

# A Quantitative Analysis of the Need for High Conversion Efficiency PV Technologies in Carbon Mitigation Strategy: Supplementary Material

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## 1 Energy storage and its impact on *mPCEs* and land areas

If a *Contribution* of 15% and a roof pitch of  $39^\circ$  are assumed, the *mPCE* over the course of the year is found to be  $(61 \pm 13)\%$  using the IBEIS Energy Trends Monthly table for 2018 [1]. Assuming then that roof-mounted PV has a PCE of precisely 61%, it is clear that, while a 15% *Contribution* could be achieved over the course of the year, an excess of energy would be generated over the six summer months, and there would be a deficit over the six winter months, as shown in Table S1. This Table also shows the numbers found assuming a PCE of 23.8%; the overall *Contribution* for this PCE, given these consumption figures, is found to be  $(6 \pm 2)\%$ .

We researched the current market of batteries for residential energy storage [2–5], and found the highest energetic capacity per battery to be 13.5 kW h, achieved using lithium-ion technology [6]. The same battery also has the highest capacity per unit volume, at  $\sim 400 \text{ kW h m}^{-3}$ . Each of these batteries weighs 114 kg and takes up  $0.127 \text{ m}^3$  [6]. Using this battery as an example, we investigate the practicality of current storage technology. Looking first at daily storage, Table S1 shows that the maximum average daily energy excess for a 61% PCE is  $\sim 31 \text{ kW h}$  in July, which would require three of these batteries. Given that these batteries are quoted as being stackable up to ten units [6], this suggests that energy generated beyond the 15% *Contribution* level could be stored on a daily timescale. For a PCE of 23.8%, there is no average daily excess, so it can be assumed that any fluctuations around this could easily be stored in one battery.

However, if the 15% *Contribution* is to be maintained year-round, then all the excess energy generated during the summer months must be stored for use during the winter months. Looking on this seasonal timescale, a 61% PCE system would generate an additional  $\sim 970 \text{ kW h}$  in July alone, which would require 72 of these batteries to store. This would have a total weight and volume of batteries of  $>8000 \text{ kg}$  and

$>9 \text{ m}^3$  respectively [6]. The total summer excess generated by a system with a 61% PCE is  $\sim 3900 \text{ kW h}$ , corresponding to 292 batteries, weighing  $>33\,000 \text{ kg}$  and requiring  $\sim 37 \text{ m}^3$ .

A 61% PCE would allow for a true 15% *Contribution*, as long as the methods used to provide the other 85% are able to provide variable amounts of energy throughout the year. Only when *Contributions* approach 100% does excess energy generation become a serious problem that must be tackled with storage solutions to avoid losses. As shown in Table S1, the summer months can produce a *Contribution* of  $>15\%$  (up to 33% in July). This means that, even though the *Contribution* from the PV system may fall over winter to just 3%, a 15% *Contribution* over the year may be upheld if other energy sources can be used to provide up to 97% over winter, but need only produce  $\sim 70\%$  in July.



S1) shows that in this case, the values for the panels laid flat are:

$$A_F = N_F D, \quad (1)$$

$$M_F = N_F D, \quad (2)$$

$$I_F = N_F D \cos \varphi, \quad (3)$$

where  $\varphi$  is the latitude of the farm. We have set the length of the solar panel in the direction parallel with the equator to be 1, as it does not affect the results. For the solar panels being tilted at an angle  $\theta$ , the values are:

$$A_T = (N_T - 1)D \cdot \frac{\cos(\varphi - \theta)}{\cos \varphi} + D \cos \theta \quad (4)$$

$$M_T = N_T D \quad (5)$$

$$I_T = N_T D \cos(\varphi - \theta). \quad (6)$$

For the two cases to be equivalent in terms of energy generation, the insolation received by the panels of farm must be equal:

$$I_F = I_T \quad (7)$$

$$N_F D \cos \varphi = N_T D \cos(\varphi - \theta). \quad (8)$$

Hence,

$$N_T = N_F \cdot \frac{\cos \varphi}{\cos(\varphi - \theta)}. \quad (9)$$

This is an exact expression, however it is here that the condition  $N_{F,T} \in \mathbb{Z}$ ; requiring that the tilted panels receive *no less* insolation than the flat panels, this gives,

$$N_T = \left\lceil N_F \cdot \frac{\cos \varphi}{\cos(\varphi - \theta)} \right\rceil. \quad (10)$$

The area of material for the tilted panel case is then given by,

$$M_T = D \cdot \left\lceil N_F \cdot \frac{\cos \varphi}{\cos(\varphi - \theta)} \right\rceil. \quad (11)$$

The expression for the land area required is,

$$\left( \left\lceil N_F \cdot \frac{\cos \varphi}{\cos(\varphi - \theta)} \right\rceil - 1 \right) \cdot D \cdot \frac{\cos(\varphi - \theta)}{\cos \varphi} + D \cdot \cos \theta. \quad (12)$$

Table S2 gives maximal values for the fractional difference in land area,  $\frac{(A_T - A_F)}{A_F}$ , and Table S3 does similarly for the fractional difference in area of solar panel material needed,  $\frac{(M_T - M_F)}{M_F}$ . The values were calculated for all  $\theta \in [0, 90^\circ]$ , and for the range  $N_F \in \mathbb{Z} \in [250, 600]$ . This lower bound was chosen as an estimate of a lower bound for the number of rows solar panels that would be needed to create a solar farm of the areas involved in our calculations ( $\sim 1200 \text{ km}^2$ ). The upper bound was arbitrarily chosen as a larger number by which point the argument has been made.

We see from Table S2 that the fractional change in land area required is  $<1\%$  for all latitudes  $<69^\circ$ . Now, in 2011 it was true that 99% of the world's population lived within  $60^\circ$  of the equator [10]; based on this, it is safe to assume that any latitudes at which such a solar farm might be constructed will be within this range, and so the maximum fractional change in land area needed with panel inclination will be  $<1\%$ .

Notice also that every maximal fractional area change in Table S2 occurs for a panel incline that is far from the latitude, and at a number of rows  $<300$ , which is near the start of the range of  $N_F$  considered.

Since the optimal tilt angle for material purposes is at  $\theta = \varphi$ , and with the number of rows likely to be significantly greater than these values, the change in area with these parameters will be smaller still. We therefore believe the invariance of spatial efficiency used in the main paper is justified.

Indeed, the results of Table S3 show that the maximal fractional decrease in solar panel material needed always comes at  $\theta = \varphi$ , with a few values missing due to rounding errors. Table S3 also shows that the change in material area is very variable, unlike the land area needed. An important point to note is that for locations nearer the equator, that is at smaller latitudes, the maximum saving is much smaller than at higher latitudes. For the calculations in the main paper, we were looking at the UK, and London in particular, with a latitude of  $\sim 52^\circ$ ; it can be seen in Table S3 that the maximum fractional decrease in material is  $\sim 38\%$ , as was quoted in the main paper.

The main shortcoming of the data in Table S3 is that the range of  $N_F$  used went only as high as 600 rows. While this was not a problem in Table S2 – where the fractional change in land area generally decreased with increasing  $N_F$  – in this case the value of  $N_F$  at which the fractional change in material is maximised varies significantly across the whole range. As such, it may be the case that greater fractional savings would be possible at higher numbers of rows beyond 600.

Latitude ( $^\circ$ )	$N_F \geq 250$			All $N_F$		
	Max. Fractional Area Change (%)	$N_F$	$\theta$ ( $^\circ$ )	Max. Fractional Area Change (%)	$N_F$	$\theta$ ( $^\circ$ )
0	0.38	250	1	99.97	1	1
1	0.37	262	11	99.82	1	3
2	0.38	254	11	99.54	1	5
3	0.38	250	9	99.15	1	7
4	0.38	252	16	98.63	1	9
5	0.38	262	5	97.99	1	11
6	0.36	250	14	97.24	1	13
7	0.37	255	17	96.36	1	15
8	0.36	273	4	95.37	1	17
9	0.39	256	7	94.26	1	19
10	0.39	256	3	93.04	1	21
11	0.39	250	7	91.70	1	23
12	0.38	256	8	90.24	1	25
13	0.39	258	1	88.68	1	27
14	0.38	256	10	87.01	1	29
15	0.40	250	4	85.23	1	31
16	0.39	250	6	83.35	1	33
17	0.38	251	8	81.37	1	35
18	0.39	252	4	79.28	1	37
19	0.38	250	9	77.10	1	39
20	0.40	251	2	74.82	1	41
21	0.39	252	4	72.45	1	43
22	0.38	251	14	69.99	1	45
23	0.38	255	8	67.44	1	47
24	0.39	251	4	64.81	1	49
25	0.40	252	1	62.10	1	51
26	0.39	254	3	59.32	1	53
27	0.38	256	5	56.45	1	55
28	0.39	251	6	53.52	1	57

Latitude (°)	$N_F \geq 250$			All $N_F$		
	Max. Fractional Area Change (%)	$N_F$	$\theta$ (°)	Max. Fractional Area Change (%)	$N_F$	$\theta$ (°)
29	0.38	258	5	50.52	1	59
30	0.38	255	10	50.00	1	60
31	0.40	250	2	53.05	1	62
32	0.39	251	8	56.16	1	64
33	0.39	253	3	59.33	1	66
34	0.39	250	13	62.54	1	68
35	0.40	251	1	65.80	1	70
36	0.39	254	3	69.10	1	72
37	0.39	251	4	72.44	1	74
38	0.39	251	10	75.81	1	76
39	0.39	251	7	79.21	1	78
40	0.40	251	2	82.64	1	80
41	0.39	253	5	86.08	1	82
42	0.39	251	3	89.55	1	84
43	0.40	250	7	93.02	1	86
44	0.39	256	3	96.51	1	88
45	0.39	253	8	98.25	1	89
46	0.40	253	80	98.25	1	89
47	0.41	251	82	98.25	1	89
48	0.44	253	88	98.25	1	89
49	0.44	251	87	98.25	1	89
50	0.46	258	86	98.25	1	89
51	0.49	250	88	98.25	1	89
52	0.50	252	83	98.25	1	89
53	0.52	250	89	98.25	1	89
54	0.53	251	86	98.25	1	89
55	0.56	250	88	98.25	1	89
56	0.57	251	84	98.25	1	89
57	0.60	256	87	98.25	1	89
58	0.63	251	84	98.25	1	89
59	0.66	252	87	98.25	1	89
60	0.67	253	87	98.25	1	89
61	0.70	250	81	98.25	1	89
62	0.74	251	86	98.25	1	89
63	0.77	252	82	98.25	1	89
64	0.79	256	82	99.13	2	89
65	0.83	257	88	99.13	2	89
66	0.87	253	88	99.13	2	89
67	0.91	258	85	99.13	2	89
68	0.95	254	84	99.13	2	89
69	1.02	252	80	99.13	2	89
70	1.07	253	88	99.13	2	89
71	1.12	251	81	99.13	2	89
72	1.20	250	87	99.42	3	89
73	1.27	251	88	99.42	3	89
74	1.35	250	84	99.42	3	89

Latitude (°)	$N_F \geq 250$			All $N_F$		
	Max. Fractional Area Change (%)	$N_F$	$\theta$ (°)	Max. Fractional Area Change (%)	$N_F$	$\theta$ (°)
75	1.43	254	80	99.42	3	89
76	1.53	253	85	99.56	4	89
77	1.71	252	83	99.56	4	89
78	1.86	251	88	99.56	4	89
79	1.98	250	85	99.65	5	89
80	2.21	253	83	99.65	5	89
81	2.44	255	85	99.71	6	89
82	2.80	250	88	99.75	7	89
83	3.19	254	86	99.78	8	89
84	3.62	258	86	99.81	9	89
85	4.50	252	88	99.84	11	89
86	5.53	258	87	99.88	14	89
87	7.07	266	81	99.91	19	89
88	10.98	257	84	99.94	28	89
89	19.95	286	86	99.97	57	89
90	100.00	1	4	100.00	1	4

**Table S2:** A summary of the maximal fractional change in land area required for a solar farm when tilting the panels rather than having them laid flat. The third and fourth columns show the values of  $N_F$  (number of rows of panels) and  $\theta$  (angle of panels to the horizontal) respectively at which these maxima occur. We have only looked at  $N_F \geq 250$ , as we know a large number of rows will be needed for the areas in question.

Latitude (°)	$N_F \geq 250$						All $N_F$					
	Max. Fractional Increase (%)	$N_F$	$\theta$ (°)	Max. Fractional Decrease (%)	$N_F$	$\theta$ (°)	Max. Fractional Increase (%)	$N_F$	$\theta$ (°)	Max. Fractional Decrease (%)	$N_F$	$\theta$ (°)
0	5630.23	258	89	0.00	250	0	5700.00	1	89	0.00	1	0
1	2765.30	268	89	0.00	250	0	2800.00	1	89	0.00	1	0
2	1809.96	251	89	0.00	250	0	1900.00	1	89	0.00	1	0
3	1331.95	266	89	0.00	250	0	1400.00	1	89	0.00	1	0
4	1044.96	258	89	0.24	411	4	1100.00	1	89	0.24	411	4
5	853.41	264	89	0.38	263	5	900.00	1	89	0.38	263	5
6	716.42	268	89	0.55	548	6	800.00	1	89	0.55	548	6
7	613.55	251	89	0.74	537	7	700.00	1	89	0.74	537	7
8	533.33	252	89	0.97	514	8	600.00	1	89	0.97	514	8
9	469.17	253	89	1.23	325	9	500.00	1	88	1.23	325	9
10	416.48	273	89	1.52	395	10	500.00	1	89	1.52	395	10
11	372.51	251	89	1.84	381	11	400.00	1	87	1.84	381	11
12	335.20	250	89	2.18	595	12	400.00	1	88	2.18	595	12
13	303.15	254	89	2.56	586	13	400.00	1	89	2.56	586	13
14	275.20	250	89	2.97	303	14	300.00	1	86	2.97	101	14
15	250.80	250	89	3.41	587	15	300.00	1	87	3.41	587	15
16	229.14	278	89	3.87	284	16	300.00	1	88	3.87	284	16
17	209.84	254	89	4.37	412	17	300.00	1	89	4.37	206	17
18	192.49	253	89	4.89	470	18	200.00	1	80	4.89	470	18
19	176.83	259	89	5.45	257	19	200.00	1	81	5.45	257	19
20	162.60	254	89	6.03	398	20	200.00	1	82	6.03	199	20
21	149.61	254	89	6.64	527	21	200.00	1	84	6.64	527	21
22	137.65	255	89	7.28	412	22	200.00	1	85	7.28	206	22
23	126.69	251	89	7.95	390	23	200.00	1	86	7.95	390	23
24	116.54	260	89	8.64	428	24	200.00	1	87	8.64	428	24
25	107.12	267	89	9.37	523	25	200.00	1	89	9.37	523	25
26	98.31	296	89	10.12	415	26	100.00	1	53	10.12	415	26
27	90.16	254	89	10.90	367	27	100.00	1	55	10.90	367	27
28	82.49	257	89	11.70	393	28	100.00	1	57	11.70	393	28

Latitude (°)	$N_F \geq 250$						All $N_F$					
	Max. Fractional Increase (%)	$N_F$	$\theta$ (°)	Max. Fractional Decrease (%)	$N_F$	$\theta$ (°)	Max. Fractional Increase (%)	$N_F$	$\theta$ (°)	Max. Fractional Decrease (%)	$N_F$	$\theta$ (°)
29	75.30	251	89	12.54	335	29	100.00	1	59	12.54	335	29
30	68.53	251	89	13.40	418	30	100.00	1	61	13.40	209	30
31	62.15	251	89	14.26	596	30	100.00	1	63	14.26	596	30
32	56.08	255	89	15.20	487	32	100.00	1	65	15.20	487	32
33	50.20	251	89	16.13	279	33	100.00	1	67	16.13	31	33
34	44.92	256	89	17.10	427	34	100.00	1	69	17.10	427	34
35	39.72	282	89	18.08	553	35	100.00	1	71	18.08	553	35
36	34.80	250	89	19.10	377	36	100.00	1	73	19.10	377	36
37	30.08	256	89	20.14	442	37	100.00	1	75	20.14	442	37
38	25.60	250	89	21.20	434	38	100.00	1	77	21.20	217	38
39	21.27	268	89	22.28	534	39	100.00	1	79	22.28	534	39
40	17.13	251	89	23.40	483	40	100.00	1	81	23.40	483	40
41	13.16	266	89	24.53	265	41	100.00	1	83	24.53	53	41
42	9.34	257	89	25.68	292	42	100.00	1	85	25.68	292	42
43	5.64	266	89	26.86	577	43	100.00	1	87	26.86	577	43
44	2.07	290	89	28.07	367	44	100.00	1	89	28.07	367	44
45	0.00	250	0	29.29	478	45	0.00	1	0	29.29	239	45
46	0.00	250	0	30.53	488	46	0.00	1	0	30.53	488	46
47	0.00	250	0	31.80	500	47	0.00	1	0	31.80	500	47
48	0.00	250	0	33.09	541	48	0.00	1	0	33.09	541	48
49	0.00	250	0	34.39	503	49	0.00	1	0	34.39	503	49
50	0.00	250	0	35.71	252	50	0.00	1	0	35.71	14	50
51	0.00	250	0	37.07	491	51	0.00	1	0	37.07	491	51
52	0.00	250	0	38.43	549	52	0.00	1	0	38.43	549	52
53	0.00	250	0	39.82	550	53	0.00	1	0	39.82	550	53
54	0.00	250	0	41.22	262	54	0.00	1	0	41.22	131	54
55	0.00	250	0	42.64	598	55	0.00	1	0	42.64	598	55
56	0.00	250	0	44.08	397	56	0.00	1	0	44.08	397	56
57	0.00	250	0	45.54	336	57	0.00	1	0	45.54	112	57



Latitude ( $^{\circ}$ )	$N_F \geq 250$						All $N_F$					
	Max. Fractional Increase (%)	$N_F$	$\theta$ ( $^{\circ}$ )	Max. Fractional Decrease (%)	$N_F$	$\theta$ ( $^{\circ}$ )	Max. Fractional Increase (%)	$N_F$	$\theta$ ( $^{\circ}$ )	Max. Fractional Decrease (%)	$N_F$	$\theta$ ( $^{\circ}$ )
58	0.00	250	0	47.01	568	58	0.00	1	0	47.01	568	58
59	0.00	250	0	48.50	565	59	0.00	1	0	48.50	565	59
60	0.00	250	0	50.00	250	60	0.00	1	0	50.00	2	60
61	0.00	250	0	51.52	264	61	0.00	1	0	51.52	33	61
62	0.00	250	0	53.05	475	62	0.00	1	0	53.05	475	62
63	0.00	250	0	54.60	500	63	0.00	1	0	54.60	500	63
64	0.00	250	0	56.16	568	64	0.00	1	0	56.16	568	64
65	0.00	250	0	57.74	336	65	0.00	1	0	57.74	168	65
66	0.00	250	0	59.33	445	66	0.00	1	0	59.33	445	66
67	0.00	250	0	60.93	540	67	0.00	1	0	60.93	540	67
68	0.00	250	0	62.54	323	68	0.00	1	0	62.54	323	68
69	0.00	250	0	64.16	466	69	0.00	1	0	64.16	466	69
70	0.00	250	0	65.80	345	70	0.00	1	0	65.80	345	70
71	0.00	250	0	67.44	258	71	0.00	1	0	67.44	43	71
72	0.00	250	0	69.10	521	72	0.00	1	0	69.10	521	72
73	0.00	250	0	70.76	472	73	0.00	1	0	70.76	236	73
74	0.00	250	0	72.44	312	74	0.00	1	0	72.44	156	74
75	0.00	250	0	74.12	255	75	0.00	1	0	74.12	85	75
76	0.00	250	0	75.81	310	76	0.00	1	0	75.81	62	76
77	0.00	250	0	77.50	529	77	0.00	1	0	77.50	529	77
78	0.00	250	0	79.21	303	78	0.00	1	0	79.21	101	78
79	0.00	250	0	80.92	283	79	0.00	1	0	80.92	283	79
80	0.00	250	0	82.63	334	80	0.00	1	0	82.63	167	80
81	0.00	250	0	84.36	505	81	0.00	1	0	84.36	505	81
82	0.00	250	0	86.08	388	82	0.00	1	0	86.08	194	82
83	0.00	250	0	87.81	599	83	0.00	1	0	87.81	599	83
84	0.00	250	0	89.55	287	84	0.00	1	0	89.55	287	84
85	0.00	250	0	91.28	436	85	0.00	1	0	91.28	218	85
86	0.00	250	0	93.02	258	85	0.00	1	0	93.02	43	85

Latitude ( $^{\circ}$ )	$N_F \geq 250$						All $N_F$					
	Max. Fractional Increase (%)	$N_F$	$\theta$ ( $^{\circ}$ )	Max. Fractional Decrease (%)	$N_F$	$\theta$ ( $^{\circ}$ )	Max. Fractional Increase (%)	$N_F$	$\theta$ ( $^{\circ}$ )	Max. Fractional Decrease (%)	$N_F$	$\theta$ ( $^{\circ}$ )
87	0.00	250	0	94.77	535	87	0.00	1	0	94.77	535	87
88	0.00	250	0	96.51	573	88	0.00	1	0	96.51	573	88
89	0.00	250	0	98.25	401	88	0.00	1	0	98.25	401	88
90	0.00	250	0	99.83	600	1	0.00	1	0	99.83	600	1

**Table S3:** A summary of the maximal fractional changes in area of solar panel material needed when going from panels laid flat to inclining them at an angle  $\theta$  to the horizontal. Values for increased and decreased areas of material have been included separately. The values of  $N^F$  (the number of rows when laid flat) and of  $\theta$  at which these extrema occur have been included. Only values for  $N^F \geq 250$  have been used due to the high number of rows that will be needed for the areas involved.

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