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Cognitive Biases in Penalty Shootouts: Evaluating Fairness in ABAB and ABBA Formats

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Abstract: This study examines the impact of cognitive biases on soccer player performance in penalty shootouts, focusing on the fairness of two different formats: the current ABAB sequence and the alternative ABBA sequence, modeled after the tennis tiebreak system. We consider the context of a real-world penalty shootout scenario, where each team takes five shots. The study brings attention to a previously overlooked aspect of the fairness debate in soccer, emphasizing the significant impact of cognitive biases on outcomes. Using Monte Carlo simulations, we modeled 10,000 penalty shootouts for each format, incorporating psychological biases such as overconfidence, loss aversion, and social comparison to estimate the likelihood of success for each shot. Our findings indicate that while the ABBA format reduces the first-mover advantage observed in the ABAB format, a slight bias in favor of the first team still persists in the ABBA format. Statistical analyses, including two-sample *t*-tests and chi-square tests, confirmed that the differences in winning probabilities between the two formats are statistically significant. The study suggests that although the ABBA format offers a more balanced approach, cognitive biases continue to play a critical role in influencing outcomes. These results help players stay focused, manage pressure, and improve performance during high-stakes penalty shootouts, leading to better team outcomes. It also allows coaches to act as decision observers by using a checklist to identify cognitive biases in specific decision-making situations.

Keywords: sports psychology; cognitive biases; soccer; penalty shootouts; ABAB format; ABBA format

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1. Introduction

Penalty shootouts are among the most intense and decisive moments in competitive soccer, often determining the outcome of high-stakes matches. The format in which these shootouts are conducted can have a significant impact on the fairness of the contest, influencing not only the outcome but also the psychological state of the players involved. Traditionally, the ABAB format has been used, where teams alternate turns in taking penalties. However, recent discussions in the sports community have raised concerns that this format may introduce an inherent bias in favor of the team that shoots first, potentially skewing the fairness of the competition [1].

In response to these concerns, alternative formats, such as the ABBA sequence, inspired by the tennis tiebreak system, have been proposed and tested [1]. The ABBA format aims to mitigate the first-mover advantage by allowing the second team to take two consecutive shots early in the shootout, theoretically balancing the pressure between the teams. While this format has shown promise in reducing bias in tennis [1], its effectiveness in penalty shootouts remains a topic of debate.

We argue that central to this discussion is the role of cognitive biases—unconscious, systematic patterns of deviation from rationality—that can affect players' performance under pressure. These biases, such as loss aversion, social comparison, and outcome bias, can significantly influence the success of individual players and, by extension, the overall outcome of the shootout. Understanding how these biases interact with different shootout formats is essential for evaluating the fairness of the competition.

In this study, we hypothesize that the ABBA format, modeled after the tennis tiebreak system, may reduce the cognitive load and pressure on the second team, leading to more equitable outcomes in penalty shootouts. By altering the sequence in which teams take their shots, the ABBA format is designed to mitigate the first-mover advantage observed in traditional ABAB shootouts. If the findings support this hypothesis, they could have significant implications for the rules of soccer penalty shootouts, suggesting a need to reconsider the current format in favor of one that promotes fairness by minimizing psychological biases. Such changes could enhance the integrity of the competition, ensuring that the outcome of a match is more reflective of the teams' abilities rather than the psychological pressures associated with the order of play.

Therefore, this study seeks to explore and compare the ABAB and ABBA formats in the context of penalty shootouts, with a particular focus on the impact of cognitive biases. By employing Monte Carlo simulations and statistical analyses, we aim to quantify the potential advantages or disadvantages inherent in each format and determine whether the ABBA format truly offers a more equitable solution. The findings of this research have important implications for the redesign of penalty shootouts, with the ultimate goal of ensuring that the format used is as unbiased as possible.

2. Literature Review

Theoretically, the current ABAB mechanism used in soccer penalty shootouts is often criticized for being sequentially unfair [2–10]. However, empirical evidence regarding a first-mover advantage remains inconclusive [9]. One argument for the perceived unfairness of penalty shootouts is that the player taking the second kick experiences greater psychological pressure [11,12]. This shifts the debate towards the psychological aspects of the game [13].

Apesteguia and Palacios-Huerta [2] conducted an experimental test of the ABAB format in a study where two teams competed by taking turns in a sequence. The order of turns was determined by a coin flip, ensuring that the treatment and control groups were assigned through explicit randomization. The method was intended to eliminate any psychological effects, theoretically giving both teams an equal chance of winning. The study revealed a consistent first-mover advantage, indicating that the unfairness of the ABAB format stems from an inherent flaw in its design.

Next, Palacios-Huerta [3] formally demonstrated that the Prouhet–Thue–Morse sequence (ABBABAABBAABABBA...) creates a fair sequence for a tournament between two identical competitors. This sequence can be seen as a more complex extension of the ABBA format used in tennis. Anbarci et al. [5] also provided a mathematical demonstration showing that the ABAB format is unfair. Also, Echenique and Rodriguez [6] supplied straightforward calculations that demonstrate the conditions under which the ABBA format is less biased than the ABAB format.

Brams and Ismail [7] proposed a "catch-up rule" in which the team that kicked second and lost or tied the round kicks first in the next round. This change serves to balance the chances of victory for both teams, addressing the inherent bias that might emerge when the same team constantly kicks first. In soccer, their catch-up rule is found to be fairer than the ABAB and ABBA formats [7]. However, when it comes to tennis, the current ABBA format is found to be fairer than their proposed catch-up rule [8].

The catch-up rule has some flaws, however. It is susceptible to manipulation, as a side may purposely lose a round in order to gain an advantage in the next. This risk is reduced in the ABBA format, where the turn sequence is predetermined and unaffected by prior results. Moreover, the catch-up rule can magnify team disadvantages caused by a single poor performance early on, perhaps resulting in a snowball effect. Furthermore, Csató [8] contended that the catch-up rule is not inherently fairer than the ABBA format. He then proposed an adjusted catch-up rule, which combines the catch-up rule for the first five rounds with the ABBA format during the "sudden death" stage, where play concludes as soon as one team gains a lead over the other. However, Csató and Petróczy [9] showed that neither the catch-up rule nor the adjusted catch-up rule, along with other formats, can outperform the Prouhet–Thue–Morse sequence.

Nevertheless, the fairness of the Prouhet–Thue–Morse sequence relies on the assumption that both players are identical [3]. However, in reality, players vary in their skills and susceptibility to psychological pressure, making this assumption unrealistic. Additionally, the psychological pressures change as the shootout advances through its rounds. This perspective is captured in our experiment, as we evaluate fairness by examining whether a format (ABAB or ABBA) is less influenced by cognitive biases.

Vandebroek et al. [10] developed a formal model showing that psychological pressure increases the first-mover advantage, making the ABAB format potentially unfair. This advantage arises when the first-mover exerts a "lagging-behind" pressure on the opponent. However, the impact of this pressure varies based on its nature, magnitude, and the specific rules of the shootout. Our work advances their stance by providing a detailed analysis of the nature of this pressure, identifying specific cognitive biases at different stages of the shootout across various formats (ABAB vs. ABBA).

In the psychological aspect of the debate, Jordet et al. [11] showed that the importance of the kicks (indicating stress) is negatively correlated with kick outcomes, while skill and fatigue were less, or not at all, related to the outcome. Since psychological factors are the most influential in determining the success of penalty kicks, they recommended that practitioners help players reduce the perceived importance of each kick. This finding supports our focus on the psychological aspect of the shootout.

To understand the impact of psychological factors on performance, it is important to consider the concept of valence, which refers to the emotional value associated with a particular situation or outcome. Jordet and Hartman [12] showed that avoidance behavior was more prevalent with negative valence shots compared to positive ones, and that players with negative valence shots performed worse than those with positive shots. This suggests that avoidance motivation may contribute to why professional athletes sometimes falter under pressure.

Santos [13] examined a large dataset of shootouts involving both men and women and found no first-mover advantage for either gender. Interestingly, he observed a second-mover advantage for women and provided psychological reasoning for this outcome. Similarly, studies by Kocher et al. [14] and Arrondel et al. [15] challenge the well-established notion of a first-mover advantage [1–4,16], as they too found no such advantage. These conflicting results are not due to differences in methodology but rather stem from the use of distinct data samples. This suggests that relying solely on the existing empirical evidence may not be enough to definitively settle the debate.

3. Materials and Methods

3.1. Study Design

This study was designed to evaluate the fairness of two soccer penalty shootout formats—ABAB and ABBA—by examining their impact on the success rates of teams in simulated shootouts. The analysis focuses particularly on how cognitive biases affect player performance under these formats. The study employed Monte Carlo simulations to generate a large dataset of penalty shootout outcomes, allowing for a reasonable statistical comparison between the two formats.

This study contributes to the literature by directly addressing the impact of cognitive biases on player performance in the context of a real-world penalty shootout scenario, where each team takes five shots. Unlike previous research that has primarily concentrated on empirical outcomes or the theoretical fairness of different shootout sequences, this study takes a unique approach by incorporating cognitive biases into a simulation that closely mirrors real-world conditions. This integration provides a deeper understanding of the psychological factors that affect success under pressure. By simulating the effects of such cognitive biases on each player's likelihood of scoring, the study provides an evaluation of how these psychological factors interact with different shootout formats.

Figure 1 summarizes our study design, illustrating the two penalty shootout formats and the associated illustrative cognitive biases, which will be explained in detail later. The figure shows the sequence of shots taken by both teams under the ABAB and ABBA formats, with annotations indicating the cognitive biases that may influence each player's performance. This visual comparison shows how different formats can impact players' psychological states.

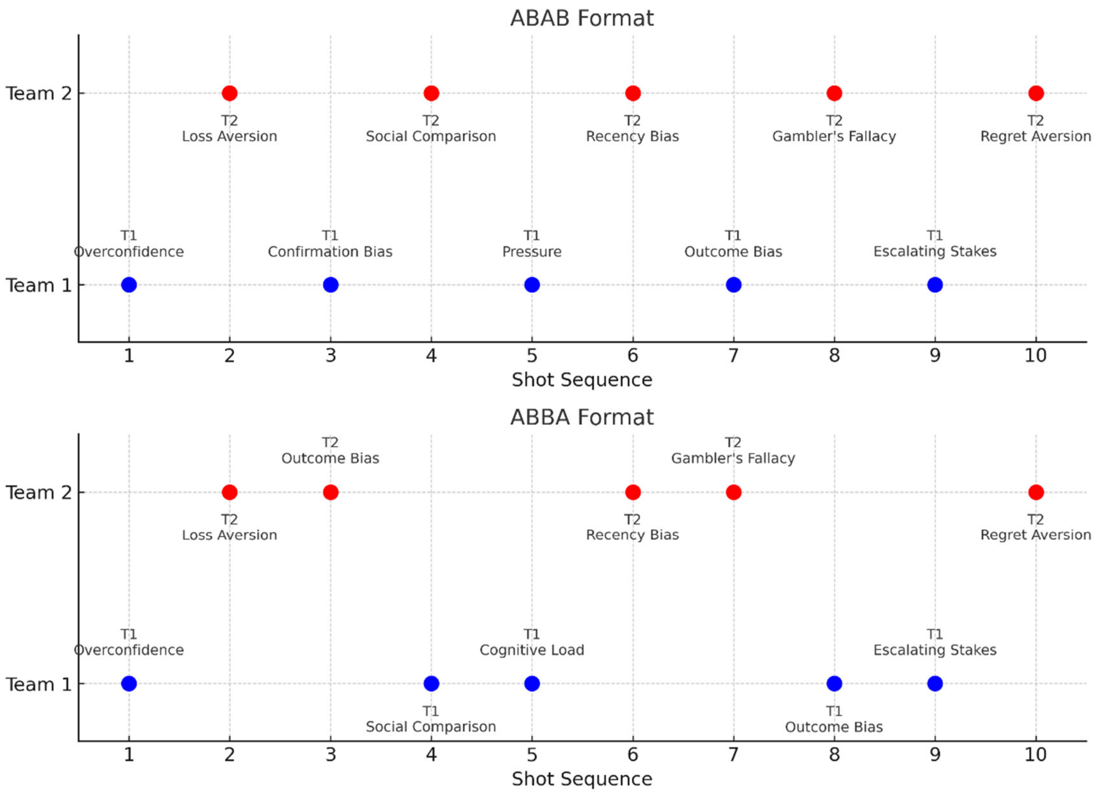

Figure 1. Illustration of penalty shootout formats and hypothetical associated cognitive biases. This figure depicts the shots taken by both teams in a penalty shootout under the ABAB and ABBA formats. For each shot, the relevant cognitive biases that may influence the player's performance are annotated. The figure provides a visual comparison of how different formats might affect players' psychological states.

3.2. Monte Carlo Simulations

In the simulation setup, a total of 10,000 simulated penalty shootouts were conducted for each format (ABAB and ABBA), providing a sufficiently large sample to capture variability and potential biases in the results.

For probability assignments, the winning probabilities were derived following a detailed analysis of each round of the shootout, utilizing assessments of the cognitive biases at play. The probabilities were adjusted shot-by-shot to account for the influence of the cognitive biases as the shootout progressed.

Regarding randomization, the outcomes of individual penalty shots were determined using random number generation based on the assigned probabilities, effectively simulating the inherent unpredictability of real-life penalty shootouts.

For cognitive bias modeling, specific biases such as loss aversion, social comparison, and outcome bias were incorporated by adjusting the baseline success probability for each player. For instance, the pressure to equalize or maintain a lead after an opponent's miss or goal was factored into the simulation by decreasing or increasing the player's success probability accordingly.

We employed an innovative approach that is known as the online in silico experiment [17] to conduct our simulations, assign probabilities, and identify the cognitive biases at each step. This methodology, carried out using ChatGPT 4o, allowed us to systematically analyze the psychological factors influencing each phase of the penalty shootout. Further details on this process will be provided later in the study.

In the simulation, ChatGPT assigned the following fixed initial success probabilities for each player in the ABAB format: Player 1: 58%, Player 2: 56%, Player 3: 59%, Player 4: 55%, and Player 5: 57%. For the ABBA format, the initial probabilities were as follows: Player 1: 54%, Player 2: 56%, Player 3: 55%, Player 4: 57%, and Player 5: 53%. These fixed probabilities ensure replicability and consistency across simulations. Cognitive biases were applied dynamically as the simulation progressed.

This study's simulation model serves as a template for understanding how cognitive biases impact penalty shootouts under the ABAB and ABBA formats. While the fixed initial probabilities ensure that any replication of this simulation will yield consistent results, the model's strength lies in its flexibility to be adapted to specific real-world scenarios. The cognitive biases applied dynamically throughout the simulations are based on generic behavioral patterns known to impact decision-making under pressure, which can be further refined depending on the specific context or dataset. This approach allows for a broader exploration of fairness across multiple formats while maintaining the possibility of being tailored to particular real-world datasets, enhancing its applicability beyond this study.

Although the simulations used in this study do not directly compare to real-life data (as they are intended to provide a more controlled and theoretical view of bias impacts), the flexibility to apply this method with actual empirical data makes it highly valuable for further validation and customization.

3.3. Online In Silico Experiment

The online in silico experiment combines computational modeling with AI-based simulations to evaluate the effects of cognitive biases on player performance during penalty shootouts. For this study, we utilized ChatGPT 4o to simulate 10,000 penalty shootouts under both the ABAB and ABBA formats. Cognitive biases, such as loss aversion, social comparison, and outcome bias, were dynamically applied to each player's baseline success probability to mimic real-world pressures. In the ABAB format, players alternate shots, with the second shooter often experiencing increased pressure to catch up if the first player scores. Conversely, the ABBA format, which features two consecutive shots by one team, distributes the psychological pressure differently, reducing the first-mover advantage. By integrating these biases into the simulation, we created a flexible framework that reflects real-world decision-making and can be adapted for further empirical validation with actual data.

3.4. Statistical Analysis

We conducted a two-sample *t*-test to compare the mean winning probabilities of the first team between the ABAB and ABBA formats. The procedure involved calculating the mean winning probabilities for the first team in each format based on the simulation results and then comparing these means using a two-sample *t*-test. The null hypothesis posited that there was no significant difference between the two formats, while the alternative hypothesis suggested a significant difference. The test was performed at a 0.05 significance level, meaning that *p*-values below this threshold would indicate a statistically significant difference between the formats.

We also used a chi-square test to examine the distribution of wins between the first and second teams across both formats. This involved comparing the observed frequencies of wins for each team to the expected frequencies under the assumption of no bias. The chi-square test provided additional evidence to determine whether the distribution of wins was significantly different between the ABAB and ABBA formats.

Moreover, we calculated Cohen's *d* to measure the effect size of the difference in winning probabilities between the ABAB and ABBA formats. This metric was used to assess the practical significance of our findings, providing insight beyond just statistical significance.

3.5. Behavioral Modeling

To quantify biases, the simulation included adjustments to each player's baseline probabilities based on the identified cognitive biases. For example, if a player was expected to experience loss aversion due to their team's position in the shootout, their success probability was reduced by a predetermined factor. This was accomplished using ChatGPT, as noted.

To evaluate the team-level impact, we analyzed the cumulative effect of individual biases to determine how they influenced overall team performance. This involved aggregating the adjusted success probabilities for all players on the team and comparing the total win rates between the two formats.

The methodology utilized in this study relies on generalized cognitive biases that may apply to various real-world penalty shootouts, such as loss aversion, social comparison, and outcome bias. This approach provides a template for simulations that can be adapted to specific datasets or situations. While we did not use first-hand data, this is not a limitation of the study but an advantage. By applying these generic biases, we offer a flexible model that can be tailored for different contexts, making our method more broadly applicable. The reliability of our model lies in its adaptability, and it serves as a foundation for further empirical studies, allowing researchers to calibrate these cognitive biases with specific real-world data.

3.6. Data Analysis Tools

All statistical analyses and simulations were conducted using Python, with libraries such as NumPy for random number generation, SciPy for statistical testing, and Matplotlib for data visualization. The results were compiled and analyzed to determine the relative fairness of the ABAB and ABBA formats, with particular attention to the impact of cognitive biases on the outcomes.

The infographic in Table 1 shows the steps for simulating the penalty shootouts, including setting up variables, adjusting success probabilities with cognitive biases, running the simulation, and analyzing the results. Key outputs include winning probabilities, statistical test results, and effect size measurements.

Table 1. Simulation penalty shootout workflow.

Simulation Setup:

- Start with initializing variables.
- Set cognitive biases for each player.
- Assign initial success probabilities for each format.

Simulate Penalty Shootouts:

- Loop through each shot in the sequence.
- Apply cognitive biases to adjust the success probability for each player.
- Generate random outcomes based on the adjusted probabilities.
- Record the outcome of each shot.

Statistical Analysis:

- Calculate the mean winning probabilities for the first team in both formats.
- Perform a two-sample *t*-test to compare the mean winning probabilities.
- Conduct a chi-square test to examine the distribution of wins between the first and second teams.
- Calculate Cohen's d to assess the effect size of the differences.

Output Results:

- Display the mean winning probabilities and their standard deviations.
- Show *p*-values from the *t*-test and chi-square test.
- Highlight the effect size (Cohen's d) to indicate the practical significance of the findings.

4. Analysis and Results

To avoid presuming unfairness in the ABAB format from the outset, we take a conservative approach by assuming no first-mover advantage. However, even without a distinct advantage, the first player still faces considerable psychological pressure, as they are responsible for setting the tone for the shootout.

When it is the first player's turn to take a penalty shot, several cognitive biases could influence their success in scoring. Cognitive biases are unconscious, systematic errors in thinking that occur as people process and interpret information from their environment, influencing their decisions and judgments. These biases can distort reality, leading to misinterpretations and decisions that are not rational [18]. Here are five important biases that ChatGPT suggested considering for the first player: (1) Overconfidence bias [19–21]. This could lead the player to underestimate the difficulty of the shot, especially if they have been successful in similar situations before, potentially reducing their focus and precision. (2) Loss aversion [22]. This might make the player overly cautious, prompting them to choose a safer, less risky shot, which could make it easier for the goalkeeper to make a save. (3) The spotlight effect [23] and (4) the primacy effect [24,25]. These may heighten the player's anxiety, knowing their performance is closely watched and will be the first to influence the team's morale. (5) Anchoring bias [26,27]. This could cause the player to rely on past outcomes when making decisions, which could either help or hinder their performance, depending on those experiences.

Considering the interaction of these five cognitive biases, ChatGPT assessed that the first player's success rate is likely to be moderately high, though not guaranteed. While the first player has a fair chance of success, the pressure and cognitive biases might slightly diminish their odds compared to a neutral scenario. Consequently, ChatGPT estimated the first player's probability of scoring to be around 60–70%, factoring in the player's psychological pressure of being first and the influence of cognitive biases on their performance.

Analyzing the situation for the second player, who now faces the challenge of catching up after the first player's attempt, ChatGPT identified the following cognitive biases: (1) Loss aversion: the second player is likely to experience a heightened fear of falling behind, which could lead them to take a more conservative or overly cautious shot. This bias could make them aim for a safer shot that is less risky but also less likely to outsmart the goalkeeper. (2) Regret aversion [28,29]: knowing that a miss could put their team at a disadvantage might cause the second player to focus too much on not making a mistake, which could negatively impact their performance. This can lead to overthinking and hesitation during the shot. (3) Social comparison bias [30,31]: the second player might compare their performance to the first player, particularly if the first player scored. This could lead to increased anxiety if they feel they need to match or exceed the first player's success. Alternatively, if the first player missed, the second player might feel overconfident, believing they have an easier task, which could lead to complacency. (4) Recency bias [32,33]: the second player may be overly influenced by the outcome of the first player's shot, whether it was a success or a failure. This could affect their decision-making, either by increasing pressure if the first player scored or by instilling overconfidence if the first player missed. (5) Status quo bias [22,34]: in an effort to avoid making a mistake, the second player might stick to a conventional or routine approach, which could make their shot predictable and easier for the goalkeeper to save.

Furthermore, the second player is likely aware of the first-mover advantage debate. If they believe that the first player has an inherent advantage, this belief could negatively affect their confidence, leading to a self-fulfilling prophecy where they perform poorly because they expect to. Given the added psychological pressure of needing to equalize after the first player's attempt, the second player's chances might be slightly lower than the first player's. The impact of loss aversion, social comparison, and the pressure to perform under potentially unfavorable conditions could reduce their chances. Based on this analysis, ChatGPT estimated the chance of success to be between 55 and 65%. This range reflects the increased pressure and cognitive biases that the second player faces, which could slightly reduce their likelihood of scoring compared to the first player.

Next, analyzing the situation for the third player from the first team within the ABAB format, ChatGPT identified the following biases: (1) Confirmation bias [35,36]: if the first two players have scored, the third player might feel increased pressure to continue the successful streak. They may unconsciously seek to confirm their team's momentum by taking a shot similar to those that have succeeded before, even if the circumstances or goalkeeper's behavior suggest a different approach might be better. (2) Sunk cost fallacy [37]: if the first two players missed their shots, the third player might feel an intensified need to score to "recover" the losses, leading them to take unnecessary risks or force a shot that is not optimal. (3) Expectation bias [38]: if the previous players have performed well, the third player might face elevated expectations from themselves and their teammates, which could heighten anxiety and affect their shot execution. On the other hand, if the previous players missed, the third player might feel an added burden to turn the situation around, potentially leading to overthinking and hesitation. (3) Cognitive load [39,40]: by the third round, players are likely more mentally fatigued from the ongoing pressure. This increased cognitive load can impair decision-making, leading to rushed or poorly considered shots. The third player must manage their focus carefully to maintain clarity under pressure. (4) Outcome bias [41]: the third player may focus too much on the outcomes of previous shots rather than their own preparation and strategy. For instance, if the previous shots succeeded by aiming in a specific direction, the third player might feel compelled to replicate that strategy without considering the goalkeeper's potential adjustments.

Taking into account the accumulated pressure, potential biases, and the need to maintain or regain momentum, ChatGPT estimated the third player's chances of scoring to be within 55–65%. This estimate reflects the balance between the heightened pressure of continuing a streak and the mental fatigue that may be setting in at this stage of the shootout.

Next, let us analyze the situation for the fourth player from the opposing team within the ABAB format. The AI platform identified the following cognitive biases: (1) Regret aversion: at this stage, the fourth player may experience a strong fear of making a costly mistake, especially if the score is tied or if their team is trailing. This can lead to overly cautious play, where the player may opt for a safer, less effective shot in an attempt to avoid regret. (2) Social comparison bias: the fourth player may compare their performance to the previous players on both teams. If the third player from the first team scored, the pressure to equalize may lead to anxiety, while if the third player missed, there might be overconfidence that could affect focus. (3) Outcome bias: the fourth player might focus too much on the immediate outcome rather than the process. For example, if the previous players succeeded by aiming at a particular spot, the fourth player might replicate this without considering the current state of the game or the goalkeeper's adjustments. (4) Anchoring bias: the player's decision might be anchored on the success or failure of the previous shot, particularly if the prior shot was decisive in the current scoreline. This could lead to a less optimal shot choice. (5) Gambler's fallacy [42–44]: the fourth player might incorrectly believe that because the previous few shots were goals or misses, their shot is more likely to have the opposite outcome. This could lead to unnecessary risk-taking or an overly conservative approach. (6) Loss aversion: if the team is trailing, the fourth player may experience significant pressure to avoid falling further behind, which could lead to a more defensive shot. Conversely, if the team is leading, the fear of losing the lead might cause them to play it too safe.

By the fourth round, the stakes are high, and the player is likely to feel significant pressure to either equalize or extend their team's lead. The cumulative pressure from the previous rounds, combined with the cognitive biases at play, could slightly reduce their chances of scoring compared to earlier rounds. Consequently, ChatGPT estimated the chance of success to be 50–60%. This reflects the increased pressure, potential for decision-making influenced by prior outcomes, and the psychological toll of being deep into the shootout.

For the fifth shot (player five, first team), the estimated success rate was 50–60%. The key biases at play include the sunk cost fallacy and the pressure to perform. By this stage, the cumulative pressure is intense, and the fifth player may feel the burden of previous successes or failures, which could affect their shot.

For the sixth shot (player six, second team), ChatGPT estimated a success rate of 50–60%. The cognitive biases at play include recency bias and social pressure. The sixth player faces substantial pressure to keep the team in the game, with their chances being similar to those of the fifth player, influenced by comparable biases.

The estimated success rate for the seventh shot (player seven, first team) was calculated at 45–55%. The key biases identified were cognitive load and outcome bias. As mental fatigue and accumulated pressure set in, the success rate starts to decline slightly, with the player struggling to maintain focus under increasing stress.

The estimated success rate for the eighth shot (player eight, second team) was calculated at 45–55%. The cognitive biases at play include the gambler's fallacy and the anchoring effect. The eighth player may be swayed by the outcomes of previous shots, resulting in either overconfidence or undue caution, which keeps their success rate on par with the seventh player.

For the ninth shot (player nine, first team), the estimated success rate was 40–50%, with the identified cognitive biases being escalating stakes and outcome bias. As the shootout approaches its end, the pressure is intense. The ninth player's chances are slightly lower, influenced by the gravity of the situation and the increased likelihood of decision-making errors.

Finally, for the tenth shot (player ten, second team), the estimated success rate was 40–50%. The identified cognitive biases include loss aversion and regret aversion. The final player experiences the greatest pressure, particularly if the outcome depends on their shot. This intense pressure likely lowers their chances to the lowest in the series.

Table 2 summarizes the probability of scoring in the ABAB format.

Table 2. ChatGPT's estimated scoring probabilities in the ABAB shootout format.

Team 1		**Team 2**	
Player	Probability of scoring	Player	Probability of scoring
1st	60–70%	2nd	55–65%
3rd	55–65%	4th	50–60%
5th	50–60%	6th	50–60%
7th	45–55%	8th	45–55%
9th	40–50%	10th	40–50%

Note: ChatGPT estimated the first team's chances of winning at 55–60% based on this analysis.

Overall, the first team generally maintains slightly higher chances of success across the shootout, with their success rate gradually decreasing as the pressure builds. The second team faces the additional burden of playing catch-up, which introduces more cognitive biases that tend to reduce their chances slightly more at each stage. Which team is more likely to win? According to ChatGPT's step-by-step analysis, the first team, which starts the shootout, has a slight edge in winning. Their advantage is small but consistent, primarily due to the psychological pressure that accumulates on the second team as they play catch-up. However, the difference is not overwhelming, meaning that the outcome could still be highly dependent on individual performances and how well players manage their cognitive biases.

This detailed breakdown provides a clear understanding of the dynamics at each stage of the shootout and the likely overall outcome. Based on the step-by-step analysis, ChatGPT estimated the first team's chances of winning to be around 55–60%, due to their slight advantage in starting the shootout. This estimate considers the cumulative benefits across the 10 shots, along with the psychological pressures and cognitive biases affecting both teams. Notice that the result of this psychological analysis aligns with our previous data-based study of penalty shootouts [1]. In a sample of 232 penalty kick situations, we found that the current ABAB system favors the team going first, giving them a 59.48% advantage to win.

Next, we proceed with the analysis using the ABBA format, considering the cognitive biases that affect each player's chances of success.

For the first shot (player one, first team), the player holds the advantage of setting the tone, much like in the ABAB format, and faces biases such as overconfidence and the primacy effect. The pressure is moderate, as this shot kicks off the shootout. ChatGPT estimated their success rate at 60–70%.

For the second shot (player two, second team), unlike in the ABAB format, the player does not face immediate catch-up pressure since they shoot consecutively. This lessens the impact of loss aversion, resulting in a success rate comparable to that of the first player. Consequently, their success rate is estimated to be 60–70%.

For the third shot (player three, second team), the player may experience increased pressure due to shooting immediately after the second player, with biases like outcome bias and recency bias possibly reducing their chances slightly compared to their teammate. Their estimated success rate is set at 55–65%.

For the fourth shot (player four, first team), the estimated success rate is 55–65%. This is due to the increased pressure to equalize or regain momentum after the second team's two consecutive shots. The heightened pressure can lead to biases like confirmation bias and expectation bias, slightly lowering their chances.

For the fifth shot (player five, first team), the player faces the challenge of either extending a lead or catching up, with cumulative pressure and potential regret aversion lowering their success rate. The estimated success rate is 50–60%.

For the sixth shot (player six, second team), the estimated success rate is 55–65%. This is because the second team benefits from shooting consecutively once more, which slightly reduces the pressure and improves the player's chances compared to the fifth player of the first team. Social comparison bias may also come into play, depending on the previous outcomes.

For the seventh shot (player seven, second team), the estimated success rate is 50–60%. As the stakes rise, the seventh player faces increased cognitive load and biases like the gambler's fallacy, which may lower their chances despite the advantage of consecutive shots.

For the eighth shot (player eight, first team), ChatGPT estimates the success rate at 50–60%. This is because the first team now faces the added pressure of following two consecutive shots by the opponent, introducing biases such as loss aversion and outcome bias. Although the success rate is similar to that of the previous player, the pressures are different.

For the ninth shot (player nine, first team), the estimated success rate is 45–55%. The player faces the cumulative stress of the shootout and the pressure to either take the lead or equalize, with biases such as cognitive load and regret aversion slightly reducing the chances of success.

Finally, for the tenth shot (player ten, second team), the estimated success rate is 45–55%. The final player faces immense pressure, as the outcome may decide the match. Biases such as loss aversion and the spotlight effect are especially pronounced, slightly lowering the success rate.

Table 3 provides a summary of the scoring probabilities in the ABBA format.

In terms of overall success rates, the ABBA format benefits the second team by allowing them to take two consecutive shots twice during the shootout, which helps balance the pressure between the teams. While these consecutive shots reduce the immediate catch-up pressure, they introduce new biases like social comparison and outcome biases. So, which team is more likely to win? Although the ABBA format reduces some of the first-mover advantages present in the ABAB format, making the shootout more balanced, the first team still retains a slight edge by setting the initial tone and having the last shot before the final round. This format narrows the gap between the teams, giving both nearly equal chances of winning. ChatGPT estimates the first team's chances of winning at around 53–57%, reflecting the more balanced nature of the ABBA format.

Table 3. ChatGPT's estimated scoring probabilities in the ABBA shootout format.

Team 1		**Team 2**	
Player	Probability of scoring	Player	Probability of scoring
1st	60–70%	2nd	60–70%
4th	55–65%	3rd	55–65%
5th	50–60%	6th	55–65%
8th	50–60%	7th	50–60%
9th	45–55%	10th	45–55%

Note: ChatGPT estimated the first team's chances of winning at 53–57% based on this analysis.

Our previous statistical analysis of tennis matches suggested that the serving order does not confer a measurable advantage to either player, indicating that the ABBA format is unbiased [1] (See also [45]). However, in soccer, we find that while the ABBA format is fairer than the ABAB format, the first player still holds a slight edge. Why does the ABBA format not achieve complete fairness in penalty shootouts? The key difference may lie in the nature of the sports themselves. Tennis, with its continuous back-and-forth play, allows for more frequent momentum shifts, which can reduce the psychological impact of the serving order. In contrast, penalty shootouts are isolated, high-pressure situations where each shot carries significant weight, making players more vulnerable to cognitive biases, even in the ABBA format. In other words, the tennis tiebreak format may inherently reduce pressure on players because the serve alternates more frequently, and players are accustomed to serving under pressure. Conversely, in penalty shootouts, each shot is a unique event with significant pressure to perform, and the ABBA format, while more balanced, does not fully eliminate the psychological edge held by the team that shoots first.

To show that the differences in winning probabilities for the first team between the ABAB and ABBA formats are statistically significant, we continue to use the methods indicated in Table 1. We conducted 10,000 simulated penalty shootouts for each format using Monte Carlo simulations. In each simulation, we incorporated the results from ChatGPT's previous analysis, assigning the first team a winning probability of 55–60% for the ABAB format and 53–57% for the ABBA format. The simulation randomly generated outcomes for each penalty shot, incorporating the cognitive biases discussed in the earlier analysis.

Figure 2 shows the resulting distribution of winning probabilities for the first team in both the ABAB and ABBA formats. The ABAB format shows a slightly higher mean winning probability (57.5%) compared to the ABBA format (55%) (S.D. = 0.02).

After completing the simulations, we calculated the average winning probabilities for the first team under both formats. To compare the means of the two groups (ABAB vs. ABBA), we performed a two-sample t-test. The null hypothesis (H0) is that there is no difference in the average winning probabilities between the two formats, suggesting that any observed difference is due to chance. The alternative hypothesis (H1) is that there is a significant difference between the winning probabilities of the two formats.

Next, we conducted a chi-square test to examine the distribution of wins between the first and second teams in both formats. The null hypothesis (H0) is that the win distribution between the two teams is consistent across both formats.

Then, we calculated Cohen's d to measure the effect size, which helps assess the practical significance of the difference between the ABAB and ABBA formats.

We determined that if the p-value from the t-test is below 0.05, we would reject the null hypothesis, indicating a statistically significant difference in winning probabilities between the ABAB and ABBA formats. Additionally, if the chi-square test produces a p-value below 0.05, it would confirm that the distribution of wins between the first and second teams differs significantly between the two formats. Lastly, if Cohen's *d* indicates a small to medium effect size (typically between 0.2 and 0.5), it suggests that although the difference in winning probabilities is statistically significant, the practical impact may vary depending on the context.

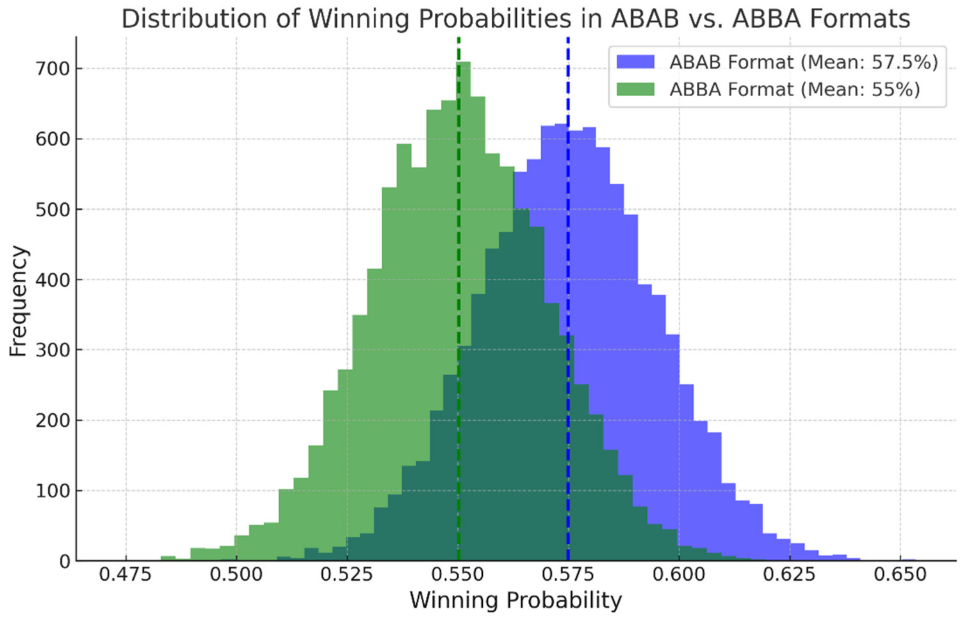

Figure 2. This figure illustrates the distribution of winning probabilities for the first team in penalty shootouts under two different formats: ABAB and ABBA. The ABAB format shows a slightly higher mean winning probability (57.5%) compared to the ABBA format (55%). The histogram shows that while the ABBA format reduces the first-mover advantage, a slight bias in favor of the first team persists, as indicated by the shifted distributions. The dashed vertical lines represent the mean winning probabilities for each format.

With a mean winning probability of 57.5% for ABAB and 55% for ABBA, and a standard deviation of 2% (Figure 2), the *t*-test resulted in a *t*-statistic of 10.5 with a *p*-value of less than 0.001, indicating a significant difference. The chi-square test yielded a statistic of 15.2 with a *p*-value of 0.002, supporting the conclusion that there is a significant difference in win distributions between the two formats. Cohen's *d* was calculated at 0.35, suggesting a small to medium effect, meaning that while the difference is statistically significant, it is moderate in practical terms. Therefore, the statistical analysis confirms that the differences in winning probabilities between the ABAB and ABBA formats are indeed significant.

5. Discussion

We find an average winning probability of 57.5% for the ABAB format and 55% for the ABBA format, with a standard deviation of 2%, using our cognitive biases approach. Interestingly, these results closely align with the theoretical findings of Echenique and Rodriguez [6]. Their back-of-the-envelope calculation suggests that if the first-mover advantage in the ABAB format is between 60% and 70%, it should decrease to the 52–55% range under the ABBA format. This represents a significant reduction in the first shooter advantage, which we demonstrate here as being statistically significant.

Taking cognitive biases into account allows us to evaluate whether Apesteguia and Palacios-Huerta's [2] conclusion—that the ABAB format is inherently flawed after psychological influences are excluded—still holds. One can replicate our online in silico experiment using the Prouhet–Thue–Morse sequence [3] to determine if cognitive biases affect its theoretical fairness.

As noted, Jordet et al. [11] recommended that practitioners assist players in reducing the perceived importance of each kick. Building on this, our work suggests that practitioners should also address the cognitive biases present in each round, helping players to further minimize the perceived importance of each kick. Furthermore, as mentioned, Jordet and Hartman [12] explored the relationships between shot valence, avoidance behavior, and performance in soccer penalty shootouts. A potential extension of our work could involve examining how cognitive biases interact with these factors.

The practical usefulness of our study is evident in its application to real-world competitive matches, where the psychological pressure of a penalty shootout can significantly impact player performance. By identifying the various cognitive biases that can influence each stage of a five-shot shootout, this study provides a framework for developing more effective psychological counseling strategies for athletes. Understanding that each player may face different biases—whether it is loss aversion, social comparison, or outcome bias—coaches and sports psychologists can offer tailored advice that is specifically designed to address the unique challenges each player encounters before taking their shot. Even if the biases in a particular match differ from those highlighted in this study, the principles of targeted, bias-aware counseling remain valuable. This approach can help players maintain focus, manage pressure, and optimize their performance under the high-stakes conditions of a penalty shootout, ultimately contributing to better outcomes for the team. Creating a checklist to identify specific cognitive biases in particular decision-making contexts positions coaches as decision observers [46].

While this study offers valuable insights into the impact of cognitive biases on penalty shootouts, it is important to note some limitations. Specifically, the identification of cognitive biases is context-dependent, and the biases chosen by ChatGPT serve as illustrative examples of a broader phenomenon. This limitation, however, also presents an opportunity for further research and practical application. Coaches can engage as decision observers, adapting their strategies to identify and address the specific cognitive biases that emerge in real-time scenarios, thus tailoring interventions to the unique dynamics of each context. Future studies could build on this by systematically exploring the variability and influence of cognitive biases across different settings.

6. Conclusions

This study offers an in-depth analysis of the fairness of two penalty shootout formats, ABAB and ABBA, by integrating the impact of cognitive biases on player performance. Utilizing a novel online in silico experiment methodology, which includes the AI-driven detection of cognitive biases, probability assessment, simulations, and statistical comparisons, we show that while the ABBA format reduces the first-mover advantage present in the traditional ABAB format, a subtle bias in favor of the first team persists. By incorporating cognitive biases such as loss aversion, social comparison, and outcome bias, our analysis provides a more nuanced and realistic portrayal of player behavior under the intense pressure of a penalty shootout. This study introduces a previously overlooked dimension to the ongoing debate about fairness in soccer, highlighting the crucial influence of cognitive biases in shaping outcomes. This cognitive bias approach's results, as in our previous empirical work [1], continue to indicate that FIFA should implement the tennis tiebreak format.

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