOT}_{\varepsilon s}|\Psi(t_1)\rangle \\
&= \mathrm{CNOT}_{\varepsilon m}\mathrm{CNOT}_{\varepsilon s}\left[\frac{1}{\sqrt{2}}\big(|+z\rangle_F|m_{+z}\rangle|\varepsilon_{+z}\rangle|+z\rangle_A + |-z\rangle_F|m_{-z}\rangle|\varepsilon_{-z}\rangle|-z\rangle_A\big)\right] \\
&= \mathrm{CNOT}_{\varepsilon m}\left[\frac{1}{\sqrt{2}}\big(|m_{+z}\rangle|\varepsilon_{+z}\rangle|+z\rangle_A + |m_{-z}\rangle|\varepsilon_{-z}\rangle|-z\rangle_A\big)\right]|s_0\rangle_F \\
&= \frac{1}{\sqrt{2}}\big(|\varepsilon_{+z}\rangle|+z\rangle_A + |\varepsilon_{-z}\rangle|-z\rangle_A\big)|s_0\rangle_F|m_0\rangle.
\end{aligned}
\tag{A2}
$$

Appendix B

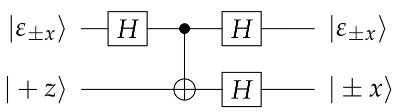
At time t_2, F creates many (say N) particles according to the outcome she observed. In the frame $\mathcal{R}$, this can be modeled unitarily as a series of CNOT gate operations, each taking the environment state as the control and a pre-existing qubit in a "ready" state $|q_i\rangle_W = |+z\rangle_W$ as the target.

$$
\begin{aligned}
|\Psi(t_2)\rangle &= \bigotimes_{i=1}^{N}\mathrm{CNOT}_{\varepsilon q_i}|\Psi(t_1^*)\rangle|q_i\rangle_W \\
&= \bigotimes_{i=1}^{N}\mathrm{CNOT}_{\varepsilon q_i}\left[\frac{1}{\sqrt{2}}\big(|\varepsilon_{+z}\rangle|+z\rangle_A + |\varepsilon_{-z}\rangle|-z\rangle_A\big)|s_0\rangle_F|m_0\rangle\right]|q_i\rangle_W \\
&= \frac{1}{\sqrt{2}}\big(|\varepsilon_{+z}\rangle|+z\rangle_A|+z\rangle_W^{\otimes N} + |\varepsilon_{-z}\rangle|-z\rangle_A|-z\rangle_W^{\otimes N}\big)|s_0\rangle_F|m_0\rangle.
\end{aligned}
\tag{A3}
$$

Now, in $\mathcal{R}'$, for modeling the friend's action, we require a unitary operation that changes the states $|q_i\rangle_W$ to $|\pm x\rangle_W$ depending on an environment state $|\varepsilon_{\pm x}\rangle$. This can be done with the help of a Hadamard gate (H). Knowing that $H|\pm z\rangle = |\pm x\rangle$ and $H|\pm x\rangle = |\pm z\rangle$, we can construct the following operation:

$$
\begin{aligned}
H_\varepsilon H_{q_i}\mathrm{CNOT}_{\varepsilon q_i}H_\varepsilon|\varepsilon_{\pm x}\rangle|q_i\rangle &= H_\varepsilon H_{q_i}\mathrm{CNOT}_{\varepsilon q_i}H_\varepsilon|\varepsilon_{\pm x}\rangle|+z\rangle \\
&= H_\varepsilon H_{q_i}\mathrm{CNOT}_{\varepsilon q_i}|\varepsilon_{\pm z}\rangle|+z\rangle \\
&= H_\varepsilon H_{q_i}|\varepsilon_{\pm z}\rangle|\pm z\rangle \\
&= H_\varepsilon|\varepsilon_{\pm z}\rangle|\pm x\rangle \\
&= |\varepsilon_{\pm x}\rangle|\pm x\rangle.
\end{aligned}
\tag{A4}
$$

In circuit notion, the operation is as follows:



Now, using Equation (A4), we can write the evolution of the lab state, as the friend creates many qubits depending on the environment state, as observed by Wigner in $\mathcal{R}'$:

$$
\begin{aligned}
|\Psi'_{\pm x}(t_2')\rangle &= H_\varepsilon\bigotimes_{i=1}^{N}H_{q_i}\mathrm{CNOT}_{\varepsilon q_i}H_\varepsilon|\Psi'_{\pm x}(t_3')\rangle|q_i\rangle_W \\
&= H_\varepsilon\bigotimes_{i=1}^{N}H_{q_i}\mathrm{CNOT}_{\varepsilon q_i}H_\varepsilon\big(|\varepsilon_{\pm x}\rangle|\pm x\rangle_A\big)|s_0\rangle_F|m_0\rangle|q_i\rangle_W \\
&= \big(|\varepsilon_{\pm x}\rangle|\pm x\rangle_A|\pm x\rangle_W^{\otimes N}\big)|s_0\rangle_F|m_0\rangle.
\end{aligned}
\tag{A5}
$$

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