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Oscillation and Decay of Neutrinos in Matter: An Analytic Treatment †

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Abstract: We present compact analytic expressions for neutrino propagation probabilities in matter, with effects from the invisible decay of the ν_3 mass eigenstate included. These will be directly relevant for long-baseline experiments. The inclusion of decay leads to a non-Hermitian effective Hamiltonian, with the Hermitian part corresponding to oscillation, and the anti-Hermitian part representing the decay. In the presence of matter, the two components invariably become non-commuting. We employ the Cayley–Hamilton theorem to calculate the neutrino oscillation probabilities in constant density matter. The analytic results obtained provide a physical understanding of the possible effects of neutrino decay on these probabilities. Certain non-intuitive features like an increase in the survival probability $P(\nu_\mu \to \nu_\mu)$ at its oscillation dips may be explained using our analytic expressions.

Keywords: neutrino oscillations; neutrino decay; long-baseline neutrino experiments

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1. Introduction

Neutrino oscillation experiments have conclusively established that neutrinos have nonzero masses and that neutrino flavor eigenstates mix. The mixing parameters have been measured to a good accuracy and can explain most of the observations [1]. However, subleading effects of new physics scenarios are still allowed. One such possible scenario is the invisible decay of neutrinos [2].

In these Proceedings, we explore the effects of the invisible decay of ν_3 vacuum mass eigenstate in the presence of matter effects. In matter, there will invariably be a mismatch between the effective mass eigenstates and decay eigenstates [3]. We employ the Cayley–Hamilton theorem to calculate the neutrino oscillation probabilities for long-baseline neutrino experiments like DUNE. The analytic expressions [4] help in understanding the nature of modifications to oscillation probabilities in the presence of neutrino decay and will aid in the interpretation of future data.

2. The Formalism

When the ν_3 mass eigenstate decays invisibly, i.e., to particles that cannot be detected, the neutrino propagation in matter may be expressed in terms of the effective Hamiltonian

$$\mathcal{H}_f^{(\gamma_3)} = \frac{1}{2E_\nu}\, U \cdot \mathrm{Diag}\Big[\Big(0,\ \Delta m^2_{21},\ \Delta m^2_{31}(1 - i\gamma_3)\Big)\Big] \cdot U^\dagger + \mathrm{Diag}\Big[(V_{\rm cc},\ 0,\ 0)\Big]\,, \quad (1)$$

where $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$, with m_i being the mass of the ν_i vacuum eigenstate. The matter potential is $V_{\rm cc} = \sqrt{2} G_F N_e$, where G_F is the Fermi constant, and N_e is the electron number density. We define γ_3 such that it is given by $\gamma_3 \Delta m^2_{31} = m_3/\tau_3$, where m_3 is the mass, and τ_3 is the lifetime of ν_3 in the rest frame. The neutrino mixing matrix is given by $U = U_{23}(\theta_{23})\, U_{13}(\theta_{13}, \delta_{\rm CP})\, U_{12}(\theta_{12})$.

We define the dimensionless quantities $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$, $A = 2E_\nu V_{cc}/\Delta m^2_{31}$ and $\Delta = \Delta m^2_{31} L/4E_\nu$ and use $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$. Since any effect of decay must be subleading to oscillations, i.e., decay length must be larger than the oscillation length scale, we have the normalized decay width $\gamma_3 \lesssim 0.1$. We can express the small parameters α, s_{13} and γ_3, in terms of the powers of a common book-keeping parameter $\lambda \equiv 0.2$, as

$$\alpha \approx 0.03 \simeq O(\lambda^2)\,, \qquad s_{13} \simeq 0.14 \simeq O(\lambda)\,, \qquad \gamma_3 \lesssim 0.1 \simeq O(\lambda)\,. \tag{2}$$

3. Neutrino Oscillation Probabilities

We employ the Cayley–Hamilton theorem to calculate the oscillation probabilities with exact dependence on the matter term A. Using the Cayley–Hamilton theorem, any function $g(\mathbb{M})$ of a matrix $\mathbb{M}$ can be expressed as

$$g(\mathbb{M}) = \sum_{i=1}^{k} M_i\, g(\Lambda_i)\,, \quad \text{with} \quad M_i \equiv \prod_{j=1, j\neq i}^{k} \frac{1}{\Lambda_i - \Lambda_j}(\mathbb{M} - \Lambda_j \mathbb{I})\,, \tag{3}$$

where values of Λ_i are distinct eigenvalues of the matrix $\mathbb{M}$. Taking $\mathbb{M} = -i\mathcal{H}_f^{(\gamma_3)} L$, the probability amplitude matrix $\mathcal{A}_f$ in the flavor basis may be calculated. The neutrino oscillation probabilities are then obtained as $P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) = |[\mathcal{A}_f]_{\beta\alpha}|^2$.

Expanding in terms of the small parameters s_{13}, α and γ_3 (and hence in terms of powers of λ) and expressing this as $P_{\alpha\beta} \equiv P^{(0)}_{\alpha\beta} + P^{(\gamma_3)}_{\alpha\beta}$, the survival and conversion probabilities are

$$\begin{aligned} P^{(0)}_{\mu\mu} &= 1 - \sin^2 2\theta_{23} \sin^2\Delta - 4 s^2_{13} s^2_{23} \frac{\sin^2[(A-1)\Delta]}{(A-1)^2} + \alpha\, c^2_{12} \sin^2 2\theta_{23}\, \Delta \sin 2\Delta \\ &\quad - \frac{2}{A-1} s^2_{13} \sin^2 2\theta_{23} \left(\sin\Delta \cos A\Delta \frac{\sin[(A-1)\Delta]}{A-1} - \frac{A}{2}\Delta \sin 2\Delta \right) + O(\lambda^3)\,, \end{aligned} \tag{4}$$

$$P^{(\gamma_3)}_{\mu\mu} = -\gamma_3 \Delta \left(\sin^2 2\theta_{23} \cos 2\Delta + 4 s^4_{23} \right) + \gamma_3^2 \Delta^2 \left(\sin^2 2\theta_{23} \cos 2\Delta + 8 s^4_{23} \right) + O(\lambda^3)\,, \tag{5}$$

$$\begin{aligned} P^{(0)}_{\mu e} &= 4 s^2_{13} s^2_{23} \frac{\sin^2[(A-1)\Delta]}{(A-1)^2} \\ &\quad + 2\alpha\, s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta + \delta_{CP}) \frac{\sin[(A-1)\Delta]}{A-1} \frac{\sin A\Delta}{A} + O(\lambda^4)\,, \end{aligned} \tag{6}$$

$$P^{(\gamma_3)}_{\mu e} = -8\gamma_3\, s^2_{13} s^2_{23}\, \Delta \frac{\sin^2[(A-1)\Delta]}{(A-1)^2} + O(\lambda^4)\,. \tag{7}$$

The probability $P_{e\mu}$ is obtained from $P_{e\mu} = P_{\mu e}(\delta_{CP} \to -\delta_{CP})$ and the antineutrino oscillation probabilities are obtained using $P_{\bar{\alpha}\bar{\beta}} = P_{\alpha\beta}(\delta_{CP} \to -\delta_{CP}\,,\ A \to -A)$. In the vacuum limit ($A \to 0$), the probabilities given above match those given in [5] to appropriate orders. The perturbative expansions in Equations (4)–(7) remain valid as long as $\alpha\Delta \lesssim 1$ and $\gamma_3\Delta \lesssim 1$.

We have also obtained the probability expressions with exact dependence on γ_3 [4]. In addition to the naively expected $e^{-\gamma_3\Delta}$ behavior, analytic expressions also involve additional terms with non-trivial dependence on γ_3. Taking into account the exact dependence on γ_3 improves the accuracy and expands the region of validity to lower energies.

4. Results

Let us now compare the accuracy of our analytic expressions against the exact numerical results. We take $\gamma_3 = 0.1$, and the neutrino mixing parameters as

$$\begin{aligned} &\theta_{12} = 33^\circ\,, \quad \theta_{23} = 45^\circ\,, \quad \theta_{13} = 8.5^\circ\,, \quad \delta_{CP} = 0^\circ\,, \\ &\Delta m^2_{21} = 7.37 \times 10^{-5}\ \text{eV}^2\,, \quad \Delta m^2_{31} = 2.56 \times 10^{-3}\ \text{eV}^2\,. \end{aligned} \tag{8}$$

The values chosen agree with the global fit [1] within 3σ for normal mass ordering.

4.1. Accuracy of the Analytic Approximations

In Figure 1, we plot the different analytic approximations and the exact numerical probabilities (calculated within the constant density approximation) to check the accuracy of our results. We plot the absolute accuracy $|\Delta P_{\alpha\beta}|$, defined by

$$\Delta P_{\alpha\beta} \equiv P_{\alpha\beta}(\text{analytic}) - P_{\alpha\beta}(\text{numerical}). \tag{9}$$

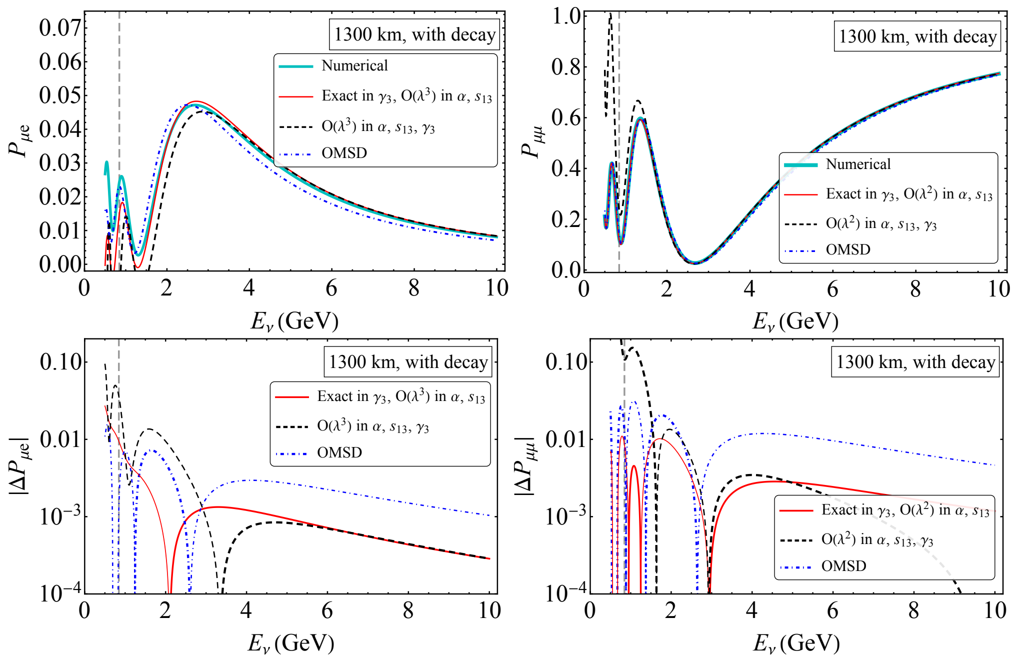

Figure 1. The top panels show the probabilities $P_{\mu e}$ and $P_{\mu\mu}$ with $\gamma_3 = 0.1$, for $L = 1300$ km, for the analytic expressions mentioned in these Proceedings, as well as for the One Mass Scale Dominance (OMSD) approximation [4]. The bottom panels show the absolute accuracy $|\Delta P_{\alpha\beta}|$ of these approximations. The thick (thin) curves indicate positive (negative) signs of $\Delta P_{\alpha\beta}$. The figure is taken from [4].

We observe that the salient features like the positions and heights of oscillation dips and peaks are predicted accurately by the analytic expressions. Note that the analytic approximations are very accurate, with the absolute accuracy $|\Delta P_{\alpha\beta}| \sim 0.001$ in the 2–4 GeV regime, especially in the conversion channel $P_{\mu e}$. Curiously, the oscillation dips for the survival and conversion probability do not reach zero in spite of satisfying the maximal mixing condition of $\theta_{23} = \pi/4$. We elaborate upon this below.

4.2. Increase in the Survival Probability at Oscillation Dips

From the probability expression with exact dependence on γ_3 in [4], we obtain the leading contribution to the survival probability $P_{\mu\mu}$ due to ν_3 decay as

$$P_{\mu\mu}^{\text{leading}} = c_{23}^4 + s_{23}^4\, e^{-4\gamma_3\Delta} + 2s_{23}^2 c_{23}^2 \cos(2\Delta) e^{-2\gamma_3\Delta}\,. \tag{10}$$

For a value of $\gamma_3 \sim O(\lambda)$, the above expression suggests significant deviations from the standard neutrino oscillation probabilities. We focus on these deviations at the first and second oscillation dips. Our leading order analytic approximation in Equation (10) predicts

$$P_{\mu\mu}(\text{first dip}) \simeq \frac{1}{4}\left(1 - e^{-\pi\gamma_3}\right)^2 \geq 0\,, \qquad P_{\mu\mu}(\text{second dip}) \simeq \frac{1}{4}\left(1 - e^{-3\pi\gamma_3}\right)^2 \geq 0 \tag{11}$$

at $\theta_{23} = \pi/4$. In the absence of decay, we would have expected $P_{\mu\mu} = 0$. Such a increase in the probabilities due to ν_3 decay is a non-intuitive feature of our analytic prediction.

For ν_3 decay with $\gamma_3 = 0.1$, Equation (11) predicts an increase of $\sim$0.02 at the first oscillation dip and $\sim$0.1 at the second oscillation dip for $P_{\mu\mu}$. In Figure 2, we test this prediction by plotting the numerical probabilities $P_{\mu\mu}$ at these dips, in scenarios with and without decay. It is observed that the increase in the probability at these dips matches our estimates, even at large baselines where matter effects are important.

The increase in the probability at the oscillation dip may be used as a novel signature of neutrino decay. At a long-baseline experiment like DUNE, the first (second) dip is expected at $\sim$2.7 GeV ($\sim$1 GeV), where identifying this signature may be possible.

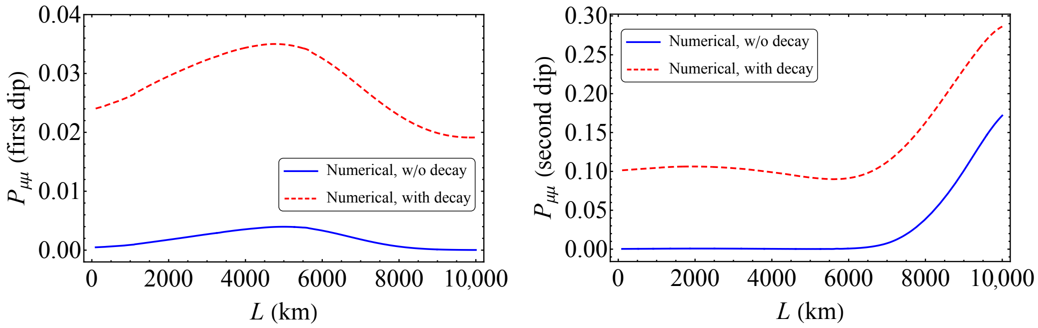

Figure 2. The survival probability $P_{\mu\mu}$ at the first (**left**) and the second (**right**) oscillation dips ($\Delta = \frac{\pi}{2}, \frac{3\pi}{2}$) for a range of baselines L, with $\theta_{23} = 45°$ and $\gamma_3 = 0.1$. The figure is taken from [4].

5. Concluding Remarks

In these Proceedings, we present the modifications to the neutrino probabilities due to the possible invisible decay of ν_3 in matter, in a compact analytic form. Furthermore, we show that our expressions are accurate enough to be of use for long-baseline neutrino experiments like DUNE. The accuracy of our analytic expressions ensures that the salient features of the modifications to the oscillation probabilities due to neutrino decay are captured.

As long as the constant matter density approximation is valid, the neutrino oscillation probabilities given in these Proceedings can be used to probe the physics of the invisible decay of ν_3 for any long-baseline and atmospheric neutrino experiment.

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