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Multi–Dimensional Data Analysis of Deep Language in J.R.R. Tolkien and C.S. Lewis Reveals Tight Mathematical Connections

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Abstract: Scholars of English Literature unanimously say that J.R.R. Tolkien influenced C.S. Lewis's writings. For the first time, we have investigated this issue mathematically by using an original multi-dimensional analysis of linguistic parameters, based on surface deep language variables and linguistic channels. To set our investigation in the framework of English Literature, we have considered some novels written by earlier authors, such as C. Dickens, G. MacDonald and others. The deep language variables and the linguistic channels, discussed in the paper, are likely due to writers' unconscious design and reveal connections between texts far beyond the writers' awareness. In summary, the capacity of the extended short-term memory required to readers, the universal readability index of texts, the geometrical representation of texts and the fine tuning of linguistic channels within texts—all tools largely discussed in the paper—revealed strong connections between *The Lord of the Rings* (Tolkien), *The Chronicles of Narnia*, *The Space Trilogy* (Lewis) and novels by MacDonald, therefore agreeing with what the scholars of English Literature say.

Keywords: alphabetical languages; extended short-term memory; human communication; human mind; sentences: mathematical modeling; universal readability index

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1. Introduction

Unanimously, in a large number of papers—some of which are recalled here [1–8] from the vast literature on the topic—scholars of English Literature state that J.R.R. Tolkien influenced C.S. Lewis's writings. The purpose of the present paper is not to review the large wealth of literature based on the typical approach used by scholars of literature—which is not our specialty—but to investigate this issue mathematically and statistically—a study that has never been conducted before—by using recent methods devised by researching the impact of the surface deep language variables [9,10] and linguistic channels [11] in literary texts. Since scholars mention the influence of George MacDonald on both, we consider some novels written by this earlier author. To set all these novels in the framework of English Literature, we consider some novels written by other earlier authors, such as C. Dickens and others.

After this introduction, in Section 2, we introduce the literary texts (novels) considered. In Section 3, we report the series of words, sentences and interpunctions versus chapters for some novels, and define an index useful to synthetically describe regularity due to what we think is a conscious design by authors. In Section 4, we start exploring the four deep language variables; to avoid misunderstanding, these variables, and the linguistic channels derived from them, refer to the "surface" structure of texts, not to the "deep" structure mentioned in cognitive theory. In Section 5, we report results concerning the extended short-term memory and a universal readability index; both topics address human short-term memory buffers. In Section 6, we represent literary texts geometrically in the Cartesian plane by defining linear combinations of deep language variables and calculate the probability that a text can be confused with another. In Section 7, we show the linear relationships existing between linguistic variables in the novels considered. In Section 8, we report the theory of linguistic channels. In Section 9, we apply it to the novels presently studied. Finally, in Section 10, we summarize the main findings and conclude. Several Appendices report numerical data.

2. Database of Literary Texts (Novels)

Let us first introduce the database of literary texts used in the present paper. Table 1 lists some basic statistics of the novels by Tolkien, Lewis and MacDonald. To set these texts in the framework of earlier English Literature, we consider novels by Charles Dickens (Table 2) and other authors (Table 3).

Table 1. Novels written by Tolkien, Lewis and MacDonald, with year of publication. Number of chapters (i.e., the number of samples considered in calculating the regression lines reported below), total number of characters contained in the words (C), total number of words W and sentences (S). Titles, footnotes and other extraneous material present in the digital texts have been deleted.

John R.R. Tolkien (1892–1973)	**Chapters (M)**	**Characters (C)**	**Words (W)**	**Sentences (S)**
The Hobbit (1937)	19	394,154	95,914	5890
The Lord of the Rings (1954–1955)	62	1,906,531	472,173	34,601
The Silmarillion (posthumous, 1977)	24	429,639	101,627	3346
Clive S. Lewis (1898–1963)				
The Screwtape Letters (1942)	31	135,204	31,040	1330
The Space Trilogy (1938–1945)	123	1,243,141	295,240	20,124
The Chronicles of Narnia (1950–1956)	110	1,318,482	322,544	23,515
George MacDonald (1824–1905)				
Phantastes: A Fairie Romance for Men and Women (1858)	25	283,676	67,551	3274
At the Back of the North Wind (1871)	38	349,041	90,697	5017
The Princess and the Goblin (1872)	32	208,325	51,090	3205
Lilith: A Romance (1895)	47	386,522	94,127	6271

Table 2. Novels by Charles Dickens, with year of publication. Number of chapters (M, i.e., the number of samples considered in calculating the regression lines reported below), total number of characters contained in the words (C), total number of words W and sentences (S).

Novel (Year of Publication)	**Chapters (M)**	**Characters (C)**	**Words (W)**	**Sentences (S)**
The Adventures of Oliver Twist (1837–1839)	53	679,008	160,604	9121
David Copperfield (1849–1850)	64	1,469,251	363,284	19,610
Bleak House (1852–1853)	64	1,480,523	350,020	20,967
A Tale of Two Cities (1859)	45	607,424	142,762	8098
Our Mutual Friend (1864–1865)	67	1,394,753	330,593	17,409

We have used the digital text of a novel (WinWord file) and counted, for each chapter, the number of characters, words, sentences and interpunctions (punctuation marks). Before doing so, we have deleted the titles, footnotes and other extraneous material present in the digital texts, a burdensome work. The count is very simple, although time-consuming. Winword directly provides the number of characters and words. The number of sentences was calculated by using WinWord to replace every full stop with a full stop: of course, this action does not change the text, but it gives the number of these substitutions and therefore the number of full stops. The same procedure was repeated for question marks and exclamation marks. The sum of the three totals gives the total number of sentences in the text analyzed. The same procedure gives the total number of commas, colons and semicolons. The sum of these latter values with the total number of sentences gives the total number of interpunctions.

Table 3. Novels by authors of English Literature, with year of publication. Number of chapters (***M***, i.e., the number of samples considered in calculating the regression lines reported below), total number of characters contained in the words (***C***), total number of words ***W*** and sentences (***S***).

Novel (Author, Year)	Chapters (*M*)	Characters (*C*)	Words (*W*)	Sentences (*S*)
Pride and Prejudice (J. Austen, 1813)	61	537,005	121,934	6013
Vanity Fair (W. Thackeray, 1847–1848)	66	1,285,688	277,716	13,007
Moby Dick (H. Melville, 1851)	132	92,2351	203,983	9582
The Mill On The Floss (G. Eliot, 1860)	57	888,867	207,358	9018
Alice's Adventures in Wonderland (L. Carroll, 1865)	12	107,452	27,170	1629
Adventures of Huckleberry Finn (M. Twain, 1884)	42	427,473	110,997	5887
The Jungle Book (R. Kipling, 1894)	9	209,935	51,090	3214
The War of the Worlds (H.G. Wells, 1897)	27	265,499	60,556	3306
The Wonderful Wizard of Oz (L.F. Baum, 1900)	22	156,973	39,074	2219
The Hound of The Baskervilles (A.C. Doyle, 1901–1902)	15	245,327	59,132	4080
Peter Pan (J.M. Barrie, 1902)	17	194,105	47,097	31,77
Martin Eden (J. London, 1908–1909)	45	601,672	139,281	9173

Some homogeneity can be noted in novels of the same author. The stories in *The Space Trilogy* and *The Chronicles of Narnia,* by Lewis are told with about the same number of chapters, words and sentences, as is also for a couple of MacDonald's novels, such as *At the Back of the North Wind* and *Lilith: A Romance*. Some homogeneity can be found in *David Copperfield*, *Bleak House* and *Our Mutual Friend* (by Dickens) and in *The Adventures of Oliver Twist* and *A Tale of Two Cities*. These numerical values, we think, are not due to chance but consciously managed by the authors, which is a topic we purse more in the next section.

3. Conscious Design of Texts: Words, Sentences and Interpunctions versus Chapters

First, we study the linguistic variables which we think the authors deliberately designed. In the specifics, we show the series of words, sentences and interpunctions versus chapter.

Let us consider a literary work (a novel) and its subdivision into disjointed blocks of text long enough to give reliable average values. Let n_S be the number of sentences contained in a text block, n_W the number of words contained in the n_S sentences, n_C the number of characters contained in the n_W words and n_I the number of punctuation marks (interpunctions) contained in the n_S sentences.

Figure 1 shows the series n_W versus the normalized chapter number for *The Lord of the Rings*, *The Chronicles of Narnia*, *The Space Trilogy*.

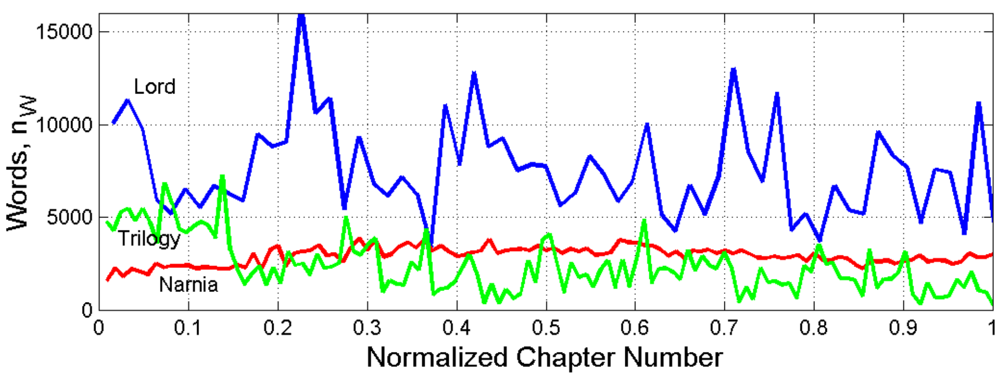


Figure 1. Series of words versus the normalized chapter number. Blue line: *The Lord of the Rings* (*Lord*); red line: *The Chronicles of Narnia* (*Narnia*); green line: *The Space Trilogy* (*Trilogy*).

For example, the normalized value of chapter 10 in *The Chronicles of Narnia*, is 10/110 = 0.09 in the x–scale of Figure 1. This normalization allows the synoptic showing of novels with a different number of chapters.

In *The Chronicles of Narnia* (in the following, *Narnia*, for brevity), we can notice a practically constant value n_W compared to *The Lord of the Rings* (*Lord*) and *The Space Trilogy* (*Trilogy*).

Let us define a synthetic index to describe the series drawn in Figure 1, namely the coefficient of variation δ, given by the standard deviation σ_{n_W} divided by the mean value $< n_W >$

$$\delta = \frac{\sigma_{n_W}}{< n_W >} \tag{1}$$

Tables 4 and 5 report δ for n_W, n_S and n_I. Since n_S and n_I are very well correlated with n_S, the three coefficients of dispersion are about the same.

Table 4. The coefficient of dispersion in the series of words, sentences and interpunctions in the indicated novels by Tolkien, Lewis and MacDonald.

Novel	Words	Sentences	Interpunctions	Average
The Hobbit	0.49	0.48	0.50	0.49
The Lord of the Rings	0.34	0.36	0.34	0.35
The Silmarillion	0.73	0.86	0.80	0.80
The Screwtape Letters	0.07	0.17	0.14	0.13
The Space Trilogy	0.60	0.61	0.58	0.59
The Chronicles of Narnia	0.16	0.20	0.20	0.19
At the Back of the North Wind	0.54	0.61	0.56	0.57
Phantastes: A Fairie Romance for Men and Women	0.66	0.73	0.63	0.67
Lilith: A Romance	0.43	0.53	0.46	0.47
The Princess and the Goblin	0.53	0.75	0.62	0.64

Table 5. The coefficient of dispersion in the series of words, sentences and interpunctions in the indicated novels.

Novel	Words	Sentences	Interpunctions	Average
The Adventures of Oliver Twist	0.31	0.33	0.32	0.32
David Copperfield	0.37	0.38	0.37	0.37
Bleak House	0.28	0.31	0.30	0.30
A Tale of Two Cities	0.52	0.57	0.52	0.54
Our Mutual Friend	0.26	0.29	0.27	0.27
Martin Eden	0.29	0.33	0.29	0.31
The Hound of The Baskervilles	0.26	0.29	0.25	0.27
Peter Pan	0.29	0.41	0.33	0.34

In Narnia $\delta = 0.16$, in Lord $\delta = 0.34$ and in Trilogy $\delta = 0.60$. Let us also notice the minimum value $\delta = 0.07$ in *The Screwtape Letters* (*Screwtape*).

The overall (words, sentences and interpunctions mixed together) mean value is $< \delta >= 0.44$ and the standard deviation $\sigma_\delta = 0.18$. Therefore, *Screwtape* is practically more than $2 \times \sigma_\delta$ from the mean, as also is *Silmarillion* on the other side, and *Narnia* is at about $1.5 \times \sigma_\delta$. In contrast, *Trilogy*, *Lord* and *The Hobbit* (*Hobbit*) are within $1 \times \sigma_\delta$.

From these results, it seems that Lewis designed the chapters of *Narnia* and *Screwtape* with an almost uniform distribution of words, sentences and interpunctions, very likely because of the intended audience in *Narnia* (i.e., kids) and the "letters" fiction tool used in *Screwtape*. In *Trilogy* the design seems very different ($\delta = 0.60$, well within $1 \times \sigma_\delta$) likely due to the development of the science fiction story narrated.

Tolkien acted differently from Lewis, because he seems to have designed chapters more randomly and within $1 \times \sigma_\delta$, as *Hobbit* and *Lord* show. An exception is *The Silmarillion*, published posthumously, which is a text far from being a "novel".

Finally, notice that the novels by MacDonald show more homogeneous values, very similar to *Hobbit* and *Trilogy* and to the other novels listed in Table 5.

In conclusion, the analysis of series of words, sentences and interpunctions per chapter does not indicate likely connections between Tolkien, Lewis and MacDonald. Each author structured their use of words, sentences and punctuation according to distinct plans, which varied not only between authors but also between different novels by the same author.

There are, however, linguistic variables that—as we have reported for modern and ancient literary texts—are not consciously designed/managed by authors; therefore, these variables are the best candidates to reveal hidden mathematical/statistical connections between texts. In the next section, we start dealing with these variables, with the specific purpose of comparing Tolkien and Lewis, although this comparison is set in the more general framework of the authors mentioned in Section 2.

4. Surface Deep Language Variables

We start exploring the four stochastic variables we called deep language variables, following our general statistical theory on alphabetical languages [9–11]. To avoid possible misunderstandings, these variables, and the linguistic channels derived from them, refer to the "surface" structure of texts, not to the "deep" structure mentioned in cognitive theory.

Contrarily to the variables studied in Section 3, the deep language variables are likely due to unconscious design. As shown in [9–11], they reveal connections between texts far beyond writers' awareness; therefore, the geometrical representation of texts [10] and the fine tuning of linguistic channels [11] are tools better suited to reveal connections. They can also likely indicate the influence of an author on another.

We defined the number of characters per chapter n_C and the number of I_P's per chapter n_{I_P}, and the four deep language variables are [9] the number of characters C_P:

$$\boldsymbol{C_P = \frac{n_C}{n_W}} \tag{2}$$

the number of words per sentence P_F:

$$\boldsymbol{P_F = \frac{n_W}{n_S}} \tag{3}$$

the number of interpunctions per word, referred to as the word interval, I_P:

$$\boldsymbol{I_P = \frac{n_I}{n_W}} \tag{4}$$

the number of word intervals per sentence M_F:

$$\boldsymbol{M_F = \frac{n_{I_P}}{n_S}} \tag{5}$$

Equation (5) can be written also as $M_F = P_F / I_P$.

Tables 6–9 reports the mean and standard deviation of these variables. Notice that these values have been calculated by weighing each chapter with its number of words to avoid the short chapters weighing as much as long ones. For example, chapter 1 of *Lord* has 10097 words; therefore, its statistical weight is $10097/472173 \approx 0.021$, not $1/62 \approx 0.016$. Notice, also, that the coefficient of dispersion used in Section 2 was calculated by weighing each chapter 1/62, not 10097/472173, to visually agree with the series drawn in Figure 1.

Table 6. John R.R. Tolkien. Mean value and standard deviation (in parentheses) of $< C_P >$, $< P_F >$, $< I_P >$, $< M_F >$ in the indicated novels. Mean and standard deviation have been calculated by weighing each chapter with its number of words.

Novel	C_p	P_F	I_P	M_F
The Hobbit	4.11 (0.06)	16.54 (2.03)	7.93 (0.98)	2.09 (0.12)
The Lord of the Rings	4.04 (0.08)	13.92 (1.98)	6.68 (0.51)	2.08 (0.20)
The Silmarillion	4.23 (0.08)	31.21 (5.32)	8.58 (0.58)	3.62 (0.42)

Table 7. Clive S. Lewis. Mean value and standard deviation (in parentheses) of $< C_P >$, $< P_F >$, $< I_P >$, $< M_F >$ in the indicated novels. Mean and standard deviation have been calculated by weighing each chapter with its number of words.

Novel	C_p	P_F	I_P	M_F
The Screwtape Letters	4.36 (0.12)	23.95 (3.82)	9.72 (1.00)	2.47 (032)
The Space Trilogy	4.21 (0.16)	15.25 (3.05)	7.47 (0.98)	2.03 (0.22)
The Chronicles of Narnia	4.09 (0.09)	13.97 (1.94)	7.10 (0.89)	1.97 (0.15)

Table 8. George MacDonald. Mean value and standard deviation (in parentheses) of $< C_P >$, $< P_F >$, $< I_P >$, $< M_F >$ in the indicated novels. Mean and standard deviation have been calculated by weighing each chapter with its number of words.

At the Back of the North Wind	3.85 (0.11)	15.52 (3.34)	6.76 (0.77)	2.29 (0.35)
Phantastes: A Fairie Romance for Men and Women	4.20 (0.12)	21.15 (3.58)	6.43 (0.49)	3.28 (0.45)
Lilith: A Romance	4.11 (0.25)	15.87 (3.89)	6.43 (0.57)	2.45 (0.42)
The Princess and the Goblin	4.08 (0.14)	17.81 (4.05)	7.09 (1.22)	2.46 (0.53)

Table 9. Other authors. Mean value and standard deviation (in parentheses) of $< C_P >$, $< P_F >$, $< I_P >$, $< M_F >$ in the indicated novels. Mean and standard deviation have been calculated by weighing each chapter with its number of words.

Novel	$<C_P>$	$<P_F>$	$<I_P>$	$<M_F>$
The Adventures of Oliver Twist	4.23 (0.09)	18.04 (3.13)	5.70 (0.52)	3.16 (0.34)
David Copperfield	4.04 (0.12)	18.83 (2.50)	5.61 (0.30)	3.35 (0.33)
Bleak House	4.23 (0.14)	16.95 (2.21)	6.59 (0.49)	2.57 (0.21)
A Tale of Two Cities	4.26 (0.12)	18.27 (4.24)	6.19 (0.46)	2.93 (0.46)
Our Mutual Friend	4.22 (0.12)	16.46 (2.01)	6.03 (0.37)	2.73 (0.27)
Pride and Prejudice	4.40 (0.14)	21.31 (5.02)	7.16 (0.46)	2.95 (0.46)
Vanity Fair	4.63 (0.08)	21.95 (3.67)	6.73 (0.63)	3.25 (0.39)
Moby Dick	4.52 (0.16)	23.82 (7.44)	6.45 (0.99)	3.64 (0.80)
The Mill On The Floss	4.29 (0.13)	23.84 (4.99)	7.09 (0.69)	3.35 (0.48)
Alice's Adventures in Wonderland	3.96 (0.08)	17.19 (3.20)	5.79 (0.55)	2.95 (0.28)
Adventures of Huckleberry Finn	3.85 (0.10)	19.39 (3.12)	6.63 (0.67)	2.94 (0.48)
The Jungle Book	4.11 (0.09)	16.46 (3.09)	7.14 (0.53)	2.29 (0.30)
The War of the Worlds	4.38 (0.18)	19.22 (4.13)	7.67 (0.92)	2.48 (0.31)
The Wonderful Wizard of Oz	4.02 (0.10)	17.90 (2.23)	7.63 (0.64)	2.34 (0.15)
The Hound of The Baskervilles	4.15 (0.12)	15.07 (3.16)	7.83 (0.94)	1.91 (0.22)
Peter Pan	4.12 (0.09)	15.65 (3.98)	6.35 (0.92)	2.44 (0.37)
Martin Eden	4.32 (0.13)	15.61 (2.71)	6.76 (0.64)	2.30 (0.26)

Specifically, let M be the number of samples (i.e., chapters), then the mean value $< P_F >$ is given by

$$< \boldsymbol{P_F} >= \sum_{k=1}^{M} \boldsymbol{P_{F,k}} \times \left(\boldsymbol{n_{W,k}} / \sum_{k=1}^{M} \boldsymbol{n_{W,k}} \right) \tag{6}$$

Therefore, notice, for not being misled, that $< P_F > \neq \frac{1}{M} \sum_{k=1}^{M} P_{F,k} \neq \sum_{k=1}^{M} n_{W,k} / \sum_{k=1}^{M} n_{S,k} = W/S$. In other words, $< P_F >$ is not given by the total number of words W divided by the total number of sentences S, or by assigning the weight $1/M$ to every chapter. The three values coincide only if all the text blocks contain the same number of words and the same number of sentences, which did not occur. The same observations apply to all other variables.

The following characteristics can be observed from Tables 6–9. *Lord* and *Narnia* share the same $< P_F >$. *Silmarillion* is distinctly different from *Lord* and *Hobbit*, which is in agreement with the different coefficient of dispersion. *Screwtape* is distinctly different from *Narnia* and *Trilogy.* There is a great homogeneity in Dicken's novels and a large homogeneity in $< C_P >$ in all novels.

In the next sections, we use $< P_F >$, $< I_P >$ and $< M_F >$ to calculate interesting indices connected to the short-term memory of readers.

5. Extended Short-Term Memory of Writers/Readers and Universal Readability Index

In this section, we deal with the linguistic variables that, very likely, are not consciously managed by writers who, of course, act also as readers of their own text. We first report findings concerning the extended short-term memory and then those concerning a universal readability index. Both topics address human short-term memory buffers.

5.1. Extended Short-Term Memory and Multiplicity Factor

In [12,13], we have conjectured that the human short-term memory is sensitive to two independent variables, which apparently engage two short-term memory buffers in series, constituents of what we have called the extended short-term memory (E–STM). The first buffer is modeled according to the number of words between two consecutive interpunctions, i.e., the variable I_P, the word interval, which follows Miller's 7 ± 2 law [14]; the second buffer is modeled according to the number of word intervals, I_P's, contained in a sentence—i.e., the variable M_F—ranging approximately from 1 to 7.

In [13], we studied the patterns (which depend on the size of the two buffers) that determine the number of sentences that theoretically can be recorded in the E–STM of a given capacity. These patterns were then compared with the number of sentences actually found in novels of Italian and English literature. We have found that most authors write for readers with short memory buffers and, consequently, are forced to reuse sentence patterns to convey multiple meanings. This behavior is quantified by the multiplicity factor α, defined as the ratio between the number of sentences in a novel and the number of sentences theoretically allowed by the two buffers, a function of I_P and M_F.

We found that $\alpha > 1$ is more likely than $\alpha < 1$ and often $\alpha \gg 1$. In the latter case, writers reuse many times the same pattern of number of words. Few novels show $\alpha < 1$; in this case, writers do not use some or most of them. The values of α found in the novels presently studied are reported in Tables 10 and 11.

Table 10. Multiplicity factor α, universal readability index $< G_U >$ and number of school years Y in the indicated novels by Tolkien, Lewis, MacDonald.

Novel	α	$<G_U>$	Y
The Hobbit	39.4	52.4	9.9
The Lord of the Rings	368.1	64.2	7.4
The Silmarillion	0.2	38.7	>13

Table 10. *Cont.*

Novel	α	$<G_U>$	Y
The Screwtape Letters	1.4	33.5	>13
The Space Trilogy	186.3	56.2	9.0
The Chronicles of Narnia	297.7	61.1	7.9
At the Back of the North Wind	26.3	63.9	7.5
Phantastes: A Fairie Romance for Men and Women	24.3	56.6	8.9
Lilith: A Romance	1.4	63.0	7.5
The Princess and the Goblin	8.5	58.2	8.4

Table 11. Multiplicity factor α, universal readability index $< G_U >$ and number of school years in the indicated novels of English Literature.

Novel	α	$<G_U>$	Y
The Adventures of Oliver Twist	9.46	63.19	7.5
David Copperfield	12.63	64.78	7.2
Bleak House	56.98	58.74	8.3
A Tale of Two Cities	11.89	59.91	8.0
Our Mutual Friend	43.41	62.68	7.6
Pride and Prejudice	5.20	50.33	10.4
Vanity Fair	5.26	49.74	10.5
Moby Dick	1.56	52.63	9.9
The Mill On The Floss	2.17	50.22	10.5
Alice's Adventures in Wonderland	2.90	59.27	8.1
Adventures of Huckleberry Finn	7.05	60.42	8.0
The Jungle Book	14.10	57.59	8.6
The War of the Worlds	6.72	49.05	10.8
The Wonderful Wizard of Oz	7.02	53.83	9.5
The Hound of The Baskervilles	43.87	54.87	9.2
Peter Pan	13.07	63.60	7.5
Martin Eden	46.33	58.53	8.2

5.2. Universal Readability Index

In Reference [14], we have proposed a universal readability index given by

$$G_U = 89–10kC_P + 300/P_F–6(I_P–6) \tag{7}$$

$$k = < C_{P,ITA} > / < C_{P,Eng} > \tag{8}$$

In Equation (8), $< C_{p,ITA} >= 4.48$, $< C_{p,ENG} >= 4.24$. By using Equations (7) and (8), the average value $< kC_P >$ of any language is forced to be equal to that found in Italian, namely 4.48. The rationale for this choice is that C_P is a parameter typical of a language which, if not scaled, would bias G_U without really quantifying the reading difficulty for readers, who in their language are used, on average, to reading shorter or longer words than in Italian. This scaling, therefore, avoids changing G_U for the only reason that a language has, on average, words shorter (as English) or longer than Italian. In any case, C_p affects Equation (7) much less than P_F or I_P.

The values of $< G_U >$—calculated as the other linguistic variables, i.e., by weighing chapters (samples) according to the number of words – are reported in Tables 10 and 11.

The reader may be tempted to calculate Equation (7) by introducing the mean values reported in Tables 6–9. This, of course, can be performed but it should be noted that the values so obtained are always less or equal (hence they are lower bounds) to the means calculated from the samples (see Appendix A). For example, for *Lord*, instead of 64.9, we would obtain 61.9.

It is interesting to "decode" these mean values into the minimum number of school years, Y necessary to make a novel "easy" to read, according to the Italian school system, which is assumed as the reference, see Figure 1 of [15]. The results are also listed in Tables 10 and 11.

5.3. Discussion

Several intriguing observations can be drawn from the results presented in the preceding subsections.

(a). *Silmarillion* with $\alpha = 0.2$ is quite diverse from other Tolkien's writings. Mathematically, this is due to its large $< M_F >= 3.62$ and $< I_P >= 8.58$. In practice, the number of theoretical sentences allowed by the E–STM to read this text is only $1/\alpha = 5$ times the number of sentence patterns actully used in the text. The reader needs a powerful E–STM and reading ability, since $G_U = 38.7$ and $Y > 13$. This does not occur for *Hobbit* ($\alpha = 39.4$, $G_U = 52.4$, Y =9.9) and *Lord* ($\alpha = 368.1, G_U = 64.2$, $Y = 7.4$) in which Tolkien reuses patterns many times, especially in *Lord*.

(b). *Lord* and *Narnia* show very large values, $\alpha = 368.1$ and $\alpha = 297.7$, and very similar G_U's and school years: $G_U = 64.2$, $Y = 7.4$ and $G_U = 61.1$, $Y = 7.9$, respectively. Sentence patterns are reused many times by Lewis in this novel, but not in *Screwtape* ($\alpha = 1.4$), which is more difficult to read ($G_U = 33.5$) and requires more years of schooling, $Y > 13$. Moreover, *Lord* and *Narnia* have practically the same $< P_F > \approx 14$.

(c). In general, *Narnia* is closer to *Lord* than to *Trilogy*, although the number of words and sentences in *Trilogy* and *Narnia* are quite similar (Table 1). This difference between *Trilogy* ($G_U = 56.2$, $Y = 9$) and *Narnia* ($G_U = 61.1$, $Y = 7.9$) might depend on the different readers addressed, kids for *Narnia* and adults for *Trilogy*, with different reading ability, as G_U indicates.

(d). The novels by MacDonald show values of α and G_U very similar to those of the other English novels.

(e). Notice the homogeneity in Dicken's novels, which require about $Y = 7 \sim 8$ years of school and readability index $< G_U >= 59 \sim 65$.

In conclusion, *Lord* and *Narnia* are the novels that address readers with very similar E–STM buffers, reuse sentence patterns in similar ways, contain the same number of words per sentence, and require the same reading ability and school years compared to other novels by Tolkien and Lewis. The mathematical connections between *Lord* and *Narnia* will be further pursued in the next section, where the four deep language parameters are used to represent texts geometrically.

6. Geometrical Representation of Texts

The mean values of Tables 6–9 can be used to assess how texts are "close", or mathematically similar, in the Cartesian coordinate plane, by defining linear combinations of deep–language variables. Texts are then modeled as vectors; the representation is discussed in detail in [9,10] and briefly recalled here. An extension of this geometrical representation of texts allows the calculation of the probability that a text may be confused with another one, an extension in two dimensions of the problem discussed in [16]. The values of the conditional probability between two texts (authors) can be considered an index indicating who influenced who.

6.1. Vector Representation of Texts

Let us consider the following six vectors of the indicated components of deep language variables: $\vec{R}_1 = (< C_P >, < P_F >)$, $\vec{R}_2 = (< M_F >, < P_F >)$, $\vec{R}_3 = (< I_P >, < P_F >)$, $\vec{R}_4 = (< C_P >, < M_F >)$, $\vec{R}_5 = (< I_P >, < M_F >)$, $\vec{R}_6 = (< I_P >, < C_P >)$ and their resulting vector sum:

$$\vec{R} = \sum_{k=1}^{6} \vec{R}_k = x\,\vec{i} + y\,\vec{j} \tag{9}$$

The choice of which parameter represents the component in the abscissa and ordinate axes is not important because, once the choice is made, the numerical results will depend on it, but not the relative comparisons and general conclusions.

In the first quadrant of the Cartesian coordinate plane, two texts are likely mathematically connected—they show close ending points of vector (9)—if their relative Pythagorean distance is small. A small distance means that texts share a similar mathematical structure, according to the four deep language variables.

By considering the vector components x and y of Equation (9), we obtain the scatterplot shown in Figure 2 where X and Y are normalized coordinates calculated by setting *Lord* at the origin $(X = 0,\ Y = 0)$ and *Silmarillion* at $(X = 1,\ Y = 1)$, according to the linear tranformations:

$$X = \frac{x - x_{Lord}}{x_{Silma} - x_{Lord}} \tag{10}$$

$$X = \frac{y - y_{Lord}}{y_{Silma} - y_{Lord}} \tag{11}$$

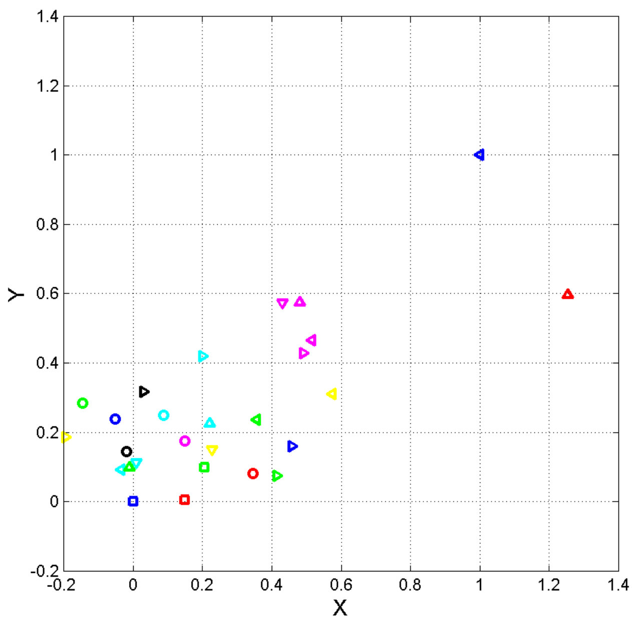


Figure 2. Normalized coordinates X and Y of the ending point of vector (5) such that *Lord*, blue square, is at (0,0) and *Silmarillion*, blue triangle pointing left, is (1,1). *Narnia*: red square; *Trilogy*: red circle; *Hobbit*: blue triangle pointing right; *Screwtape*: red triangle pointing upward; *Back*: cyan triangle pointing left; *Lilith*: cyan triangle pointing downward; *Back*: cyan triangle pointing left; *Phantastes*: cyan triangle pointing right; *Princess*: cyan triangle pointing upward; *Oliver*: blue circle; *David*: green circle; *Tale*: cyan circle; *Bleak*: magenta circle; *Mutual*: black circle; *Pride*: magenta triangle pointing right; *Vanity*: magenta triangle pointing left; *Moby*: magenta triangle pointing downward; *Mill*: magenta triangle pointing upward; *Alice*: yellow triangle pointing right; *Jungle*: yellow triangle pointing downward; *War*: yellow triangle pointing right; *Oz*: green triangle pointing left; *Bask*: green triangle pointing right; *Peter*: green triangle pointing upward; *Martin*: green square; *Finn*: black triangle pointing right.

From Figure 2, we can notice that *Silmarillion* and *Screwtape* are distinctly very far from all other texts examined, marking their striking diversity, as already remarked; therefore, in the following analyses, we neglect them. Moreover, *Pride*, *Vanity*, *Moby* and *Floss* are grouped together and far from *Trilogy*, *Narnia* and *Lord*; therefore, in the following analyses, we will not consider them further.

The complete set of the Pythagorean distance d between pairs of texts is reported in Appendix B. These data synthetically describe proximity of texts and may indicate to scholars of literature connections between texts not considered before.

Figure 3 shows example of these distances concerning *Lord*, *Narnia* and *Trilogy*. By referring to the cases in which $d < 0.2$, we can observe the following:

(a). The closest texts to *Lord* are *Narnia*, *Back*, *Lilith*, *Mutual* and *Peter*.
(b). The closest texts to *Narnia* are *Lord*, *Lilith*, *Bleak*, *Martin* and *Peter*.
(c). The closest texts to *Trilogy* are *Hobbit*, *Martin* and *Peter*.

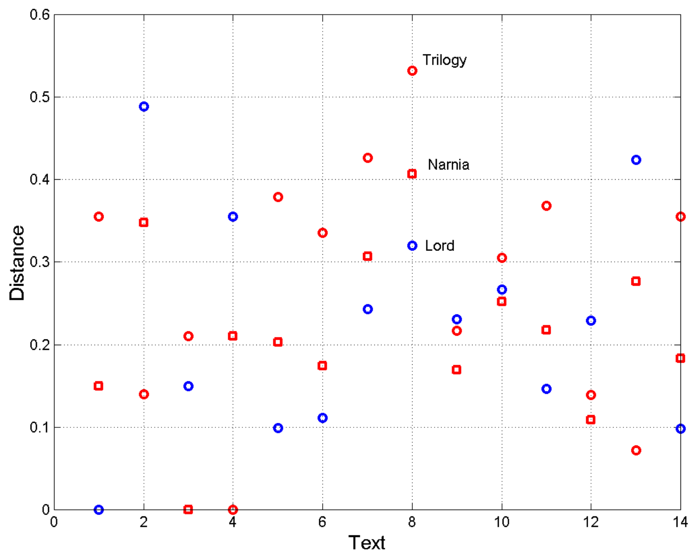


Figure 3. Pythagorean distance d between pairs of texts considering *Lord* (the distances referring to this case are labeled with blue circles), *Narnia* (red squares) and Trilogy (red circles). Key: *Lord* 1, *Hobbit* 2, *Narnia* 3, *Trilogy* 4, *Back* 5, *Lilith* 6, *Oliver* 7, *David* 8, *Bleak* 9, *Tale* 10, *Mutual* 11, *Martin* 12, *Bask* 13, *Peter* 14.

Besides the proximity with earlier novels, *Lord* and *Narnia* show close proximity with each other and with two novels by MacDonald.

These remarks, however, refer to the "average" display of vectors whose ending point depends only on mean values. The standard deviation of the four deep language variables, reported in Tables 6–9, do introduce data scattering; therefore, in the next subsection, we study and discuss this issue by calculating the probability (called "error" probability) that a text may be mathematically confused with another one.

6.2. Error Probability: An Index to Assess Who Influenced Who

Besides the vector $\vec{R}$ of Equation (9)—due to mean values—we can consider another vector $\vec{\rho}$, due to the standard deviation of the four deep language variables that adds to $\vec{R}$. In this case, the final random vector describing a text is given by

$$\vec{T} = \vec{R} + \vec{\rho} \tag{12}$$

Now, to obtain some insight into this new description, we consider the area of a circle centered at the ending point of $\vec{R}$.

We fix the magnitude (radius) ρ as follows. First, we add the variances of the deep language variables that determine the components x and y of $\vec{R}$, let them be σ_x^2, σ_y^2. Then, we calculate the average value $\sigma_\rho^2 = 0.5 \times \left(\sigma_x^2 + \sigma_y^2\right)$ and finally, we set

$$\rho = \sigma_\rho \tag{13}$$

Now, since in calculating the coordinates x and y of $\vec{R}$ a deep language variable can be summed twice or more, we add its standard deviation (referred to as sigma) twice or more times before squaring. For example, in the x–component, I_P appears three times; therefore, its contribution to the total variance in the x–axis is 9 times the variance calculated from the standard deviation reported in Tables 6–9. For *Lord*, for example, it is 9×0.51^2. After these calculations, the values of the 1–sigma circle are transformed into the normalized coordinates X, Y according to Equations (10) and (11).

Figure 4 shows a significant example involving *Lord*, *Narnia*, *Trilogy*, *Back* and *Peter*. We see that *Lord* can be almost fully confused with *Narnia*, and partially with *Trilogy*, but not vice versa. *Lord* can also be confused with *Peter* and *Back, therefore* indicating strong connections with these earlier novels.

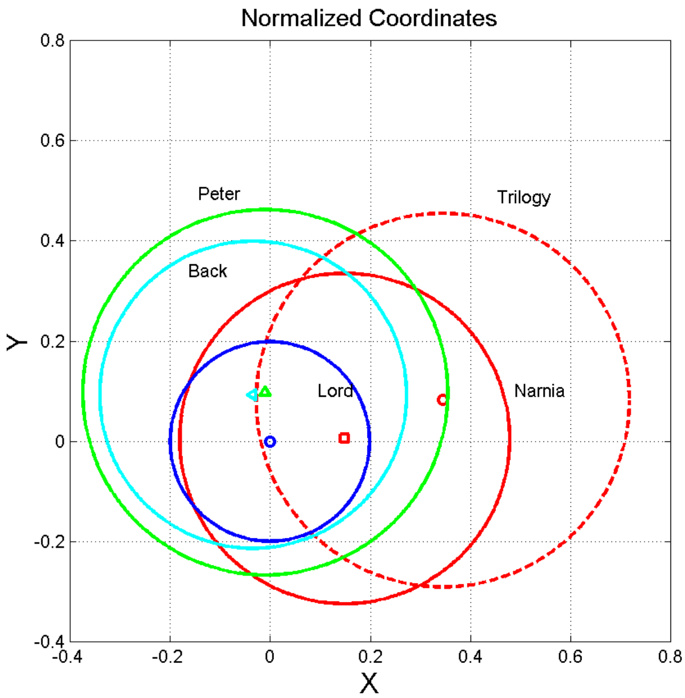


Figure 4. Normalized coordinates X and Y of the ending point of vector (5) and 1–sigma circles, such that *Lord*, blue square, is at (0,0) and *Silmarillion*, blue triangle pointing left, is (1,1). *Lord*: blue square (blue 1–sigma circle); *Narnia*: red square (red 1–sigma circle); *Trilogy*: red circle (dashed red 1–sigma circle); *Back*: cyan triangle pointing left (cyan 1–sigma circle); *Peter*: green triangle pointing upward (green 1–sigma circle).

Now, we can estimate the (conditional) probability that a text is confused with another by calculating the ratio of areas. This procedure is correct if we assume that the bivariate density of the normalized coordinates ρ_X, ρ_Y, centered at $\vec{R}$, is uniform. By assuming this hypothesis, we can calculate probabilities as the ratio of areas [17,18].

The hypothesis of substantial uniformity around $\vec{R}$ should be justified by noting that the coordinates X, Y are likely distributed according to a log-normal bivariate density because the logarithm of the four deep language variables, which combine in Equation (9) linearly, can be modeled as a Gaussian. For the central limit theorem, we should expect approximately a Gaussian model on the linear values, but with a significantly larger standard deviation that that of the single variables. Therefore, in the area close to $\vec{R}$, the bivariate density function should not be peaked, hence the uniform density modeling.

Now, we can calculate the following probabilities. Let A be the common area of two 1–sigma circles (i.e., the area proportional to the joint probability of two texts), let A_1 be the area of 1–sigma circle of text 1 and A_2 the area of 1–sigma circle of text 2. Now, since probabilities are proportional to areas, we obtain the following relationships:

$$\frac{A}{A_1} = \frac{P(A_1, A_2)}{P(A_1)} = \frac{P(A_2/A_1)P(A_1)}{P(A_1)} = P(A_2/A_1) \tag{14}$$

$$\frac{A}{A_2} = \frac{P(A_1, A_2)}{P(A_2)} = \frac{P(A_1/A_2)P(A_2)}{P(A_2)} = P(A_1/A_2) \tag{15}$$

In other words, A/A_1 gives the conditional probability $P(A_2/A_1)$ that part of text 2 can be confused (or "contained") with text 1; A/A_2 gives the conditional probability $P(A_1/A_2)$ that part of text 1 can be confused with text 2. Notice that these conditional probabilities depend on the distance between two texts and on the 1–sigma radii (Appendix C).

Of course, these joint probabilities can be extended to three or more texts, e.g., in Figure 4 we could calculate the area shared by *Lord*, *Narnia* and *Trilogy* and the corresponding joint probability, which is not conducted in the present paper.

We think that the conditional probabilities and the visual display of 1–sigma circles give useful clues to establish possible hidden connections between texts and, maybe, even between authors, because the variables involved are not consciously managed by them.

In Table 12, the conditional probability $P(A_2/A_1)$ is reported in the columns; therefore, A_1 refers to the text indicated in the upper row. $P(A_1/A_2)$ is reported in the rows; therefore, A_2 refers to the text indicated in the left column.

Table 12. Conditional probability between the indicated novels. $P(A_2/A_1)$ is reported in the columns; therefore, A_1 refers to the text indicated in the upper row. $P(A_1/A_2)$ is reported in the rows; therefore, A_2 refers to the text indicated the left column. For example, assuming *Lord* as text 1 (column 1 of Table 12) and *Narnia* as text 2 (row 3), we find $P(A_2/A_1) = 0.974$ and vice versa. If we assume *Narnia* as text 1 (column 3) and *Lord* as text 2 (row 1), we find $P(A_2/A_1) = 0.356$.

	Novel	***Lord***	***Hobbit***	***Narnia***	***Trilogy***	***Back***	***Lilith***	***Oliver***	***David***	***Bleak***	***Tale***	***Mutual***	***Martin***	***Bask***	***Peter***
1	*Lord*	1	0.031	0.356	0.142	0.423	0.511	0.277	0.041	0.307	0.231	0.619	0.301	0.078	0.299
2	*Hobbit*	0.099	1	0.421	0.731	0.171	0.225	0.074	0	0.592	0.376	0.060	0.665	0.833	0.227
3	*Narnia*	0.974	0.354	1	0.550	0.647	0.781	0.489	0.166	0.927	0.625	0.886	0.949	0.462	0.602
4	*Trilogy*	0.498	0.786	0.704	1	0.400	0.510	0.297	0	0.921	0.608	0.473	0.978	0.908	0.421
5	*Back*	1	0.124	0.559	0.270	1	0.997	0.866	0.796	0.763	0.692	1	0.566	0.179	0.707
6	*Lililth*	0.891	0.121	0.498	0.254	0.735	1	0.741	0.559	0.762	0.661	1	0.546	0.169	0.521
7	*Oliver*	0.352	0.029	0.227	0.108	0.466	0.540	1	0.913	0.444	0.579	0.917	0.239	0.043	0.378
8	*David*	0.024	0	0.035	0	0.195	0.186	0.416	1	0.029	0.173	0.262	0.001	0	0.168
9	*Bleak*	0.307	0.182	0.339	0.264	0.323	0.437	0.350	0.051	1	0.598	0.526	0.558	0.208	0.296
10	*Tale*	0.330	0.165	0.327	0.248	0.417	0.541	0.650	0.427	0.852	1	0.774	0.484	0.176	0.385
11	*Mutual*	0.390	0.012	0.204	0.085	0.266	0.361	0.454	0.285	0.331	0.342	1	0.205	0.031	0.188
12	*Martin*	0.490	0.333	0.565	0.455	0.389	0.509	0.307	0.004	0.906	0.551	0.529	1	0.396	0.384
13	*Bask*	0.250	0.826	0.545	0.838	0.244	0.312	0.110	0	0.669	0.397	0.160	0.785	1	0.289
14	*Peter*	1	0.234	0.736	0.403	1	1	0.996	0.968	0.988	0.904	1	0.790	0.300	1

Notice that $P(A_2/A_1) = 1$ means $A = A_1$; therefore, text 1 can be fully confused with text 2. $P(A_1/A_2) = 1$ means $A = A_2$; therefore, text 2 can be fully confused with text 1.

For example, assuming *Lord* as text 1 (column 1 of Table 12) and *Narnia* as text 2 (row 3), we find $P(A_2/A_1) = 0.974$ and vice versa. If we assume *Narnia* as text 1 (column 3) and *Lord* as text 2 (row 1), we find $P(A_2/A_1) = 0.356$. These data indicate that *Lord* can be confused with *Narnia* with a probability close to 1, but not vice versa. In other words, in the data bank considered in this paper, if a machine randomly extracts a chapter from *Lord*, another machine, unaware of this choice, could attribute it to *Lord*, but also with decreasing probability to *Back*, *Peter*, *Narnia* and *Lilith.*

On the contrary, if the text is extracted from *Narnia*, then it is more likely attributed to *Peter* or *Trilogy* than to *Lord* or other texts.

We think that these conditional probabilities indicate who influenced who more. In other words, Tolkien influenced more Lewis that the opposite.

Now, we can define a synthetic parameter which highlights how much, on the average, two texts can be erroneously confused with each other. The parameter is the average conditional probability (see [16] for a similar problem):

$$p_e = P(A_2/A_1)P(A_1) + P(A_1/A_2)P(A_2) \tag{16}$$

Now, since in comparing two texts we can assume $P(A_1) = P(A_2) = 0.5$, we receive

$$p_e = 0.5 \times [P(A_2/A_1) + P(A_1/A_2)] \tag{17}$$

If $p_e = 0$, there is no intersection between the two 1–sigma circles. The two texts cannot be each other confused; therefore, there is no mathematical connection involving the deep language parameters (this happens for *Screwtape* and *Silmarillion*, which can be each other confused, but not with the other texts). If $p_e = 1$, the two texts can be totally confused, and the two 1–sigma circles coincide. Appendix D reports the values of p_e for all the pairs of novels.

Now, just to allow some rough analysis, it is reasonable to assume $p_e = 0.5$ as a reference threshold, i.e., the probability of obtaining heads or tails in flipping a fair coin. If $p_e > 0.5$, then two texts can be confused not by chance; if $p_e \leq 0.5$, then two texts cannot likely be confused.

To visualize p_e, Figure 5 draws p_e when text 1 is *Lord* (column 1 of Table 12), *Narnia* (column 3) or *Trilogy* (column 4). We notice that $p_e > 0.5$ in the following cases:

(a). *Lord* as text 1: *Narnia*, *Back*, *Lilith*, *Mutual*, *Peter*.
(b). *Narnia* as text 1: *Lord*, *Trilogy*, *Back*, *Lilith*, *Bleak*, *Mutual*, *Martin*, *Peter*.
(c). *Trilogy* as text 1: *Hobbit*, *Narnia*, *Bleak*, *Martin*, *Bask*.

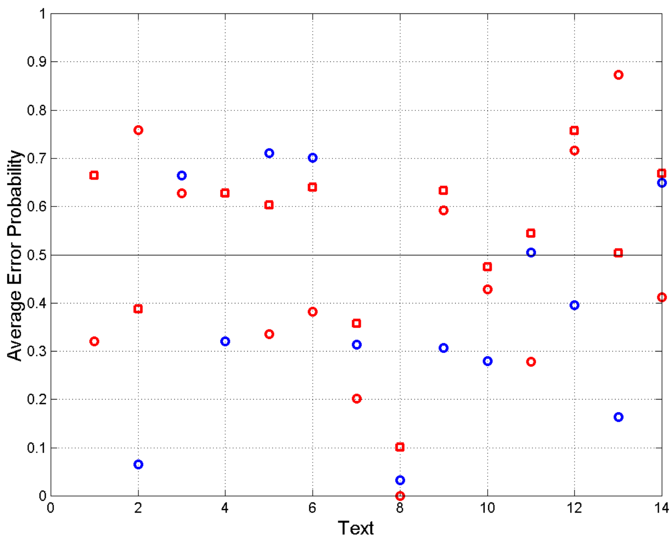


Figure 5. Error probability p_e versus text 2. *Lord* (the probabilities referring to this case are labeled with blue circles), *Narnia* (red squares) and *Trilogy* (red circles). Text key: *Lord* 1, *Hobbit* 2, *Narnia* 3, *Trilogy* 4, *Back* 5, *Lilith* 6, *Oliver* 7, *David* 8, *Bleak* 9, *Tale* 10, *Mutual* 11, *Martin* 12, *Bask* 13, *Peter* 14.

We can reiterate that Tolkien (*Lord*) appears significantly connected to Lewis (*Narnia*), to MacDonald (*Back*, *Lilith*) and Barrie (*Peter*), but not to Dicken's novels where, on the contrary, Lewis appears connected.

In the next section, the four deep language variables are singled out to consider linguistic channels existing in texts. This is the analysis we have called the "fine tuning" of texts [11].

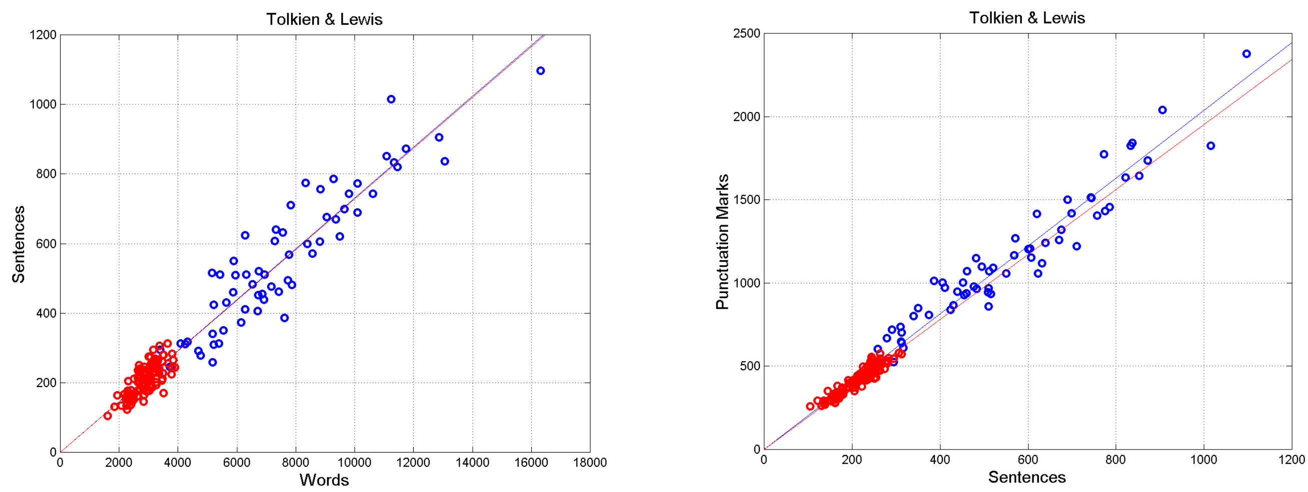
7. Linear Relationships in Literary Texts

The theory of linguistic channels, which we will be revisited in the next section, is based on the regression line between linguistic variables:

$$y = mx \tag{18}$$

Therefore, we show examples of these linear relationships found in *Lord* and *Narnia*.

Figure 6a shows the scatterplot of n_S versus n_W of *Lord* and *Narnia*. In *Narnia*, the slope of the regression line is $m = 0.0729$ and the correlation coefficient $r = 0.7610$. In *Lord*, $m = 0.0731$ and $r = 0.9199$. Since the average relationships—i.e., Equation (18)—are practically identical—see also the values of $< P_F >$ in Tables 6 and 7—while the correlation coefficients—i.e., the scattering of the data—are not, this fact will impact the sentence channel discussed in Section 9.



(**a**) (**b**)

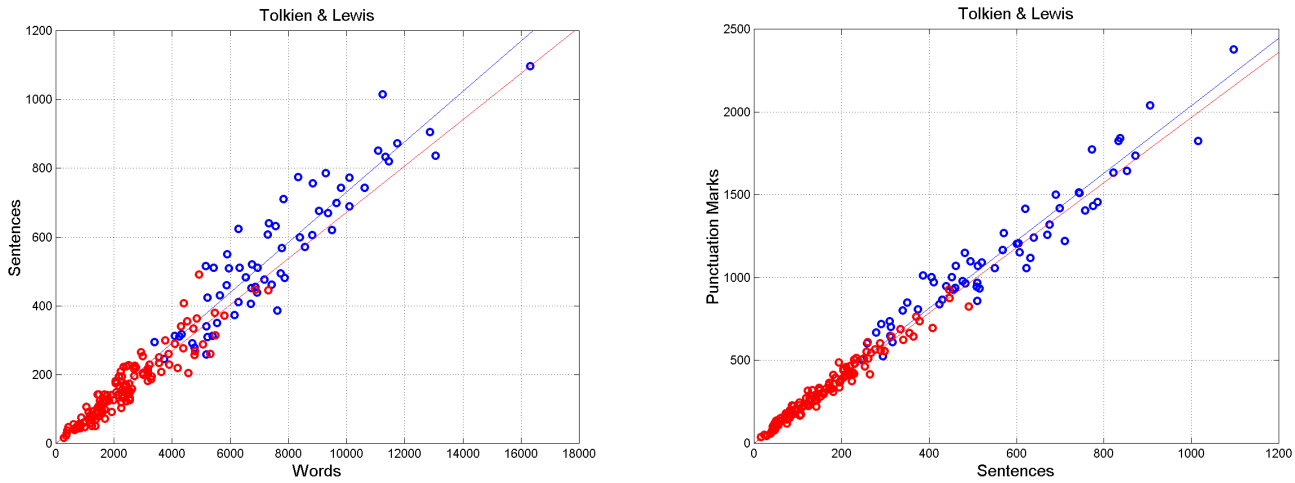
Figure 6. (**a**) Scatterplot of n_S versus n_W in *Lord* (blue) and *Narnia* (red); (**b**) n_I versus n_S in *Lord* (blue) and *Narnia* (red).

Similar observations can be carried out for Figure 6b, which shows n_I versus n_S in *Lord* and *Narnia*. We find $m = 2.0372$, $r = 0.9609$ in *Lord*, and $m = 1.9520$ and $r = 0.9384$ in *Narnia*. Appendix E reports the complete set of these parameters.

Figure 7 shows the scatterplots of *Lord* and *Trilogy*. In *Trilogy*, for n_S versus n_W m = 0.0672, $r = 0.9325$; for n_I versus n_S $m = 1.9664, r = 0.9830$.

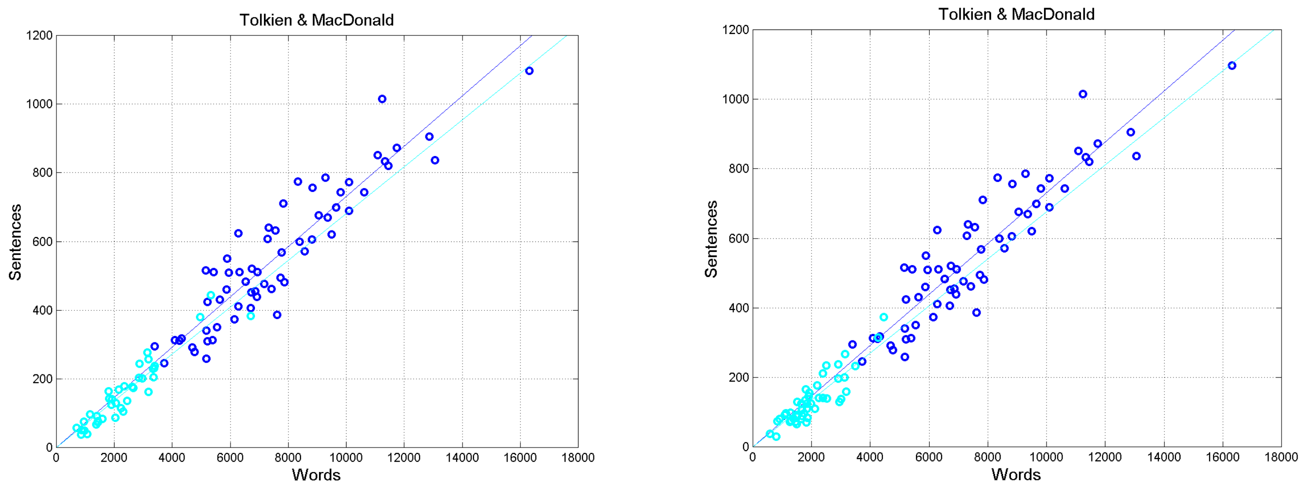
Figure 8 shows the scatterplots for *Lord* and *Back* or *Lilith*. We see similar regression lines and data scattering. In *Back* (left panel), the regression line between n_S and n_W gives $m = 0.0681$, $r = 0.9416$; in *Lilith* (right panel), $m = 0.0676$, $r = 0.8890$. These results likely indicate the influence of MacDonald on Tolkien's writings because they are different from most other novels.

In conclusion, the regression lines of *Lord*, *Narnia* and *Trilogy* are very similar, but they can differ in the scattering of the data. Regression lines, however, describe only one aspect of the relationship, namely the relationship between conditional average values in Equation (18); they do not consider the other aspect of the relationship, namely the scattering of data, which may not be the same even when two regression lines almost coincide, as shown above. The theory of linguistic channels, discussed in the next section, on the contrary, considers both slopes and correlation coefficients and provides a "fine tuning" tool to compare two sets of data by singling out each of the four deep language parameters.



(**a**) (**b**)

Figure 7. (**a**) Scatterplot of n_S versus n_W in *Lord* (blue) and *Trilogy* (red); (**b**) n_I versus n_S in in *Lord* (blue) and *Trilogy* (red).



(**a**) (**b**)

Figure 8. Scatterplot of the number of sentences n_S versus the number of words n_W: (**a**) *Lord* (blue) and *Back* (cyan); (**b**) *Lord* (blue) and *Lilith* (cyan).

8. Theory of Linguistic Channels

In this section, we recall the general theory of linguistic channels [11]. In a literary work, an independent (reference) variable x (e.g., n_W) and a dependent variable y (e.g., n_S) can be related by the regression line given by Equation (18).

Let us consider two different text blocks Y_k and Y_j, e.g., the chapters of work k and work j. Equation (18) does not give the full relationship between two variables because it links only conditional average values. We can write more general linear relationships, which take care of the scattering of the data—measured by the correlation coefficients r_k and r_j, respectively—around the average values (measured by the slopes m_k and m_j):

$$y_k = m_k x + n_k \tag{19}$$

$$y_j = m_j x + n_j \tag{20}$$

The linear models Equations (19) and (20) introduce additive "noise" through the stochastic variables n_k and n_j , with zero mean value [9,11,15]. The noise is due to the correlation coefficient $|r| \neq 1$.

We can compare two literary works by eliminating x; therefore, we compare the output variable y for the same number of the input variable x. For example, we can compare the number of sentences in two novels—for an equal number of words—by considering not only the average relationship, Equation (18), but also the scattering of the data, measured by the correlation coefficient, Equations (19) and (20). We refer to this communication channel as the "sentences channel", S–channel, and to this processing as "fine tuning" because it deepens the analysis of the data and can provide more insight into the relationship between two literary works or any other texts.

By eliminating x from Equations (19) and (20), we obtain the linear relationship between the input number of sentences in work Y_k (now the reference, input text) and the number of sentences in text Y_j (now the output text):

$$y_j = \frac{m_j}{m_k} y_k - \frac{m_j}{m_k} n_k + n_j \tag{21}$$

Compared to the new reference work Y_k, the slope m_{jk} is given by

$$\boldsymbol{m_{jk} = m_j / m_k} \tag{22}$$

The noise source that produces the correlation coefficient between Y_k and Y_j is given by

$$\boldsymbol{n_{jk} = -\frac{m_j}{m_k} n_k + n_j = -m_{jk} n_k + n_j} \tag{23}$$

The "regression noise–to–signal ratio", R_{m}, due to $\left|m_{jk}\right| \neq 1$, of the new channel is given by

$$\boldsymbol{R_m = \left(m_{jk} - 1\right)^2} \tag{24}$$

The unknown correlation coefficient r_{jk} between y_j and y_k is given by

$$\boldsymbol{r_{jk} = \cos\left|\arccos\left(r_j\right) - \arccos\left(r_k\right)\right|} \tag{25}$$

The "correlation noise–to–signal ratio", R_{r}, due to $\left|r_{jk}\right| < 1$, of the new channel from text Y_k to text Y_j is given by

$$\boldsymbol{R_r = \frac{1 - r_{jk}^2}{r_{jk}^2} m_{jk}^2} \tag{26}$$

Because the two noise sources are disjoint and additive, the total noise-to-signal ratio of the channel connecting text Y_k to text Y_j is given by

$$\boldsymbol{R = R_m + R_r} \tag{27}$$

Notice that Equation (27) can be represented graphically [10]. Finally, the total and the partial signal-to-noise ratios are given by

$$\Gamma_{dB} = -10 \times \log_{10} \mathrm{R} \tag{28}$$

$$\Gamma_{m,dB} = -10 \times \log_{10} R_m \tag{29}$$

$$\Gamma_{r,dB} = -10 \times \log_{10} R_r \tag{30}$$

Of course, we expect that no channel can yield $\left|r_{jk}\right| = 1$ and $\left|m_{jk}\right| = 1$; therefore, $\Gamma_{dB} = \infty$, a case referred to as the ideal channel, unless a text is compared with itself. In practice, we always find $\left|r_{jk}\right| < 1$ and $\left|m_{jk}\right| \neq 1$. The slope m_{jk} measures the multiplicative "bias" of the dependent variable compared to the independent variable; the correlation coefficient r_{jk} measures how "precise" the linear best fit is.

In conclusion, the slope m_{jk} is the source of the regression noise R_m, and the correlation coefficient r_{jk} is mostly the source of the correlation noise of the channel R_r.

9. Linguistic Channels

In long texts (such as novels, essays, etc.), we can define at least four linguistic linear channels [11], namely:

(a). Sentence channel (S–channel)
(b). Interpunctions channel (I–channel)
(c). Word interval channel, WI–channel
(d). Characters channel (C–channel).

In S–channels, the number of sentences of two texts is compared to the *same* number of words. These channels describe how many sentences the author of text j writes, compared to the writer of text k (reference text), by using the same number of words. Therefore, these channels are more linked to P_F than to other parameters. It is very likely they reflect the style of the writer.

In I–channels, the number of word intervals of two texts is compared for the *same* number of sentences. These channels describe how many short texts between two contiguous punctuation marks (of length I_P) two authors use; therefore, these channels are more linked to M_F than to other parameters. Since M_F is very likely connected with the E–STM, I–channels are more related to the second buffer of readers' E–STM than to the style of the writer.

In WI–channels, the number of words contained in a word interval (i.e., I_P) is compared for the *same* number of interpunctions. These channels are more linked to I_P than to other parameters. Since I_P is very likely connected with the E–STM, WI–channels are more related to the first buffer of readers' E–STM than to the style of the writer.

In C–channels, the number of characters of two texts is compared to the same number of words. They are more related to the language used, e.g., English, than to the other parameters, unless essays or scientific/academic texts are considered because these latter texts use, on average, longer words [9].

As an example, Table 13 reports the total and the partial signal-to-noise ratios Γ_{dB}, $\Gamma_{m,dB}$, $\Gamma_{r,dB}$ in the four channels by considering *Lord* as reference (input) text. In other words, text j is compared to text k (reference text, i.e., *Lord*).

Table 13. Total and the partial signal-to-noise ratios Γ_{dB}, $\Gamma_{m,dB}$, $\Gamma_{r,dB}$ in the four channels by considering *Lord* as reference (input) text.

	S–Channel			**I–Channel**			**WI–Channel**			**C–Channel**		
Novel	Γ_{dB}	$\Gamma_{m,dB}$	$\Gamma_{r,dB}$	Γ_{dB}	$\Gamma_{m,dB}$	$\Gamma_{r,dB}$	Γ_{dB}	$\Gamma_{m,dB}$	$\Gamma_{r,dB}$	Γ_{dB}	$\Gamma_{m,dB}$	$\Gamma_{r,dB}$
Hobbit	14.60	15.48	21.94	15.04	30.08	15.18	15.61	15.77	30.19	28.87	35.90	29.83
Narnia	10.12	51.26	10.12	21.83	27.57	23.18	7.94	27.08	7.99	23.14	37.47	23.30
Trilogy	21.27	21.86	30.24	20.07	29.18	20.64	18.74	19.21	28.63	27.61	27.85	40.30
Back	21.10	23.30	25.11	20.96	24.12	23.82	21.96	57.42	21.96	26.52	26.68	41.05
Lililth	19.92	22.47	23.44	18.47	19.15	26.87	20.86	26.28	22.33	19.17	33.98	19.31
Oliver	12.87	12.93	31.51	5.72	5.74	29.30	16.54	16.56	40.83	26.50	26.63	41.73
David	11.43	11.52	28.37	4.18	4.19	30.77	14.43	16.10	19.37	32.88	51.53	32.94
Bleak	14.91	14.93	38.01	12.10	12.29	25.80	29.00	36.33	29.88	21.76	26.69	23.45
Tale	12.66	13.31	21.25	8.26	8.56	19.99	16.13	22.76	17.20	25.88	25.89	55.01
Mutual	16.13	16.22	33.05	9.97	10.00	31.20	19.40	20.33	26.55	23.24	26.87	25.70
Martin	16.00	20.01	18.20	18.04	20.74	21.38	29.30	40.14	29.68	21.11	23.16	25.34
Bask	10.92	23.84	11.14	18.77	20.81	23.03	11.77	15.70	14.02	26.34	32.54	27.53
Peter	18.10	24.41	19.25	17.31	17.53	30.45	16.67	22.23	18.09	31.19	34.62	33.81

Appendix F reports Γ_{dB} for all novels considered in the paper.

Let us make some fundamental remarks on Table 13, applicable to whichever is the reference text. The signal-to-noise ratios of C–channels are practically the largest ones, ranging from 19.17 dB (*Lilith*) to 31.19 dB (*Back*). These results are simply saying that all authors use the same language and write texts of the same kind, which is novels, not essays or scientific/academic papers. These channels are not apt to distinguish or assess large differences between texts or authors.

In the three other channels, we can notice that *Trilogy*, *Back* and *Lilith* have the largest signal-to-noise ratios, about ~ 19 to ~ 22 dB; therefore, these novels are very similar to *Lord*. In other words, these channels seem to confirm the likely influence by MacDonald on both *Lord* and *Trilogy* and the connection between *Lord* and *Trilogy*.

On the contrary, *Narnia* shows poor values in the S–Channel (10.12 dB) and WI–Channel (7.94 dB). These low values are determined by the correlation noise because $R = R_m + R_r \approx R_r$. If we consider only $\Gamma_{m,dB}$—i.e., only the regression line—then we notice a strong connection with *Lord* since $\Gamma_{m,dB} = 51.26$ dB. As we have already observed regarding Figure 6, the regression lines are practically identical but the spreading of the data is not. Lewis in *Narnia* is less "regular" than in *Trilogy* or Tolkien in *Lord* in shaping (unconsciously) these two linguistic channels.

10. Summary and Conclusions

Scholars of English Literature unanimously say that J.R.R. Tolkien influenced C.S. Lewis's writings. For the first time, we have investigated this issue mathematically by using an original multi-dimensional analysis of linguistic parameters, based on the surface deep language variables and linguistic channels.

To set our investigation in the framework of English Literature, we have also considered some novels written by earlier authors, such as Charles Dickens and others, including George MacDonald, because scholars mention his likely influence on Tolkien and Lewis.

In our multi-dimensional analysis, only the series of words, sentences and interpunctions per chapter, in our opinion, were consciously planned by the authors and, specifically, they do not indicate strong connections between Tolkien, Lewis and MacDonald. Each author distributed words, sentences and interpunctions differently from author to author and, sometimes, even from novel to novel by the same author.

On the contrary, the deep language variables and the linguistic channels, discussed in the paper, are likely due to unconscious design and can reveal connections between texts far beyond writers' awareness.

In summary, the buffers of the extended short-term memory required to readers, the universal readability index of texts, the geometrical representation of texts and the fine tuning of linguistic channels—all tools largely discussed in the paper—have revealed strong connections between *The Lord of the Rings* (Tolkien), *The Chronicles of Narnia* and *The Space Trilogy* (Lewis) on one side, and the strong connection also with some novels by MacDonald on the other side, therefore substantially agreeing with what scholars of English Literature say.

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Appendix A. Universal Readability Mean Index Lower Bound

The mean value of G_U is given by

$$< G_U >= 89 – 10k < C_P > +300 < 1/P_F > –6(< I_P > –6) \tag{A1}$$

The value calculated by introducing the mean of the variables is given by

$$G_{U,mean} = 89–10k< C_P > +300/< P_F >–6(< I_P > –6) \tag{A2}$$

Therefore

$$< G_U > –G_{U,mean} = 300 \times \left(\frac{1}{< P_F >} – < \frac{1}{P_F} > \right) \tag{A3}$$

Now, it can be proved with the Cauchy–Schwarz inequality that $1/< x > \geq < 1/x >$; therefore $< G_U > –G_{U,mean} \geq 0$; hence $< G_U > \geq G_{U,mean}$.

Appendix B. Pythagorean Distance *d* between Pairs of Texts

Table A1. Pythagorean distance *d* between pairs of texts.

Novel	Lord	Hobbit	Narnia	Trilogy	Back	Lilith	Oliver	David	Bleak	Tale	Mutual	Martin	Bask	Peter
Lord	0													
Hobbit	0.488	0												
Narnia	0.150	0.348	0											
Trilogy	0.355	0.140	0.211	0										
Back	0.099	0.499	0.203	0.379	0									
Lililth	0.112	0.453	0.174	0.336	0.048	0								
Oliver	0.244	0.518	0.307	0.426	0.146	0.141	0							
David	0.320	0.620	0.407	0.532	0.222	0.234	0.106	0						
Bleak	0.231	0.312	0.170	0.217	0.201	0.153	0.211	0.316	0					
Tale	0.267	0.381	0.252	0.305	0.201	0.161	0.143	0.239	0.096	0				
Mutual	0.146	0.479	0.218	0.369	0.055	0.044	0.099	0.190	0.170	0.151	0			
Martin	0.230	0.2612	0.109	0.139	0.241	0.197	0.294	0.400	0.096	0.192	0.230	0		
Bask	0.424	0.0964	0.277	0.072	0.451	0.408	0.496	0.602	0.286	0.371	0.441	0.211	0	
Peter	0.098	0.4744	0.183	0.355	0.025	0.024	0.146	0.232	0.177	0.182	0.048	0.217	0.427	0

Appendix C. Common Area between Circles

We list the Matlab code to calculate the common area between text, downloaded from https://it.mathworks.com/matlabcentral/answers/273066%E2%80%93overlapping%E2%80%93area%E2%80%93between%E2%80%93two%E2%80%93circles (accessed on 15 June 2024).

Let the distance between the centers of two circles be d and their two radii be r1 and r2. Then, the area, A, of the overlap region of the two circles can be calculated as follows using Matlab's a'tan2' function:

```
t = sqrt((d+r1+r2)*(d+r1-r2)*(d-r1+r2)*(-d+r1+r2));

A = r1^2*atan2(t,d^2+r1^2-r2^2)+r2^2*atan2(t,d^2-r1^2+r2^2)-t/2;
```

Appendix D. Conditional Error Probability

Table A2. Error probability between the indicated texts.

	Lord	Hobbit	Narnia	Trilogy	Back	Lilith	Oliver	David	Bleak	Tale	Mutual	Martin	Bask	Peter
Lord	—													
Hobbit	0.065	—												
Narnia	0.665	0.387	—											
Trilogy	0.320	0.759	0.627	—										
Back	0.711	0.148	0.603	0.335	—									
Lililth	0.701	0.173	0.639	0.382	0.866	—								
Oliver	0.314	0.051	0.358	0.203	0.666	0.641	—							
David	0.033	0	0.101	0	0.496	0.372	0.664	—						
Bleak	0.307	0.387	0.633	0.593	0.543	0.560	0.397	0.040	—					
Tale	0.280	0.270	0.475	0.428	0.554	0.601	0.615	0.300	0.725	—				
Mutual	0.505	0.036	0.545	0.279	0.633	0.681	0.686	0.273	0.428	0.558	—			
Martin	0.395	0.499	0.757	0.716	0.478	0.528	0.273	0.003	0.732	0.517	0.367	—		
Bask	0.164	0.830	0.504	0.873	0.211	0.241	0.077	0	0.439	0.286	0.095	0.591	—	
Peter	0.649	0.231	0.669	0.412	0.853	0.761	0.687	0.568	0.642	0.644	0.594	0.587	0.294	—

Appendix E. Slope and Correlation Coefficient of the Regression Lines

Table A3. Slope/correlation coefficient of the regression line $y = mx$, Equation (18), modeling the indicated variables (dependent/independent). We keep four digits because some novels differ only at the third and fourth digit.

Novel	Sentences/Words	Interpunctions/Sentences	Words/Interpunctions	Characters/Words
Lord	0.0731/0.9199	2.0372/0.9609	6.6134/0.9609	4.0367/0.9982
Hobbit	0.0608/0.9532	2.1010/0.9936	7.6902/0.9532	4.1014/0.9996
Narnia	0.0729/0.7610	1.9520/0.9384	6.9062/0.7991	4.0907/0.9919
Trilogy	0.0672/0.9325	1.9664/0.9830	7.3380/0.9696	4.2002/0.9976
Back	0.0681/0.9416	2.1640/0.9759	6.6045/0.9799	3.8496/0.9976
Lililth	0.0676/0.8890	2.2619/0.9488	6.2926/0.9800	4.1174/0.9863
Oliver	0.0566/0.9059	3.0893/0.9544	5.6302/0.9638	4.2248/0.9977
David	0.0537/0.9390	3.2949/0.9657	5.5775/0.9882	4.0474/0.9966
Bleak	0.0600/0.9258	2.5324/0.9715	6.51250.9694	4.2235/0.9923
Tale	0.0573/0.9574	2.7972/0.9785	6.1323/0.9912	4.2417/0.9983
Mutual	0.0618/0.9299	2.6814/0.9549	5.9766/0.9740	4.2197/0.9940
Martin	0.0658/0.8583	2.2243/0.9364	6.6785/0.9514	4.3171/0.9939
Bask	0.0684/0.7706	1.8517/0.9366	7.6984/0.9005	4.1320/0.9949
Peter	0.0687/0.8686	2.3080/0.9679	6.1018/0.9152	4.1117/0.9968

Appendix F. Total Signal-to-Noise Ratios Γ_{dB}, in the Four Linguistic Channels

Tables A4–A7 report the signal-to-noise ratio Γ_{dB} in the channels between the input text k (reference) reported in the first row, and the output text j reported in the left column. For example, in Table A4, if the input text is *Lord* and the output text is *Trilogy* then $\Gamma_{dB} = 21.27$ dB; vice versa, $\Gamma_{dB} = 20.44$. A slight asymmetry is typical of linguistic channels [12,15].

Table A4. Total signal-to-noise ratios Γ_{dB}, S–Channels.

Novel	*Lord*	*Hobbit*	*Narnia*	*Trilogy*	*Back*	*Lilith*	*Oliver*	*David*	*Bleak*	*Tale*	*Mutual*	*Martin*	*Bask*	*Peter*
Lord	∞	12.65	10.08	20.44	20.23	18.92	10.61	8.68	13.19	10.18	14.63	14.51	9.79	17.14
Hobbit	14.60	∞	8.21	19.11	19.02	14.75	15.97	17.00	21.64	24.02	23.05	12.73	8.50	13.08
Narnia	10.12	5.30	∞	8.21	7.70	11.54	6.81	4.19	6.85	4.15	7.11	13.32	23.39	13.54
Trilogy	21.27	18.00	9.58	∞	30.77	19.49	13.81	11.96	18.29	14.21	21.14	15.10	9.69	16.57
Back	21.10	17.94	8.87	30.56	∞	17.45	12.67	11.43	16.83	14.07	19.31	13.65	8.86	15.12
Lililth	19.92	13.16	12.79	19.39	17.58	∞	13.99	10.37	15.86	10.98	16.86	23.03	13.29	26.92
Oliver	12.87	17.07	10.19	15.51	14.60	15.61	∞	19.50	22.67	16.83	19.93	15.65	11.20	14.50
David	11.43	18.20	8.42	13.92	13.49	12.83	20.29	∞	19.17	21.60	17.53	12.38	9.09	11.86
Bleak	14.91	21.86	9.79	19.30	18.05	17.27	21.95	18.12	∞	19.17	30.17	15.67	10.43	15.34
Tale	12.66	24.57	7.84	15.85	15.69	13.21	16.61	20.78	19.89	∞	19.45	11.91	8.23	11.93
Mutual	16.13	22.78	9.70	21.87	20.24	18.07	18.92	16.27	29.88	18.43	∞	15.64	10.19	15.76
Martin	16.00	11.43	14.86	15.46	14.23	23.46	13.93	9.79	14.29	9.78	14.62	∞	16.34	26.65
Bask	10.92	6.53	23.97	9.38	8.78	13.09	8.49	5.56	8.33	5.38	8.51	15.69	∞	15.21
Peter	18.10	11.21	14.52	16.19	14.96	26.66	12.56	9.04	13.59	9.38	14.24	26.20	15.13	∞

Table A5. Total signal-to-noise ratios Γ_{dB}, I–Channels.

Novel	*Lord*	*Hobbit*	*Narnia*	*Trilogy*	*Back*	*Lilith*	*Oliver*	*David*	*Bleak*	*Tale*	*Mutual*	*Martin*	*Bask*	*Peter*
Lord	∞	15.57	21.18	19.50	21.74	19.50	9.35	8.36	14.05	11.16	12.37	19.14	17.61	18.44
Hobbit	15.04	∞	11.25	19.77	19.33	13.61	9.23	8.52	13.97	11.74	11.60	12.29	10.07	16.07
Narnia	21.83	12.48	∞	15.46	16.18	17.10	8.65	7.72	12.20	9.93	11.23	18.24	25.28	15.11
Trilogy	20.07	20.66	15.33	∞	20.28	15.03	8.61	7.83	12.86	10.53	11.05	14.25	14.16	15.95

Table A5. *Cont.*

Novel	*Lord*	*Hobbit*	*Narnia*	*Trilogy*	*Back*	*Lilith*	*Oliver*	*David*	*Bleak*	*Tale*	*Mutual*	*Martin*	*Bask*	*Peter*
Back	20.96	18.83	14.72	19.35	∞	19.45	10.31	9.26	16.69	12.90	13.81	17.18	12.61	23.09
Lililth	18.47	12.40	15.76	13.22	18.74	∞	11.43	10.00	17.75	13.45	16.06	27.65	12.92	23.22
Oliver	5.72	5.21	4.61	4.42	7.06	8.72	∞	22.73	12.64	16.56	16.36	8.04	3.42	9.25
David	4.18	4.22	3.04	3.25	5.57	6.66	22.01	∞	10.38	14.45	12.63	5.99	1.96	7.38
Bleak	12.10	11.84	9.57	10.56	15.30	16.42	14.53	12.69	∞	20.10	21.84	14.23	7.91	20.13
Tale	8.26	9.01	6.36	7.46	10.66	11.20	17.84	16.02	19.15	∞	19.40	9.88	5.04	13.18
Mutual	9.97	8.68	8.40	8.02	11.71	14.57	17.59	14.48	21.08	20.07	∞	13.28	6.83	15.34
Martin	18.04	11.32	17.10	12.46	16.71	27.92	10.97	9.59	15.87	12.48	15.04	∞	13.93	19.38
Bask	18.77	12.04	25.74	15.14	14.57	14.71	7.92	7.11	10.98	9.06	10.12	15.52	∞	13.36
Peter	17.31	14.69	13.28	14.34	22.40	22.88	11.85	10.47	20.95	14.94	16.77	18.77	11.10	∞

Table A6. Total signal-to-noise ratios Γ_{dB}, WI–Channels.

Novel	*Lord*	*Hobbit*	*Narnia*	*Trilogy*	*Back*	*Lilith*	*Oliver*	*David*	*Bleak*	*Tale*	*Mutual*	*Martin*	*Bask*	*Peter*
Lord	∞	16.96	8.69	19.72	21.94	20.11	15.14	12.42	28.75	14.96	18.33	29.47	13.83	15.46
Hobbit	15.61	∞	7.80	22.03	13.72	11.77	8.65	7.24	14.25	9.46	10.34	16.38	16.86	10.64
Narnia	7.94	9.59	∞	7.96	6.03	5.48	5.46	3.05	6.96	3.80	5.44	8.92	13.77	10.57
Trilogy	18.74	22.69	6.92	∞	18.24	15.14	10.32	9.40	17.94	12.40	12.81	18.24	13.92	10.84
Back	21.96	15.48	6.80	19.30	∞	26.10	14.37	14.32	26.03	19.46	19.22	19.02	11.69	12.07
Lililth	20.86	13.90	7.07	16.59	26.52	∞	17.01	17.17	24.86	22.59	24.34	18.32	11.15	12.89
Oliver	16.54	11.40	8.63	12.64	15.98	18.25	∞	18.55	17.28	16.43	23.12	15.86	10.46	16.18
David	14.43	10.55	6.50	12.03	15.89	18.35	18.72	∞	15.66	20.68	20.29	13.45	9.04	11.74
Bleak	29.00	15.86	7.97	18.98	26.27	24.36	15.99	13.95	∞	17.22	20.71	23.33	12.70	14.34
Tale	16.13	12.16	5.82	14.41	20.40	23.01	15.15	19.82	18.16	∞	19.83	14.60	9.60	10.71
Mutual	19.40	12.72	7.83	14.60	20.15	24.90	22.43	19.37	21.49	20.26	∞	17.75	10.94	14.62
Martin	29.30	17.61	9.50	19.33	18.82	17.40	14.28	11.19	22.92	13.25	16.43	∞	14.97	16.69
Bask	11.77	16.84	12.10	13.11	9.38	8.32	7.06	4.92	10.38	6.29	7.77	13.14	∞	11.52
Peter	16.67	13.01	12.48	13.17	13.36	13.42	14.94	10.26	15.40	10.80	14.27	17.92	13.59	∞

Table A7. Total signal-to-noise ratios Γ_{dB}, C–Channels.

Novel	*Lord*	*Hobbit*	*Narnia*	*Trilogy*	*Back*	*Lilith*	*Oliver*	*David*	*Bleak*	*Tale*	*Mutual*	*Martin*	*Bask*	*Peter*
Lord	∞	29.12	23.37	27.97	26.10	19.51	26.91	32.93	22.42	26.31	23.84	21.90	26.70	31.43
Hobbit	28.87	∞	20.03	26.66	22.08	17.21	26.33	24.95	20.20	27.13	21.51	20.64	22.77	25.73
Narnia	23.14	20.07	∞	24.11	21.11	28.25	23.63	26.63	30.01	22.43	29.10	25.22	31.08	26.48
Trilogy	27.61	26.30	23.69	∞	20.81	19.94	44.44	27.92	25.21	36.70	27.87	26.30	28.81	32.34
Back	26.52	22.80	21.89	21.57	∞	19.06	21.03	25.94	19.85	20.63	20.44	18.83	22.56	23.80
Lililth	19.17	17.15	28.14	20.28	18.09	∞	20.09	21.25	26.43	19.28	24.45	23.08	23.79	21.31
Oliver	26.50	25.91	23.13	44.38	20.22	19.65	∞	26.67	24.97	39.76	27.56	26.57	27.81	30.39
David	32.88	25.17	26.81	28.28	25.48	21.54	27.08	∞	24.77	25.80	26.30	23.38	31.25	36.01
Bleak	21.76	19.71	29.73	25.11	18.84	26.05	24.98	24.22	∞	23.63	36.70	31.88	29.79	25.53
Tale	25.88	26.71	21.87	36.57	19.78	18.78	39.69	25.31	23.56	∞	25.70	25.32	25.80	28.24
Mutual	23.24	21.04	28.76	27.79	19.51	24.05	27.59	25.83	36.72	25.79	∞	32.93	32.78	27.93
Martin	21.11	19.88	24.71	25.90	17.72	22.43	26.23	22.73	31.63	25.03	32.73	∞	26.77	24.53
Bask	26.34	22.64	30.92	29.07	21.84	23.73	28.13	30.99	30.08	26.19	32.99	27.17	∞	33.27
Peter	31.19	25.68	26.39	32.56	23.22	21.33	30.67	35.87	25.93	28.59	28.28	25.08	33.36	∞

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