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Article

On the Value of the Cosmological Constant in Entropic Gravity

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Abstract: We explicitly calculate the value of the cosmological constant, Λ, based on the recently developed theory connecting entropic gravity with quantum events induced by transactions, called transactional gravity. We suggest a novel interpretation of the cosmological constant and rigorously show its inverse proportionality to the squared radius of the causal universe $\Lambda \sim R_U^{-2}$.

Keywords: cosmological constant; transaction; entropic gravity; Hubble radius; particle horizon

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1. Introduction

The cosmological constant Λ has played a pivotal role in cosmology ever since Einstein introduced it in 1917. He invented it with the desire to describe a static universe and for this purpose to counterbalance the effects of gravity. As explained in detail in [1], several phases in the history of Λ can be discerned. Hubble's discovery of the expanding universe led Einstein to dismiss the cosmological constant in 1931. However, in 1927, Lamaître already incorporated the cosmological constant into his non-static model of the universe, interpreting it as a sort of vacuum energy density, which is still today's standard interpretation [2]. More precise measurements of the Hubble constant in the 1930s again undermined the case for a non-zero Λ. In the 1960s, there was a short-lived revival of a non-zero Λ due to the observation of quasars, which seemed to suggest a non-conventional expansion of the universe. Afterward, physicists thought for a long time that Λ should be exactly zero, but observations by Perlmutter, Riess, and collaborators in 1998 [3,4] on Type Ia supernovae definitively showed that the expansion of the universe is accelerating. This discovery is empirical evidence that $\Lambda > 0$, with a value of $\Lambda \sim 10^{-52}\text{m}^{-2}$ (the dimension is hence an inverse area like in the Ricci tensor), as calculated by the Planck collaboration [5]. However, there is a significant mismatch between the theoretical expectations and the empirical facts [6], since what would be a natural value of the quantum vacuum energy, given the theories we have, lies many orders of magnitude away from the measured reality [7,8]. This raises the question of the true nature of Λ. A second mystery is the question as to where the striking relation to the age (size) (time is expressed by $\tau = ct$ and hence has the dimension of length), R_U, of the causal universe, namely $\Lambda \sim R_U^{-2}$, comes from [9]. Attempts have been made to explain the origin and value of the constant in, e.g., [10–15]. Yet, the topic is far from conclusively settled.

Recently, a connection between the relativistic transactional interpretation of quantum mechanics and entropic gravity has been found and a transactional theory of gravity has been developed [16,17]. This theory gives a different physical interpretation of the cosmological constant, Λ, which, as we will show in this paper, directly leads to the relation $\Lambda \sim R_U^{-2}$. To this end, we first introduce the basics of transactional gravity and then, in Section 2, give the main result of this paper, namely the calculation of Λ.

Transactional Gravity

In [16,17], it is shown how empirical spacetime together with its metric structure emerge from quantum events that happen pairwise by transactions. Quantum systems are elements of a realm of potentialities that empirically become actualized by transactions, which consist of the emission and absorption of (on-shell) photons between these systems.

The choice of a propagator thereby defines the direction of time. Empirical spacetime thus becomes the connected set of emission and absorption events, between which spacetime intervals are being created through the four-momentum of the exchanged photons. Let us look at this in some more detail.

Quantum amplitudes of closed, isolated systems are represented as unit vectors in a Hilbert space, $\psi_x \epsilon H$, also called quantum states. In the transactional interpretation [17], a quantum state, ψ_x, is launched as an "offer-wave" by an emitter and obtains possible responses by "confirmation-waves", represented by dual vectors, ψ_y^*, launched by possible absorbers. The selection of a specific "response", ψ_y^*, is fundamentally indeterministic and leads to a "transaction", which is the actualization of absorption and emission as real events in spacetime, and whose probability (density) is $|\delta_{(y-x)} * \psi_x|^2 = |\psi_y|^2$. The relativistic transactional interpretation in addition offers the reason why offer waves (and confirmation waves) are actually being created, by focusing on the electromagnetic interaction (it is unknown to what extent a transactional theory of different bosons, belonging to the weak and strong forces, can be constructed). While emitters correspond to the retarded solutions of the wave equation and to creation operators, absorbers are connected to advanced solutions and to annihilation operators. Relativistic electromagnetic interactions can be thought of as the mutual exchange of virtual photons by quantum fields, creating possibilities in a pre-spacetime process. Transactions, in turn, are characterized by the exchange of real photons and their four-momenta between emitter and absorber. While virtual photons correspond to the Coulomb force and such interactions are unitary, real photons correspond to radiative processes, which are non-unitary interactions ([18], Chapter 5). The general amplitude, α, for the emission and absorption of real photons is the coupling amplitude between matter and gauge fields, and the non-unitary transactional process can arise if the conservation laws are satisfied. By this non-unitary exchange of four-momentum, the quantum states of emitter and absorber collapse, and the physical systems are localized at the corresponding spacetime points (regions). Empirical spacetime thus becomes the connected set of emission and absorption points, between which space-time (null)intervals are being created through the four-momenta of the exchanged photons. It is here, where the transactional view touches on the causal-set theory [19], in which events spread in spacetime by a stochastic Poisson process. Boson exchange, understood as a decay process in quantum field theory, is then a special case in this model [20] (the transactional interpretation thinks differently of spacetime than the causal-set approach does, which is unimportant for our purposes). Of note is that the actualization of a spacetime interval amounts to spontaneously breaking the unitary evolution of the quantum states. At the same time, the exchanged four-momentum selects a space direction, whereas a time direction is a priori determined, since only positive energy is being transferred (this amounts to the choice of the Feynmann propagator as opposed to the Dyson propagator). Because there is no preferred emission direction, the process is spatially isotropic, and because the whole mechanism is indifferent to specific locations, it is also homogeneous. Observed inhomogeneities of the universe are then the consequence of a possible anisotropic distribution of initial transactions and the spatiotemporal variation of some other parameter, as we will see in Section 2.

So far, we have motivated the idea that the formalism of quantum physics is suited to explain the emergence of empirical space and time as unified, yet distinct, dimensions by the mechanism of transactions. Real photon exchange creates metric relations between emitters and absorbers and the mechanism contains therefore an intrinsic way to measure time intervals by means of the exchanged photons, as described in [17]. This approach clearly lends itself to the relational view of spacetime as it emerges from pairs of emitters and absorbers, and there is no spacetime without matter (although the matter itself is not a component of metrical spacetime; [18], Chapter 8). Mathematically, quantum states can be described as fields parametrized by spacetime coordinates. Note that there is no circularity here, since this description does not suggest that spacetime has a real a priori existence of its own. It is only a continuous model representation of our observations, where we never measure standalone spacetime points. Hence, the term "empirical spacetime" is used in order to distinguish observed reality from continuous mathematical models. We also note that the conception of quantum states as fields over spacetime uses the spacetime parameters as possibilities for localization relative to a particular inertial frame, and that such quantum states are not physically 'in spacetime', where the latter is understood as the emergent manifold of connected events.

By a transaction, the involved systems become localized which, in order to keep the entropic balance, leads to a weak limit to Newton's gravitational force. It is mathematically shown in [16,17,21] that this entropic process leads more generally to a gauge of the length in temporal direction, which together with the light-cone structure is sufficient to derive the Einstein equations and hence to govern the four-metric of spacetime [22]. The three-momenta of the exchanged photons in particular lead to a cosmological term in the Einstein equations, which is what we are going to show next.

2. The Cosmological Constant

In transactional gravity, the energy of the transferred photons gauges the rhythm of becoming as it defines the period of a natural light clock which, together with the light-cone structure, leads to Einstein's equation [17,22]. The three-momenta of the transferred photons enter this equation in the form of a cosmological constant, Λ, as we will now show. This fact is intuitively plausible, since the momenta of the photons exercise a repulsive pressure on the material systems involved in the transactions. Note that in transactional gravity, it is the energies and momenta of matter involved in transactions that add to the local energy–stress tensor and there is no basis for the inclusion of expectation values of quantum fields or alike [16]. This fact excludes the vacuum energies of the different fields from being the cause of expansion. Concretely, there arises pressure from photon three-momenta, emitted in all the spatial directions equiprobably, which defines at a given point the Laue-scalar T:

$$T = \sum_{i=1}^{3} T_{ii} = \lim_{A_i \to 0} \sum_{i=1}^{3} \frac{F_i}{A_i} = \lim_{A_i \to 0} \sum_{i=1}^{3} \frac{1}{A_i} \frac{dp_i}{dt}. \tag{1}$$

Let $N_R(t)$ be the number of actualizations within (spatial) volume V_R at a time t. We have with $x_0 = ct$, $N_R(t) = N_R\left(\frac{x_0}{c}\right) = \widetilde{N}_R(x_0)$ and with the de Broglie relation $\left|\overrightarrow{p}\right| = \frac{h}{R}$

$$T = -3\frac{dN_R(t)}{dt} \cdot \frac{1}{A_R} \cdot \frac{h}{R} = -3\frac{c \cdot h}{3} \cdot \frac{d\widetilde{N}_R(x_0)}{dx_0 V_R} = -c \cdot h \cdot \frac{d\lambda(x_0)}{dx_0}. \tag{2}$$

The negative sign indicates the repulsive effect and the function $\lambda(x_0) = \frac{\widetilde{N}_R(x_0)}{V_R}$ denotes the number of transactional events per spatial volume at time x_0, which we simply call transaction density. The term $\frac{d\lambda(x_0)}{dx_0}$ is therefore the change rate of the transaction density and there holds to first order $\lambda(x_0 + \Delta x_0) = \lambda(x_0) + \frac{d\lambda(x_0)}{dx_0} \cdot \Delta x_0$. In (2), we assumed that $\lambda(x_0)$ is constant over space, which also amounts to the homogeneity and isotropy of space with respect to transactional events. Equation (2) also tacitly assumes that $\lambda(x_0)$ is a differentiable function in x_0. This is an assumption, which cannot hold in quantum mechanics, since quantum events represent discrete sets and are not deterministic, but obey a random process. The only known Lorentz-invariant stochastic process for the spreading of quantum events in Minkowski space, such that the number of events is proportionate to the volume, is a Poisson process with constant transaction density rate ϱ_γ [23]. Hence, in analogy to the above terminology, there holds for the average transaction density $\overline{\lambda}(x_0)$ and for $\Delta x_0 > 0$:

$$\overline{\lambda}(x_0 + \Delta x_0) = \overline{\lambda}(x_0) + \varrho_\gamma \cdot \Delta x_0. \tag{3}$$

So, by (3), we can define, in analogy to (2), a scalar T_γ:

$$T_\gamma = -3\frac{c \cdot h}{3} \cdot \frac{\Delta\overline{\lambda}(x_0)}{\Delta x_0} = -c \cdot h \cdot \varrho_\gamma. \tag{4}$$

Remember the Einstein equation:

$$R_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right),\ 0 \leq \mu,\nu \leq 3. \tag{5}$$

Equation (4) leads on the right-hand side of Einstein's equation, with $l_P = \sqrt{\frac{G\hbar}{c^3}}$ denoting the Planck length, to the term

$$\frac{4\pi G}{c^4}T_\gamma = -\frac{4\pi Gh}{c^3}\varrho_\gamma = -8\pi^2 l_P^2 \varrho_\gamma. \tag{6}$$

The right-hand side of Equation (5) consists of the local energy–momentum distribution, whereas Expression (6) is global and independent of any local fields. It hence represents a structural component of Equation (5). Consequently, putting it to the left-hand side of the Einstein equation leaves us to interpret it as the cosmological constant (note that $[\Lambda] = \mathrm{m}^{-2}$):

$$\Lambda = 8\pi^2 l_P^2 \varrho_\gamma. \tag{7}$$

Einstein's equation hence takes the form of:

$$R_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right),\ 0 \leq \mu,\nu \leq 3. \tag{8}$$

2.1. Spatial Information

Since transactions localize emitters and absorbers, there is an entropy production in the process (see Appendix A). The resulting information will, of course, depend upon the physical systems involved. In order to estimate the number of transactional events in the absence of concrete knowledge of the matter fields, we can define the spatial information content and then calculate the number of bits residing in a region of space $\Omega \subset \mathbb{R}^3$, as performed in [17] and shown next.

We assume a local inertial frame throughout the following exposition. Let there be a bounded region, $\Omega \subset \mathbb{R}^3$, on a spatial hyperplane and a partition by balls:

$$\mathcal{B} = \{B_{\varepsilon_n}(x_n)\}_{x_n \epsilon \Omega,\ \varepsilon_n > 0},\ \bigcup_{x_n} B_{\varepsilon_n}(x_n) = \Omega. \tag{9}$$

Relative to the partition, $\mathcal{B}$, position information can be attributed to a quantum system in terms of square-integrable functions over Ω, $\psi(x)\epsilon L^2(\Omega)$, by

$$I^{\mathcal{B}}(\psi) = -\sum_{x_n \epsilon \Omega} p_{x_n} ln(p_{x_n}),\ \ p_{x_n} = \int_{B_{\varepsilon_n}(x_n)} |\psi(x)|^2 dx. \tag{10}$$

By multiplication with the Boltzmann constant, k_B, we obtain

$$S^{\mathcal{B}}(\psi) = I^{\mathcal{B}}(\psi)k_B. \tag{11}$$

We can ask whether it is possible to take a different perspective and attribute information not to material systems, but to regions or idealized single points ($x_0 \epsilon \mathbb{R}^3$). A point, $x_0 \epsilon \Omega$, can empirically be associated with matter or not and hence represents in this sense one bit of information. Given a single physical system, $\psi(x)\epsilon L^2(\Omega)$, we can therefore state that the information of the one bit, $x_0 \epsilon \Omega$, with respect to $\psi(x)$ and the partition $\mathcal{B}$ (9) (we picked the ball $B_{\varepsilon_{\tilde{n}}}(x_{\tilde{n}})$ with $x_0 \epsilon\, B_{\varepsilon_{\tilde{n}}}(x_{\tilde{n}})$ and minimal $|x_0 - x_{\tilde{n}}|$), is

$$I^{\mathcal{B}}_{\psi}(x_0) = -[p_{x_0} ln(p_{x_0}) + (1 - p_{x_0}) ln(1 - p_{x_0})]. \tag{12}$$

To find a generic definition, we have to account for all possible partitions ($\mathcal{B}$), which requires taking into account all probabilities, $0 \leq p_{x_0} \leq 1$. Since it is always possible to find a bounded $\Omega \subset \mathbb{R}^3$ with $x_0 \epsilon \Omega$, we can define the information $I(x_0)$, $x_0 \epsilon \mathbb{R}^3$, by

$$I(x_0) = -2 \int_0^1 p_{x_0} ln(p_{x_0}) dp_{x_0} = \frac{1}{2}. \tag{13}$$

Evidently, (13) is not only independent of a chosen partition $\mathcal{B}$, but also of the particular material system $\psi(x)$. While the choice of a particular $\mathcal{B}$ is, of course, frame dependent, the described process will lead to the definition of $I(x_0)$ by Equation (13) in every local inertial frame.

2.2. Transactional Density

In order to investigate the behavior of Λ, it is, by Equation (7), necessary to understand the transaction density rate ϱ_γ. Since ϱ_γ is a constant and the transaction density at the beginning of the period is zero, $\overline{\lambda}(0) = 0$, we choose the direct approach and estimate the spatial density of the expected number of transactional events in today's universe and divide it by the age of the universe to obtain the average transaction density rate. In order to make closed calculations possible, we employ a simple model of an expanding, flat universe where, due to the early rapid decay of the matter and radiation densities, expansion dominates. Furthermore, we have evidence for a decreasing Hubble parameter $H(t)$ and, if t_U is the age of the universe today, we set today's Hubble parameter as $H(t_U) \stackrel{\text{def}}{=} H_0$. Note that in this situation the first Friedmann equation implies $H(t)^2 \sim \frac{\Lambda(t)c^2}{3}$. We use today's Hubble radius, $R_{H_0} = \frac{c}{H_0}$, to express the age, t_U, of the universe, which is $t_U = \frac{1}{H_0}$. We further know that for the expansion factor, $a(t)$, we have $\frac{\dot{a}(t)}{a(t)} = H(t) : a(t) = a_0 e^{\int_0^t H(\tau)d\tau}$ and $a(0) = a_0$, chosen such that $a(t_U) = 1$. The causal universe at any time, $t \leq t_U$, is bounded by the particle horizon, $R_P(t)$, which is defined by

$$R_P(t) = a(t) \int_0^t \frac{c d\tau}{a(\tau)} = a(t) \int_0^t \frac{c d\tau}{a_0 e^{\int_0^\tau H(s)ds}}. \tag{14}$$

Since $H(t) \geq H_0$, there holds

$$R_P(t) \leq a(t) \int_0^t \frac{c d\tau}{a_0 e^{H_0 \tau}} = \frac{a(t)}{a_0} R_{H_0} \left(1 - e^{-H_0 t}\right). \tag{15}$$

In particular, for today's particle horizon, $R_P(t_U)$, we have with $\epsilon \stackrel{\text{def}}{=} \left(1 - e^{-1}\right)$:

$$R_P(t_U) \leq \frac{1}{a_0} R_{H_0} \left(1 - e^{-1}\right) = \frac{\epsilon}{a_0} R_{H_0}. \tag{16}$$

We further remember, as calculated above (13), that a single point, x_0, in space represents one bit of information with information content $I(x_0) = \frac{1}{2}$. Assuming the Planck length to be a minimal length in nature, the total number of bits, n_R, on the surface of a ball of radius R with surface area A_R amounts to

$$n_R = \frac{A_R}{2 l_P^2}. \tag{17}$$

By the holographic principle [24], Expression (17) represents the maximum information encoded within the ball, B_R. Furthermore, the fine-structure constant, α^2 (with q denoting the elementary electric charge and ε_0 the dielectrical constant, $\alpha^2 = \frac{q^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$), is the base probability for a transaction to happen (at most reduced by a factor stemming from additional amplitudes) and we can reasonably assume that the number of transactional events at any time (x_0) is (maximally) proportionate to the available spatial information (this holds because emitting or absorbing material systems are much larger in size than the Planck length). Hence, the (maximum) expected number of transactional events within a ball of radius R, B_R, is

$$\overline{N}_R = n_R \cdot \alpha^2 = \frac{A_R \alpha^2}{2l_P^2}. \tag{18}$$

Since the causal universe has been growing to today's particle horizon, $R_P(t_U)$, we can assume that the total amount of transactional events today is coded on the surface of the radius $R_P(t_U)$ and it is possible by Equation (18) to set the total expected number of transactional events in the causal universe today, $\overline{N}_{R_P(t_U)}$, as

$$\overline{N}_{R_P(t_U)} = \frac{4\pi R_P^2(t_U)\alpha^2}{2l_P^2}. \tag{19}$$

To obtain the average transaction density today, $\overline{\lambda}(t_U)$, we need to divide Expression (19) by the volume of the causal universe today to obtain

$$\overline{\lambda}(t_U) = \frac{\overline{N}_{R_P(t_U)}}{\frac{4}{3}\pi R_P^3(t_U)} = \frac{3\pi\alpha^2}{2l_P^2 R_P(t_U)}. \tag{20}$$

Since $\overline{\lambda}(0) = 0$, we have together with Equations (3) and (16):

$$\varrho_\gamma = \frac{\overline{\lambda}(t_U) - \overline{\lambda}(0)}{ct_U} \leq \frac{\epsilon\overline{\lambda}(t_U)}{a_0 R_P(t_U)} = \frac{3\pi\epsilon\alpha^2}{2a_0 l_P^2 R_P^2(t_U)}. \tag{21}$$

By setting $C_0 \stackrel{\text{def}}{=} \left(\frac{3\pi\epsilon}{2a_0}\right)$, which is a dimensionless number, and by Equation (7), we obtain the (maximal) cosmological constant:

$$\Lambda = 8\pi^2 C_0 \frac{\alpha^2}{R_P^2(t_U)}. \tag{22}$$

In Equation (22), we directly recover the measured fact that $\Lambda \sim R_U^{-2}$ (we have $R_P(t_U) = 46.5Gly \approx 4.2 \cdot 10^{26}$ m today). Also, the key role of the fine-structure constant becomes clear. It is a governing factor of the expected number of transactional events in the universe and as such enters the formula for its expansion.

3. Conclusions

The problem of the empirically found tiny value of the cosmological constant has been bothering physicists for a long time. In addition, the proportionality to the squared inverse of the age of the universe seemed a coincidence, albeit an intriguing one. In the theory of transactional gravity, where spacetime and its metric emerge from quantum events, called transactions, the cosmological constant arises very naturally as the repulsive pressure generated by the three momenta of event radiation, i.e., of the photons constituting transactions. By the same entropic considerations that lead to an entropic force, i.e., gravity, we also arrive at a natural expression for the cosmological constant which turns out to have exactly the desired behavior. The vacuum energy of the diverse quantum fields plays no role anymore, since in transactional gravity non-transacting parts do not enter the equations. In our theory, Λ is at any stage related to the age of the universe but does not necessarily become infinite as time returns back to the origin, since the number of transactions also decreases. In addition, it might be the case that the transaction density rate regionally differs, which would, next to the distribution of initial transactional events, lead to observable inhomogeneous structures at large scales of the universe. It remains to be seen in the future whether transactional gravity is the model that nature actually follows. In any case, the theory very naturally produces a number of explanations for so far rather elusive facts around gravity.

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Appendix A

Let there be a bound state $\mathcal{B}$ in equilibrium with an environment of temperature T_0 and a photon γ with energy $E_\gamma = h\nu$ before absorption by the bound state. To model the situation as simply as possible and reasonable, we assume that the wave function $\Psi\left(x, \vec{x}_j\right)$, $j\epsilon J \subset \mathbb{N}$, $\left(x, \vec{x}_j\right)\epsilon\left(\mathbb{R}, \mathbb{R}^3\right)$, of the bound state is factorizable as the product of a center of mass component $\psi(x)$ and an orbital component $Y\left(\vec{x}_j\right)$, $j\epsilon J \subset \mathbb{N}, \Psi\left(x, \vec{x}_j\right) = \psi(x)Y\left(\vec{x}_j\right)$ [25]. Since the main contribution of the mass $m > 0$ stems from the nucleus, we may assume that the center of mass component $\psi_{p_0}(x)$ "carries" linear kinetic energy, whereas orbital energy components reside in $Y\left(\vec{x}_j\right)$. The bound state together with the photon form a closed system, Σ_0, and under the assumption that the bound state "moves freely" with small momentum uncertainty, the function $\psi_{p_0}(x)$ can be assumed to be Gaussian (i.e., $\left|\psi_{p_0}(x)\right|^2 \sim \mathcal{N}(\mu_x, \sigma_x)$). We will use $\psi_{p_0}(x)$ in order to model and analyze the entropic situation before and after absorption.

Let us define any wave function $\psi\epsilon L^2(]-\infty,\infty[,\mathbb{C})$, with the information entropy I_ψ to be

$$I_\psi = -\int_{-\infty}^{\infty} |\psi(s)|^2 ln|\psi(s)|^2 ds. \tag{A1}$$

The total information entropy $I^{tot}_{\psi_{p_0}}$ of $\psi_{p_0}(x)$ is then defined by

$$I^{tot}_{\psi_{p_0}} = I_{\psi_{p_0}(x)} + I_{\varphi_{p_0}(p)}, \tag{A2}$$

where $\varphi_{p_0}(p) = \hat{\psi}_{p_0}(p)$ is the conjugate state (the Fourier-transformed state). As a result of Leipnik [26], there holds for any pair of conjugate variables $\psi(x)$ and $\varphi(p)$ and with Planck's constant, h,

$$I^{tot}_\psi = I_{\psi(x)} + I_{\varphi(p)} \geq ln\left(\frac{he}{2}\right), \tag{A3}$$

with equality in the case of Gaussian functions, which we may assume to be a good representation of systems in an equilibrium situation, as mentioned above. Since the bound state is supposed to move freely at a definite momentum, $\varphi_{p_0}(p)$ is highly concentrated around a mean value, $\mu_p = p_0$, and there is hence a negative entropy contribution $I_{\varphi_{p_0}(p)} < 0$ (note that the differential entropy, $I_\mathcal{N}$, of a Gaussian $\mathcal{N}(\mu,\sigma)$ is $I_\mathcal{N} = ln\left(\sqrt{2\pi}\sigma\right) + \frac{1}{2}$ and hence $\lim_{\sigma\searrow 0} I_\mathcal{N} = -\infty$). By (A2), there holds

$$I_{\varphi_{p_0}(p)} = ln\left(\frac{he}{2}\right) - I_{\psi_{p_0}(x)}. \tag{A4}$$

At the same time, the momentum of the photon γ is known to be $p_\gamma = \frac{h\nu}{c}$ and its position is undefinable since there is no rest-frame. So, we set (consistent with $\mu(p_\gamma) = 1$)

$$I_\gamma^{tot} = 0. \tag{A5}$$

For the total system entropy $I_{\Sigma_0}^{tot}$ before absorption, we therefore have

$$I_{\Sigma_0}^{tot} = I_{\psi_{p_0}}^{tot}. \tag{A6}$$

Let us finally define, in analogy to Boltzmann's H-function, the thermodynamic entropy of the system Σ_0 by

$$S^{\Sigma_0} = k_B I_{\varphi_{p_0}(p)} = -k_B \int_{-\infty}^{\infty} \left|\varphi_{p_0}(p)\right|^2 ln\left|\varphi_{p_0}(p)\right|^2 dp, \tag{A7}$$

where k_B denotes the Boltzmann constant. If, initially, we have $\psi_{p_0}(x) = \psi_{p_0}(x,0)$ and $\varphi_{p_0}(p) = \varphi_{p_0}(p,0)$, respectively, then a free evolution leads after some time $t > 0$ to new states $\psi_{p_0}(x,t)$ and $\varphi_{p_0}(p,t)$, still conjugates of each other (note that $\psi_{p_0}(x,t)$ is no more a function with real variance). The evolution is unitary and causes an increasing dispersion, $\sigma_x(t)$, of the density $\left|\psi_{p_0}(x,t)\right|$ around some evolving position mean value $\mu_x(t)$, while the density $\left|\varphi_{p_0}(p,t)\right|$ remains equally concentrated around p_0 and $\left|\varphi_{p_0}(p,t)\right| = \left|\varphi_{p_0}(p,0)\right|$. Therefore, there holds by Definition (A7) for $t \geq 0$:

$$S^{\Sigma(t)} = const. \tag{A8}$$

In other words, the entropy of the unitarily evolving free bound state remains constant, as expected from a reversible process (time reversal $t \to -t$ demands $\psi \to \psi^*$).

Let us now look at the situation after the absorption. The absorption at some time $t_1 > 0$ does two things at once: it annihilates the photon and localizes the center of mass component and thus transforms system Σ_0 into a spatially localized system, Σ_1. This leaves us with a state, $\psi_{x_1}(x)$, which is a Gaussian well concentrated around some spatial mean value, $\mu_x = x_1$. So, there is now a negative entropy contribution, $I_{\psi_{x_1}(x)} < 0$, to total entropy (A2). But because of (A3), the entropy contribution of the conjugate Gaussian $\varphi_{x_1}(p)$ must compensate and we have in analogy to (A4) the following:

$$I_{\varphi_{x_1}(p)} = ln\left(\frac{he}{2}\right) - I_{\psi_{x_1}(x)}. \tag{A9}$$

So, by (A4), (A8) and (A9), the transition $\Sigma(t) \to \Sigma_1$ induces for $0 \leq t < t_1$ an entropy difference of

$$\begin{aligned} \Delta_{\Sigma(t)}^{\Sigma_1} S = k_B\left(I_{\varphi_{x_1}(p)} - I_{\varphi_{p_0}(p,t)}\right) = k_B\left(I_{\varphi_{x_1}(p)} - I_{\varphi_{p_0}(p,0)}\right) = \\ k_B\left(I_{\psi_{p_0}(x)} - I_{\psi_{x_1}(x)}\right) > 0. \end{aligned} \tag{A10}$$

After the measurement, the bound state Σ_1 will again develop freely $(\Sigma_1 \to \Sigma_1(t))$ and by Equation (A8) the entropy, $S^{\Sigma_1(t)}$, remains constant, while the position state disperses around a moving mean position.

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